Doublet Splitting and the Low-Field Evolution of the Real Squashing Modes in Superfluid ^{3}He-*B*

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We consider the effects of textures on the collective-mode spectrum of ^{3}He. Our result for the frequency splitting of the real-squashing-mode *doublet* agrees well with the measurements of Shivaram *et al.* The smooth disappearance of the doublet and the nonlinear evolution of the real-squashing-mode frequencies at low magnetic fields are explained in terms of a crossover to the dispersion-dominated regime when $(qv_f)^2/\Delta > \gamma H$.

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The superfluid phases of ^{3}He support a rich spectrum of collective excitations which have been studied extensively by use of NMR and ultrasound.[1] Most theoretical work on the high-frequency ($\omega \sim \Delta$) collective modes assumes that the equilibrium superfluid is spatially uniform. In this Letter we describe the theory of collisionless collective excitations in superfluid ^{3}He when the scale of the inhomogeneity is much longer than the coherence length $\xi_0 \sim \hbar v_f/\pi\Delta \sim 500$ Å. We discuss the effects of *textures* of the order parameter on the gap modes in ^{3}He and focus on the real-squashing- (RSQ-) mode *doublet* observed by Shivaram *et al.*[2]

The observation by Shivaram *et al.*[2] of *six* anomalies in the absorption of ultrasound, as shown in the inset of Fig. 1, appears to disagree with the identification of these anomalies as excitations of the five RSQ modes with angular momentum $J=2$. Volovik[4] proposed a phenomenological theory for the splitting of the central RSQ mode substate into a *doublet* in terms of the textural bending of the quantization axis of the RSQ modes. However, a quantitative comparison with the experiment is not possible without a microscopic theory. Using a gradient expansion of the *quasiclassical* equations[3,5] of ^{3}He, we derive the textural shifts of the RSQ modes, as well as the local oscillator form for the sound absorption proposed by Volovik.[4] Our main results are the following: (i) the calculated doublet frequency ω_d agrees with the measurements of Ref. 2; (ii) at low fields ($H < 200$ G), where few measurements have been reported, the RSQ modes evolve nonlinearly with field due to the competition between the dispersion energy ($\sim v_f^2 q^2/\Delta$) and Zeeman energy (γH); (iii) the *smooth* disappearance of the doublet below 200 G is caused by the crossover from the texture-dominated to the dispersion-dominated regime; and (iv) at high fields ($H > 1.5$ kG), where no measurements have been reported, the doublet splitting decreases with field due to gap distortion.

The superfluid phases of ^{3}He are condensates of spin-triplet ($S=1$), p-wave ($L=1$) Cooper pairs, which are described by a 3×3 complex order parameter $A_{\alpha i}$. The simplest form for the B-phase order parameter is the isotropic tensor $A_{\alpha i} \sim \delta_{\alpha i}$. However, all states obtained from the isotropic state by a relative rotation of spin and orbital coordinates, as well as by a uniform gauge trans-

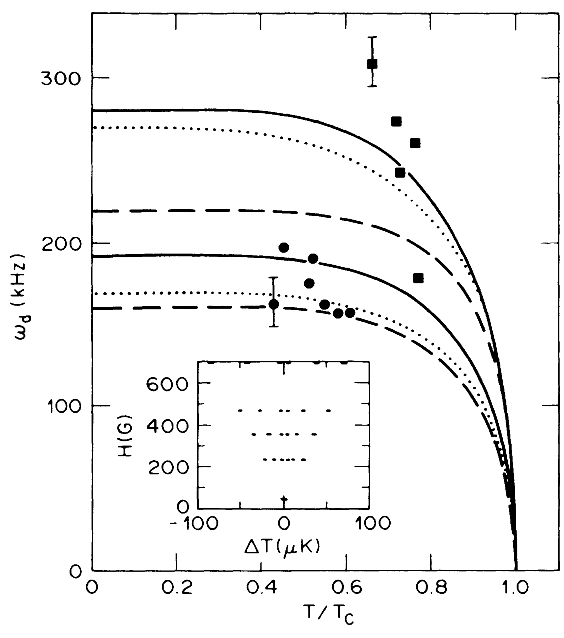

FIG. 1. The observed (Ref. 2) doublet splitting for 1 (squares) and 5 (circles) bars. The theoretical 1-bar curves use $T_c = 1.061$ mK, $F_0^a = -0.711$, $F_1^a = -0.73$, and (solid) $F_2^a = x_3^{-1} = 0$; (dashed) $F_2^a = -1.20$, $x_3^{-1} = -0.28$; and (dotted) $F_2^a = 0.53$, $x_3^{-1} = -0.28$. The 5-bar curves use $T_c = 1.477$ mK, $F_0^a = -0.733$, $F_1^a = -1.24$, and (solid) $F_2^a = x_3^{-1} = 0$; (dashed) $F_2^a = -0.40$, $x_3^{-1} = -0.10$; and (dotted) $F_2^a = 0.05$, $x_3^{-1} = -0.22$. Greywall's values (Ref. 17) are used for T_c and F_0^a. The values for F_2^a and x_3^{-1} were obtained from fits to the theory of Ref. 3. Inset: The temperature shifts of the RSQ modes at 63.8 MHz and 5.3 bars, with ^{3}He cooled in a field.

formation, have the same bulk condensation energy. Thus, the general representative of this class of states is

$$A_{ai} = \Delta e^{i\Phi} R[\hat{\mathbf{n}},\theta]_{ai}\,, \tag{1}$$

where the matrix $\mathbf{R}[\hat{\mathbf{n}},\theta]$ generates a rotation about $\hat{\mathbf{n}}$ by θ.[6]

The condensation energy fixes the gap Δ and the weak dipolar energy fixes the Leggett angle $\theta = \cos^{-1}(-\frac{1}{4})$ for small fields. The long-wavelength spatial variations of the phase Φ and the axis $\hat{\mathbf{n}}$, which cost little energy compared with the consensation energy, are determined by walls and magnetic fields. While a spatially varying phase $\Phi(\mathbf{R})$ defines the superfluid velocity $\mathbf{v}_s(\mathbf{R}) \sim \nabla\Phi$, a spatially varying axis of rotation $\hat{\mathbf{n}}(\mathbf{R})$ defines the *texture* and generates a spin supercurrent with a velocity field $\omega_i^a = \epsilon_{a\mu\nu} R_{\mu l} \nabla_i R_{\nu l}$. A magnetic field tends to align $\hat{\mathbf{n}}$ along field lines and a wall typically orients $\hat{\mathbf{n}}$ at an oblique angle to the surface. The axis $\hat{\mathbf{n}}$ heals from the wall over the length scale $\xi_H \approx 46(1-T/T_c)$ cm G/H, which is usually much longer than the coherence length.[7]

Before discussing the effects of $\hat{\mathbf{n}}$ textures, we summarize the spectrum of collective modes for uniform ^{3}He-B.[1] The Goldstone modes of ^{3}He-B are zero sound (oscillations of Φ) and spin waves (oscillations of R_{ai}). The *gap modes,* which are unrelated to the equilibrium order parameter by a gauge, spin, or space rotation, have excitation energies of order Δ. Several of these modes have a simple physical interpretation as excitations out of the condensate into an *excited state of bound Cooper pairs.* Ultrasound has proven to be an excellent probe of these excited pair states.[1]

The equilibrium order parameter A_{ai} in Eq. (1) is an eigenstate of the *twisted* angular momentum, $\mathbf{J} = \mathbf{L} + \mathbf{S}\cdot\mathbf{R}$, with $J=0$. The collective excitations above this ground state are classified by the eigenvalues (J,m), where m is the projection of $\mathbf{J}$ along a suitably chosen quantization axis. The eighteen collective modes that are associated with the p-wave, spin-triplet order parameter are grouped into multiplets with $J=0$ (nondegenerate), $J=1$ (threefold degenerate), and $J=2$ (fivefold degenerate). Two sets of collective modes for each quantum number J correspond to excitations of the *real* $(+)$ and *imaginary* $(-)$ parts of the order parameter.

Most of the order-parameter modes have been observed: the $J=0^-$ mode is observable as zero sound; the $J=1^+$ modes are observed in NMR; the $J=1^-$ gap modes have been observed very close to the gap edge 2Δ in a magnetic field[8]; and the $J=2^{\pm}$ modes are observed as sharp features in the attenuation and group velocity of zero sound.[9] When the zero-sound mode dispersion relation $\omega = c_0 q$ intersects that of the $J=2^{\pm}$ modes, at frequencies of $\omega_{2+} \simeq 1.1\Delta$ and $\omega_{2-} \simeq 1.5\Delta$, the resonant excitation of Cooper pairs strongly damps the sound mode. The most convincing demonstration of this classification was the observation by Avenel, Varoquaux, and Ebisawa[10] of the fivefold splitting of the $J=2^+$ modes (the RSQ modes) in a magnetic field. The Zeeman splitting of the $J=2^-$ modes has also recently been observed.[11]

The observation of a *sixth* RSQ mode,[2] lying close in frequency to the central $m=0$ substate for fields greater than 200 G, does not fit into this symmetry classification. The splitting of the central mode into a *doublet* survives down to a field of 100 G for field-cooled ^{3}He-B and appears only above 500 G when ^{3}He is first cooled below T_c in zero field. This hysteresis suggests the role of a texture. Here we use the quasiclassical theory to derive the texture-induced splitting of the doublet.

The central objects of the quasiclassical theory are the propagator $\hat{g}(\hat{\mathbf{p}},\epsilon;\mathbf{R},t)$ and gap matrix $\hat{\Delta}(\hat{\mathbf{p}},\mathbf{R},t)$, which are 4×4 matrices in particle-hole and spin space. The diagonal elements of $\hat{g}$ in particle-hole space describe the evolution of the quasiparticles (with excitation energy ϵ and Fermi velocity $v_f\hat{\mathbf{p}}$), and the off-diagonal elements represent the time-dependent pair amplitude (with center-of-mass position $\mathbf{R}$). Acoustic and magnetic modes couple to oscillations of the time-dependent, spin-triplet gap matrix, written in terms of the vector $\mathbf{d}$ as $\hat{\Delta}_{12}(\hat{\mathbf{p}},\mathbf{R},t) = \mathbf{d}\cdot(i\sigma\sigma_2)$. The *inhomogeneous,* equilibrium gap matrix is obtained by local unitary transformation applied to a spatially uniform, *reference* order parameter $\hat{\Delta}_0(\hat{\mathbf{p}})$.

A gradient expansion of the gap equation to leading order in ξ_0/ξ_H yields the eigenvalue equation for the real part of $\mathbf{d}$,

$$\int \frac{d\Omega'}{4\pi}\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}'\left\{\frac{\omega^2}{4\Delta^2}\mathbf{d}(\hat{\mathbf{p}}') - \Delta_0(\hat{\mathbf{p}}')\cdot\mathbf{d}(\hat{\mathbf{p}}')\Delta_0(\hat{\mathbf{p}}')/\Delta^2\right\} = \int \frac{d\Omega'}{4\pi}\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}'\left\{\left(\frac{\mathbf{q}\cdot\mathbf{v}_{\hat{\mathbf{p}}'}}{\Delta}\right)^2 [\Pi_1\mathbf{d}(\hat{\mathbf{p}}') + \Pi_2\Delta_0(\hat{\mathbf{p}}')\cdot\mathbf{d}(\hat{\mathbf{p}}')\Delta_0(\hat{\mathbf{p}}')/\Delta^2] + i\Pi_3\frac{\gamma H}{\Delta^3}\hat{\mathbf{H}}\cdot\mathbf{R}[\hat{\mathbf{n}},\theta]\cdot\Delta_0(\hat{\mathbf{p}}')[\Delta_0(\hat{\mathbf{p}}')\times\mathbf{d}(\hat{\mathbf{p}}')] + i\Pi_4\frac{\mathbf{v}_{\hat{\mathbf{p}}'}\cdot\mathbf{q}}{\Delta^2}(\mathbf{v}_{\hat{\mathbf{p}}'})i[\omega_i\times\mathbf{d}(\hat{\mathbf{p}}')] + \cdots\right\}, \tag{2}$$

where Π_i are dimensionless functions of frequency and temperature of order 1 and Fermi-liquid corrections are suppressed. The perturbative corrections to the eigenvalue equation for the RSQ modes are given on the right-hand side of Eq. (2). The texture appears here in the form $\hat{\mathbf{h}}(\mathbf{R}) = \hat{\mathbf{H}}\cdot\hat{\mathbf{R}}[\hat{\mathbf{n}}(\mathbf{R}),\theta]$.[12] The quantization axis of the collective modes is determined by the largest correction on the right-hand side of Eq. (2). In the high-field limit, $\gamma H \gg (qv_F)^2/\Delta$, the Zeeman energy dominates the dispersion energy, $(qv_F)^2/\Delta$, so that the quantization axis is fixed along $\hat{\mathbf{h}}(\mathbf{R})$; but in the low-field limit the quantization axis is fixed along $\mathbf{q}$, *independent* of the texture. In high fields, Eq. (2) is solved by

$$\omega_m(T,\mathbf{R})^2 = \omega_{\mathrm{RSQ}}(T)^2 + c_{1m}^2 q^2 + c_{2m}^2[\hat{\mathbf{h}}(\mathbf{R})\cdot\mathbf{q}]^2\,, \tag{3}$$

where the velocities c_{1m} and c_{2m} are of order v_F. We have calculated these mode velocities as functions of temperature and pressure within weak-coupling theory, including Fermi-liquid interactions.

The measurements of Ref. 2 were performed in a parallel-wall geometry with sound propagating perpendicular to the walls ($\hat{\mathbf{q}}=\hat{\mathbf{x}}$) and to the magnetic field ($\mathbf{H}=H\hat{\mathbf{z}}$). The texture in the cell for high fields is described in Ref. 4. The quantization axis changes from $\hat{\mathbf{h}}=\hat{\mathbf{x}}$ at the walls to $\hat{\mathbf{h}}=\hat{\mathbf{z}}$ in the interior, so that the RSQ-mode frequencies ω_m vary with position along the sound trajectory. The RSQ modes are excited when the zero-sound frequency ω and wave vector q match the RSQ-mode frequencies and wave vectors. For wavelengths $2\pi/q$ smaller than the healing length ξ_H, a gradient expansion yields the local oscillator form for the sound absorption, which was proposed by Volovik[4] to explain the doublet:

$$\alpha(\omega)=\sum_m \frac{1}{L}\int_{-L/2}^{L/2} dx\,\lambda_m(x)\delta(\omega-\omega_m(x))=\sum_m \frac{\lambda_m(x)}{L}\left|\frac{d\omega_m}{dx}\right|^{-1}_{x=x_m(\omega)}, \tag{4}$$

where λ_m is the coupling strength to the mth RSQ mode and L is the path length of sound. In the high-field texture, $\hat{\mathbf{h}}\cdot\hat{\mathbf{q}}=1$ at $x=\pm L/2$ and $\hat{\mathbf{h}}\cdot\hat{\mathbf{q}}=0$ at $x=0$. Hence, the sound absorption is dominated by excitation of the RSQ modes at the walls and in the center of the cell, where $d\omega_m/dx=0$. It can be shown that the coupling strength λ_m is finite for the $m_h=\hat{\mathbf{h}}\cdot\mathbf{J}=\pm 2$ and 0 modes in the center of the cell and nonzero *only* for the $m_h=0$ mode at the walls.[13]

The doublet splitting is the frequency difference of the $m_h=0$ modes at the walls and in the center of the cell:

$$\omega_d=\frac{c_{20}^2 q^2}{2\omega_{\text{RSQ}}}[\hat{\mathbf{h}}(\pm L/2)\cdot\hat{\mathbf{q}}]^2=\frac{1}{7}\left(\frac{qv_F}{\Delta}\right)^2\frac{\Delta^2}{\omega_{\text{RSQ}}}\left(1+\frac{3}{10}\frac{f(\lambda)}{\lambda}\right), \tag{5}$$

where $f(\lambda)/\lambda$ is a function of temperature and frequency given in Ref. 3 and Fermi-liquid corrections are omitted. Since $q=\omega/c_0$, $\omega_d \propto (v_F/c_0)^2\Delta$ decreases with increasing pressure, as shown in Fig. 1 and in agreement with Ref. 2. We find[14] that ω_d is sensitive to the Fermi-liquid parameter F_2^a and the f-wave pairing parameter x_3, which are used in the theoretical curves of Fig. 1. It also follows from Eqs. (83) and (84) of Ref. 3 that ω_d is equal to the difference between the *zero-field* dispersion shifts of the $m_q=\hat{\mathbf{q}}\cdot\mathbf{J}=\pm 2$ and ± 1 modes, in agreement with Volovik.[4]

Below 1 kG, two effects explain the field dependence of ω_d and the hysteresis of the doublet. When ^{3}He is cooled below T_c in zero field, the doublet appears *abruptly* at about 500 G. The zero-field-cooled texture is nearly uniform with $\hat{\mathbf{h}}(\mathbf{R})$ nonparallel to $\mathbf{H}$ throughout the cell. At $H\approx 500$ G, Volovik[4] argues that a textural transition aligns $\hat{\mathbf{h}}$ along the field in the center of the cell, causing the sudden appearance of the doublet. In field-cooled ^{3}He-*B* the doublet survives to fields as low as 100 G but ω_d decreases *smoothly* below 200 G. For the field-cooled texture, $\hat{\mathbf{h}}(\mathbf{R})$ is parallel to $\mathbf{H}$ in the center of the cell.[15] However, for $H<200$ G the quantization axis crosses over from $\hat{\mathbf{h}}(\mathbf{R})$ to the sound propagation direction $\hat{\mathbf{q}}$ as the dispersion energy dominates the Zeeman energy, *everywhere* in the experimental cell. Thus, if the field-cooled texture is metastable to very low fields, as we expect, the frequency splitting of the doublet will decrease smoothly below 200 G, in qualitative agreement with experiment.

The crossover from the field-dominated regime to the dispersion-dominated regime also explains the low-field *nonlinear* evolution of the RSQ modes shown in Fig. 2, where the RSQ-mode frequencies are plotted versus field

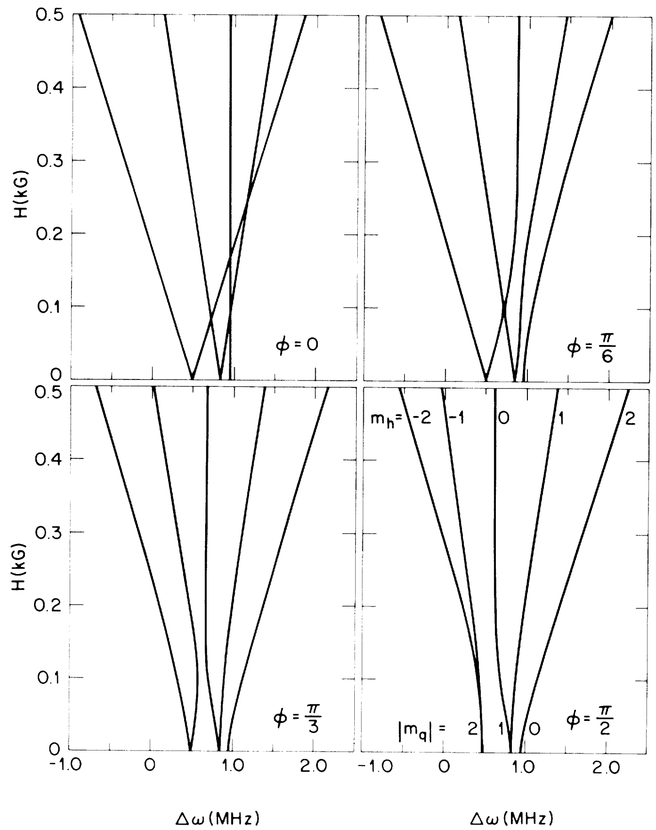

FIG. 2. The low-field evolution of the RSQ modes at $T=0$ and 0 bar for various values of $\phi=\cos^{-1}(\hat{\mathbf{z}}\cdot\mathbf{R}\cdot\hat{\mathbf{q}})$, with Greywall's values (Ref. 17) for $F_0^a=-0.700$ and $T_c=0.929$ mK, but all other material parameters set to zero.

for different values of $\phi=\cos^{-1}(\hat{\mathbf{h}}\cdot\hat{\mathbf{q}})$. In all four cases, the zero-field mode frequencies are split by dispersion with the quantization axis along $\hat{\mathbf{q}}$. The modes evolve linearly with H only if $\hat{\mathbf{h}}=\hat{\mathbf{q}}$. If $\phi\neq 0$, the magnetic field mixes the zero-field eigenmodes and the frequencies evolve nonlinearly with H because of mode repulsion. At sufficiently high fields ($\gamma H \gg q^2 v_F^2/\Delta$), the quantization axis switches from $\hat{\mathbf{q}}$ to $\hat{\mathbf{h}}$ and the linear Zeeman effect is recovered. Note that the $m_q=0$ state evolves into the $m_h=+2$ magnetic states, the $|m_q|=1$ states evolve into the $m_h=+1,0$ magnetic states, and the $|m_q|=2$ states evolve into the $m_h=-1,-2$ magnetic states.

At very high fields, above 1.5 kG, we expect the doublet to exhibit a weak-field dependence, which has not yet been observed. The field dependence of the Leggett angle θ and of the texture, which are due to gap distortion, both act to decrease $\hat{\mathbf{h}}\cdot\hat{\mathbf{q}}$ at the cell walls and hence to decrease ω_d. If only the variation of the Leggett angle is considered, then

$$\hat{\mathbf{h}}(\pm L/2)\cdot\hat{\mathbf{q}}=(1-\cos\theta)/5+\sqrt{3/5}\sin\theta. \qquad (6)$$

Using Eq. 5 and the field dependence[16] of θ, we estimate that ω_d decreases by $\sim$20% at 2.5 kG.

The agreement between ultrasonic measurements in ^{3}He and weak-coupling theory leaves little doubt that the doublet in the RSQ-mode spectrum is due to the textural shift of the $m_h=0$ mode at the wall relative to the center of the cell. Additional experiments on textural effects in the B-phase collective mode spectrum can provide new information on the quasiparticle interactions, as well as test our predictions for the low-field, nonlinear evolution of the RSQ modes.

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