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QUASICLASSICAL THEORY OF THE JOSEPHSON EFFECT IN SUPERFLUID ^{3}He*

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In an ideal Josephson junction the current is purely sinusoidal in the phase difference φ across the junction; $J = J_c \sin\varphi$, where J_c is the critical current. In narrow channels or orifices, the current-phase relation is not a simple sinusoidal function of φ, and may be a multi-valued, but periodic function of φ. The importance of the precise form of $J(\varphi)$ has been recently discussed in the context of weak links in superfluid ^{3}He. It is argued that dissipative events are associated with phase-slip events and are evidence for multi-valued current-phase relation [1]. We are interested in the theoretical constraints of geometry and surface scattering on the operation of a Josephson weak link in superfluid ^{3}He, and more generally in the current-phase relation and the onset of dissipation. Although superfluid ^{3}He is well described by the BCS pairing theory there are important features of ^{3}He that differentiate it from conventional superconductors and are important to the study of critical currents in narrow channels and the Josephson effect in weak links. The order parameter $A_{i\alpha}$, describing the spin (i) and orbital (α) degrees of freedom of the Cooper pairs, is particularly sensitive to boundaries. Even a specular boundary suppresses the components $A_{i\alpha}n_\alpha$ normal to the surface; it is this boundary condition that locks the l vector normal to the wall in ^{3}He-A and leads to the vanishing of the supercurrent in A-phase when the l vectors are antiparallel on the two sides of the weak link [2]. Surface roughness and magnetism introduce additional pairbreaking [3]. There is evidence from measurements of the critical currents in channels and thin films that surface roughness plays an important role in modifying the structure of the order parameter near a surface [4]. Realistic calculations of the Josephson effect must take account of these effects. The problem is particularly delicate also because the components of the order parameter that survive near a wall are typically distinct from those that survive inside a channel. It has been recently shown that a qualitatively new form of the current-phase relation arises in finite length channels, suggesting that the Josephson effect may be useful in identifying a many-component order parameter [5].

A simple weak link is a small orifice in a wall between two vessels of ^{3}He. The Josephson effect in such a geometry has been recently observed by Avenel and Varoquaux [1]. For a theoretical investigation of weak links the quasiclassical theory [6] is particularly suitable. It allows the calculation of the order parameter and current-phase relation in weak links of various shapes and sizes, circular or rectangular, short or long. It can handle the spatial dependence of the order parameter near surfaces as well as the dependence on external variables such as temperature, superflow parallel to the wall and magnetic fields. All results presented here are restricted to thin walls (*i.e.* thickness $<< \xi_0 = \hbar v_F/2\pi k_B T_c$). Results for more general weak links than pinholes are moreover restricted to specular scattering. We study the effects of a finite size radius aperture on the order parameter and Josephson current, and compare with the pinhole in the infinitely thin wall (*i.e.* radius $<< \xi_0$), which can be calculated straight-forwardly for both specular and diffuse surfaces.

The quasiclassical transport equation is a first-order differential equation for a propagator defined along classical trajectories of quasiparticles. It can be solved efficiently with the explosion method. It must be augmented by the self-consistency equation for the order parameter and Landau molecular fields, and boundary conditions describing surface scattering; a brief review of different boundary conditions is given in [3]. For a specular surface the boundary condition is simply the continuity of the propagator

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along the specularly reflected trajectory; rough surfaces are more difficult to treat. We use the 'thin dirty layer' model (original Refs. in [3]), but we have recently tested an alternative linear boundary condition for fully diffuse scattering which is more efficient in calculations and has so far reproduced the results of the 'thin dirty layer model.' The calculation of the Josephson current is relatively easy for a pinhole ($a \ll \xi_0$). Since the hole does not to distort the order parameter beyond what is already present due to the wall, the current is obtained by a straight-forward integration of the quasiclassical transport equation with the order parameter for a wall without a hole as input. In Fig. 1 we show the maximum Josephson current as a function of the temperature for a pinhole in a specular wall and a pinhole in a diffusely scattering wall.

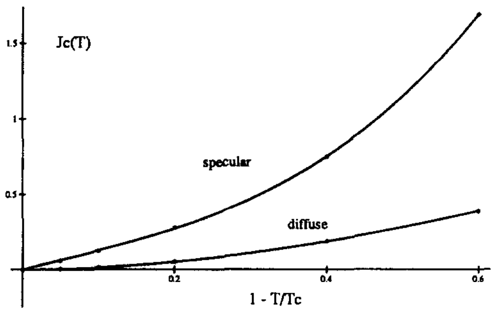

Figure 1. Josephson critical current for a pinhole.

The critical current for a diffuse-wall is quadratic in $(T_c - T)$ as predicted by Kopnin [7]. Also $J(\varphi)$ is asymmetric at lower temperatures, but remains single-valued. The asymmetry is slightly more pronounced in the specular case than in the diffuse case. At $T = 0.4T_c$ the maximum in the current is at $\varphi = 1.96$ and $\varphi = 1.77$, respectively.

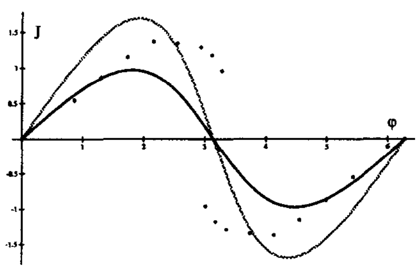

Figure 2. Josephson current: (gray) specular pinhole, (black) diffuse pinhole, (dots) specular aperture scaled by 3.

For finite size apertures the current and order parameter must be calculated self-consistently for each value of the phase difference. For small apertures the current-phase relation is qualitatively similar to that of the pinhole. However, for a hole of diameter $a = 3\xi_0$, $J(\varphi)$ develops a second branch at low temperatures signaling the onset of dissipation via phase slip (Fig. 2). The current is very large near the edge of the orifice, which leads to a very deformed order parameter. For phase differences near $\varphi = \pi$ the order parameter develops a structure which appears similar to the core of a B-phase vortex [8], including an induced magnetization density shown in Fig. 3. The vortex-like order parameter and magnetization move toward the center of the channel and their magnitudes increase as the phase difference increases, suggesting that this static structure is the source of phase slip in a dissipative Josephson weak link for ^{3}He-B.

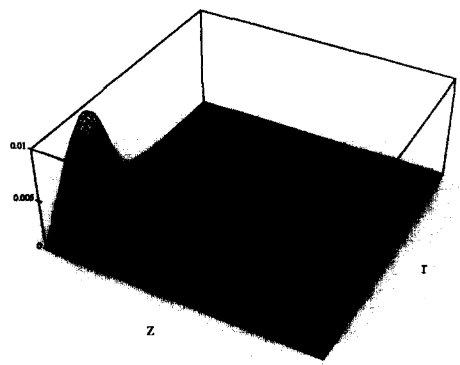

Figure 3. Magnetization density in the vicinity of the orifice; z — normal to wall, r — radial.

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