NONLINEAR ACOUSTICS IN SUPERFLUID ^{3}He-B

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ABSTRACT

The nonlinear interaction of zero sound with the order parameter collective modes in superfluid ^{3}He-B is considered within perturbation theory in the amplitude of the sound field. Selection rules for nonlinear excitation of the order parameter modes are determined by the approximate particle-hole symmetry of the ^{3}He Fermi liquid. A diagrammatic algorithm, based on the quasiclassical theory of superfluid ^{3}He, is used to calculate nonlinear coupling constants. These nonlinearities are sufficiently large that it should be possible to observe two phonon absorption and stimulated Raman scattering of zero sound by the real squashing ($J = 2^+$) mode. Finally, we discuss the possibility of using these nonlinearities to produce zero sound with 'squeezed' noise.

INTRODUCTION

The order parameter collective modes of superfluid ^{3}He have been studied extensively with zero sound. Most of these studies have been devoted to the linear response of the superfluid.[1] Interesting phenomena should also be observable in the nonlinear acoustic response.

The order parameter for the superfluid phases of ^{3}He is the Cooper pair amplitude $<\psi_\alpha\psi_\beta>$ which for a p-wave ($\ell = 1$), spin triplet ($s = 1$) state can be written in terms of a 3x3 complex matrix A_{ij}.[2] The order parameter collective modes in the B phase are oscillations A_{ij} about is equilibrium value $\Delta(T)\delta_{ij}$ (where $\Delta(T)$ is the temperature dependent energy gap), and can be classified[3] by the quantum numbers J^ζ and M where $J = 0,1,2$ is the total angular momentum, $M = \{-J,\cdots,0,\cdots J\}$ is the magnetic quantum number, and $\zeta = +, -$ is the 'parity' under particle-hole symmetry (discussed below) for the real and imaginary parts of the order parameter, respectively.

The $J = 2^+$ and $J = 2^-$ modes, which are also known as the real and imaginary squashing modes, respectively, are of particular interest because they couple to zero sound. These modes lie below the pair breaking edge $2\Delta(T)$ ($\omega_{2^+} \simeq 1\cdot 1\Delta(T)$ and $\omega_{2^-} \simeq 1.5\Delta(T)$) and are weakly damped by quasiparticle collisions. Consequently, the $J = 2^+, 2^-$ modes result in sharp features in the zero sound attenuation, phase velocity and group velocity whenever the frequency and wave vector of sound is equal to that of one of the collective modes. In a magnetic field, H, a five-fold Zeeman splitting of the $J = 2^+$ mode is observed,[4] the splitting becomes nonlinear as a function of H for large magnetic fields[5] as a result of gap distortion and level repulsion.[6] Recently, the Zeeman splitting of the $J = 2^-$ modes has been observed in the group velocity spectrum.[7] In general, the acoustic spectroscopy of ^{3}He-B involves phenomena similar to those seen in the optical spectroscopy of atoms, molecules and solids. Here we show that it should also be possible to observe nonlinear acoustic processes in ^{3}He-B analogous to the well known nonlinear optical effects of two photon absorption and stimulated Raman scattering. Both of these processes in ^{3}He involve quanta of the real squashing mode (*real squashons* hereafter). The first process is the excitation of a real squashon by two zero sound phonons. This occurs if the frequencies ω_1 and ω_2 and wavevectors $\vec{q}_1$ and $\vec{q}_2$ of the phonons satisfy the conditions

$$\begin{aligned} \omega_1 \pm \omega_2 &= \omega_M \\ \vec{q}_1 \pm \vec{q}_2 &= \vec{q}_M \end{aligned} \qquad (1)$$

with positive signs, where ω_M and $\vec{q}_M$ are the frequency and wavevector respectively of the real squashon. The second process is the decay of a zero sound phonon into a real squashon and a second zero sound phonon, which occurs if eq. (1) is satisfied with the negative sign. Whether or not these processes are observable depends on the answers to two questions. Are the processes allowed by the selection rules that are implied by the symmetries of ^{3}He? And, if so, what acoustic energy density is needed to detect them?

Liquid ^{3}He at low temperatures has an approximate symmetry under the interchange of quasiparticle and quasihole states near the Fermi surface. An important selection rule is imposed by exact particle-hole symmetry which is represented by a unitary operator C that maps quasiparticle states just above the Fermi surface into quasihole states just below the Fermi surface (and vice-versa).[9,10] Particle-hole symmetry determines the selection rules for the coupling of zero-sound to the order parameter collective modes because the real (imaginary) components of the order parameter are even (odd) under C, whereas the density fluctuations are odd under C.[9] Thus, with exact particle-hole symmetry the $J = 2^+$ modes do not couple linearly to sound. However, particle-hole symmetry is weakly broken in ^{3}He because the density of states just above and just below the Fermi surface differ slightly. Consequently, there is a weak linear coupling between the $J = 2^+$ modes and sound. In the linear response limit the dynamical equations for the $J = 2^+$ modes are

$$\bar{\lambda}(\omega)\ [(\omega+i\Gamma)^2 - \omega_{M+}(q)^2]\ D_+^M(\omega,\vec{q}) = \frac{6}{1+F_0^s}\,\beta_M\ \delta n(\omega,\vec{q}), \qquad (2)$$

where $D_+^M(\omega,\vec{q})$ is the amplitude of the mode with magnetic quantum number M, $\frac{1}{\Gamma}$ is the lifetime of the mode due to quasiparticle collisions, $\bar{\lambda}(\omega)$ is the Tsuneto function and $\delta n(\omega,\vec{q})$ is the density fluctuation. The coupling constant β_M is small, of order $\eta = N'(E_F)\Delta/N(E_F)$, where $N(E_F)$ and $N'(E_F)$ are the density of states and its' slope at the Fermi surface.[11]

The propagation of sound in superfluid ^{3}He is described by a wave equation

$$\left[\frac{\partial^2}{\partial t^2} - c_1^2\nabla^2 \right] \delta n = 2c_1^2\nabla^2\delta\Pi , \qquad (3)$$

where $\delta\Pi(\vec{R},t)$ is related to the stress tensor of the superfluid, and c_1 is the hydrodynamic sound velocity. Equation (3) is a consequence of the mass and momentum conservation laws. It is important to note that although this equation is linear in δn and $\delta\Pi$ it describes nonlinear sound propagation because the longitudinal stress $\delta\Pi$ is in general a nonlinear functional of the density fluctuation and, in general, the amplitudes of the collective modes of the system which couple to zero-sound. The relationship between the fluctuating stress $\delta\Pi$ and the density fluctuation δn must be obtained from a more microscopic theory than hydrodynamics. Under certain conditions the constitutive relation is of the form

$$\delta\Pi \simeq \chi^{(1)}\delta n + \chi^{(2)}(\delta n)^2 + \chi^{(3)}(\delta n)^3 + \cdots \qquad (4)$$

In the linear response limit the frequency dependent attenuation $\alpha(\omega)$ and the phase velocity $c(\omega)$ of sound are given by

$$\alpha = -q\,\mathrm{Im}\ \chi^{(1)}, \qquad \frac{c(\omega)-c_1}{c_1} = \mathrm{Re}\ \chi^{(1)}. \qquad (5)$$

For exact particle-hole symmetry the second order susceptibility $\chi^{(2)}$ vanishes. Analogues between the above situation and that in optics[8] have been pointed out recently.[12]

Although the $J = 2^+$ modes can only be excited by a single zero sound phonon via the small intrinsic particle-hole asymmetry in ^3He, the excitation of the $J = 2^+$ mode by two phonon processes is not forbidden by particle-hole symmetry selection rules. Thus, at higher sound amplitudes the right-hand side of eq. (2) contains a driving term which is second order in the density; and the stress tensor has a term which is bilinear in δn and the amplitude D^M_+ for the $J = 2^+$ mode. These nonlinear couplings have been calculated from microscopic theory.[12] The results are

$$\delta\Pi(\omega) = \frac{1}{(1 + F^s_0)\Delta} \sum_M \int d\nu \; A^M(\omega,\nu,\omega - \nu) \; \delta n(\nu) \; D^M_+ \; (\omega-\nu), \qquad (6)$$

$$\bar{\lambda}(\omega) \left[(\omega+i\Gamma)^2 - \omega_{M+}(q)^2\right] D^M_+(\omega,\vec{q}) = \frac{6}{(1 + F^s_0)^2\Delta} \times$$

$$\int d\nu \; A^M(\nu-\omega,\nu,-\omega)^* \; \delta n(\nu) \; \delta n(\omega-\nu), \qquad (7)$$

where A^M is a dimensionless function of order one. These are the central equations describing the interaction of the $J = 2^+$ modes with two zero sound waves.

TWO PHONON PROCESSES

Equations (2) and (3) are now applied to the nonlinear interaction of two zero sound waves with the $J = 2^+$ mode. The density fluctuation is written in the form

$$\delta n(\vec{R},t) = \mathrm{Re}[N_1(\vec{R},t) + N_2(\vec{R},t)] \qquad (8)$$

where $N_j(\vec{R},t) = \tilde{N}_j(\vec{R},t)e^{i[\omega_j t - \vec{q}_j \cdot \vec{R}]}$, $j = 1,2$ and it is assumed that the wave amplitudes $\tilde{N}_j(\vec{R},t)$ vary slowly on the time scale of the mode lifetime $1/\Gamma$. In this quasi-steady-state approximation eq. (6) can be solved for $D^M_+(\vec{R},t)$. It is straightforward to show that the $J = 2^+$ mode amplitudes contains terms oscillating with frequencies, 0, $2\omega_1$, $2\omega_2$, $\omega_1 + \omega_2$, and $\omega_1 - \omega_2$. If these solutions, together with eq. (8), are substituted in eq. (6) it is found that $\delta\Pi(\vec{R},t)$ contains terms oscillating with frequencies ω_1, ω_2, $3\omega_1$, $3\omega_2$, $2\omega_1 \pm \omega_2$, and $2\omega_2 \pm \omega_1$.

The term with frequency ω_1 is written as

$$\frac{\delta\Pi_1(\vec{R},t)}{N_1} = [\chi^{(3)}(\omega_1,-\omega_2,\omega_1+\omega_2) + \chi^{(3)}(\omega_1,\omega_2,\omega_1-\omega_2)]|N_2|^2, \quad (9)$$

where the nonlinear susceptibility is

$$\chi^{(3)}(\omega,\nu,\omega-\nu) = \frac{6}{5\Delta^2(1+F_0^s)^3}\sum_M \frac{|A^M(\omega,\nu,\omega-\nu)|^2}{\bar{\lambda}(\omega-\nu)[(\omega-\nu+i\Gamma)^2 - \omega_{M+}^2]} . \quad (10)$$

If the wave with frequency ω_2 is of much higher intensity than the wave with frequency ω_1, then the intensity $|N_2|^2$ can be treated as a constant. The attenuation and shift in phase velocity of the sound wave with frequency ω_1, due to the nonlinear interaction with the sound wave with frequency ω_2 and the $J = 2^+$ collective modes is then calculated from eqs. (3), (9) and (10). Well defined features in the spectrum occur whenever one of the resonance conditions $\omega_1 \pm \omega_2 = \omega_{M+}$ is satisfied. These resonance features correspond to two phonon absorption (+) and stimulated Raman scattering (-) of phonons by the $J = 2^+$ modes.

Figures 1 (a) and (b) show the change in the phase velocity in a zero sound wave of frequency ω_1 due to its linear and nonlinear interaction with the $J = 2^+$ modes in the presence of a second wave of high intensity and frequency ω_2. The features on the left ($T_+ \simeq 0.62T_c$) are due to *two-phonon absorption* by the $J = 2^+$ mode and occur at a temperature such that $\omega_1+\omega_2 = \sqrt{8/5}\ \Delta(T_+)$. The features on the right ($T_- \simeq 0.76T_c$) are due to *stimulated Raman scattering* of phonons by the $J = 2^+$ mode and occur at a temperature such that $|\omega_1 - \omega_2| = \sqrt{8/5}\ \Delta(T_+)$. The large central feature in Figure 1 (a) is due to the *linear* coupling of the sound to the $J = 2^+$ mode as a result of particle-hole asymmetry and occurs at a temperature T_o such that $\omega_1 = \sqrt{8/5}\ \Delta(T_o)$. The linear resonance is not shown in Fig. 1 (b) because it occurs at a temperature greater than $0.8T_c$. Amplification of the low-frequency sound wave is possible at the resonance T_- for the same parameters as Fig. 1(b).[12] An amplification occurs as the high-frequency phonons decay into real squashons and low frequency phonons. In order to reduce heating effects in the experimental cell, it may be desirable to use a smaller sound energy density than the value $U/U_c = 0.2$ used in Fig. 1 (a) and (b). Although a smaller value of U/U_c reduces the size of the nonlinear features they should still be observable since changes in the phase velocity of order one part in 10^6 are presumably detectable.

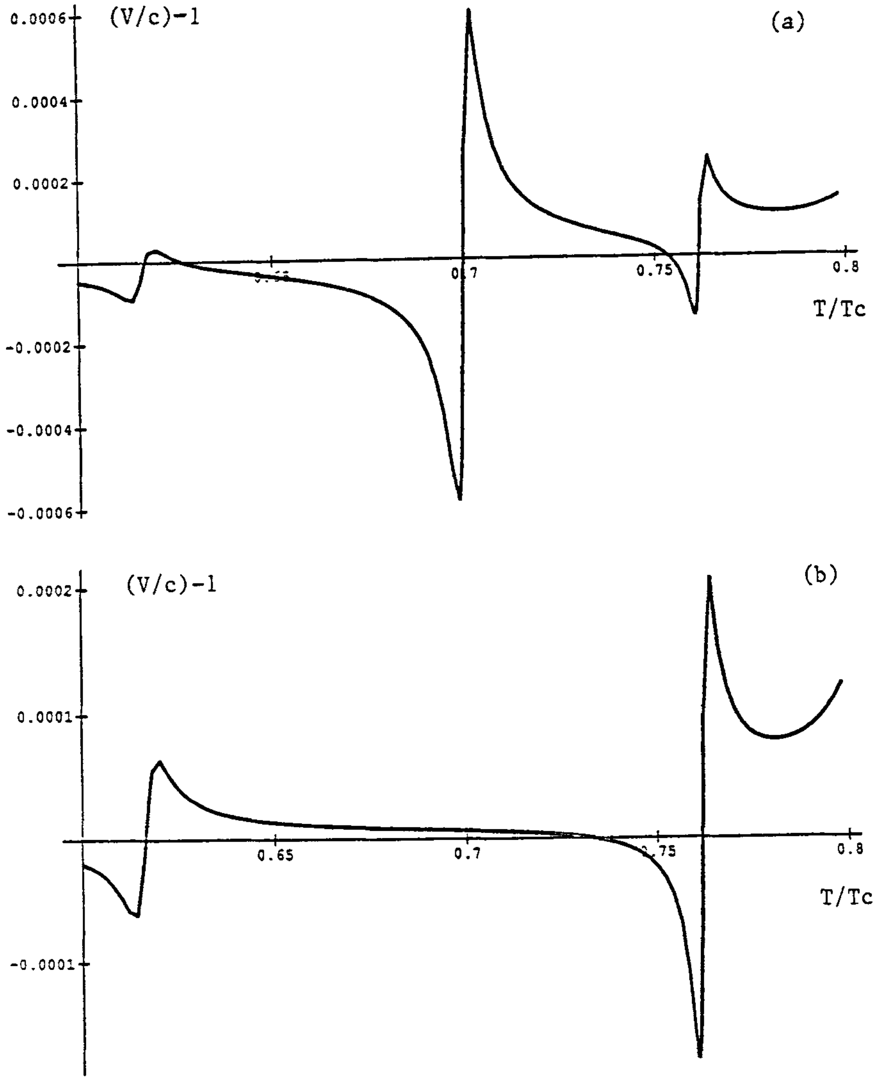

Figure 1. The predicted temperature dependence at zero pressure of $v_{2+}(T)/c_1$, where v_{2+} is the contribution of the $J = 2^+$ mode to the phase velocity of a zero sound wave of frequency ω_1 in the presence of a parallel wave of frequency ω_2 and energy density $0.2U_c$. In (a) ω_1=35.4 MHz and ω_2=2.87 MHz and in (b) ω_1=2.87 MHz and ω_2=35.4 MHz The features at $T/T_c \sim 0.62$ and 0.76 in both graphs are the nonlinear resonances.

SQUEEZING OF ACOUSTIC NOISE

The density oscillation produced by a sound mode of frequency ω can be written in the form

$$\delta n = P_1 \cos \omega t + P_2 \sin \omega t \tag{11}$$

where for simplicity the spatial dependence is omitted. In general, there are fluctuations in the amplitudes P_1 and P_2 caused by noise of various sources. For example, there is noise in the electrical signal that drives the sound transducer, as well as thermal fluctuations of the density. Generally, the noise will be randomly distributed in phase and the fluctuations ΔP_1 and ΔP_2 in two quadratures will be equal.

We now consider the possibility of producing "squeezed" sound which has unequal noise in the two quadratures. In analogy with nonlinear optics[13] this can be done by four-wave mixing, which makes use of the large nonlinear susceptibility $\chi^{(3)}$ in ^{3}He-B. Two high intensity counter-propagating waves (referred to as pump waves) interact with a second pair of counter propagating waves (signal waves) in a cavity oriented at an oblique angle to the pump waves. The four waves usually have the same frequency. It can be shown that the fluctuations ΔP_1 and ΔP_2 in the quadratures of the signal waves leaving the cavity are related to the initial fluctuations ΔP_o in the signal waves entering the cavity by

$$\begin{aligned}(\Delta P_1)^2 &= (\Delta P_o)^2\left[e^{-2s} \cos^2 \tfrac{1}{2}\theta + e^{2s} \sin^2 \tfrac{\theta}{2}\right]\\ (\Delta P_2)^2 &= (\Delta P_o)^2\left[e^{-2s} \sin^2 \tfrac{1}{2}\theta + e^{2s} \cos^2 \tfrac{\theta}{2}\right]\end{aligned} \tag{12}$$

where $se^{i\theta} \equiv \chi^{(3)} A^2 L$, A is the amplitude of the pump waves, and L is the interaction length for the cavity. In order to observe significant squeezing of the noise it is desirable to tune $\theta \approx 0$ or π which implies that the $|\mathrm{Im}\chi^{(3)}| \ll |\mathrm{Re}\ \chi^{(3)}|$. In addition the squeezing parameter s should be of order unity or larger. This requires a large nonlinear, susceptibility, large pump-wave energy densities and a long interaction length. For zero sound in ^{3}He-B with frequency about half the $J = 2^+$ mode frequency $|\chi^{(3)}|$ is sufficiently large that values of $s \sim 1$ are possible for interaction lengths of order centimeters and pump wave energy densities U several orders of magnitude smaller than the superfluid condensation energy density U_c. In addition the frequency is far enough from resonance that $|\mathrm{Im}\chi^{(3)}| \ll |\mathrm{Re}\chi^{(3)}|$.

The squeezing of noise can be measured by *homodyne detection*: the signal wave is mixed with another sound wave (known as the local oscillator) of much higher intensity and with a phase ϕ relative to the signal wave. The noise power of this mixed wave is then measured. Suppose that P_A is the noise power in the absence of both signal and pump waves and P_o+P_A is the noise power measured in the absence of the pump wave. Then P_s is given by

$$P_s = P_A + P_o \left[1-\eta+\eta\left(e^{-2s}\sin^2(\phi-\frac{\theta}{2})+e^{2s}\cos^2(\phi-\frac{\theta}{2})\right)\right] \tag{13}$$

where $\eta = 1$ for classical noise. Thus, as the local oscillator phase ϕ is varied squeezing results in oscillations in the noise power, and so the squeezing of classical acoustic noise should be observable.

Even if the thermal sources of noise mentioned above can be eliminated there will be noise due to quantum fluctuations in the superfluid. It can be shown that the variances ΔP_1 and ΔP_2 must satisfy the uncertainty principle,

$$\Delta P_1 \Delta P_2 \geq 2n \frac{\hbar\omega}{Vmc_o{}^2} \tag{14}$$

where n is the equilibrium density of the superfluid, m is the mass of the ^3He atom, c_o is the velocity of sound and V the volume of the sound mode. It is an interesting question as to whether it would be possible to reduce the classical noise in superfluid ^3He-B to the extent that the equality in (14) is satisfied. In optical systems it is possible to generate *coherent states of* light using stable lasers in which the noise is dominated by quantum fluctuations. Moreover, it has been possible to use four-wave mixing and homodyne detection to produce squeezed quantum states of light.[13] The noise power that is measured is similar to (13) with η equal to the quantum efficiency of the photodetector, i.e. the ratio of the number of incident photons to the number of photons detected. In optical experiments η is usually greater than 0.5 so the oscillations in P_s with ϕ are large enough to be observed. Even if it is possible to produce coherent phonon states, squeezed quantum states of sound will not be observable unless high efficiency phonon detectors are developed. The quartz transducers commonly used have a large acoustic impedance mismatch with superfluid ^3He, resulting in a low value for η.

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