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Sound Propagation and Transport Properties of Liquid ^{3}He in Aerogel

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Abstract

Superfluid ^{3}He confined in aerogel offers a unique chance to study the effects of a short mean free path on the properties of a well defined superfluid Fermi liquid with anisotropic pairing. Transport coefficients and collective excitations, e.g. longitudinal sound, are expected to react sensitively to a short mean free path and to offer the possibility for testing recently developed models for quasiparticle scattering at aerogel strands. Sound experiments, together with a theoretical analysis based on Fermi liquid theory for systems with short mean free paths, should give valuable insights into the interaction between superfluid ^{3}He and aerogel.

A model for liquid ^{3}He in aerogel based on a random distribution of short-ranged potentials acting on ^{3}He quasiparticles has been shown to account semi-quantitatively for the reduction of the transition temperature, T_c, and the suppression of the superfluid density, $\rho_s(T)$.[1] Although the order parameter for ^{3}He in aerogel is not firmly identified, measurements of the magnetization indicate that the low pressure phase is an ESP state.[2] A transverse NMR shift is observed and is roughly consistent with that of an axial state even though the B-phase is stable in pure ^{3}He at these pressures. The addition of a small concentration of ^{4}He, which coats the aerogel strands, induces a suppression of the magnetization for $T < T_c^{\text{aerogel}}$, indicative of a non-ESP state like the BW phase.[2] Thus, the stability of the superfluid state of ^{3}He in aerogel is quite sensitive to the detailed interaction between ^{3}He and the aerogel strands. Calculations of the free energy for ^{3}He in aerogel which include anisotropic and magnetic scattering, and the effects of orientational correlations of the aerogel strands, confirm the sensitivity of the superfluid phases to the interaction between ^{3}He and the aerogel.[1,3]

The high porosity of the aerogel implies that the silica structure does not significantly modify the bulk properties of normal ^{3}He. The dominant effect of the aerogel structure is to scatter ^{3}He quasiparticles moving at the bulk Fermi velocity. If the coherence length, $\xi_0 = \hbar v_f / 2\pi T_{c0}$, is sufficiently long compared to the average distance between scattering centers then a reasonable starting point is to treat the aerogel as a homogenous scattering medium (HSM) described by a mean-free path ℓ.[1] In this article we discuss the effects of a short mean free path on some of the transport properties of liquid ^{3}He. The calculations presented below for sound propagation assume the BW phase is stable; however, the propagation and damping of low-frequency sound are expected to be qualitatively similar for other phases.

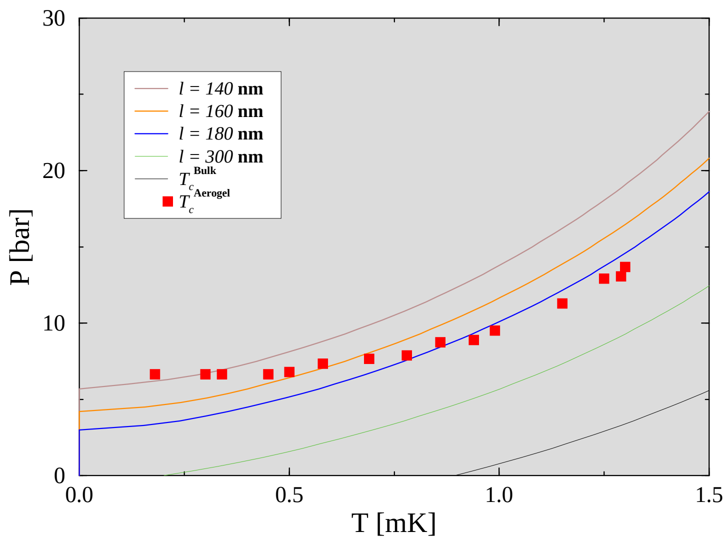

Fig. 1 - p-T phase diagram vs. ℓ. The geometric mean-free path is estimated to be 1750 $\AA$. The data is from Ref. 4.

The superfluid transition temperature for ^{3}He in aerogel is suppressed by quasiparticle scattering off the aerogel structure. In the HSM model the suppression of T_c is given by the Abrikosov-Gorkov formula,

$$\ln(T_{c0}/T_c) = \Psi\left(\tfrac{1}{2} + \tfrac{1}{2}\tfrac{\xi_0}{\ell}\tfrac{T_{c0}}{T_c}\right) - \Psi\left(\tfrac{1}{2}\right) , \tag{1}$$

with the pair-breaking parameter given by ξ_0/ℓ.[1] Figure 1 shows the calculated aerogel transition temperature for several values of the mean-free path. The pressure dependence of the calculated phase diagram is determined by the pressure dependence of the bulk transition temperature and the Fermi velocity via $\xi_0 = \hbar v_f / 2\pi T_{c0}$. The experimental data for $T_c(p)$ is from Matsumoto, et al.[4] for an aerogel with $\approx 98\%$ open volume. For this porosity the typical diameter of the aerogel strands is $d \simeq 30\,\AA$ and the mean distance between strands is $D \simeq 325\,\AA$, which should be compared with the bulk coherence length $\xi_0 \simeq 700\,\AA$ at $p = 1\,$bar. The geometric mean-free path of the aerogel determined from the surface to volume ratio, $S \simeq 2.6 \times 10^5\,\text{cm}^{-1}$, is $\ell_{\text{geom}} = \frac{3\pi}{2}/S \simeq 1750\,\mathring{A}$. The calculations show a mean-free-path of this order gives good agreement with the phase diagram at low temperatures and low pressures, i.e. for $\xi(p) = \hbar v_f / 2\pi k_B T_c(p, \ell) > D \simeq 325\,\mathring{A}$, which corresponds to pressures $p \lesssim 15\,\text{bar}$. Also note that the mean-field phase diagram shows a zero-temperature phase transition as a function of pressure determined by $\xi_0(p_c) = 0.28\,\ell$.

The transport properties of ^{3}He should also be strongly affected by scattering from the aerogel. In the normal state the quasiparticle distribution function, $\phi(\hat{\mathbf{p}}, \mathbf{R}; \epsilon, t)$, satisfies the Boltzmann-Landau transport equation,

$$\frac{\partial \phi}{\partial t} + \mathbf{v}_f \cdot \boldsymbol{\nabla} \phi + \left(\frac{\partial \phi_0}{\partial \epsilon} \right) \frac{\partial \mathcal{E}}{\partial t} = I[\phi] \,, \tag{2}$$

where $\mathcal{E}$ is the effective potential acting on the quasiparticles and $I[\phi]$ is the collision integral. The effective potential consists of the external driving potentials and the internal potentials resulting from quasiparticle interactions. In pure liquid ^{3}He the quasiparticle lifetime is determined by inelastic collisions between quasiparticles and is of order $\tau_{\text{in}} \simeq 1\mu\text{s-mK}^2/T^2$ at $p = 15\,\text{bar}$. However, in aerogel the mean time between scatterings by the aerogel is $\tau_{\text{el}} = \ell / v_f \simeq 4\,\text{ns}$ at the same pressure. Thus, for temperatures below $T_* \simeq 16\,\text{mK}$ the collision rate is dominated by quasiparticle scattering by the aerogel: $I_{el} = -\frac{1}{\tau_{el}} (\phi - \langle \phi \rangle_{\text{FS}})$. This leads to strong reduction in the thermal conductivity and viscosity, and to an increase in the damping of hydrodynamic sound. The static transport coefficients exhibit cross-over behavior dictated by the scattering rate; the thermal conductivity and shear viscosity scale as

$$\kappa = \frac{1}{3} C_v v_f^2 \, \tau \sim \left\{ \begin{array}{l} 1/T \\ T \end{array} \right. , \; \eta = \frac{1}{15} N_f v_f^2 p_f^2 \, \tau \sim \left\{ \begin{array}{lll} 1/T^2 & , & T > T_* \\ \text{const} & , & T < T_* \end{array} \right. , \tag{3}$$

above and below T_*. Recent speculations that the low-temperature phase of ^{3}He in aerogel for $p < p_{cr}$ is not described by Fermi-liquid theory[4,5] can be tested by measuring these transport coefficients.

At low frequencies a compressible aerogel will move nearly in phase with the ^{3}He density and longitudinal current mode; sound propagates but it is damped by the viscous coupling of the ^{3}He to the aerogel, $\alpha_1 = \frac{\omega^2}{\rho c_1^3} \eta$, where c_1 is the hydrodyanmic sound velocity and ρ is the mass density of ^{3}He. The viscous damping of hydrodynamic sound saturates for $T < T^*$ at $\alpha_1 / q \simeq \frac{3}{5} (\omega \tau_{el}) / (1 + F_0^s)$. At higher frequencies the impedance mismatch between ^{3}He and the aerogel sound mode leads to an increasingly out-of-phase motion of ^{3}He excitations and aerogel. Hydrodynamic sound may become overdamped and reemerge as a diffusive mode. The frequency at which the cross-over from damped hydrodynamic sound to an overdamped diffusive mode occurs depends on the elastic compliance of the aerogel and the microscopic details of the coupling between ^{3}He and the aerogel strands. Here we assume the frequency is above this cross-over, in which case the hydrodynamic mode is an overdamped diffusive mode, $\omega = -i\mathcal{D}_s\, q^2$, where $\mathcal{D}_s = c_1^2\,\tau$ is the acoustic diffusion constant with $1/\tau = (1 + F_1^s/3)/\tau_{el}$. At still higher frequencies, $\omega\tau \gg 1$, zero sound can propagate, albeit with relatively high attenuation, $\alpha_0 \simeq 1/c_0\tau \sim 10^4\,\text{cm}^{-1}$ at $p = 10\,\text{bar}$. The regime for propagating zero sound is also pushed to higher frequencies, $\omega/2\pi > v_f/2\pi\ell \simeq 42\,\text{MHz}$ at $p = 10\,\text{bar}$.

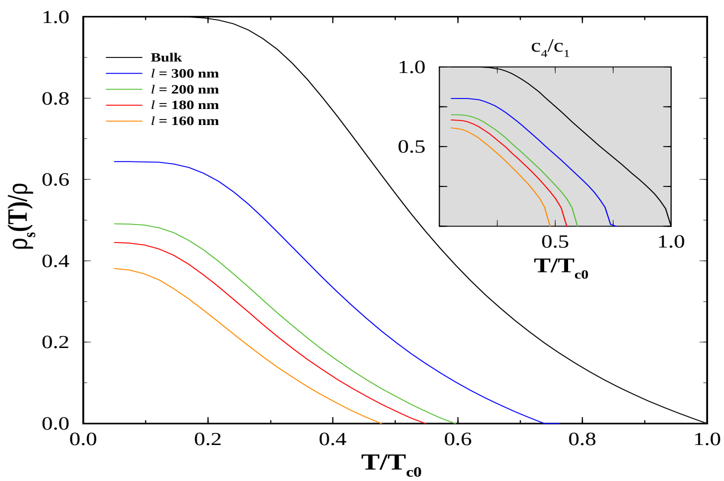

Fig. 2 - Suppression of $\rho_s(T)$ by scattering at $p = 10\,\text{bar}$ with $F_1^s = 8.6$. Results in Born approximation are shown for bulk ^{3}He-B and mean-free paths $\ell = 1600$ to $3000\,\mathring{A}$. The inset shows the 4^{th} sound velocity for the same values of the mean-free path.

The collective mode spectrum of superfluid ^{3}He in aerogel is also expected to show significant changes compared to bulk ^{3}He.[6–8] A typical example is low frequency sound in superfluid ^{3}He ($\omega \ll \Delta/\hbar$). In bulk ^{3}He one has collisionless zero sound and hydrodynamic first sound with nearly the same velocities, $c_0^2 \approx c_1^2 = \frac{1}{3}(1 + F_0^s)(1 + \frac{1}{3}F_1^s)v_f^2$, and small damping by quasiparticle-quasiparticle scattering (for reviews on sound and collective modes in ^{3}He see e.g. Refs. 9–11). In aerogel, on the other hand, one expects a behavior which is similar to sound in ^{3}He confined to small channels.[12] Sound will be weakly damped for $q\ell \gg 1$, ceases to be a well defined mode for $q\ell \approx 1$, and reappears for $q\ell \ll 1$ as fourth sound with a temperature dependent velocity, $c_4 \simeq \sqrt{\rho_s(T)/\rho}\, c_1$. Fig. 2 shows the effects of elastic scattering on the superfluid density.[13] Note the reduction in ρ_s/ρ at $T = 0$. For a mean-free path of $\ell = 1800\,\mathring{A}$ less than $50\,\%$ of the ^{3}He mass density contributes to $\rho_s(T=0)$. The inset shows the fourth sound velocity neglecting damping.

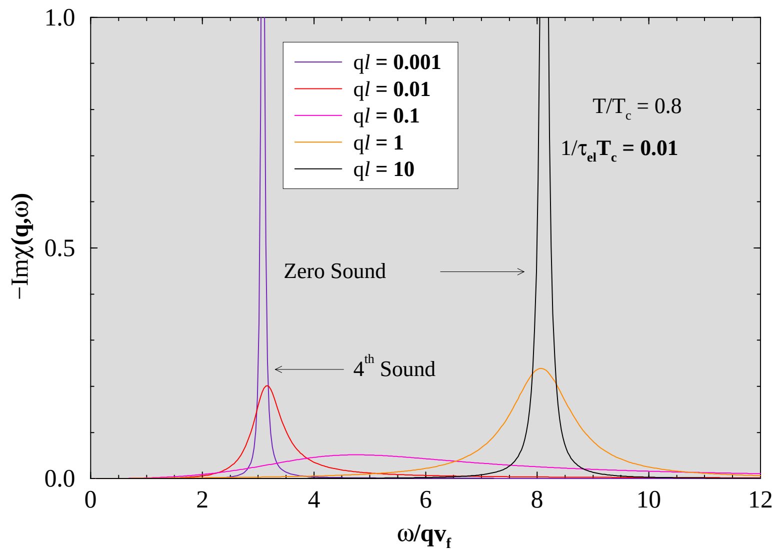

Fig. 3 - Spectral function of the density response $\chi(\mathbf{q},\omega)$ for $q\ell = 0.001 \ldots 10$. The results are for $T/T_c = 0.8$, $1/\tau_{el}T_c = 0.01$, $F_0^s = 50$ and $F_1^s = 8.6$; the corresponding zero sound mode is at $\omega/v_f q = c_0/v_f = 8.1$.

Measurements of sound propagation in ^{3}He-aerogel should provide a sensitive test of the HSM model. We study the spectrum of longitudinal sound in this model by calculating the linear response of the ^{3}He density, $\rho(\mathbf{q};\omega)$, to a driving force of wavevector $\mathbf{q}$ and frequency ω. Our driving force will be a scalar field described by a potential, $\delta U_{ext}(\mathbf{q};\omega)$, which couples to the ^{3}He density. More realistic "experimental driving forces" require more elaborate calculations, which are not called for given the present experimental status. The calculation follows the quasiclassical version[14] of the method of Betbeder-Matibet and Nozières.[15] In the low frequency limit one has to solve Boltzmann-Landau transport equations for the branches of particle-like and hole-like excitations with distribution functions $\delta\phi_{B1,B2}(\hat{\mathbf{p}},\mathbf{R};\epsilon,t)$. We keep the dominant Landau parameters, F_0^s and F_1^s, and obtain an effective scalar potential, $\delta\tilde{u}(\mathbf{q};\omega)$, and longitudinal vector potential, $\delta\tilde{a}(\mathbf{q};\omega) = \mathbf{v}_f \cdot \delta\mathbf{A}(\mathbf{q};\omega)$. The collision terms in the HMS model have the form, $I_{B1,B2} = -\frac{1}{\tau(\epsilon)}\left(\delta\phi_{B1,B2} - \left\langle\delta\phi_{B1,B2}\right\rangle_{FS}\right)$, where

$\tau(\epsilon) = \ell/v(\epsilon)$, and $v(\epsilon) = v_f\sqrt{\epsilon^2 - |\Delta|^2}/\epsilon$ is the energy dependent quasiparticle velocity in the superfluid state. In the low frequency limit the transport equations have to be supplemented by Landau's self-consistency equations for the effective potentials, $\delta\tilde{u}$ and $\delta\tilde{a}$, and the particle conservation law, $\dot{\rho} + \boldsymbol{\nabla}\cdot\mathbf{j} = 0$. In this limit we can ignore the less important self-consistency equation for the amplitude, $\delta \mid \Delta \mid$, of the order parameter. The five coupled linear equations for the distribution functions, the effective potentials, and the phase, $\delta\Psi(\mathbf{q};\omega)$, of the order parameter can be solved, and will be described elsewhere.

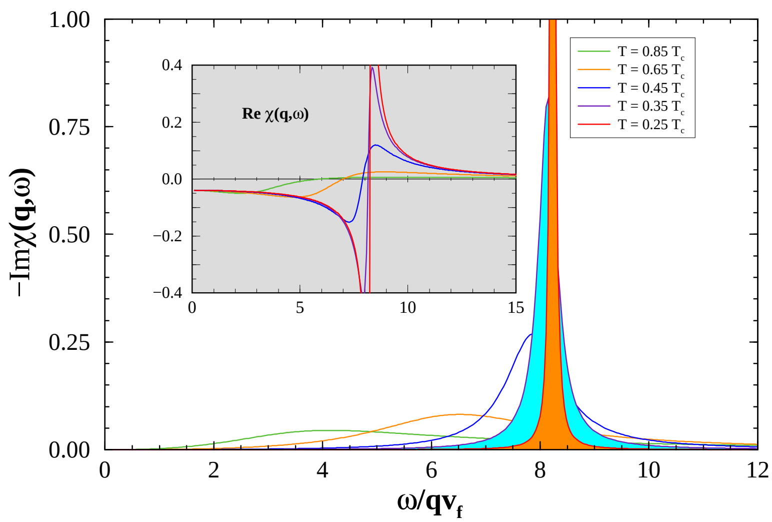

Fig. 4 - Temperature dependence of the imaginary and real (inset) parts of the spectral function $\chi(\mathbf{q},\omega)$ for $T/Tc = 0.25\ \dots\ 0.85$. The results are shown for $q\ell = 0.1$, $qv_f = 0.001T_c$, $F_0^s = 50$ and $F_1^s = 8.6$.

The calculated crossover from weakly damped zero sound to weakly damped fourth sound is shown in Fig. 3. We display the frequency dependent spectral function, $-\mathcal{I}\mathrm{m}\chi(q,\omega)$, for various wavelengths at fixed temperature and elastic scattering time. One can see clearly the transition from zero sound at $v_f q \gg 1/\tau_{el}$ to fourth sound at $v_f q \ll 1/\tau_{el}$. Fig. 4 shows the real and imaginary parts of the response function, $\chi(q,\omega)$, at various temperatures and fixed τ_{el}. Because of the short mean free path, $q\ell = 0.1$, the zero sound resonance with wavevector $\mathbf{q}$ is overdamped in the normal state and just below T_c. The damping decreases exponentially in the superfluid state for $T \ll \Delta$ because of the freezing out of thermally excited quasiparticles; and, as can be seen from Fig. 4, a well defined zero sound mode develops.

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