

Charge dynamics of vortex cores in layered chiral triplet superconductors

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Abstract. In an accompanying paper (Sauls and Eschrig 2009 *New J. Phys.* **11** 075008) we have studied the equilibrium properties of vortices in a chiral quasi-two-dimensional triplet superfluid/superconductor. Here we extend our studies to include the dynamical response of a vortex core in a chiral triplet superconductor to an external ac electromagnetic field. We consider in particular the response of a doubly quantized vortex with a homogeneous core in the time-reversed phase. The external frequencies are assumed to be comparable in magnitude to the superconducting gap frequency, such that the vortex motion is nonstationary but can be treated by linear response theory. We include broadening of the vortex-core bound states due to impurity scattering and consider the intermediate-clean regime, with a broadening comparable to or larger than the quantized energy level spacing. The response of the order parameter, impurity self-energy, induced fields and currents are obtained by a self-consistent calculation of the distribution functions and the excitation spectrum. Using these results we obtain the self-consistent dynamically induced charge distribution in the vicinity of the core. This charge density is related to the nonequilibrium response of the bound states and order parameter collective mode, and dominates the electromagnetic response of the vortex core.

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1. Introduction

Transport and optical properties in type II superconductors are closely related to the response of Abrikosov vortices [1] to external electromagnetic fields. Abrikosov vortices that form a lattice in sufficiently strong magnetic fields give rise to a finite resistance when a voltage is applied to the superconducting sample. This flux flow resistance appears, once collective vortex pinning [2] is overcome, as a result of motion of the vortex lattice in an electric field [3] with a resistance independent of the field strength for small fields [4]–[6]. In superconductors with moderate impurity disorder the vortex lattice moves in a constant electric field with constant velocity v_L predominantly in the direction transverse to the transport current,

$$\mathbf{v}_L = \frac{\mu}{c} \mathbf{j}_{\text{tr}} \times \Phi_0 \quad (1)$$

with $\Phi_0 = (hc/2|e|) \mathbf{n}_B$ being the magnetic flux quantum multiplied with the unit vector $\mathbf{n}_B = \mathbf{B}_{\text{av}}/B_{\text{av}}$ that describes the direction of the average vortex unit cell magnetic field, B_{av} , and μ quantifies the vortex mobility. This viscous flux flow is in contrast to superclean superconductors or to neutral superfluids, where vortices move predominantly together with the superfluid velocity. The electric field averaged over the unit cell, \mathbf{E}_{av} is given by

$$\mathbf{E}_{\text{av}} = \frac{1}{c} \mathbf{B}_{\text{av}} \times \mathbf{v}_L. \quad (2)$$

On a phenomenological level, vortex motion has been discussed within the Bardeen–Stephen model [7], the Nozières–Vinen model [8] and within time-dependent Ginzburg–Landau theory [9]. The microscopic theory for flux flow resistance has been pioneered by Gor’kov and Kopnin [4, 6] and by Larkin and Ovchinnikov [2, 5], [10]–[12]. In extremely clean superconductors vortices move predominantly in the direction of the transport current, giving rise to an electric field perpendicular to the transport current, and thus a Hall effect [13]–[20].

Vortices also provide valuable information about the nature of low-lying excitations in the superconducting state. In clean s-wave BCS superconductors the low-lying excitations

Table 1. Three degrees of disorder are distinguished for vortices in superconductors with impurities. Here, $1/\tau = v_f/\ell$, with the normal state Fermi velocity v_f .

Superclean case	Moderately clean	Dirty limit
$\ell \gg \xi_0 \frac{E_f}{\Delta}$	$\xi_0 \frac{E_f}{\Delta} \gg \ell \gg \xi_0$	$\xi_0 \gg \ell \gg \xi_0 \frac{\Delta}{E_f}$
$\frac{\hbar}{\tau} \ll \frac{\Delta^2}{E_f}$	$\frac{\Delta^2}{E_f} \ll \frac{\hbar}{\tau} \ll \Delta$	$\Delta \ll \frac{\hbar}{\tau} \ll E_f$

in the core are the bound states of Caroli *et al* [21]. These excitations have superconducting as well as normal metallic properties. For example, these states are the source of circulating supercurrents in the equilibrium vortex core, and they are strongly coupled to the condensate by Andreev scattering [22, 23]. Furthermore, the response of the vortex-core states to an electromagnetic field is generally very different from that of normal electrons.

In the following we will discuss the response of a vortex to an ac electromagnetic field with frequency ω . We are interested in frequencies ω that are comparable to the frequencies set by the superconducting gap, Δ/\hbar , i.e. 1 GHz–1 THz. In this region the vortex motion is nonadiabatic and is not described by the low-frequency limit of flux flow motion. We consider *layered* superconductors with a weak Josephson coupling between the conducting planes. In this case the vortices are quasi-two-dimensional objects, called *pancake vortices*. The peculiarities of the vortex phases in layered superconductors has been reviewed in [24] (for recent developments see also [25]).

Disorder plays a central role in the dissipative dynamics of the mixed state of type II superconductors. Impurities and defects are a source of scattering that limits the mean free path, ℓ , of carriers, thus increasing the resistivity. Defects also provide ‘pinning sites’ that inhibit vortex motion and suppress the flux-flow resistivity [2]. However, for ac fields even pinned vortices are sources for dissipation. The magnitude and frequency dependence of this dissipation depends on the electronic structure and dynamics of the core states of the pinned vortex. Experimental studies of dissipation in vortex cores include [26]–[28].

When discussing vortex-core states in the context of impurity disorder, one needs to distinguish three degrees of disorder, which we summarize in table 1. In the dirty limit, $\hbar/\tau \gg \Delta$, the Bardeen–Stephen model [7] of a normal-metal spectrum with the local Drude conductivity in the core provides a reasonable description of the dissipative dynamics of the vortex-core. The opposite extreme is the ‘superclean limit’, $\hbar/\tau \ll \Delta^2/E_f$ (where E_f is the Fermi energy), in which the quantization of the vortex-core bound states must be taken into account. In this limit a single impurity and its interaction with the vortex-core states must be considered. The ac electromagnetic response is then controlled by selection rules governing transition matrix elements for the quantized core levels and the level structure of the core states in the presence of an impurity [29]–[34]. In the case of d-wave superconductors in the superclean limit, *minigap nodes* in the spectrum of bound states lead to a finite dissipation from Landau damping for $T \rightarrow 0$ [35].

The superclean limit is difficult to achieve even for short coherence length superconductors; weak disorder broadens the vortex-core levels into a quasi-continuum. We investigate the

intermediate-clean regime, $\Delta^2/E_f \ll \hbar/\tau \ll \Delta$, where the discrete level structure of the vortex-core states is broadened and the selection rules are broken due to strong overlap between the resulting resonances. However, the vortex-core states remain well defined on the scale of the superconducting gap, Δ . In this regime we can take advantage of the power of the quasiclassical theory of nonequilibrium superconductivity [10], [36]–[39]. Previous results for s-wave and d-wave vortices in layered superconductors [40]–[43] have shown that in the intermediate-clean regime electrodynamics of the vortex state is nonlocal and largely determined by the response of the vortex-core states. Transitions involving the vortex-core states, and their coupling to the collective motion of the condensate requires dynamically self-consistent calculations of the order parameter, self-energies, induced fields, excitation spectra and distribution functions. In particular, it has been found that branches of localized vortex bound states that cross the chemical potential (the so-called anomalous branches) are of crucial importance for the dissipation in the vortex core [41]. The relaxation of localized excitations takes place via their interaction with impurities. The importance of anomalous branches for the flux flow resistance in moderately clean superconductors has been clarified by Kopnin and Lopatin [17].

An interesting aspect of vortex dynamics is the charge dynamics associated with moving vortices. It is known that even for a static vortex charge transfer takes place from the vortex-core region to outer regions of the vortex, leading to a depletion of charge carriers in the vortex-core regions [18], [44]–[55]. Associated with this charge transfer is a spatially varying electrostatic potential that is typically of the order of Δ^2/E_f . The smallness of the parameter Δ/E_f means that the electrostatic potential can be neglected in the self-consistent determination of the spatially varying order parameter and magnetic field.

There are two main sources for vortex charges. The first contribution comes from the lowering of free energy by the pairing, which leads to a force on the unpaired electrons in the vortex core by the condensate around the vortices [56]. This force varies on the coherence length scale. It arises due to the change of the chemical potential as function of the modulus of the order parameter, and is in general proportional to $\partial\gamma/\partial n$ and $\partial T_c/\partial n$, where γ is the normal state-specific heat coefficient, T_c the superconducting transition temperature and n the density of conduction charge carriers. It includes contributions from the condensation energy E_{con} , which is proportional to γT_c^2 . For example, in a Gorter–Casimir two-fluid picture [57], using an extended version [58] of the Ginzburg–Landau approximation [59] the corresponding contribution reads [51]

$$e\Phi_{\text{Therm}}(\mathbf{r}) \approx \frac{n_s(\mathbf{r})}{n} \frac{\partial E_{\text{con}}}{\partial n} + \sqrt{1 - \frac{n_s(\mathbf{r})}{n}} \frac{T^2}{2} \frac{\partial \gamma}{\partial n}, \quad (3)$$

where $n_s(\mathbf{r})/n$ is the condensate fraction of the density. This term is referred to as the thermodynamic contribution to the electrostatic potential [60, 61]. A second contribution comes from the inertial forces and the Lorentz force that acts on the circulating supercurrent around the vortex [62]–[64]. This force decays on the scale of the London penetration depth, and is comparable in magnitude with the thermodynamic contribution near the vortex core. It arises due to the kinetic energy of the superflow, and also leads to a depletion of charge carriers in the vortex-core region. In the intervortex regions, where the supercurrent momentum is determined by the phase gradient of the order parameter, $\nabla\vartheta$, the corresponding contribution to the electrostatic potential reads [64]

$$e\Phi_{\text{Bernoulli}}(\mathbf{r}) \approx -\frac{n_s}{n} \frac{1}{2m^*} (\hbar\nabla\vartheta(\mathbf{r}) - e^*\mathbf{A}(\mathbf{r}))^2, \quad (4)$$

where $e^* = 2e$ and m^* are the effective charge and mass of the Cooper pair, and \mathbf{A} is the vector potential. The corresponding electrostatic potential is referred to as the Bernoulli potential. The electrostatic potential resulting from both contributions leads to a charge depletion in the vortex core of order

$$\delta Q_{\text{static}} \sim e \left(\frac{\Delta}{E_f} \right)^2. \quad (5)$$

In general, the electrostatic potential is determined by a screened Poisson equation [65]. However, the spatial variations of $n(\mathbf{r})$, $\Delta(\mathbf{r})$ and $\mathbf{A}(\mathbf{r})$ occur on long-wavelength superconducting scales, while screening takes place on the short-wavelength Thomas–Fermi screening length, λ_{TF} . Thus, to leading order in the quasiclassical expansion parameter, $\lambda_{\text{TF}}/\xi_0$, the superconductor maintains local charge neutrality [6, 66]. The first corrections to the charge density are then given by

$$-\nabla^2 \Phi = 4\pi \rho(\mathbf{r}) \quad (6)$$

with $\rho(\mathbf{r}) = n(\mathbf{r}) - n_0$, and n_0 is the charge density in the normal state. We note that a possible charge pileup in a region of size λ_{TF}^2 around the vortex center *induced* by the presence of the superconducting state is expected to be small, because the corresponding overlap region between such a charge distribution and the superconducting order parameter is small by the ratio $(\lambda_{\text{TF}}/\xi_0)^2$ and in addition the order parameter is small by a factor $\lambda_{\text{TF}}/\xi_0$.⁴ Consequently, any possible charge induced within the area λ_{TF}^2 in the vortex center must be small by at least the ratio $(\lambda_{\text{TF}}/\xi_0)^3$. This allows us to neglect such charges and leads to an effective decoupling between charge variations on the Thomas–Fermi screening length and the superconducting order parameter. Charge fluctuations on the Thomas–Fermi length scale are entirely determined by the normal state charge fluctuations, and the leading order charge variations induced by the superconducting state are varying on the long-wavelength (superconducting) scales.

The entire picture discussed so far for the equilibrium vortex lattice changes dramatically when considering a time-dependent perturbation on a vortex. In order to reduce the Coulomb energy associated with the charge accumulation, an internal electro-chemical potential, $\Phi(\mathbf{r}; t)$, develops in response to an external electric field [6]. This potential produces an internal electric field, $\mathbf{E}^{\text{int}}(\mathbf{r}; t)$, which is of the same order of magnitude as the external field. Even though the external field may vary on a scale that is large compared to the coherence length, ξ_0 , the internal field develops on the coherence length scale. The source of the internal field is a charge density $\rho(\mathbf{r}, t)$ that accumulates inhomogeneously over length scales of the order of the coherence length. It is necessary to calculate the induced potential self-consistently from the spatially varying order parameter, spectral function and distribution function for the electronic states in the vicinity of the vortex core. An order of magnitude estimate shows that to produce an induced field of the order of the external field, the dynamically induced charge is of order

$$\delta Q_{\text{dynamic}} \sim e \frac{\Delta}{E_f} \frac{\delta v_\omega}{\Delta}, \quad (7)$$

where $\delta v_\omega \sim e E^{\text{ext}} \xi_0 \Delta / \omega$ is the typical energy scale set by the strength of the external field. This charge density accumulates predominantly in the vortex-core region and creates a dipolar field around the vortex core [7, 41]. For a pinned vortex the charge accumulates near the interface

⁴ This last statement needs to be modified if vortex-core shrinking takes place at low temperatures in superclean materials, however the general conclusions will still hold.

separating the metallic inclusion from the superconductor [43]. The charge in equation (7) exceeds the dynamical charge that would result from a dynamic motion of the vortex charges in equation (5), provided $\delta v_\omega \gg \Delta^2/E_f$. In this case, we can neglect the contributions coming from the response of the charges that are present already for a static vortex, and concentrate on the new contributions (7). Note that the dynamical charges are absent in the superclean case, where the electric field is purely inductual [16].

In the next two sections we provide a discussion of the expansion in small quasiclassical parameters, and a summary of the nonequilibrium quasiclassical equations, including the transport equations for the quasiparticle distribution and spectral functions, and constitutive equations for the order parameter, impurity self-energy and electromagnetic potentials. We then apply this theory to the response of a vortex in a chiral, spin-triplet superconductor to an ac electromagnetic perturbation. The equilibrium properties of such a vortex have been discussed in detail in an accompanying paper [67]. In particular, we discuss the ac response of singly quantized and doubly quantized vortices in section 4.

2. Expansion in small parameters

The physics of inhomogeneous metals and superconductors described by the quasiclassical approximation is governed by well-defined small expansion parameters. We will assume that there is one such parameter that describes all small quantities in the system, and will assign to this parameter the notation small [68]–[71]. The typical microscopic length scales of the problem, in short denoted by a_0 , are the Bohr radius a_B , the Fermi wavelength λ_f and the Thomas–Fermi wavelength λ_{TF} . The mesoscopic, superconducting length scales are the coherence length ξ_0 , and the penetration depth λ . Correspondingly, large energy scales are the Fermi energy E_f , and the Coulomb energy e^2/a_0 , whereas small, quasiclassical energy scales are the gap, Δ , the energy of the external perturbation, $\hbar\omega$, and the energy related to the transition temperature, $k_B T_c$. We assume that spatial variations near the vortex core are on the scale ξ_0 , and time variations on the scale $1/\omega$. The normal state density of states at the Fermi level, N_f , is of order $N_f \sim 1/E_f a_0^3$. With this notation we have the following assignments that we need to estimate the electromagnetic fields, charges and currents:

small ⁰	small ¹	small ²
$e^2 N_f \sim \frac{1}{a_0^2}$		
$\hbar v_f \sim \xi_0 \Delta$	$\frac{v_f}{c} \sim \frac{a_0}{\lambda}$	
	$\hbar v_f \cdot \nabla \sim \Delta$	$\nabla^2 \sim \frac{1}{\xi_0^2}$
	$\hbar \partial_t \sim \hbar \omega$	$\frac{1}{c} \partial_t \sim \frac{a_0}{\lambda \xi_0} \frac{\hbar \omega}{\Delta}$

2.1. Electrostatic fields and potentials

In equilibrium, the electrostatic fields are given in terms of the potentials by $\mathbf{E} = -\nabla\Phi$, $\mathbf{B} = \nabla \times \mathbf{A}$ and fulfill the Maxwell's equations $\nabla \cdot \mathbf{B} = 0$, $\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j}$, $\nabla \cdot \mathbf{E} = 4\pi\rho$, $\nabla \times \mathbf{E} = \mathbf{0}$.

The sources for the static fields in the superconductor are the Meissner currents \mathbf{j} and the small charges ρ discussed in the introduction. The Meissner current density in a superconductor can be estimated by noting that $\frac{e}{c}|\mathbf{j}| \sim e^2 v_f N_f \Delta / c \sim \Delta / a_0 \lambda \sim \text{small}^2$. The electrostatic potentials induced in the superconducting state are of order $e\Phi \sim \Delta^2 N_f \sim \Delta^2 / E_f a_0^3$, and estimating $E_f \sim v_f p_f / 2 \sim \hbar v_f / \lambda_f \sim \Delta \xi_0 / a_0$ we arrive at $e\Phi \sim a_0 \Delta / \xi_0 \sim \text{small}^2$. The averaged static magnetic field \bar{B}_0 is given by the condition that one flux quantum penetrates the vortex unit cell. In contrast, the variation of the magnetic field on the coherence length scale, δB_0 , is a factor ξ_0 / λ smaller than the averaged field. This leads to the following estimates for physical parameters:

small	small ²	small ³	small ⁴
$e\bar{B}_0 \sim \frac{\Delta}{a_0}$	$\frac{e}{c} \mathbf{j} \sim \frac{\Delta}{a_0 \lambda}$		$e\rho \sim \frac{a_0 \Delta}{\xi_0^3}$
$\frac{e}{c} v_f A \sim \Delta$	$e\Phi \sim \frac{a_0 \Delta}{\xi_0}$		
$e\delta B_0 \sim \frac{\xi_0}{\lambda a_0} \Delta$		$eE \sim \frac{a_0}{\xi_0^2} \Delta$	

In particular, for the free energy density the contributions from the magnetic field energy and the Coulomb energy are

$$\frac{\delta j \delta A}{2} \sim \Delta^2 N_f \frac{\xi_0^2}{\lambda^2} \sim \text{small}^2, \quad \frac{\rho \Phi}{2} \sim \Delta^2 N_f \frac{a_0^4}{\xi_0^4} \sim \text{small}^6. \quad (8)$$

For the magnetic field energy the relevant quantity considered here is the contribution that results from the deviations $\delta \mathbf{B}_0$ from the averaged field \bar{B}_0 and its corresponding current $\delta \mathbf{j} = \frac{c}{4\pi} \nabla \times \delta \mathbf{B}_0$. Note that in the strong type II limit and for short coherence length superconductors (this is the case for example for cuprate superconductors, where $\xi_0 / \lambda \sim 0.01$ and $a_0 / \xi_0 \sim 0.2$), the scales can conspire such that $(\xi_0 / \lambda)^2$ is of the same order as $(a_0 / \xi_0)^4$. In this case, the Coulomb energy contribution might become important for the vortex lattice structure [72].

2.2. Electrodynamical response

For the dynamical response it is useful to split the vector potential into transverse and longitudinal parts,

$$\mathbf{A} = \mathbf{A}_L + \mathbf{A}_T, \quad \nabla \cdot \mathbf{A}_T = 0, \quad \nabla \times \mathbf{A}_L = \mathbf{0}. \quad (9)$$

The longitudinal part can be written as $\mathbf{A}_L = \frac{c}{2e} \nabla \zeta$, and can be gauged away by changing the phase of the order parameter θ into $\theta + \zeta$ and the electrical potential Φ into $\Phi' = \Phi + \frac{1}{2e} \partial_t \zeta$. Thus, we use a gauge where $\mathbf{A}' = \mathbf{A}_T$. The fields are given in terms of the potentials as

$$\mathbf{B} = \nabla \times \mathbf{A}', \quad \mathbf{E} = -\frac{1}{c} \partial_t \mathbf{A}' - \nabla \Phi', \quad (10)$$

and the Maxwell equations read (with the corresponding estimates of the various terms)

$$\underbrace{-\nabla^2 \Phi'}_{\text{small}^3} = 4\pi\rho, \quad \underbrace{-\nabla^2 \mathbf{A}' + \frac{1}{c^2} \partial_t^2 \mathbf{A}' + \frac{1}{c} \partial_t \nabla \Phi'}_{\text{small}^4} = \underbrace{\frac{4\pi}{c} \mathbf{j}}_{\text{small}^2}. \quad (11)$$

We drive the superconducting vortices out of equilibrium with an external ac electromagnetic field, $\delta \mathbf{A}_\omega^{\text{ext}}(t) = \delta \mathbf{A}_0 e^{-i\omega t}$, of frequency $\omega \sim \Delta/\hbar$. The perturbing potential then is of the form $\delta v_\omega^{\text{ext}} = -\frac{e}{c} \mathbf{v}_f \cdot \delta \mathbf{A}_\omega^{\text{ext}}(t)$. For the linear response approximation to hold, we assume that $\delta v_\omega^{\text{ext}} \sim \hbar \omega \delta$ with δ a small parameter that defines the region of applicability of linear response. This means that for small frequencies the vortex oscillation amplitude will be $\sim \xi_0 \delta$, and the vortex velocity $v_L \sim v_f (\hbar \omega / \Delta) \delta$. We can then estimate the fields to be of order:

δ	small $\cdot \delta$	small ² $\cdot \delta$	small ³ $\cdot \delta$
$e A_\omega^{\text{ext}} \sim \frac{\lambda}{a_0} \hbar \omega \delta$		$e E_\omega^{\text{ext}} \sim \frac{(\hbar \omega)^2}{\Delta \xi_0} \delta$	
$e A_\omega^{\text{int}} \sim \frac{\xi_0^2}{\lambda a_0} \Delta \delta$	$e \Phi^{\text{int}} \sim \hbar \omega \delta$		
	$e B_\omega^{\text{int}} \sim \frac{\xi_0}{\lambda a_0} \Delta \delta$	$e E_\omega^{\text{int}} \sim \frac{\hbar \omega \delta}{\xi_0}$	$e \nabla \times \mathbf{E}_\omega^{\text{int}} \sim \frac{\hbar \omega \delta}{\lambda^2}$
	$e j_\omega \sim \frac{v_f}{a_0^2} \Delta \delta$	$\frac{e}{c} j_\omega \sim \frac{\Delta \delta}{\lambda a_0}$	$e \rho_\omega \sim \frac{\hbar \omega \delta}{\xi_0^2}$

Below, we use these estimates to simplify the coupled system of Maxwell's equations and transport equations for the superconducting charge carriers. In particular, we note that because

$$\nabla \cdot \mathbf{j} \sim \frac{\Delta^2 \delta}{e \hbar a_0^2} \sim \text{small}^2 \cdot \delta, \quad \partial_t \rho \sim \frac{\Delta^2 \delta}{e \hbar \xi_0^2} \sim \text{small}^4 \cdot \delta, \quad (12)$$

the condition of local charge neutrality is fulfilled in linear response to within two orders in small. Furthermore, the terms

$$\frac{1}{c^2} \partial_t^2 \mathbf{A}' + \frac{1}{c} \partial_t \nabla \Phi' \sim \text{small}^4 \quad (13)$$

can be neglected compared to the terms $\nabla^2 \mathbf{A}' \sim \text{small}^2$ and $\frac{4\pi}{c} \mathbf{j} \sim \text{small}^2$.

3. Nonequilibrium transport equations

The quasiclassical theory describes equilibrium and nonequilibrium properties of superconductors on length scales that are large compared to microscopic scales (i.e. the lattice constant, Fermi wavelength, k_f^{-1} , Thomas–Fermi screening length, λ_{TF} etc) and energies that are small compared to the atomic scales (e.g. Fermi energy, E_f , plasma frequency, conduction band width, etc). Thus, the expansion parameter small defines the limits of validity of the quasiclassical theory. In particular, we require $k_f \xi_0 \gg 1$, $k_B T_c / E_f \ll 1$ and $\hbar \omega \ll E_f$, where the ac frequencies of interest are typically of order $\Delta/\hbar \sim k_B T_c / \hbar$, or smaller, and the length scales of interest are of the order of the coherence length, $\xi_0 = \hbar v_f / 2\pi k_B T_c$, or longer. Hereafter we use units in which $\hbar = k_B = 1$, and adopt the sign convention $e = -|e|$ for the electron charge.

In quasiclassical theory quasiparticle wavepackets move along nearly straight, classical trajectories at the Fermi velocity. The classical dynamics of the quasiparticle excitations is

governed by semi-classical transport equations for their phase-space distribution function. The quantum mechanical degrees of freedom are the ‘spin’ and ‘particle–hole degree of freedom’, described by 4×4 density matrices (Nambu matrices). The quantum dynamics is coupled to the classical dynamics of the quasiparticles in phase space through the matrix structure of the quasiclassical transport equations.

The nonequilibrium quasiclassical transport equations [10], [36]–[39] are formulated in terms of a quasiclassical Nambu–Keldysh propagator $\check{g}(\mathbf{p}_f, \mathbf{R}; \epsilon, t)$, which is a matrix in the combined Nambu–Keldysh space [73], and is a function of position \mathbf{R} , time t , energy ϵ and momenta \mathbf{p}_f on the Fermi surface. We denote Nambu–Keldysh matrices by a ‘check’, and their 4×4 Nambu submatrices by a ‘widehat’. Thus, the Nambu–Keldysh matrices for the quasiclassical propagator and self-energy have the form

$$\check{g} = \begin{pmatrix} \widehat{g}^R & \widehat{g}^K \\ 0 & \widehat{g}^A \end{pmatrix}, \quad \check{\sigma} = \begin{pmatrix} \widehat{\sigma}^R & \widehat{\sigma}^K \\ 0 & \widehat{\sigma}^A \end{pmatrix}, \quad (14)$$

where $\widehat{g}^{R,A,K}$ are the retarded (R), advanced (A) and Keldysh (K) quasiclassical propagators, and similarly for the self-energy functions. Each of these components of \check{g} and $\check{\sigma}$ are 4×4 Nambu matrices in combined particle–hole–spin space. For a review of the methods and an introduction to the notation we refer to [12, 68, 69]. In the compact Nambu–Keldysh notation the transport equations and the normalization conditions read

$$\left[\left(\epsilon + \frac{e}{c} \mathbf{v}_f \cdot \mathbf{A} \right) \check{\tau}_3 - e Z_0 \Phi \check{1} - \check{\Delta}_{\text{mf}} - \check{\nu}_{\text{mf}} - \check{\sigma}_i, \check{g} \right]_{\otimes} + i \mathbf{v}_f \cdot \nabla \check{g} = \check{0}, \quad (15)$$

$$\check{g} \otimes \check{g} = -\pi^2 \check{1}, \quad (16)$$

where $\check{1} = \text{diag}[1, 1, 1, 1]$, $\check{\tau}_3 = \text{diag}[1, -1, 1, -1]$, the commutator is $[\check{A}, \check{B}]_{\otimes} = \check{A} \otimes \check{B} - \check{B} \otimes \check{A}$, and

$$\check{A} \otimes \check{B}(\epsilon, t) = e^{(i/2)(\partial_{\epsilon}^A \partial_t^B - \partial_t^A \partial_{\epsilon}^B)} \check{A}(\epsilon, t) \check{B}(\epsilon, t). \quad (17)$$

Here $\partial_{\epsilon}^A = \partial^A / \partial_{\epsilon}$ etc, and ∂^A indicates that the derivative acts only on \check{A} but not on \check{B} . The vector potential, $\mathbf{A}(\mathbf{R}; t)$, includes $\mathbf{A}_0(\mathbf{R})$ which generates the static magnetic field, $\mathbf{B}_0(\mathbf{R}) = \nabla \times \mathbf{A}_0(\mathbf{R})$, as well as the nonstationary vector potential describing the time-varying electromagnetic field; $\check{\Delta}_{\text{mf}}(\mathbf{p}_f, \mathbf{R}; t)$ is the mean-field order parameter matrix, $\check{\nu}_{\text{mf}}(\mathbf{p}_f, \mathbf{R}; t)$ describes diagonal mean fields due to quasiparticle interactions (Landau interactions) and $\check{\sigma}_i(\mathbf{p}_f, \mathbf{R}; \epsilon, t)$ is the impurity self-energy. The electrochemical potential $\Phi(\mathbf{R}; t)$ includes the field generated by the induced charge density, $\rho(\mathbf{R}; t)$. The coupling of quasiparticles to the external potential involves virtual high-energy processes, which result from polarization of the nonquasiparticle background. The interaction of quasiparticles with both the external potential Φ and the polarized background can be described by coupling to an effective potential $Z_0 \Phi$ [68]. The high-energy renormalization factor Z_0 is defined below in equation (25). The coupling of the quasiparticle current to the vector potential in equation (15) is given in terms of the quasiparticle Fermi velocity. No additional renormalization is needed to account for the effective coupling of the charge current to the vector potential because the renormalization by the nonquasiparticle background is accounted for by the effective potentials that determine the band structure, and therefore the quasiparticle Fermi velocity.

In quasiclassical theory the description in terms of the variables $(\mathbf{p}_f, \mathbf{R}; \epsilon)$ is related to the (\mathbf{p}, \mathbf{R}) phase-space description by the transformation of distribution functions $f(\mathbf{p}_f, \mathbf{R}; \epsilon, t)$ to $n(\mathbf{p}, \mathbf{R}; t) = f(\mathbf{p}_f, \mathbf{R}; \epsilon(\mathbf{p}, \mathbf{R}; t), t)$, with $\epsilon(\mathbf{p}, \mathbf{R}; t) = \mathbf{v}_f(\mathbf{p}_f) \cdot (\mathbf{p} - \mathbf{p}_f) + \nu_{\text{mf}}(\mathbf{p}_f, \mathbf{R}; t) + e Z_0 \Phi(\mathbf{R}; t) - \frac{e}{c} \mathbf{v}_f(\mathbf{p}_f) \cdot \mathbf{A}(\mathbf{R}; t)$, see [68].

3.1. Constitutive equations

Equations (15) and (16) must be supplemented by Maxwell's equations for the electromagnetic potentials, and by self-consistency equations for the order parameter and the impurity self-energy. We use the weak-coupling gap equation to describe the superconducting state, including unconventional pairing states. The mean field self-energies are then given by

$$\widehat{\Delta}_{\text{mf}}^{\text{R,A}}(\mathbf{p}_f, \mathbf{R}; t) = N_f \int_{-\epsilon_c}^{+\epsilon_c} \frac{d\epsilon}{4\pi i} \langle V(\mathbf{p}_f, \mathbf{p}'_f) \widehat{f}^{\text{K}}(\mathbf{p}'_f, \mathbf{R}; \epsilon, t) \rangle, \quad (18)$$

$$\widehat{v}_{\text{mf}}^{\text{R,A}}(\mathbf{p}_f, \mathbf{R}; t) = N_f \int_{-\epsilon_c}^{+\epsilon_c} \frac{d\epsilon}{4\pi i} \langle A(\mathbf{p}_f, \mathbf{p}'_f) \widehat{g}^{\text{K}}(\mathbf{p}'_f, \mathbf{R}; \epsilon, t) \rangle, \quad (19)$$

$$\widehat{\Delta}_{\text{mf}}^{\text{K}}(\mathbf{p}_f, \mathbf{R}; t) = 0, \quad \widehat{v}_{\text{mf}}^{\text{K}}(\mathbf{p}_f, \mathbf{R}; t) = 0. \quad (20)$$

The impurity self-energy,

$$\check{\sigma}_i(\mathbf{p}_f, \mathbf{R}; \epsilon, t) = n_i \check{t}(\mathbf{p}_f, \mathbf{p}_f, \mathbf{R}; \epsilon, t), \quad (21)$$

is specified by the impurity concentration, n_i , and impurity scattering t -matrix, which is obtained from the self-consistent solution of the t -matrix equations,

$$\check{t}(\mathbf{p}_f, \mathbf{p}'_f, \mathbf{R}; \epsilon, t) = \check{u}(\mathbf{p}_f, \mathbf{p}'_f) + N_f \langle \check{u}(\mathbf{p}_f, \mathbf{p}'_f) \otimes \check{g}(\mathbf{p}'_f, \mathbf{R}; \epsilon, t) \otimes \check{t}(\mathbf{p}'_f, \mathbf{p}'_f, \mathbf{R}; \epsilon, t) \rangle, \quad (22)$$

where the Fermi surface average is defined by

$$\langle \dots \rangle = \frac{1}{N_f} \int \frac{d^2 \mathbf{p}'_f}{(2\pi)^3 |\mathbf{v}'_f|} (\dots), \quad N_f = \int \frac{d^2 \mathbf{p}'_f}{(2\pi)^3 |\mathbf{v}'_f|}, \quad (23)$$

and N_f is the Fermi surface average of the normal state density of states at the Fermi level. The Nambu matrix $\widehat{f}^{\text{K}}(\widehat{g}^{\text{K}})$ is the off-diagonal (diagonal) part of \widehat{g}^{K} in particle-hole space. The other material parameters that enter the self-consistency equations are the dimensionless pairing interaction, $N_f V(\mathbf{p}_f, \mathbf{p}'_f)$, the dimensionless Landau interaction, $N_f A(\mathbf{p}_f, \mathbf{p}'_f)$, the impurity concentration, n_i , the impurity potential, $\check{u}(\mathbf{p}_f, \mathbf{p}'_f)$ and the Fermi surface data: \mathbf{p}_f (Fermi surface), $\mathbf{v}_f(\mathbf{p}_f)$ (Fermi velocity). We eliminate both the magnitude of the pairing interaction and the cut-off, ϵ_c , in favor of the transition temperature, T_c , using the linearized, equilibrium form of the mean-field gap equation (18).

The quasiclassical equations are supplemented by constitutive equations for the charge density, the current density and the induced electromagnetic potentials. The formal result for the nonequilibrium charge density, to linear order in Δ/E_f , is given in terms of 4×4 matrix trace (Tr) over the Keldysh propagator by

$$\rho^{(1)}(\mathbf{R}; t) = e N_f \int_{-\epsilon_c}^{+\epsilon_c} \frac{d\epsilon}{4\pi i} \langle Z(\mathbf{p}'_f) \text{Tr} [\widehat{g}^{\text{K}}(\mathbf{p}'_f, \mathbf{R}; \epsilon, t)] \rangle - 2e^2 N_f Z_0 \Phi(\mathbf{R}; t), \quad (24)$$

with the renormalization factors given by

$$Z(\mathbf{p}_f) = 1 - \langle N_f A(\mathbf{p}'_f, \mathbf{p}_f) \rangle, \quad Z_0 = \langle Z(\mathbf{p}_f) \rangle. \quad (25)$$

The high-energy renormalization factor is related to an average of the scattering amplitude on the Fermi surface by a Ward identity that follows from the conservation law for charge [68]. The charge current induced by $\mathbf{A}(\mathbf{R}; t)$, calculated to leading order in Δ/E_f , is also obtained from the Keldysh propagator (here and in the following $\widehat{\tau}_3 = \text{diag}[1, -1]$),

$$\mathbf{j}^{(1)}(\mathbf{R}; t) = e N_f \int \frac{d\epsilon}{4\pi i} \text{Tr} \langle \mathbf{v}_f(\mathbf{p}'_f) \widehat{\tau}_3 \widehat{g}^{\text{K}}(\mathbf{p}'_f, \mathbf{R}; \epsilon, t) \rangle. \quad (26)$$

There is no additional high-energy renormalization of the coupling to the vector potential because the quasiparticle Fermi velocity already includes the high-energy renormalization of the charge-current coupling in equation (26). Furthermore, the *self-consistent* solution of the quasiclassical equations for \widehat{g}^K ensures that the continuity equation for charge conservation,

$$\partial_t \rho^{(1)}(\mathbf{R}; t) + \nabla \cdot \mathbf{j}^{(1)}(\mathbf{R}; t) = 0, \quad (27)$$

is satisfied. An estimate of the contribution to the charge density from the integral in equation (24) leads to the condition of ‘local charge neutrality’ [6, 66]. A charge density given by the elementary charge times the number of states within an energy interval Δ around the Fermi surface implies $\rho^{(1)} \sim 2eN_f\Delta$. Such a charge density cannot be maintained within a coherence volume because of the cost in Coulomb energy. This can be seen in the estimate in equation (11), where up to third order in the parameter small the charge fluctuations ρ are suppressed. This suppression, and the associated suppression in Coulomb energy, must be taken care of by requiring the leading order contribution to the charge density to vanish, i.e. $\rho^{(1)}(\mathbf{R}; t) = 0$. Thus, the spatially varying renormalized electro-chemical potential, $Z_0\Phi$, is determined by

$$2eZ_0\Phi(\mathbf{R}; t) = \int_{-\epsilon_c}^{+\epsilon_c} \frac{d\epsilon}{4\pi i} \text{Tr} \langle Z(\mathbf{p}'_f) \widehat{g}^K(\mathbf{p}'_f, \mathbf{R}; \epsilon, t) \rangle. \quad (28)$$

The continuity equation implies $\nabla \cdot \mathbf{j}^{(1)}(\mathbf{R}; t) = 0$. We discuss violations of the charge neutrality condition (28), which are higher order in Δ/E_f , in section 4.1. Finally, Ampere’s equation, with the current given by equation (26), determines the vector potential in the quasiclassical approximation,

$$\nabla \times (\nabla \times \mathbf{A}(\mathbf{R}; t)) = \frac{8\pi e N_f}{c} \int \frac{d\epsilon}{4\pi i} \text{Tr} \langle \mathbf{v}_f(\mathbf{p}'_f) \widehat{\tau}_3 \widehat{g}^K(\mathbf{p}'_f, \mathbf{R}; \epsilon, t) \rangle. \quad (29)$$

Equations (15)–(22) and (28)–(29) constitute a complete set of equations for calculating the electromagnetic response of vortices in the quasiclassical limit. For high- κ superconductors we can simplify the self-consistency calculations to some degree. Since quasiparticles couple to the vector potential via $\frac{e}{c} \mathbf{v}_f \cdot \mathbf{A}$, equation (29) shows that this quantity is of order $8\pi e^2 N_f v_f^2 / c^2 = 1/\lambda^2$, where λ is the magnetic penetration depth. Thus, for $\kappa = \lambda/\xi_0 \gg 1$, as in the layered cuprates, the feedback effect of the current density on the vector potential is small by factor $1/\kappa^2$.

3.2. Linear response

For sufficiently weak fields we can calculate the electromagnetic response to linear order in the external field. The propagator and the self-energies are separated into unperturbed equilibrium parts and terms that are first order in the perturbation,

$$\check{g} = \check{g}_0 + \delta\check{g}, \quad \check{\Delta}_{\text{mf}} = \check{\Delta}_0 + \delta\check{\Delta}_{\text{mf}}, \quad \check{\sigma}_i = \check{\sigma}_0 + \delta\check{\sigma}_i, \quad (30)$$

and similarly for the electromagnetic potentials, $\mathbf{A} = \mathbf{A}_0 + \delta\mathbf{A}$, $\Phi = \delta\Phi$. The equilibrium propagators obey the matrix transport equation,

$$\left[\left(\epsilon + \frac{e}{c} \mathbf{v}_f \cdot \mathbf{A}_0 \right) \check{t}_3 - \check{\Delta}_0 - \check{\sigma}_0, \check{g}_0 \right] + i\mathbf{v}_f \cdot \nabla \check{g}_0 = \check{0}. \quad (31)$$

These equations are supplemented by the self-consistency equations for the mean fields, equations (18) and (19), the impurity self-energy, equations (21) and (22), the local charge-neutrality condition for the scalar potential, equation (28), Ampère's equation for the vector potential, equation (29), the equilibrium normalization conditions,

$$\check{g}_0^2 = -\pi^2 \check{1}, \quad (32)$$

and the equilibrium relation between the Keldysh function and equilibrium spectral density,

$$\widehat{g}_0^K = \tanh\left(\frac{\epsilon}{2T}\right) [\widehat{g}_0^R - \widehat{g}_0^A]. \quad (33)$$

The first-order correction to the matrix propagator obeys the linearized transport equation,

$$\left[\left(\epsilon + \frac{e}{c} \mathbf{v}_f \cdot \mathbf{A}_0 \right) \check{t}_3 - \check{\Delta}_0 - \check{\sigma}_0, \delta \check{g} \right]_{\otimes} + i \mathbf{v}_f \cdot \nabla \delta \check{g} = \left[\delta \check{\Delta}_{\text{mf}} + \delta \check{\sigma}_i + \delta \check{v}, \check{g}_0 \right]_{\otimes}, \quad (34)$$

with source terms on the right-hand side from both the external field ($\delta \check{v}$) and the internal fields ($\delta \check{\Delta}_{\text{mf}}$, $\delta \check{\sigma}_i$). In addition, the first-order propagator satisfies the 'orthogonality condition',

$$\check{g}_0 \otimes \delta \check{g} + \delta \check{g} \otimes \check{g}_0 = \check{0} \quad (35)$$

obtained from linearizing the full normalization condition. Note that the convolution product between an equilibrium and a nonequilibrium quantity simplifies after Fourier transforming $t \rightarrow \omega$:

$$\check{A}_0 \otimes \check{B}(\epsilon, \omega) = \check{A}_0(\epsilon + \omega/2) \check{B}(\epsilon, \omega), \quad \check{B}(\epsilon, \omega) \otimes \check{A}_0 = \check{B}(\epsilon, \omega) \check{A}_0(\epsilon - \omega/2). \quad (36)$$

The system of linear equations is supplemented by the equilibrium and first-order self-consistency conditions for the order parameter,

$$\widehat{\Delta}_0^{\text{R,A}}(\mathbf{p}_f, \mathbf{R}) = N_f \int_{-\epsilon_c}^{+\epsilon_c} \frac{d\epsilon}{4\pi i} \langle V(\mathbf{p}_f, \mathbf{p}'_f) \widehat{f}_0^K(\mathbf{p}'_f, \mathbf{R}; \epsilon) \rangle, \quad (37)$$

$$\delta \widehat{\Delta}_{\text{mf}}^{\text{R,A}}(\mathbf{p}_f, \mathbf{R}; t) = N_f \int_{-\epsilon_c}^{+\epsilon_c} \frac{d\epsilon}{4\pi i} \langle V(\mathbf{p}_f, \mathbf{p}'_f) \delta \widehat{f}^K(\mathbf{p}'_f, \mathbf{R}; \epsilon, t) \rangle, \quad (38)$$

and the impurity self-energy,

$$\check{\sigma}_0(\mathbf{p}_f, \mathbf{R}; \epsilon) = n_i \check{t}_0(\mathbf{p}_f, \mathbf{p}_f, \mathbf{R}; \epsilon), \quad (39)$$

$$\check{t}_0(\mathbf{p}_f, \mathbf{p}''_f, \mathbf{R}; \epsilon) = \check{u}(\mathbf{p}_f, \mathbf{p}''_f) + N_f \langle \check{u}(\mathbf{p}_f, \mathbf{p}'_f) \check{g}_0(\mathbf{p}'_f, \mathbf{R}; \epsilon) \check{t}_0(\mathbf{p}'_f, \mathbf{p}''_f, \mathbf{R}; \epsilon) \rangle, \quad (40)$$

$$\delta \check{\sigma}_i(\mathbf{p}_f, \mathbf{R}; \epsilon, t) = n_i N_f \langle \check{t}_0(\mathbf{p}_f, \mathbf{p}'_f, \mathbf{R}; \epsilon) \otimes \delta \check{g}(\mathbf{p}'_f, \mathbf{R}; \epsilon, t) \otimes \check{t}_0(\mathbf{p}'_f, \mathbf{p}_f, \mathbf{R}; \epsilon) \rangle. \quad (41)$$

The Keldysh matrix components of the last equation read explicitly

$$\delta \widehat{\sigma}_i^{\text{R,A}}(\mathbf{p}_f, \mathbf{R}; \epsilon, t) = n_i N_f \langle \widehat{t}_0^{\text{R,A}}(\mathbf{p}_f, \mathbf{p}'_f, \mathbf{R}; \epsilon) \otimes \delta \widehat{g}^{\text{R,A}}(\mathbf{p}'_f, \mathbf{R}; \epsilon, t) \otimes \widehat{t}_0^{\text{R,A}}(\mathbf{p}'_f, \mathbf{p}_f, \mathbf{R}; \epsilon) \rangle, \quad (42)$$

$$\delta \widehat{\sigma}_i^{\text{a}}(\mathbf{p}_f, \mathbf{R}; \epsilon, t) = n_i N_f \langle \widehat{t}_0^{\text{R}}(\mathbf{p}_f, \mathbf{p}'_f, \mathbf{R}; \epsilon) \otimes \delta \widehat{g}^{\text{a}}(\mathbf{p}'_f, \mathbf{R}; \epsilon, t) \otimes \widehat{t}_0^{\text{A}}(\mathbf{p}'_f, \mathbf{p}_f, \mathbf{R}; \epsilon) \rangle, \quad (43)$$

with the 'anomalous' propagator and self-energy

$$\delta \widehat{\sigma}^{\text{a}} = \delta \widehat{\sigma}^{\text{K}} - \delta \widehat{\sigma}^{\text{R}} \tanh\left(\frac{\epsilon - \omega/2}{2T}\right) + \tanh\left(\frac{\epsilon + \omega/2}{2T}\right) \delta \widehat{\sigma}^{\text{A}}, \quad (44)$$

$$\delta\hat{g}^a = \delta\hat{g}^K - \delta\hat{g}^R \tanh\left(\frac{\epsilon - \omega/2}{2T}\right) + \tanh\left(\frac{\epsilon + \omega/2}{2T}\right) \delta\hat{g}^A. \quad (45)$$

In general, the diagonal mean fields also contribute to the response. However, we do not expect Landau interactions to lead to qualitatively new phenomena for the vortex dynamics, so we have neglected these interactions in the following analysis and set $A(\mathbf{p}_f, \mathbf{p}'_f) = 0$ (i.e. $\check{v}_{\text{mf}} = 0$). As a result the local charge neutrality condition for the electro-chemical potential becomes

$$2e\delta\Phi(\mathbf{R}; t) = \int_{-\epsilon_c}^{+\epsilon_c} \frac{d\epsilon}{4\pi i} \text{Tr} \langle \delta\hat{g}^K(\mathbf{p}_f, \mathbf{R}; \epsilon, t) \rangle. \quad (46)$$

In what follows we work in a gauge in which the induced electric field, $\mathbf{E}^{\text{ind}}(\mathbf{R}; t)$, is obtained from $\delta\Phi(\mathbf{R}; t)$ and the uniform external electric field, $\mathbf{E}_\omega^{\text{ext}}(t)$, is determined by the vector potential $\delta\mathbf{A}_\omega(t)$. For $\lambda/\xi_0 \gg 1$ we can safely neglect corrections to the vector potential due to the induced current. Thus, in the Nambu–Keldysh matrix notation the electromagnetic coupling to the quasiparticles is given by

$$\delta\check{v} = -\frac{e}{c} \mathbf{v}_f \cdot \delta\mathbf{A}_\omega(t) \check{\tau}_3 + e\delta\Phi(\mathbf{R}; t) \check{1}. \quad (47)$$

The validity of linear response theory requires the external perturbation $\delta\check{v}$ to be sufficiently small and the induced vortex motion to respond to the external field at the frequency set by the external field. At very low frequencies frictional damping of the vortex motion, arising from the finite mean free path of quasiparticles scattering from impurities, gives rise to a nonlinear regime in the dynamical response of a vortex. This regime is discussed extensively in the literature [10], and is not the subject of our study. However, for sufficiently small field strengths the vortex motion is nonstationary over any time interval, although it may be regarded as quasistationary at low enough frequencies. The nonstationary motion of the vortex can be described by linear response theory if $\delta\check{v} \ll 1/\tau$ for $\omega \lesssim 1/\tau$, and $\delta\check{v} \ll \omega$ for $\omega \gtrsim 1/\tau$. Note that the frequency of the perturbation, ω , is not required to be small compared to the gap frequency; it is only restricted to be small compared to atomic scale frequencies, e.g. $\omega \ll E_f/\hbar$.

Self-consistent solutions of equations (38), (41) and (46) for the self-energies and scalar potential are fundamental to obtaining a physically sensible solution for the electromagnetic response. The dynamical self-energy corrections are equivalent to ‘vertex corrections’ in the Kubo formulation of linear response theory. They are particularly important in the context of nonequilibrium phenomena in inhomogeneous superconductors. Vertex corrections often vanish in homogeneous superconductors because of translational and rotational symmetries. Inhomogeneous states break these symmetries and typically generate nonvanishing vertex corrections. In our case these corrections are of vital importance; the self-consistency conditions enforce charge conservation. In particular, equations (39)–(41) imply charge conservation in scattering processes, whereas (37) and (38) imply charge conservation in particle–hole conversion processes; any charge which is lost (gained) in a particle–hole conversion process is compensated by a corresponding gain (loss) of condensate charge. It is the coupled quasiparticle and condensate dynamics which conserves charge in superconductors. Neglecting the dynamics of either component, or using a nonconserving approximation for the coupling leads to unphysical results. Self-consistent calculations for the local excitation spectrum (spectral density) also provides key information for the interpretation of the dynamical response. Because of particle–hole coherence the spectral density is sensitive to the phase winding and symmetry of the order parameter, as well as material properties such as the transport mean-free path and impurity cross section.

In the limit $\omega \rightarrow 0$ the equations above have to be modified, in order to eliminate the zero modes associated with the stationary motion of the vortex lattice. However, it is possible also to obtain solutions for this limit in quasiclassical theory. For completeness, we provide the expression for the transport current density that arises in the low-frequency limit, in the flux flow regime [4, 12, 17]:

$$\frac{1}{c} \mathbf{j}_{\text{tr}} \times \Phi_0 = -N_f \int_{\text{cell}} d^2 \mathbf{R} \int \frac{d\epsilon}{8\pi i} \text{Tr} \left\langle \left[\left(\nabla - 2i \frac{e}{c} \mathbf{v}_f \cdot \mathbf{A}_0 \hat{\tau}_3 \right) \hat{\Delta}_0 + \frac{e}{c} \mathbf{v}_f \times (\nabla \times \mathbf{A}_0) \hat{\tau}_3 \right] \delta \hat{g}^{\text{nst}} \right\rangle \quad (48)$$

with $\delta \hat{g}^{\text{nst}} = \lim_{\omega \rightarrow 0} [\delta \hat{g}^{\text{K}} - \tanh(\epsilon/2T)(\delta \hat{g}^{\text{R}} - \delta \hat{g}^{\text{A}})]$, that is obtained from a systematic expansion for small ω . The leading terms are proportional to the vortex velocity v_L .

4. Nonequilibrium response of chiral vortices

The dynamics of the electronic excitations of the vortex core plays a key role in the dissipative processes in type II superconductors. Except in the dirty limit, $\ell \ll \xi_0$, the response of the core states to an electromagnetic field is generally very different from that of normal electrons. It is energetically unfavorable to maintain a charge density of the order of an elementary charge over a region with diameter of order of the coherence length. Instead, an electrochemical potential is induced which ensures that almost no net charge accumulates in the core region. On the other hand, a dipolar-like charge distribution develops which generates an internal electric field in the core. The internal field varies on the scale of the coherence length. This leads to a nonlocal response of the quasiparticles to the total electric field, even when the applied field varies on a much longer length scale and can be considered homogeneous. The dynamical response of the vortex core includes the collective mode of the inhomogeneous order parameter. This mode couples to the electro-chemical potential, $\delta\Phi$, in the vortex-core region. This potential is generated by the charge dynamics of vortex-core states and gives rise to internal electric fields which in turn drive the current density and the order parameter near the vortex-core region. The induced electric fields in the core are the same order of magnitude as the external field. The dynamics of the core states are strongly coupled to the charge current and collective mode of the order parameter. Thus, the determination of the induced order parameter, as well as the spectrum and distribution function for the core states and nonequilibrium impurity scattering processes requires dynamical self-consistency. Numerous calculations of the ac response neglect the self-consistent coupling of the collective mode and the spectral dynamics, or concentrate on the $\omega \rightarrow 0$ limit [17, 29, 31, 74]. Presently, quasiclassical theory is the only formulation of the theory of nonequilibrium superconductivity capable of describing the nonlocal response of the order parameter and quasiparticle dynamics in the presence of mesoscopic inhomogeneities and disorder. The numerical solution to the self-consistency problem was presented for unpinned s-wave and d-wave vortices in [40]–[42] and for pinned s-wave vortices in [43]. In the following we present our results for a vortex in a chiral, spin-triplet superconductor. As discussed in our accompanying paper [67], the order parameter we consider is of the form

$$\vec{\Delta}(\mathbf{p}_f, \mathbf{R}) = \vec{d} \left[|\Delta_+(\mathbf{R})| e^{im\phi} \frac{(\hat{p}_x + i\hat{p}_y)}{\sqrt{2}} + |\Delta_-(\mathbf{R})| e^{ip\phi} \frac{(\hat{p}_x - i\hat{p}_y)}{\sqrt{2}} \right]. \quad (49)$$

We have shown, that stable vortex structures can occur for $m = -1$, $p = 1$ (singly quantized vortex) and for $m = -2$, $p = 0$ (doubly quantized vortex with nearly homogeneous core).

We will in the following discuss for these two vortex structures the dynamical charge response, the induced current density and the magnetic field response. Our calculations were performed with impurity scattering included in the Born scattering limit for a mean free path of $\ell = 10\xi_0$, where $\xi_0 = \hbar v_f / 2\pi k_B T_c$ is the coherence length. The Fermi surface parameters are assumed to be isotropic, and the temperature was chosen to be $T = 0.2T_c$. For simplicity we also assumed the high- κ limit, where the penetration depth is large compared to the coherence length. Our calculations of the vortex structure are appropriate to the low-field limit near H_{c1} where vortices are well separated from each other. The order parameter, impurity self-energy and the equilibrium spectra and current densities were obtained self-consistently using the Riccati formulation of the quasiclassical transport theory with impurity and pairing self-energies [67, 75].

4.1. Dynamical charge response

The charge dynamics of layered superconductors has two distinct origins. The c -axis dynamics is determined by the Josephson coupling between the conducting planes. Here we are concerned with the in-plane electrodynamics associated with the response of the order parameter and quasiparticle states bound to the vortex core. We assume strong Josephson coupling between different layers, and neglect variations of the response between different layers. This requires that the polarization of the electric field be in-plane, so that there is no coupling of the in-plane dynamics to the Josephson plasma modes. The external electromagnetic field is assumed to be long wavelength compared to the size of the vortex core, $\lambda_{EM} \gg \xi_0$. In this limit we can assume the ac electric field to be uniform and described by a vector potential, $\mathbf{E}_\omega(t) = -\frac{1}{c}\partial_t \mathbf{A}_\omega$. We can also neglect the response to the ac magnetic field in the limit $\lambda \gg \xi_0$. In this case the spatial variation of the induced electric field occurs mainly within each conducting layer on the scale of the coherence length, ξ_0 . Poisson's equation implies that induced charge densities are of order $\delta\Phi/\xi_0^2$, where $\delta\Phi$ is the induced electrochemical potential in the core. This leads to a dynamical charge of order $e(\Delta/E_f)$ in the vortex core. Once the electrochemical potential is calculated from equation (46) we can calculate the charge density fluctuations of order $(\Delta/E_f)^3$ from Poisson's equation,

$$\rho^{(3)}(\mathbf{R}; t) = -\frac{1}{4\pi} \nabla^2 \delta\Phi(\mathbf{R}; t). \quad (50)$$

Figure 1 shows the charge distribution for $\omega = 0.2\Delta$ which oscillates out of phase with the external field. We consider two vortex structures. On the left-hand side we show results for a singly quantized vortex with quantum numbers $m = -1$, $p = 1$, and on the right-hand side the corresponding results for the doubly quantized vortex with quantum numbers $m = -2$, $p = 0$. For the singly quantized vortex we observe a charge dipole that oscillates with the external frequency. The dipole vector is parallel to the external electric field and the charge accumulation concentrated in the vortex-core area. For the doubly quantized vortex, shown on the right-hand side in figure 1, a very different picture emerges. A dipolar charge distribution accumulates at the interface between the dominant and the subdominant order parameter components, oscillating at the frequency of the external field. The induced charge which accumulates is of the order of $e\Delta/E_f$ within a region of order ξ_0^2 in each conducting layer. As discussed in the introduction, this charge is a factor of $(\delta v_\omega E_f)/\Delta^2$ larger than the static charge of a vortex that arises from particle-hole asymmetry [44]–[46].

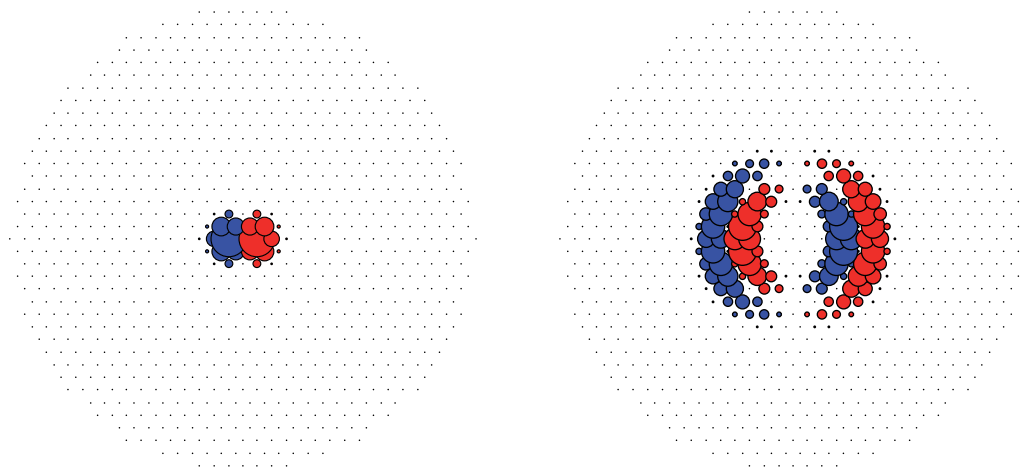


Figure 1. Out of phase charge response for the case of an $m = -1$, $p = 1$ vortex (left) and for the case of an $m = -2$, $p = 0$ vortex (right) to an ac electric field with $\omega = 0.2\Delta$ polarized in the x -direction (horizontal to the right in the figure). Red denotes negative values for the response and blue positive.

4.2. Induced current density

In the static vortex structure a circulating supercurrent is present due to the screening of the quantized magnetic field penetrating the vortex. This equilibrium current has been calculated in [67]. For a doubly quantized vortex the circulating current of the vortex has a nontrivial structure, with currents in the vortex core that flow counter to the circulating supercurrent at large distances from the vortex center. Here, we discuss the additional, dynamical part of the current that is induced in the presence of the external ac electromagnetic field, and concentrate on the absorptive component of the current response, which is *in phase* with the external field. Results for the ac component of the current density are shown in figure 2 for $\omega = 0.2\Delta$. The in-phase current response for a singly quantized vortex, shown on the left-hand side, shows a dipolar pattern, indicating a region of strong absorption ($\delta\mathbf{j} \parallel \delta\mathbf{E}^{\text{ext}}$) in the vortex core, and emission ($\delta\mathbf{j} \cdot \delta\mathbf{E}^{\text{ext}} < 0$) in the region roughly perpendicular to the direction of the applied field several coherence lengths away from the vortex center. The picture here is similar to that for an s-wave vortex in an external ac field, discussed in our previous work [40]–[43]. Energy absorbed in the core is transported away from the vortex center by the vortex-core excitations in directions predominantly perpendicular to the applied field. The net absorption is ultimately determined by inelastic scattering and requires integrating the local absorption and emission rate over the vortex array. Note that the long-range dipolar component does not contribute to the *total* dissipation. Far from the vortex core the current response is out of phase with the electric field and predominantly a nondissipative supercurrent.

For the doubly quantized vortex, the response of which is shown on the right-hand side in figure 2, we find that the main absorption results from the regions where the two time-reversed order parameter phases are overlapping. The current response here is strongly linked to the order parameter response. Absorptive currents are flowing predominantly in the region of the ‘domain wall’ between the two components. Note that the leading contribution in a multipole expansion with respect to the vortex center is again the dipolar term, although the pattern here has strong quadrupolar and higher order contributions.

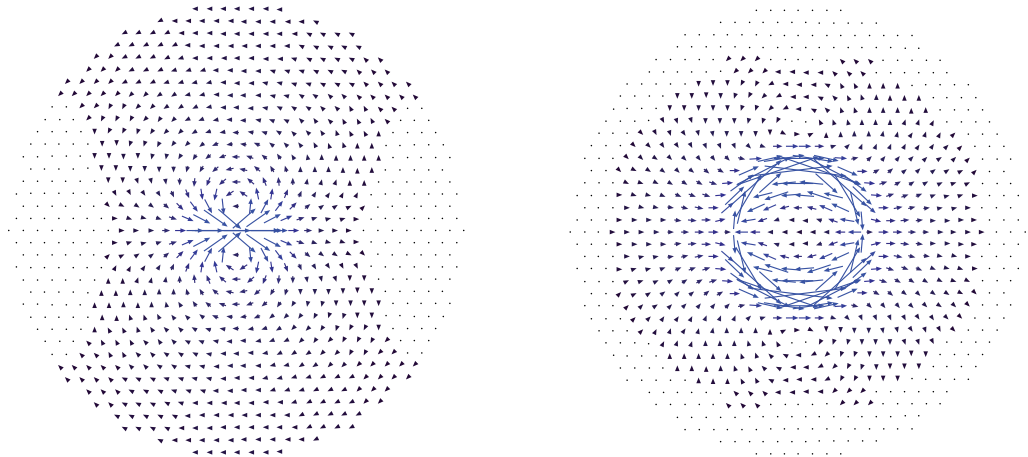


Figure 2. Absorptive current response for the case of an $m = -1$, $p = 1$ vortex (left) and for the case of an $m = -2$, $p = 0$ vortex (right) to an ac electric field with $\omega = 0.2\Delta$ polarized in the x -direction (horizontal to the right in the figure).

4.3. Magnetic field response

In the high- κ limit we can calculate the ac corrections to the local magnetic field in the vortex core directly from equation (29), using

$$\delta\mathbf{B}(\mathbf{R}, t) = \nabla \times \delta\mathbf{A}(\mathbf{R}, t). \quad (51)$$

We compare again the singly quantized and the doubly quantized vortex structures in figure 3. The time-dependent oscillating magnetic field adds to the static, time-independent magnetic field that penetrates the vortex in a quantized manner. We find that the dynamical magnetic field response is predominantly out of phase, and resembles the patterns expected for an oscillation of the magnetic field lines perpendicular to the polarization of the external electric field. For the doubly quantized vortex, shown on the right-hand side in figure 3, the magnetic response is concentrated to the region of overlap between the two time-reversed order parameter components.

5. Conclusions

We have discussed the electromagnetic response of a chiral, spin-triplet superconducting vortex for two vortex structures, a singly quantized vortex and a doubly quantized vortex. The vortex response is nonlocal and largely determined by the response of the vortex-core states. We have performed dynamically self-consistent calculations, taking into account that transitions involving the vortex-core states, and their coupling to the collective motion of the condensate are closely related to the dynamical response of the order parameter, self-energies, induced fields, excitation spectra and distribution functions.

The vortex response roughly consists of an oscillation of the magnetic field lines perpendicular to the electric field polarization, and an oscillation of a charge dipole parallel to the electric field polarization. Both the charge response and the magnetic response are linked to a current flow pattern that show absorptive response patterns in the vortex core. Particularly interesting is the response for the doubly quantized vortex. Here we find that the important

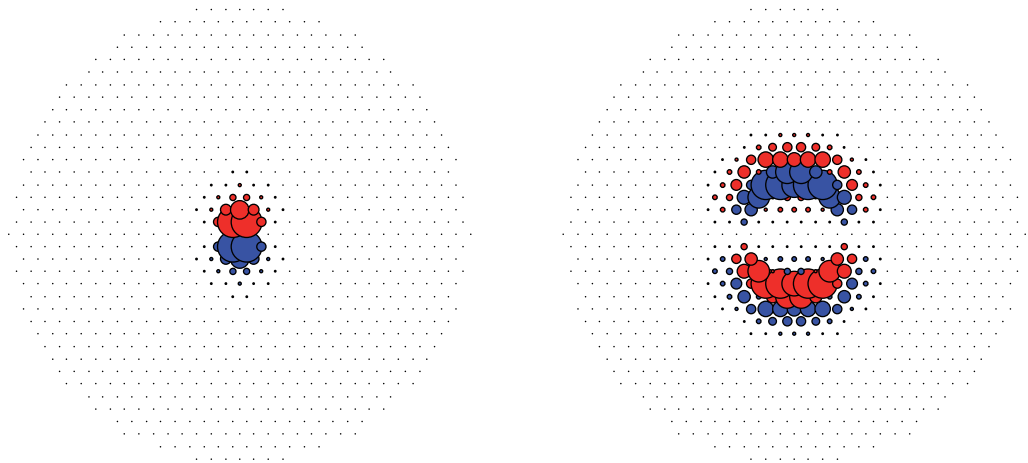


Figure 3. Out of phase magnetic field response for the case of an $m = -1$, $p = 1$ vortex (left) and for the case of an $m = -2$, $p = 0$ vortex (right) to an ac electric field with $\omega = 0.2\Delta$ polarized in the x -direction (horizontal) in the limit of high κ . Red denotes negative values for the response and blue positive.

region is the region where the time-reversed order parameter phases overlap, which happens away from the vortex center in a ‘domain wall’ region.

Based on our results, we see interesting future extensions of our work in the following directions: (i) more detailed study of the collective order parameter response for the doubly quantized vortex; here in particular the ‘optical’ mode (oscillation of the two vortex components against each other) is of interest; (ii) to include interactions between the vortices by performing calculations on a vortex lattice; and (iii) to study the interplay between the charge response and the magnetic response in a vortex lattice.

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