Influence of strong-coupling corrections on the equilibrium phase for $^3P_2$ superfluid neutron-star matter

V. Z. Vulovic* and J. A. Sauls
Joseph Henry Laboratories of Physics, Princeton University, Princeton, New Jersey 08544
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We calculate strong-coupling corrections to the $^3P_2$ neutron-star-matter Ginzburg-Landau functional including spin-orbit and central forces. Based on a two-parameter approximation for the spin-orbit scattering amplitude and typical estimates for the neutron-matter Landau parameters we conclude that the most likely equilibrium phase of $^3P_2$ neutron matter is described by a unitary order parameter. Better calculations of neutron-matter parameters, particularly the spin-orbit scattering amplitude, would allow a stronger conclusion.

INTRODUCTION AND SUMMARY

Recent theoretical work on superfluidity in neutron stars has concentrated on the structure and hydrodynamics of the rotating $^3P_2$ superfluid interior.[1,2] The novel properties of the $^3P_2$ neutron superfluid that distinguish it from the more conventional *s*-wave superfluid are a consequence of spontaneously broken spin-orbit symmetry. In particular, the structure of vortices in the $^3P_2$ neutron superfluid, which play a central role in theories of the rotational dynamics of pulsars, depends implicitly on the equilibrium-phase order parameter, or condensate amplitude for the $^3P_2$ neutron pairs.[3] The problem of determining the equilibrium-state order parameter $^3P_2$ pairing separates into two parts. The first part is to determine the possible phases by minimizing the general fourth-order Ginzburg-Landau (GL) free-energy functional over the space of $^3P_2$ order parameters for arbitrary values of the parameters that define the functional. This problem has been solved by Sauls and Serene[4] and Mermin.[5] The second part of the problem, which is the subject of this paper, is to calculate the parameters which define the GL functional from a microscopic theory and thereby determine the equilibrium phase. The calculation presented below extends the earlier work of Ref. 4 to include spin-orbit scattering in the strong-coupling corrections to BCS theory for $^3P_2$ pairing. Our conclusion that the equilibrium phase of $^3P_2$ neutron matter is described by a unitary order parameter agrees with the tentative conclusion of Sauls and Serene; we emphasize that this conclusion is significantly strengthened by our calculations which include spin-orbit scattering. In the Introduction we briefly review the GL theory of $^3P_2$ pairing and pay particular attention to the relevance of corrections to the BCS theory, discuss the importance of the spin-orbit forces to the properties of neutron matter at high density, and summarize our results for the equilibrium phase diagram for $^3P_2$ neutron matter. The rest of the paper summarizes the calculation of strong-coupling corrections with spin-orbit scattering for $^3P_2$ pairing.

The GL theory of $^3P_2$ pairing is discussed by several authors; for our purpose we use the notation of Sauls and Serene.[4] The order parameter $A_{\mu\nu}$ for $^3P_2$ pairing is a three-dimensional complex matrix which is both traceless and symmetric. The equilibrium order parameter is determined by minimizing the homogeneous mean-field free-energy functional over the space of $^3P_2$ order parameters. This functional, expanded through fourth-order in $A_{\mu\nu}$, is

$$\Delta\Omega[A]=\tfrac{1}{3}\alpha\,\mathrm{Tr}AA^* +\bar\beta_1\,|\,\mathrm{Tr}A^2\,|^2+\bar\beta_2(\mathrm{Tr}AA^*)^2 +\bar\beta_3\,\mathrm{Tr}A^2A^{*2}\,. \qquad (1.1)$$

The important result is that all the minima of this functional can be found for any set of parameters $\{\bar\beta_1\}$. There are three classes of minima corresponding to the three labeled regions of the phase diagram (Fig. 1). In the region

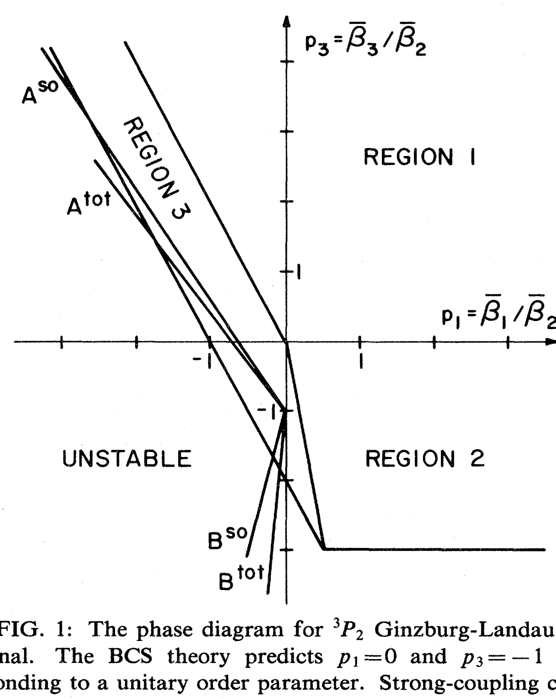

FIG. 1: The phase diagram for $^3P_2$ Ginzburg-Landau functional. The BCS theory predicts $p_1=0$ and $p_3=-1$ corresponding to a unitary order parameter. Strong-coupling corrections give $p_1<0$ and a phase point between the half-lines A^{so} and B^{so} (with slopes of -1.6 and 4.0) in the limit when spin-orbit forces dominate, or between the half-lines A^{tot} and B^{tot} (with slopes of -1.4 and 11), when we use Landau parameter values of Bäckman *et al.*[11] The phase point moves away from nonunitary regions 1 and 2.

1 the superfluid state (referred to as "type 1" hereafter) is described by

$$A^{(1)}_{\mu\nu} \propto (u_\mu + iv_\mu)(u_\nu + iv_\nu) \quad (1.2)$$

with $\hat{u}\cdot\hat{v}=0$, corresponding to a condensate of neutron pairs in a pure $M_J=+2$ state along $\hat{w}=\hat{u}\times\hat{v}$. The type-1 phase is a ferromagnetic superfluid with magnetization

$$M \sim \left[\frac{\gamma_n \hbar}{2}\right]\frac{k_F{}^3}{3\pi^2}\left[\frac{\Delta}{E_F}\right]^2 ,$$

which is of the same order of magnitude as the magnetic field in the interior of neutron stars. The type-1 phase would also have interesting rotational dynamics because the order parameter allows for vortex structures without singular cores.

The type-2 phase has an order parameter

$$A^{(2)}_{\mu\nu} \propto u_\mu u_\nu + e^{i2\pi/3} v_\mu v_\nu + e^{-i2\pi/3} w_\mu w_\nu , \quad (1.3)$$

where $(\hat{u},\hat{v},\hat{w})$ is an orthonormal triad. There is no net spin polarization in this phase even though time-reversal symmetry is broken by the type-2 order parameter. Because of the complex phase factors in Eq. (1.3) the type-2 phase has interesting topologically stable line defects that carry circulation; however, this phase does not allow for the coreless vortex structures associated with the type-1 phase.

In region 3 the GL free-energy functional is minimized by any real, traceless and symmetric order parameter

$$A^{(3)}_{\mu\nu} \propto u_\mu u_\nu + r v_\mu v_\nu - (1+r) w_\mu w_\nu , \quad (1.4)$$

where $(\hat{u},\hat{v},\hat{w})$ is an orthonormal triad and $-1 \le r \le -\frac{1}{2}$ parametrizes the accidental degeneracy of the type-3 phases. In particular, the state with $r=-\frac{1}{2}$,

$$A^{(3)}_{\mu\nu} \propto (u_\mu u_\nu - \tfrac{1}{3}\delta_{\mu\nu}) , \quad (1.5)$$

describes ${}^3P_2$ Cooper pairs in a pure $M_J=0$ state with u as the quantization axis, and is the most probable candidate for the uniform equilibrium ${}^3P_2$ phase if a type-3 phase is energetically stable.[4]

The type-3 phases are likely candidates for the equilibrium state because the BCS theory values of the GL parameters $\{\bar{\beta}_i\}$ lie in region 3. However, relatively small corrections to the BCS parameters could stabilize the type-2 phase. Much larger modifications to the BCS-theory values could stabilize the ferromagnetic type-1 phase or destabilize all possible phases within the fourth-order GL theory.

The corrections to the BCS free-energy functional were systematically examined by Rainer and Serene.[6] There it was shown that the free energy has an expansion in the parameter T_c/T_F, the ratio of the transition temperature to the Fermi temperature. Estimates of this ratio for the ${}^3P_2$ neutron superfluid vary between 10^{-3} and 10^{-1}.[15,4] The BCS free energy is of the order $(T_c/T_F)^2$, while the strong-coupling corrections to the free energy are of the order $(T_c/T_F)^3|T|^2$, where $|T|$ is the normalized quasiparticle-scattering amplitude. The important conclusion of Rainer and Serene is that to leading order in T_c/T_F the strong-coupling corrections are given by weighted angular averages of the normal-state scattering amplitude for quasiparticles at the Fermi surface. Thus, with a good approximation for the quasiparticle-scattering amplitude the leading-order strong-coupling corrections can be calculated. The form of the dimensionless quasiparticle-scattering amplitude $T_{\alpha\beta,\gamma\rho}(\hat{\kappa}_1,\hat{\kappa}_2;\hat{\kappa}_3,\hat{\kappa}_4)$ is dictated by the microscopic forces among particles. When only central forces are present the T amplitude has the form

$$T^{(\mathrm{cen})}_{\alpha\beta,\gamma\rho}(\hat{\kappa}_1,\hat{\kappa}_2;\hat{\kappa}_3,\hat{\kappa}_4) = T^{(s)}(\theta,\phi)\delta_{\alpha\gamma}\delta_{\beta\rho} + T^{(a)}(\theta,\phi)\vec{\sigma}_{\alpha\gamma}\cdot\vec{\sigma}_{\beta\rho} , \quad (1.6)$$

where (θ,ϕ) are the Abrikosov-Khalatnikov[7] angles for four unit vectors $\hat{\kappa}_1$, $\hat{\kappa}_2$, $\hat{\kappa}_3$, $\hat{\kappa}_4$ for the directions of the quasiparticle momenta which satisfy the momentum conservation law $\hat{\kappa}_1+\hat{\kappa}_2=\hat{\kappa}_3+\hat{\kappa}_4$. Specifically, $\cos\theta=\hat{\kappa}_1\cdot\hat{\kappa}_2$ and $\cos\phi=\hat{\kappa}_1\cdot(\hat{\kappa}_3-\hat{\kappa}_4)/(1-\hat{\kappa}_1\cdot\hat{\kappa}_2)$. Nucleon-nucleon scattering phase-shift data at laboratory energies $E_L \ge 300$ MeV (corresponding to Fermi energies $E_F \ge 75$ MeV) suggest that at the densities $\rho \ge 5.10^{13}$ g/cm^3 (i.e., inside neutron stars) spin-orbit forces between neutron excitations at the Fermi surface are large, while central forces are smaller and repulsive.[8] Thus, we suggest that the T amplitude at high densities in neutron-star matter is dominated by a spin-orbit scattering term

$$T^{(\mathrm{so})}_{\alpha\beta,\gamma\rho}(\hat{\kappa}_1,\hat{\kappa}_2;\hat{\kappa}_3,\hat{\kappa}_4) = L(q,q')\hat{q}\times\hat{q}'\cdot(\delta_{\alpha\gamma}\vec{\sigma}_{\beta\rho}+\vec{\sigma}_{\alpha\gamma}\delta_{\beta\rho}) . \quad (1.7)$$

Here $\vec{q}=\vec{\kappa}_3-\vec{\kappa}_1$ and $\vec{q}\,'=\vec{\kappa}_4-\vec{\kappa}_1$ are the momentum transfers and $\vec{\kappa}_i=\kappa_F\hat{\kappa}_i$, $i=1,2,3,4$. In a potential approximation the function $L(q,q')$ is given by

$$L(q,q') = \frac{i}{4}\left[q'\frac{d}{dq}\tilde{f}(q) + q\frac{d}{dq'}\tilde{f}(q')\right] , \quad (1.8)$$

where $\tilde{f}(q)$ is proportional to the Fourier transform of the radial factor $V(r)$ in the spin-orbit interaction $V(r)\vec{\mathrm{L}}\cdot\vec{\mathrm{S}}/\hbar^2$.

The total T amplitude is given by the sum of $T^{(\mathrm{cen})}$ and $T^{(\mathrm{so})}$ in Eqs. (1.6) and (1.7).[9] The strong-coupling corrections $\Delta\bar{\beta}_i$ in the GL free-energy functional are weighted angular averages of the total T amplitude. In order to evaluate these quantities we use the s-p wave approximation of Dy and Pethick,[10] which relates $T^{(\mathrm{cen})}$ to the $l=0,1$ Landau-Fermi liquid parameters, while for $T^{(\mathrm{so})}$ it is most convenient to expand $(d/dq)\tilde{f}(q)$ in the Legendre polynomials of $x_2=\hat{\kappa}_1\cdot\hat{\kappa}_2$ $[q=2\kappa_F((1-x_2)/2)^{1/2}]$. After retaining only the $l=0,1$ terms in $(d/dq)\tilde{f}(q)$, the resulting expressions for $\Delta\beta_i$ are functions of three Landau parameters A^s_0, A^a_0, and A^s_1, and two spin-orbit parameters a_0 and a_1. When spin-orbit forces dominate we show that the corrections to the BCS theory do not lead to nonunitary phases; spin-orbit scattering moves the phase point away from regions 1 and 2. Very large values of T_c/T_F and the spin-orbit coupling strength may lead to breakdown of the stability conditions on the fourth-order GL functional.

To discuss the phase point $(\bar{\beta}_1/\bar{\beta}_2,\bar{\beta}_3/\bar{\beta}_2)$ with both central and spin-orbit scattering included, we fix $T^{(\mathrm{cen})}$ with the Landau parameters evaluated by other authors[11,12] and vary the spin-orbit parameter a_0 (it turns out that contributions proportional to a_1 can be neglected). For a range of values of a_0 determined from nucleon phase-shift data, we find that the phase point moves away from regions 1 and 2, so that spin-orbit scattering is not expected to stabilize either of these phases.

II. CALCULATION AND ANALYSIS OF STRONG-COUPLING CORRECTIONS

The GL functional is a functional of the off-diagonal self-energy $\Delta(\hat{\kappa})$, which is related to the 3×3-matrix order parameter by

$$\Delta(\hat{\kappa})=i\vec{\sigma}\sigma^2\cdot\vec{\Delta}(\hat{\kappa}) ,$$
$$\Delta_\mu(\hat{\kappa})=\sum_{j=1}^{3} A_{\mu j}(\hat{\kappa})_j . \quad (2.1)$$

The GL functional can be written as the sum of the BCS term plus strong-coupling corrections,

$$\Delta\Omega_{\rm GL}[\Delta]=\Delta\Omega_{\rm BCS}[\Delta]+\Delta\phi_{\rm SC}[\Delta] , \quad (2.2)$$

where $\Delta\phi_{\rm SC}$ has a diagrammatic expansion. The leading terms for $\Delta\phi_{\rm SC}$ are determined by the normal-state quasiparticle-scattering amplitude and T_c/T_F.[6] Using the notation of Rainer and Serene,[6] $\Delta\phi_{\rm SC}=\Delta\phi_B+\Delta\phi_C+\Delta\phi_D+\Delta\phi_F$, the expressions for $\Delta\phi_\alpha$ ($\alpha=B,C,D,F$) are calculated with the scattering amplitude $T=T^{(\rm cen)}+T^{(\rm so)}$ given by Eqs. (1.6) and (1.7) and are listed in Appendix A. After doing the spin traces the resulting expressions contain angular integrals of the form

$$\int\frac{d\Omega_1}{4\pi}\int\frac{d\Omega_2}{4\pi}\int\frac{d\Omega_3}{4\pi}\delta(||\hat{\kappa}_1+\hat{\kappa}_2-\hat{\kappa}_3||-1)\times A(\hat{\kappa}_1,\hat{\kappa}_2,\hat{\kappa}_3)B(\hat{\kappa}_1,\hat{\kappa}_2,\hat{\kappa}_3) , \quad (2.3)$$

where $A(\hat{\kappa}_1,\hat{\kappa}_2,\hat{\kappa}_3)$ depends only on $T^{(\alpha)}=T^{(\alpha)}(\theta,\phi)$, $\tilde{T}^{(\alpha)}=T^{(\alpha)}(\tilde{\theta},\tilde{\phi})$, $L=L(q,q')$, and $\tilde{L}=L(q,\kappa)$ with $\alpha=s$, a, $\vec{\kappa}=\vec{\kappa}_1+\vec{\kappa}_2$, and $(\tilde{\theta},\tilde{\phi})$ are Abrikosov-Khalatnikov angles for $(\hat{\kappa}_3,-\hat{\kappa}_2,\hat{\kappa}_1,-\hat{\kappa}_4)$. All these amplitudes are functions of (θ,ϕ). The function $B(\hat{\kappa}_1,\hat{\kappa}_2,\hat{\kappa}_3)$ contains products of different energy-gap vectors $\Delta_\mu(\hat{\kappa}_i)$ and projections of $\hat{q}$, $\hat{q}'$, and $\hat{\kappa}$, and therefore it depends on other variables besides (θ,ϕ). In Appendix A we integrate out these extra variables expressing $\Delta\phi_\alpha$ in terms of weighted averages over the angles (θ,ϕ). Our result for the strong-coupling corrections

$$\Delta\bar{\beta}_i=\Delta\bar{\beta}_i^{B+C}+\Delta\bar{\beta}_i^{D}+\Delta\bar{\beta}_i^{C}$$

in terms of these averages is

$$\Delta\bar{\beta}_i^{B+C}=-\eta\frac{6.84}{16}\langle\bar{w}_i^{B+C}T^{(s)2}+\bar{v}_i^{B+C}T^{(a)2}+\bar{u}_i^{B+C}L^2+\bar{X}_i^{(s)B+C}iLT^{(s)}+\bar{X}_i^{(a)B+C}iLT^{(a)}\rangle ,$$
$$\Delta\bar{\beta}_i^{D}=-\eta\frac{10.15}{4}\langle\bar{w}_i^{D}(T^{(s)}\tilde{T}^{(s)}+T^{(a)}\tilde{T}^{(a)})+\bar{v}_i^{D}(T^{(s)}\tilde{T}^{(a)}+T^{(a)}\tilde{T}^{(s)})+\bar{u}_i^{D}L\tilde{L}+\bar{X}_i^{(s)D}iL\tilde{T}^{(s)}+\bar{X}_i^{(a)D}iL\tilde{T}^{(a)}\rangle , \quad (2.4)$$
$$\Delta\bar{\beta}_i^{F}=-\eta\frac{30.44}{16}\langle\bar{w}_i^{F}T^{(s)2}+\bar{v}_i^{F}T^{(a)2}+\bar{u}_i^{F}L^2+\bar{X}_i^{(s)F}iLT^{(s)}+\bar{X}_i^{(a)F}iLT^{(a)}\rangle .$$

TABLE I. The weighting functions $\bar{w},\bar{v},\bar{u},\bar{X}^{(s)(a)}$ expressed over momentum-transfer variables t_2 and t_3, where $x'=[(1-t_2-t_3)t_3]^{1/2}$.

	$B+C$	D	F
$\bar{w}_1$	$10(t_2-t_2^2)$	$-1+2t_2+6(t_3-t_2t_3-t_3^2)$	$20(t_2-t_2^2)-40t_2t_3$
$\bar{w}_2$	$8-8(t_2-t_2^2)$	$-4+8t_2+4(t_3-t_2t_3-t_3^2)$	$8-8(t_2-t_2^2)-8(t_3-t_3^2)-32t_2t_3$
$\bar{w}_3$	$-8-12(t_2-t_2^2)$	$2-4t_2-12(t_3-t_2t_3-t_3^2)$	$-8-52(t_2-t_2^2)+28(t_3-t_3^2)+112t_2t_3$
$\bar{v}_1$	$4-34(t_2-t_2^2)$	$1-2t_2-16(t_3-t_2t_3-t_3^2)$	$8-68(t_2-t_2^2)-48(t_3-t_3^2)+168t_2t_3$
$\bar{v}_2$	$8-8(t_2-t_2^2)$	$-4+8t_2+4(t_3-t_2t_3-t_3^2)$	$-8+8(t_2-t_2^2)+8(t_3-t_3^2)+32t_2t_3$
$\bar{v}_3$	$60(t_2-t_2^2)$	$6-12t_2+24(t_3-t_2t_3-t_3^2)$	$24+36(t_2-t_2^2)-44(t_3-t_3^2)-176t_2t_3$
$\bar{u}_1$	$(-21t_2-21t_3+28t_2t_3)/7$	$x'(1-\frac{4}{7}t_2)$	$[-16+72(t_2+t_3)-48(t_2^2+t_3^2)-248t_2t_3]/7$
$\bar{u}_2$	$(-96+96t_2+16t_3-120t_2^2-96t_2t_3-40t_3^2)/7$	$x'(-\frac{8}{7}t_2)$	$[16-16(t_2+t_3)+48(t_2^2+t_3^2)-256t_2t_3]/7$
$\bar{u}_3$	$(24+18t_2+66t_3+72t_2^2+24t_2t_3+24t_3^2)/7$	$x'(-2)$	$[16-128(t_2+t_3)+48(t_2^2+t_3^2)+752t_2t_3]/7$
$\bar{X}_1^{(s)}$	$(12-24t_2)(t_2t_3)^{1/2}$	$[16-16(t_2+t_3)](t_2t_3)^{1/2}$	$(-20+8t_2+32t_3)(t_2t_3)^{1/2}$
$\bar{X}_2^{(s)}$	0	$[-16(t_2+t_3)](t_2t_3)^{1/2}$	$(-32t_2+32t_3)(t_2t_3)^{1/2}$
$\bar{X}_3^{(s)}$	$(-24+48t_2)(t_2t_3)^{1/2}$	$[-20+48(t_2+t_3)](t_2t_3)^{1/2}$	$(40+16t_2-96t_3)(t_2t_3)^{1/2}$
$\bar{X}_1^{(a)}$	$(-12+24t_2)(t_2t_3)^{1/2}$	$[-26+32(t_2+t_3)](t_2t_3)^{1/2}$	$(60-72t_2-48t_3)(t_2t_3)^{1/2}$
$\bar{X}_2^{(a)}$	0	$[-16(t_2+t_3)](t_2t_3)^{1/2}$	$(-32t_2+32t_3)(t_2t_3)^{1/2}$
$\bar{X}_3^{(a)}$	$(24-48t_2)(t_2t_3)^{1/2}$	$[52-48(t_2+t_3)](t_2t_3)^{1/2}$	$(-124+176t_2+72t_3)(t_2t_3)^{1/2}$

The coefficient $\eta \equiv N(0)(30k_B T_c v_F p_F)^{-1}$ is related to the BCS value of $\bar{\beta}_2$ by $\eta = 1.173(T_c/T_F)\bar{\beta}_2^{\rm BCS}$ and the strong-coupling GL coefficients are defined by

$$\Delta\phi_\alpha = \Delta\bar{\beta}_i^\alpha \,|\,{\rm Tr}A^2\,|^2 + \Delta\bar{\beta}_2^\alpha({\rm Tr}AA^*)^2 + \Delta\bar{B}_3^\alpha {\rm Tr}A^2A^{*2} ,$$

while $\Delta\bar{\beta}_i^{B+C} = \Delta\bar{\beta}_i^B + \Delta\bar{\beta}_i^C$ and

$$\langle \cdots \rangle \equiv \int_0^1 d(\cos\theta/2) \int_0^{2\pi} \frac{d\phi}{2\pi}(\cdots) .$$

In Table I we give weighting functions $\bar{w}$, $\bar{v}$, $\bar{u}$, $\bar{\chi}^{(s),(a)}$ expressed in terms of the momentum-transfer variables $t_2 = (1-x_2)/2$ and $t_3 = (1-x_3)/2$, where

$$x_2 = \hat{\kappa}_1\cdot\hat{\kappa}_3 = \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\cos\phi ,$$

and

$$x_3 = \hat{\kappa}_1\cdot\hat{\kappa}_4 = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\cos\phi .$$

To complete the calculation we need a reasonable approximation for the scattering amplitudes. For the central force we use the s-p wave approximation of Dy and Pethick.[10] In this case $T^{(\alpha)}$ and $\tilde{T}^{(\alpha)}$ become

$$\begin{aligned}
T^{(s)} &= A_0^s + A_1^s + t_2(-3A_0^s - 3A_0^a - 2A_1^s) + t_3(-2A_1^s) ,\\
T^{(a)} &= -A_0^s - A_1^s + t_2(A_0^s + A_0^a + 2A_1^s) + t_3(2A_0^s + 2A_0^a + 2A_1^s) ,\\
\tilde{T}^{(s)} &= A_0^s - A_1^s + t_2(-3A_0^s - 3A_0^a) + t_3(2A_1^s) ,\\
\tilde{T}^{(a)} &= A_0^s + 2A_0^a + A_1^s + t_2(-A_0^s - A_0^a) + t_3(-2A_0^s - 2A_0^a - 2A_1^s) .
\end{aligned} \tag{2.5}$$

We have used the forward-scattering sum rule (FWSSR)[13] to write $A_1^a = -(A_0^s + A_1^s + A_0^a)$ in the s-p wave approximation. We use a potential approximation for the spin-orbit amplitudes L and $\tilde{L}$ and parametrize

$$\frac{d}{dq}\tilde{f}(q) = -\frac{q}{k_F^2}\frac{d}{dx_2}\tilde{f}(q) = \frac{q}{k_F^2}\sum_{l=0}^{\infty} a_l P_l(x_2)$$

and similarly for $\tilde{f}(q')$ and $\tilde{f}(\kappa)$; P_l is a Legendre polynomial of order l. The effective spin-orbit potential $\tilde{f}(q)$ is real and therefore so are all a_l's. Although little is known about the spin-orbit interaction in neutron matter, we assume that the effective potential $V(r)$ is very attractive at short distances, while for $r \geq 1$ fm $V(r)$ is assumed unimportant. Numerical estimates of $\{a_l\}$ based on several short-range attractive potentials for a Fermi wave vector $\kappa_F = 1.8$ fm^{-1}, which is typical for neutron-star interiors, show that $a_0 > 0$, $0 \lesssim a_1 \leq a_0$ with a typical value $a_1 \sim a_0/2$, and $|a_l| \leq a_0/5$ for $l \geq 2$. In general, once $a_1 \notin [\sim 0, a_0]$ which happens for potentials of longer range (or at larger κ_F's) a_l's with $l \geq 2$ also become of order a_0 and our approximation breaks down. With these assumptions, the spin-orbit amplitudes are approximately

$$\begin{aligned}
L &= i(t_2t_3)^{1/2}\sum_{l=0}^{\infty} a_l[P_l(x_2)+P_l(x_3)]\\
&\cong 2i(t_2t_3)^{1/2}[a_0 + a_1(1-t_2-t_3)] ,\\
\tilde{L} &= i[(1-t_2-t_3)t_2]^{1/2}\sum_{l=0}^{\infty} a_l[P_l(x_2)+P_l(-x_1)]\\
&\cong 2i[(1-t_2-t_3)t_2]^{1/2}[a_0 + a_1t_3] .
\end{aligned} \tag{2.6}$$

Inspection of Eqs. (2.4)—(2.6) and the weighting functions given in Table I shows that $\Delta\bar{\beta}_i^\alpha$ are linear combinations of basic angular averages $C_{mn} = \langle t_2^m t_3^n\rangle$ for $m,n = 1,2,\ldots$, which can be easily evaluated. We then find that the spin-orbit contribution to $\Delta\bar{\beta}_i$ is

$$\begin{aligned}
\Delta\bar{\beta}_1^{\rm so} &= -\eta a_0^2(1.061 + 0.104x + 0.008x^2) ,\\
\Delta\bar{\beta}_2^{\rm so} &= -\eta a_0^2(2.491 + 0.517x + 0.066x^2) ,\\
\Delta\bar{\beta}_3^{\rm so} &= \eta a_0^2(3.863 + 0.472x + 0.040x^2) ,
\end{aligned} \tag{2.7}$$

where $x = a_1/a_0$. The cross products between the spin-orbit and the central terms give the following contribution to $\Delta\bar{\beta}_i$:

$$\begin{aligned}
\Delta\bar{\beta}_i^{{\rm so}/c} &= -\eta a_0(-1.29A_0^s - 5.73A_0^a - 0.71A_1^s)\\
&\quad - \eta a_1(0.09A_0^s - 0.61A_0^a + 0.21A_1^s) ,\\
\Delta\bar{\beta}_2^{{\rm so}/c} &= \eta a_0[6.18(A_0^s + A_0^a)]\\
&\quad + \eta a_1[0.37(A_0^s + A_0^a)] ,\\
\Delta\bar{\beta}_3^{{\rm so}/c} &= -\eta a_0(8.80A_0^s + 17.70A_0^a + 1.34A_1^s)\\
&\quad - \eta a_1(-0.15A_0^s + 1.63A_0^a + 0.17A_1^s) .
\end{aligned} \tag{2.8}$$

Finally, the central-force contributions to the $\Delta\bar{\beta}_i$, calculated in the s-p wave approximation with A_1^a eliminated by the forward-scattering sum rule, are[14]

$$\Delta\bar{\beta}_1^{\rm cen}=-\eta[4.47(A_0^s)^2+26.07(A_0^a)^2+8.87(A_1^s)^2+19.83A_0^sA_0^a+11.07A_0^sA_1^s+27.40A_0^aA_1^s] ,$$
$$\Delta\bar{\beta}_2^{\rm cen}=-\eta[11.73(A_0^s)^2+17.40(A_0^a)^2+2.80(A_1^s)^2+23.20A_0^sA_0^a+3.21A_0^sA_1^s+6.99A_0^aA_1^s] , \quad (2.9)$$
$$\Delta\bar{\beta}_3^{\rm cen}=\eta[3.76(A_0^s)^2+2.83(A_0^a)^2-15.20(A_1^s)^2+8.68A_0^sA_0^a-16.39A_0^sA_1^s-20.87A_0^aA_1^s] .$$

To analyze the position of the phase point (p_1,p_3), where $p_1\equiv\bar{\beta}_1/\bar{\beta}_2$ and $p_3\equiv\bar{\beta}_3/\bar{\beta}_2$, it is convenient to normalize the strong-coupling parameters to $\bar{\beta}_2^{\rm BCS}$ by writing $b_i\equiv\Delta\bar{\beta}_i/\bar{\beta}_2^{\rm BCS}$. The coordinates of the phase point in Fig. 1 are then $p_1=b_1/(1+b_2)$ and $p_3=(b_3-1)/(1+b_2)$. First we consider the case of very strong spin-orbit forces when $\Delta\bar{\beta}_i$ can be approximated by $\Delta\bar{\beta}_i^{\rm so}$.

From Eq. (2.7), $\Delta\bar{\beta}_i^{\rm so}$ is negative for any value of $x=a_1/a_0$, which means that the phase point moves away from region 2. The slope of the line which connects the phase point (p_1,p_3) with the BCS phase point $(0,-1)$ is given by $S=(b_2+b_3)/b_1$ and depends only on x if we neglect the central terms. The minimum slope $S=-1.55>-2$ for $x=-2.64$ shows that region 1 is also excluded. Finally, we check if strong spin-orbit scattering violates the stability conditions on the fourth-order GL free-energy functional. In our case $\bar{\beta}_1<0$ and $S>-2$ imply that the relevant stability requirements are $\bar{\beta}_2>0$ and $p_3>-2(p_1+1)$. The first condition $b_2>-1$ for typical values $a_0=2$ (see Appendix B), $x=\frac{1}{2}$, and $T_c/T_F=4\times10^{-3}$ is satisfied by a factor of 20. The second condition gives $a_0{}^2T_c/T_F\leq0.13$ using $x=\frac{1}{2}$; for the above estimates of a_0 and T_c/T_F this inequality is satisfied by a factor of 8. However, a_0 and T_c/T_F are not well known. A transition temperature as high as $T_c/T_F\sim10^{-1}$ is not ruled out. A violation of the stability conditions presumably implies that higher-order terms in the GL functional determine the equilibrium phase.

To estimate $\Delta\bar{\beta}_i$ with both spin-orbit and central forces included, we use the available calculations of neutron-matter Fermi liquid parameters.[11,12] For $\kappa_F=1.8$ fm^{-1}, from Bäckman *et al.*[11] follows $A_0^s=0.14$, $A_0^a=0.50$, and $A_1^s=-0.57$, which gives

$$\Delta\bar{\beta}_1=(-2.560+3.101a_0-1.245a_0{}^2)\frac{T_c}{T_F}\bar{\beta}_2^{\rm BCS} ,$$
$$\Delta\bar{\beta}_2=(-5.707+4.643a_0-2.922a_0{}^2)\frac{T_c}{T_F}\bar{\beta}_2^{\rm BCS} , \quad (2.10)$$
$$\Delta\bar{\beta}_3=(4.347-10.932a_0+4.531a_0{}^2)\frac{T_c}{T_F}\bar{\beta}_2^{\rm BCS} .$$

We have neglected the a_1 terms since they are an order of magnitude smaller than the a_0 terms. $\Delta\bar{\beta}_1$ given by Eq. (2.10) is always negative which implies that the phase point moves away from region 2. The minimum slope $S(-5.4)\cong-1.43>-2$ shows the phase point also moves away from region 1. For values of a_0 between $\frac{1}{2}$ and 2 the slope is large and positive ($S\sim10$) and the phase point may cross the $p_3=-2(p_1+1)$ stability line if T_c/T_F is sufficiently large. For $a_0=2$ and $T_c/T_F=4\times10^{-3}$ (a typical estimate for this ratio[15,4]) the phase point is close to the BCS phase point ($p_1\cong-5.5\times10^{-3}$, $p_3+1\cong-3.1\times10^{-2}$).

The qualitative results are rather insensitive on particular values of Landau parameters. This suggests that spin-orbit scattering will not stabilize a nonunitary $^3P_2$ phase. It also appears unlikely that strong spin-orbit scattering violates the stability conditions of the fourth-order GL functional. Better estimates of T_c and spin-orbit scattering amplitudes would decide both questions.

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APPENDIX A

Let $\Delta\phi_\alpha$ ($\alpha=B,C,D,F$) be the free-energy contribution of diagram α of Rainer and Serene.[6] Near T_c, $\Delta\phi_\alpha$ is fourth order in $A_{\mu\nu}$ and has the form

$$\Delta\phi_\alpha=f_\alpha\frac{N(0)}{k_BT_Cv_FP_F}\times\int\frac{d\Omega_1}{4\pi}\int\frac{d\Omega_2}{4\pi}\int\frac{d\Omega_3}{4\pi}\delta(||\hat{\kappa}_4||-1)S_\alpha(\hat{\kappa}_1,\hat{\kappa}_2,\hat{\kappa}_3) . \quad (A1)$$

The constants f_α come from frequency sums and combinational coefficients and are given by $f_B=\frac{1}{2}f_c\cong-6.84/16$, $f_D\cong10.15/2$, and $f_F\cong-30.44/8$. The S_α are functions of $\Delta(\hat{\kappa}_i)$ and $\bar{\Delta}(\hat{\kappa}_i)\equiv-i\sigma^2\vec{\sigma}\cdot\vec{\Delta}(\hat{\kappa}_i)^*$,

$$S_B=\tfrac{1}{4}T_{\alpha\beta,\gamma\rho}(\text{set }1)T_{\gamma\rho,\alpha'\beta'}(\text{set }2)(\Delta(\hat{\kappa}_1)\bar{\Delta}(\hat{\kappa}_1))_{\alpha'\alpha}(\Delta(\hat{\kappa}_2)\bar{\Delta}(\hat{\kappa}_2))_{\beta'\beta} ,$$
$$S_C=\tfrac{1}{4}T_{\alpha\beta,\gamma\rho}(\text{set }1)T_{\gamma'\rho,\alpha'\beta}(\text{set }2)(\Delta(\hat{\kappa}_1)\bar{\Delta}(\hat{\kappa}_1))_{\alpha'\alpha}(\Delta(\hat{\kappa}_3)\bar{\Delta}(\hat{\kappa}_3))_{\gamma\gamma'} ,$$
$$S_D=\tfrac{1}{4}T_{\alpha\beta,\gamma\rho}(\text{set }1)T_{\gamma\beta',\alpha'\rho'}(\text{set }3)(\Delta\hat{\kappa}_1)\bar{\Delta}(\hat{\kappa}_1))_{\alpha'\alpha}\Delta(\hat{\kappa}_4)_{\rho\rho'}\bar{\Delta}(\hat{\kappa}_2)_{\beta\beta'} , \quad (A2)$$
$$S_F=\tfrac{1}{4}T_{\alpha\beta,\gamma\rho}(\text{set }1)T_{\alpha'\beta',\gamma'\rho'}(\text{set }4)\bar{\Delta}(\hat{\kappa}_1)_{\alpha\alpha'}\bar{\Delta}(\hat{\kappa}_2)_{\beta\beta'}\Delta(\hat{\kappa}_3)_{\gamma\gamma'}\Delta(\hat{\kappa}_4)_{\rho\rho'} ,$$

where set 1, set 2, set 3, and set 4 denote ordered quadruples of unit vectors $(\hat{\kappa}_1,\hat{\kappa}_2;\hat{\kappa}_3,\hat{\kappa}_4)$, $(\hat{\kappa}_3,\hat{\kappa}_4;\hat{\kappa}_1,\hat{\kappa}_2)$, $(\hat{\kappa}_1,-\hat{\kappa}_2;\hat{\kappa}_3,-\hat{\kappa}_4)$, and $(-\hat{\kappa}_1,-\hat{\kappa}_2;-\hat{\kappa}_3,-\hat{\kappa}_4)$. Summation over repeated spin indices is assumed. After performing the spin sums in (A2) and using the invariance of the domain of integration in (A1) under $\hat{\kappa}_1\leftrightarrow\hat{\kappa}_2$, $\hat{\kappa}_3\leftrightarrow\hat{\kappa}_4$, $(\hat{\kappa}_3,-\hat{\kappa}_2;\hat{\kappa}_1,-\hat{\kappa}_4)\leftrightarrow(\hat{\kappa}_1,\hat{\kappa}_2;\hat{\kappa}_3,\hat{\kappa}_4)$, and the antisymmetry property of the T amplitude, we express $\Delta\phi_\alpha$ in the form (A1) with S_α now given by

$$
\begin{aligned}
S_B &= (||\vec{\Delta}_1||^2||\vec{\Delta}_2||^2-\vec{u}_1\cdot\vec{u}_2)T^{(s)2}+(3||\vec{\Delta}_1||^2||\vec{\Delta}_2||^2+5\vec{u}_1\cdot\vec{u}_2)T^{(a)2}+2[(\vec{u}_1\cdot\hat{\kappa})(\vec{u}_2\cdot\hat{\kappa})-||\vec{\Delta}_1||^2||\vec{\Delta}_2||^2]L^2 ,\\
S_C &= (||\vec{\Delta}_1||^2||\vec{\Delta}_3||^2-\vec{u}_1\cdot\vec{u}_3)T^{(s)2}+(3||\vec{\Delta}_1||^2||\vec{\Delta}_3||^2+\vec{u}_1\cdot\vec{u}_3)T^{(a)2}+2[(\vec{u}_1\cdot\hat{\kappa})(\vec{u}_3\cdot\hat{\kappa})-||\vec{\Delta}_1||^2||\vec{\Delta}_3||^2]L^2\\
&\quad +2[(\vec{u}_1\times\vec{u}_3)\cdot\hat{q}\times\hat{q}\,']iL(T^{(s)}-T^{(a)}) ,\\
S_D &= (||\vec{\Delta}_1||^2\Delta_{42})(T^{(s)}\tilde{T}^{(s)}+T^{(a)}\tilde{T}^{(a)})+(-\vec{u}_1\cdot\vec{u}_{42})(T^{(s)}\tilde{T}^{(a)}+T^{(a)}\tilde{T}^{(s)})\\
&\quad +\{||\vec{\Delta}_1||^2[(\vec{\Delta}_4\cdot\hat{\kappa})(\vec{\Delta}^*_2\cdot\hat{q}\,')+(\vec{\Delta}_4\cdot\hat{q}\,')(\vec{\Delta}^*_2\cdot\hat{\kappa})]+(\vec{u}_1\cdot\hat{q}\,')(\vec{u}_{42}\cdot\hat{\kappa})+(\vec{u}_1\cdot\hat{\kappa})(\vec{u}_{42}\cdot\hat{q}\,')+\Delta_{42}\vec{u}_1\cdot\hat{\kappa}\times\hat{q}\,'\}L\tilde{L}\\
&\quad +\{[(||\vec{\Delta}_1||^2+||\vec{\Delta}_3||^2)\vec{u}_{42}+\Delta_{42}(\vec{u}_1+\vec{u}_3)]\cdot\hat{q}\times\hat{q}\,'\}iL\tilde{T}^{(s)}\\
&\quad +\{[(||\vec{\Delta}_1||^2+||\vec{\Delta}_3||^2)\vec{u}_{42}+\vec{\Delta}_{42}(\vec{u}_1+\vec{u}_3)-2(\vec{\Delta}_4\cdot\vec{u}_1)\vec{\Delta}^*_2-2(\vec{\Delta}^*_2\cdot\vec{u}_3)\vec{\Delta}_4]\cdot\hat{q}\times\hat{q}\,'\}iL\tilde{T}^{(a)} ,\\
S_F &= (\Delta_{31}\Delta_{42}+\Delta_{32}\Delta_{41}-\Delta_{34}\Delta^*_{12})T^{(s)2}+(-5\Delta_{31}\Delta_{42}+3\Delta_{32}\Delta_{41}+5\Delta_{34}\Delta^*_{12})T^{(a)2}\\
&\quad +2[\Delta_{31}\Delta_{42}-2\Delta_{31}(\vec{\Delta}_4\cdot\hat{\kappa})(\vec{\Delta}^*_2\cdot\hat{\kappa})-(\vec{u}_{31}\cdot\hat{\kappa})(\vec{u}_{42}\cdot\hat{\kappa})]L^2+4[\Delta_{42}(\vec{u}_{31}\cdot\hat{q}\times\hat{q}\,')]iLT^{(s)}\\
&\quad +4\{[\Delta_{42}\vec{u}_{31}-(\vec{\Delta}_4\cdot\vec{u}_{31})\vec{\Delta}^*_2-(\vec{\Delta}^*_2\cdot\vec{u}_{31})\vec{\Delta}_4]\cdot\hat{q}\times\hat{q}\,'\}iLT^{(a)} .
\end{aligned}
\tag{A3}
$$

The notation in Eqs. (A3) is $\vec{\Delta}_i=\vec{\Delta}(\hat{\kappa}_i)$, $\vec{u}_i=\vec{\Delta}_i\times\vec{\Delta}^*_i$ for $i=1,2,3,4$, $\Delta_{ij}=\vec{\Delta}_i\cdot\vec{\Delta}^*_j$, and $\vec{u}_{ij}=\vec{\Delta}_i\times\vec{\Delta}^*_j$ for $i=3,4$ and $j=1,2$, and $\Delta^*_{12}=\vec{\Delta}^*_1\cdot\vec{\Delta}^*_2$ and $\Delta_{34}=\vec{\Delta}_3\cdot\vec{\Delta}_4$. Also

$$
\begin{aligned}
T^{(\alpha)} &= T^{(\alpha)}(\theta,\phi)=T^{(\alpha)}(\hat{\kappa}_1,\hat{\kappa}_2;\hat{\kappa}_3,\hat{\kappa}_4) ,\\
\tilde{T}^{(\alpha)} &= T^{(\alpha)}(\tilde{\theta},\tilde{\phi})=T^{(\alpha)}(\hat{\kappa}_3,-\hat{\kappa}_2;\hat{\kappa}_1,-\hat{\kappa}_4)
\end{aligned}
$$

for $\alpha=s,a$, and $L=L(q,q')$, $\tilde{L}=L(q,\kappa)$.

In order to simplify these expressions for $\Delta\phi_\alpha$ we use the identity

$$
\int\frac{d\Omega_1}{4\pi}\int\frac{d\Omega_2}{4\pi}\int\frac{d\Omega_3}{4\pi}\delta(||\hat{\kappa}_1+\hat{\kappa}_2-\hat{\kappa}_3||-1)=\tfrac{1}{2}\int_0^1 d(\cos\theta/2)\int_0^{2\pi}\frac{d\phi}{2\pi}\int\frac{d\Omega_\kappa}{4\pi}\int_0^{2\pi}\frac{d\psi}{2\pi} . \tag{A4}
$$

Rainer and Serene,[6] show that for fixed (θ,ϕ) the triad $(\hat{\kappa}_1,\hat{\kappa}_2,\hat{\kappa}_3)$ can be thought of as a rigid body whose orientation is given by a unit vector $\hat{\kappa}$ and the angle ψ, by which $\hat{z}\times\hat{\kappa}$ has to be rotated around $\hat{\kappa}$ to align it with $\hat{\kappa}_1-\hat{\kappa}_2$. The $(\hat{\kappa},\psi)$ integrals of the functions $S_\alpha(\hat{\kappa}_1,\hat{\kappa}_2,\hat{\kappa}_3)$, with $\Delta_\mu(\hat{\kappa}_i)=A_{\mu\nu}(\hat{\kappa}_i)_\nu$, become linear combinations of two basic integrals:

$$
\begin{aligned}
M_4^{\mu_1\cdots\mu_4}(\{\vec{l}\}) &= \int\frac{d\Omega_\kappa}{4\pi}\int_0^{2\pi}\frac{d\psi}{2\pi}\prod_{i=1}^{4}\vec{l}_i^{\,\mu_i} ,\\
M_6^{\mu_1\cdots\mu_6}(\{\vec{l}\}) &= \int\frac{d\Omega_\kappa}{4\pi}\int_0^{2\pi}\frac{d\psi}{2\pi}\prod_{i=1}^{6}\vec{l}_i^{\,\mu_i} ,
\end{aligned}
\tag{A5}
$$

where the vectors $\vec{l}_i$ are linear combinations of $(\hat{\kappa}_1,\hat{\kappa}_2,\hat{\kappa}_3)$. The functions M_4 and M_6 are rotationally invariant tensors of ranks 4 and 6, and can be written as

$$
\begin{aligned}
M_4^{\mu_1\cdots\mu_4}(\{\vec{l}\}) &= \delta_{\mu_1\mu_2}\delta_{\mu_3\mu_4}x_4((\vec{l}_1,\vec{l}_2),(\vec{l}_3,\vec{l}_4))+\text{two other pairings} ,\\
M_6^{\mu_1\cdots\mu_6}(\{\vec{l}\}) &= \delta_{\mu_1\mu_2}\delta_{\mu_3\mu_4}\delta_{\mu_5\mu_6}x_6((\vec{l}_1,\vec{l}_2),(\vec{l}_3,\vec{l}_4),(\vec{l}_5,\vec{l}_6))+\text{fourteen other pairings} ,
\end{aligned}
\tag{A6}
$$

where

$$
\begin{aligned}
x_4((\vec{l}_1,\vec{l}_2),(\vec{l}_3,\vec{l}_4)) &= z_1(\vec{l}_1\cdot\vec{l}_2)(\vec{l}_3\cdot\vec{l}_4)+z_2[(\vec{l}_1\cdot\vec{l}_3)(\vec{l}_2\cdot\vec{l}_4)+(\vec{l}_1\cdot\vec{l}_4)(\vec{l}_2\cdot\vec{l}_3)] ,\\
x_6((\vec{l}_1,\vec{l}_2),(\vec{l}_3,\vec{l}_4),(\vec{l}_5,\vec{l}_6)) &= y_1(\vec{l}_1\cdot\vec{l}_2)(\vec{l}_3\cdot\vec{l}_4)(\vec{l}_5\cdot\vec{l}_6)\\
&\quad +y_2\{(\vec{l}_1\cdot\vec{l}_2)[(\vec{l}_3\cdot\vec{l}_5)(\vec{l}_4\cdot\vec{l}_6)+(\vec{l}_3\cdot\vec{l}_6)(\vec{l}_4\cdot\vec{l}_5)]+\text{four other products}\}\\
&\quad +y_3[(\vec{l}_1\cdot\vec{l}_3)(\vec{l}_2\cdot\vec{l}_5)(\vec{l}_4\cdot\vec{l}_6)+\text{seven other products}] .
\end{aligned}
\tag{A7}
$$

The coefficients in (A7) are determined by selecting special choices $\{\vec{l}\}$ and contracting M_4 and M_6 with various Kronecker symbols. Specifically,

$$
\begin{aligned}
z_1 &= \tfrac{4}{30},\quad z_2=-\tfrac{1}{30} ,\\
y_1 &= \tfrac{16}{210},\ y_2=-\tfrac{5}{210},\ y_3=\tfrac{2}{210} ,
\end{aligned}
\tag{A8}
$$

and the weighting functions in the table follow directly from Eqs. (A3) and (A6)—(A8).

APPENDIX B

Let $\delta(^3P_J)$ be an isospin-1 and orbital angular momentum-1 scattering phase shift for the scattering of two nucleons with center-of-mass energies $\hbar^2\kappa_F{}^2/2m$ (m is the neutron mass). Then the quantity

$$\delta_{11}^{so}(\kappa_F) = -[2\delta(^3P_0)+3\delta(^3P_1)-5\delta(^3P_2)]/12 \qquad \text{(B1)}$$

is approximately equal to the Born scattering phase shift in the $^3P_2$ state if only spin-orbit forces were present.[8,16]

The $^3P_2$ scattering phase shift is given by

$$\exp[2i\delta(^3P_2)] = 1 - i\pi^3 N'(0) \times \int d\Omega_b \int d\Omega_a Y_1^1(\hat{b})^* R_{ba} Y_1^1(\hat{a}) \ , \qquad \text{(B2)}$$

where $N'(0)$ is the single-spin free-neutron density of states at the Fermi energy and the transition matrix element R_{ba} describes scattering from the two-particle state $|a\rangle$ with particle momenta $\kappa_F\hat{a}$ and $-\kappa_F\hat{a}$ and both spins up into a state $|b\rangle$ with particle momenta $\kappa_F\hat{b}$ and $-\kappa_F\hat{b}$ and both spins up. In the Born approximation R_{ba} is is given by

$$R_{ba} = (2\pi)^{-3}\Gamma^{(0)}_{\uparrow\uparrow,\uparrow\uparrow}(\kappa_F\hat{b},0,-\kappa_F\hat{b},0;-\kappa_F\hat{a},0,-\kappa_F\hat{a},0) \ , \qquad \text{(B3)}$$

where $\Gamma^{(0)}$ is the bare four-point vertex. In order to express R_{ba} over the dimensionless quasiparticle-scattering amplitude T in neutron-star matter, we use the relation

$$T(\hat{\kappa}_1,\hat{\kappa}_2;\hat{\kappa}_3,\hat{\kappa}_4) \equiv [2N(0)/z^2]\Gamma(1,2;3,4) = [2N(0)/z^2]\Gamma^{(0)}(1,2;3,4) \ , \qquad \text{(B4)}$$

where the second equality follows from the Born approximation for the full four-point function Γ, $i \equiv (\kappa_F\hat{\kappa}_i,0)$ for $i=1,2,3,4$, and spin arguments have been suppressed. The factor z describes the renormalization of the quasiparticle pole $(0 \le z \le 1)$ and $N(0)$ is the single-spin quasiparticle density of states at the Fermi energy.

From Eqs. (B2)—(B4) it follows that

$$e^{2i\delta(^3P_2)} = 1 - \frac{iz^2N'(0)}{16N(0)} \times \int d\Omega_b \int d\Omega_a Y_1^1(\hat{b})^* T_{\uparrow\uparrow,\uparrow\uparrow}(\hat{b},-\hat{b};\hat{a},-\hat{a}) Y_1^1(\hat{a}) . \qquad \text{(B5)}$$

Substituting the expression (1.7) for the dimensionless quasiparticle-scattering amplitude and using the parametrization of L explained below Eq. (2.5), we obtain

$$e^{2i\delta_{11}^{so}(\kappa_F)} = 1 + \frac{i\pi z^2 N'(0)}{6N(0)}(a_0 - a_2/5) \ , \qquad \text{(B6)}$$

recalling that in the Born approximation $\delta(^3P_2)$ equals δ_{11}^{so} when only spin-orbit forces are present. Neglecting the a_2 term in the last equation and expanding the exponential on the left-hand side to terms linear in δ_{11}^{so}, we obtain

$$a_0 \simeq (12/\pi)\delta_{11}^{so}(m^*/m)(1/z^2) \ . \qquad \text{(B7)}$$

From the nucleon-scattering data, Signell[8] obtains $\delta_{11}^{so} \simeq 17°$ for $\kappa_F = 1.8\ \text{fm}^{-1}$. This value for δ_{11}^{so} and the value for the neutron effective mass ratio[12] $N(0)/N'(0) \equiv m^*/m \simeq 0.9$ give $a_0 \simeq 1/z^2$, and we take $a_0 = 2$ as a typical value.

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*Present address: Department of Physics and Astronomy, University of Maryland, College Park, MD 20742.

[1] H. Brand and H. Pleiner, Phys. Rev. D 24, 3048 (1981).

[2] J. A. Sauls, D. L. Stein, and J. W. Serene, Phys. Rev. B 25, 967 (1982).

[3] P. Muzikar, J. A. Sauls, and J. W. Serene, Phys. Rev. D 21, 1494 (1980).

[4] J. A. Sauls and J. W. Serene, Phys. Rev. D 17, 1524 (1978).

[5] N. D. Mermin, Phys. Rev. A 9, 868 (1974).

[6] D. Rainer and J. W. Serene, Phys. Rev. B 13, 4745 (1976).

[7] A. A. Abrikosov and I. M. Khalatnikov, Rep. Prog. Phys. 22, 329 (1959).

[8] P. Signell, in *The Two-Body Force in Nuclei*, edited by S. M. Austin and G. M. Crawley (Plenum, New York, 1972), p. 9.

[9] In principle the T amplitude contains also a term $T^{(\text{ten})}$ characteristic for tensor forces since such a term appears even if the potential term in the Bethe-Salpeter equation includes only a central term and a spin-orbit term. However, the scattering data of Signell[8] indicate that the tensor term in the T amplitude for the free neutron-neutron scattering is negligible. An indirect indication that the same is true in neutron matter is that the tensor contribution to the Landau parameters are large in symmetric (isospin-zero) nuclear matter, but almost negligible in neutron (isospin-one) matter (Ref. 12).

[10] K. S. Dy and C. J. Pethick, Phys. Rev. 185, 373 (1969).

[11] S. O. Bäckman, C. G. Källman, and O. Sjöberg, Phys. Lett. 43B, 263 (1973).

[12] J. A. Niskanen and J. A. Sauls, report, 1981 (unpublished).

[13] D. Hone, Phys. Rev. 125, 1494 (1962). Notice that the spin-orbit term (1.7) does not contribute to the forward-scattering amplitude so that the FWSSR for $T = T^{(\text{cen})} + T^{(\text{so})}$ reduces to the FWSSR for $T^{(\text{cen})}$.

[14] Equation (2.9) can also be obtained from Table III of Rainer and Serene[6] by using the identities $\text{Tr}(AA^\dagger)^2 = \frac{1}{2}|\text{Tr}A^2|^2 + (\text{Tr}AA^*)^2 - 2\,\text{Tr}A^{2*2}$ and $\text{Tr}AA^\dagger(AA^\dagger)^* = \text{Tr}A^2A^{*2}$, which hold for any traceless and symmetric 3×3 matrix A,[5] to relate the general $l=1$, $s=1$, $\Delta\beta_i^{\text{cen}}$ to the $^3P_2$ $\Delta\bar{\beta}_i^{\text{cen}}$.

[15] M. Hoffberg, A. E. Glassgold, R. W. Richardson, and M. Ruderman, Phys. Rev. Lett. 24, 775 (1970).

[16] L. Heller and M. S. Sher, Phys. Rev. 182, 1031 (1969).