

**Comment on the Coupling of Zero Sound
to the $J = 1^-$ Modes of $^3\text{He-B}$**

Ross H. McKenzie* and J. A. Sauls

Department of Physics and Astronomy
Northwestern University
Evanston, IL 60208

ABSTRACT

Features in the zero sound attenuation near the pair-breaking edge in superfluid $^3\text{He-B}$ have been observed in large magnetic fields. Schopohl and Tewordt [*J. Low Temp. Phys.* **57**, 601 (1984)] claim that the $J = 1^-$, $M = \pm 1$ order-parameter collective modes couple to zero sound as a result of the distortion of the equilibrium order parameter by a magnetic field; they identify the new features with these modes. However, we show that, when the effect of gap distortion on the collective modes is properly taken into account, the collective mode equations of Schopohl and Tewordt yield no direct coupling of zero sound to the $J = 1^-$ modes. Thus, the identification of the absorption features reported by Ling, Saunders and Dobbs [*Phys. Rev. Lett.* **59**, 461 (1987)] near the pair-breaking edge with the $J = 1^-$ modes is not clearly established.

* Present address: Department of Physics, Ohio State University, 174 West 18th Ave., Columbus, OH 43210, USA.

Two features, a “peak” and an “anti-peak”, have been observed^{1,2,3} in the attenuation spectrum of zero sound in superfluid ³He-B near the pair-breaking edge ($\omega = 2\Delta$) in strong magnetic fields. Schopohl and Tewordt⁴ (ST) have identified these features with the order-parameter collective modes that are odd under the particle-hole transformation, have total angular momentum $J = 1$ and magnetic quantum numbers $M = \pm 1$. In zero field elementary symmetry arguments show that the $J = 1^-$ modes do not couple to zero sound.⁵ To identify the observed features with the $J = 1^-, M = \pm 1$ collective modes it must be shown that these modes have frequencies corresponding to the observed positions in the spectrum, and that they couple to zero sound in the presence of a magnetic field.

Schopohl and Tewordt show that in a magnetic field, $\vec{H} = H\hat{z}$, the $J = 1^-$ modes have frequencies

$$\omega_{1^-,M} = 2\Delta - M g \omega_L, \quad (1)$$

where ω_L is the effective Larmor frequency and g is the Landé g-factor for these modes. The value for g calculated by ST agrees with the value deduced from the splitting of the two features seen by Ling, Saunders and Dobbs.² In addition, ST argue that the $J = 1^-, M = \pm 1$ modes couple to zero sound in a magnetic field. In a field the B-phase order parameter is no longer the isotropic Balian-Werthamer (BW) state with $J = 0$, but has the form

$$\vec{\Delta}(\hat{p}) = (\Delta_1 \hat{p}_x, \Delta_1 \hat{p}_y, \Delta_2 \hat{p}_z) \quad (2)$$

where the anisotropy (“gap distortion”) is quadratic in the field, *i.e.* $\Delta_1 - \Delta_2 \sim H^2$. Schopohl and Tewordt claim that the $J = 1^-, M = \pm 1$ modes couple to zero sound in a magnetic field because of this gap distortion. However, ST neglected the distortion of the collective modes by the field. When gap distortion is properly included in the time-dependent gap equation the coupling between zero sound and the $J = 1^-$ modes considered by ST vanishes. Thus, the identification of the attenuation features observed by Ling, *et al.* with the $J = 1^-$ modes is not established. Further inconsistencies between the observations of Ref. 2 and the theory of ST are pointed out in Ref. 3.

The $J = 1^-$ modes are excitations of the the $\ell = 1$ component of the imaginary part of the order parameter, $d_{\mu}^-(\hat{p}, \omega, \vec{q}) = d_{\mu j}^-(\omega, \vec{q})\hat{p}_j$, the dynamics of which are described by

the time-dependent gap equation⁶

$$\begin{aligned} \int \frac{d\Omega}{4\pi} \lambda(\omega, \eta) \left[(\omega^2 - \eta^2 - 4\Delta^2) d_{\mu}^{-} + 4(\vec{\Delta} \cdot \vec{d}^{-}) \Delta_{\mu} + G\omega_L(\hat{z} \cdot \vec{\Delta})(\vec{\Delta} \times \vec{d}^{-})_{\mu} \right] \hat{p}_j \\ = \int \frac{d\Omega}{4\pi} \lambda(\omega, \eta) \Delta_{\mu} \hat{p}_j \left[\omega \varepsilon^{+} + \eta \varepsilon^{-} \right] \end{aligned} \quad (3)$$

where $\eta = v_f \hat{p} \cdot \vec{q}$, $\lambda(\omega, \eta)$ is the Tsuneto function, G is a function which determines the Landé factors of the modes, and ε^{+} and ε^{-} are mean fields related to density and current oscillations, respectively. The left side of (3) determines the frequencies of the order parameter collective modes and the right side determines how these modes couple to the density and current oscillations induced by zero sound.⁷ Equation (3) is equivalent to eq. (15) of ST in Ref. 8 and is the starting point for our discussion of ST's calculation in Ref. 4.

First consider the zero-field case. The order parameter is the isotropic BW state, $\vec{\Delta}(\hat{p}) = \Delta \hat{p}$, and eq. (3) is solved by decomposing $d_{\mu j}^{-}$ in terms of the spherical tensors, $t_{\mu j}^{J,M}$, defined by eqs. (108) and (109) in Ref. 6,

$$d_{\mu j}^{-}(\omega, \vec{q}) = \sum_{J=0}^2 \sum_{M=-J}^{+J} D_{J,M}(\omega, \vec{q}) t_{\mu j}^{J,M} . \quad (4)$$

If we neglect the dispersion of the modes, then eq. (3) decomposes into nine independent equations with left side of the form

$$(\omega^2 - \omega_{J,M}^2) D_{J,M}(\omega, 0) .$$

This decomposition is equivalent to ST's diagonalization of their matrix R by means of a unitary transformation. Thus, J and M are good quantum numbers for the collective modes of the BW state at zero wavevector. However, when the gap $\vec{\Delta}$ is distorted in a strong magnetic field, the equilibrium order parameter, when represented as a tensor, $\Delta_{\mu} = \Delta_{\mu i} \hat{p}_i$, is composed of $J = 0$ and $J = 2$ spherical tensors. Similarly, J is no longer a good quantum number for the collective modes. This is an important point because ST neglected this change in symmetry of the collective modes in their calculation of the coupling to zero sound; they assumed that in a strong magnetic field the modes near 2Δ are described by the same $J = 1$ tensor as in zero field.

To examine the solutions of eq. (3) when the gap is distorted, we expand $d_\mu^-(\hat{p})$ as

$$d_\mu^-(\hat{p}, \omega, \vec{q}) = \mathcal{D}_{\mu j}(\omega, \vec{q}) \Delta_j(\hat{p}) , \quad (5)$$

and then separate $\mathcal{D}_{\mu j}$ into symmetric, $S_{\mu j}$, and antisymmetric, $A_{\mu j}$, parts,

$$\mathcal{D}_{\mu j} = S_{\mu j} + A_{\mu j} . \quad (6)$$

Furthermore, we can write $A_{\mu j} = \sum_{M=-1}^{+1} A^M t_{\mu j}^{1,M}$, where $t_{\mu j}^{1,M} = \frac{1}{\sqrt{2}} \epsilon_{\mu j k} u_k^M$ with $u_k^0 = \hat{z}_k$ and $u_k^{\pm 1} = \frac{1}{\sqrt{2}}(\hat{x} \pm i\hat{y})_k$, so that

$$d_\mu^- = \sum_{M=-1}^{M=+1} A^M (\vec{u}^M \times \vec{\Delta})_\mu + S_{\mu j} \Delta_j . \quad (7)$$

To obtain the equation describing the dynamics of the antisymmetric part, $A_{\mu j}$, we contract (3) with $t_{\mu k}^{1,-M} \Delta_{kj}$. This gives our main result:

$$\begin{aligned} & \int \frac{d\Omega}{4\pi} \lambda(\omega, \eta) \left\{ \omega^2 - 2g\omega\omega_L M - \eta^2 - 4\Delta^2 \right\} A^M |\vec{u}^M \times \vec{\Delta}|^2 + F^M \\ & = \int \frac{d\Omega}{4\pi} \lambda(\omega, \eta) \{ (\vec{u}^M \times \vec{\Delta}) \cdot \vec{\Delta} \} (\omega\varepsilon^+ + \eta\varepsilon^-) \equiv 0 , \end{aligned} \quad (8)$$

where F^M describes mixing with the $J = 2^-, M \pm 1$ modes. The right side of eq. (8) shows that the anti-symmetric parts, which have frequencies near 2Δ , are *not* driven by longitudinal zero sound. Schopohl and Tewordt obtained a nonzero coupling because they incorrectly assumed that the order-parameter oscillations were parallel to $\vec{u}^M \times \hat{p}$; thus, neglecting the field distortion of the collective modes. On the right side of eq. (8) ST have a term proportional to $\vec{\Delta} \cdot (\hat{p} \times \hat{u}^M)$, which is proportional to $\Delta_1 - \Delta_2 \propto H^2$, for $M = \pm 1$. However, the driving term is precisely zero because the order-parameter oscillations are parallel to $\vec{u}^M \times \vec{\Delta}$ and thus orthogonal to the equilibrium order parameter, $\vec{\Delta}$, even when it is distorted by strong magnetic fields.

There is an indirect coupling of sound to the $J = 1^-$ modes, which was not considered by ST, that arises from the off-resonant excitation of the $J = 2^-, M = \pm 1$ modes represented by F^M . However, this term gives a much smaller coupling, and a field dependence for the coupling that is in disagreement with experiment. To leading order in q^2 ,

$$F^{\pm 1} = \frac{i}{6} \lambda(\omega) (\omega^2 - 4\Delta^2) (\Delta_2^2 - \Delta_1^2) D_{2,\pm 1}(\omega, \vec{q}) . \quad (9)$$

The $J = 2^-, M \pm 1$ modes couple to longitudinal sound for fields at an oblique angle relative to the sound propagation direction. However, because of the prefactor $(\omega^2 - 4\Delta^2)$ in $F^{\pm 1}$ this indirect coupling of sound to the $J = 1^-$ modes is of order H^3 .

To summarize, we have shown that the collective modes of the imaginary part of the order parameter with frequency near 2Δ do not couple to zero sound by the mechanism proposed by Schopohl and Tewordt. However, there may be other mechanisms which couple these modes to longitudinal sound, *e.g.* the indirect coupling via the $J = 2^-, M = \pm 1$ modes. Thus, the $J = 1^-$ modes may still be identifiable with the two features in the sound attenuation observed near the pair-breaking edge^{1,2,3}; however, the theory of the coupling mechanism remains to be worked out.

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5. J. W. Serene, in *Quantum Fluids and Solids -1983* (American Institute of Physics, New York, 1983), edited by E. D. Adams and G. G. Ihas, p. 305.
6. See R. H. McKenzie and J. A. Sauls, in *Helium Three* (Elsevier, Amsterdam, 1990), edited by W.P. Halperin and L.P. Pitaevskii, for the notation and a derivation of eq. (3).
7. We note that for $q \neq 0$ the collective modes which couple to zero sound are driven not just by the density and current oscillations on the right side of (3), but also by the oscillations in the phase of the order parameter (the $J = 0^-$ mode). ST neglected the latter in both Refs. 4 and 8 because they incorrectly assumed that for $q \neq 0$ modes with different total angular momentum do not couple. Consequently, the coupling constants which they obtained disagree with the results of other authors. For example, in Ref. 8, ST obtain a coupling constant for the $J = 2^-$ modes, to first order in q^2 , that is proportional to $\left. \frac{\partial \lambda(\omega, \eta)}{\partial \eta^2} \right|_{\eta=0}$, whereas Wölfle [*Phys. Rev. B* **14**, 89 (1976)] and Maki [*J. Low Temp. Phys.* **16**, 465 (1974)] independently find the coupling to be proportional to $\lambda(\omega, 0)$ because they include the coupling to phase oscillations.
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