Published in Physical Review B 68, 224502 (2003) [doi:10.1103/PhysRevB.68.224502]

Impurity Effects on the A_1-A_2 Splitting of Superfluid ^{3}He in Aerogel

J. A. Sauls[1,2,*] and Priya Sharma[1]

[1] *Department of Physics and Astronomy, Northwestern University, Evanston, IL, USA*
[2] *Centre de Recherches sur les Très Basses Températures, Centre National de la Recherche Scientifique, Laboratoire Associé à l'Université J. Fourier, BP 166, 38042 Grenoble Cedex 9, France*

(Dated: October 8, 2016)

When liquid ^{3}He is impregnated into silica aerogel a solid-like layer of ^{3}He atoms coats the silica structure. The surface ^{3}He is in fast exchange with the liquid on NMR timescales. The exchange coupling of liquid ^{3}He quasiparticles with the localized ^{3}He spins modifies the scattering of ^{3}He quasiparticles by the aerogel structure. In a magnetic field the polarization of the solid spins gives rise to a splitting of the scattering cross-section of for 'up' vs. 'down' spin quasiparticles, relative to the polarization of the solid ^{3}He. We discuss this effect, as well as the effects of non-magnetic scattering, in the context of a possible splitting of the superfluid transition for $\uparrow\uparrow$ vs. $\downarrow\downarrow$ Cooper pairs for superfluid ^{3}He in aerogel, analogous to the $A_1 - A_2$ splitting in bulk ^{3}He. Comparison with the existing measurements of T_c for $B < 5\,\mathrm{kG}$, which show no evidence of an $A_1 - A_2$ splitting, suggests a liquid-solid exchange coupling of order $J \simeq 0.1\,\mathrm{mK}$. Measurements at higher fields, $B \gtrsim 20\mathrm{kG}$, should saturate the polarization of the solid ^{3}He and reveal the $A_1 - A_2$ splitting.

One focus of experimental investigations of ^{3}He in aerogel has been the determination of the phase diagram. Torsional oscillator, NMR, vibrating wire and sound attenuation experiments on ^{3}He in $\approx 98\%$ porosity aerogels, suggest that there is just one superfluid phase in zero magnetic field over the pressure range, $34\,\mathrm{atm} < p \lesssim 6\,\mathrm{atm}$.[1–3] At lower pressures there appears to be only normal ^{3}He down to zero temperature.[4] In short, a B-like phase, with reduced susceptibility, is the only stable superfluid phase observed in zero field. An A-like phase, if it exists in zero field, may be stable only in a very small region $\Delta T \leq 20\mu\mathrm{K}$ near T_c. As in bulk ^{3}He an A-like phase can be stabilized in a magnetic field; the region of stability, $T_{\rm AB} \leq T < T_c$, increases quadratically with field, $T_c - T_{\rm AB} = -g_{\rm AB} B^2$. However, a splitting of the transition for $\uparrow\uparrow$ and $\downarrow\downarrow$ pairs, analogous to the $A_1 - A_2$ splitting in bulk ^{3}He, has so far not been observed for fields up to $B \approx 5\,\mathrm{kG}$.[3, 5]

In pure ^{3}He, on application of a magnetic field, the A_1 phase, characterized by spin-polarized Cooper pairs composed of only $\uparrow\uparrow$ spins nucleates at a temperature slightly higher than the zero-field transition, $T_c^{A_1} = T_c + \lambda^{\rm A_1} B$.[29] This transition is followed by a second transition, shifted below the zero-field transition, at $T_c^{A_2} = T_c - \lambda^{\rm A_2} B$, in which the $\downarrow\downarrow$ pairs nucleate. The region of stability of *pure* $\uparrow\uparrow$ pairs increases linearly with field, $\Delta T_{A_1-A_2} \equiv (T_c^{A_1} - T_c^{A_2}) = (\lambda^{\rm A_1} + \lambda^{\rm A_2}) B$, at a rate of $\lambda^{\rm A_1} + \lambda^{\rm A_2} \simeq 6.1\mu\mathrm{K/kG}$ at $p = 33.4\,\mathrm{bar}$.[6, 7]

This splitting of the zero-field transition originates from the combined effect of the nuclear Zeeman coupling to the ^{3}He spin, and particle-hole asymmetry in the normal-state density of states and pairing interaction. The original estimate of $\lambda^{\rm A_1}$ by Ambegaokar and Mermin[8] was based on the asymmetry of the density of states for $\uparrow$ vs. $\downarrow$ quasiparticles at the Fermi level. More involved calculations include the effects of spin-polarization of the Fermi liquid on the pairing interaction.[9, 10] Estimates of the splitting of the A_1 transition are of order

$$\lambda^{\rm A_1} = \Lambda \left| \frac{\gamma\hbar}{2} \right| \left(\frac{k_B T_c}{E_f} \right) , \qquad (1)$$

where γ is the gyromagnetic ratio for ^{3}He, $k_B T_c / E_f \sim 10^{-3}$ and $\Lambda \sim \mathcal{O}(1-10)$ is a "high-energy" vertex. A first principles calculation of Λ requires a solution of the many-body problem for the pairing interaction in ^{3}He. Alternatively, we treat Λ on the same level as other high-energy vertices in Fermi liquid theory; Λ is a Fermi-liquid parameter, which can be determined by comparing physical predictions of the Fermi liquid theory with experiment - in this case the $A_1 - A$ splitting. Once determined, the effects of Λ on other low-energy properties of ^{3}He can be calculated. Thus, the A_2 transition can be calculated in terms of Λ, and corrections to $T_c^{A_2}$ from the $\uparrow\uparrow$ pair condensate.[11]

The disorder introduced by the aerogel structure into liquid ^{3}He is, on average, weak on the high-energy scale, $\hbar/\tau E_f \ll 1$. Thus, the Fermi-liquid interactions are essentially unaffected by the aerogel, and we can calculate the effects of aerogel on the low-energy excitations and superfluid properties within Fermi liquid theory. The main effect of the aerogel structure is to scatter ^{3}He quasiparticles moving with the Fermi velocity. At temperatures below $T^* \approx 10\,\mathrm{mK}$ elastic scattering by the aerogel dominates inelastic quasiparticle-quasiparticle collisions.[12] This limits the mean free path of normal ^{3}He quasiparticles to $\ell \simeq 130 - 180\,\mathrm{nm}$. In p-wave superfluids quasiparticle scattering is intrinsically pairbreaking and leads to renormalization of nearly all properties of the superfluid phases, including the A_1 and A_2 transition temperatures. The suppression of T_c, as well as pair-breaking effects on observable properties of the superfluid phases, have been analyzed theoretically for non-magnetic scattering.[13–18] Here we analyze the effects of scattering by the aerogel on the A_1 and A_2 transitions.

The aerogel has a strong effect on the *short-distance*, high-energy properties of the liquid locally near the silica strands. The first few atomic layers of ^{3}He are adsorbed on the silica structure and form a highly polarizable solid-like phase, observable as a Curie-like component of the magnetization of ^{3}He-aerogel.[19] The surface ^{3}He is in fast exchange with the liquid on typical NMR timescales, implying a liquid-solid

exchange interaction, $|J|/h \gtrsim 0.66\,\mathrm{MHz}$ ($|J| \gtrsim 0.03\,\mathrm{mK}$).[19, 20] The exchange coupling of liquid ^{3}He quasiparticles with the localized ^{3}He spins, J, may modify the scattering of ^{3}He quasiparticles by the aerogel structure.[21] Here we include the effect of magnetic scattering of ^{3}He quasiparticles by polarizable ^{3}He spins coating the aerogel strands. The differential scattering of $\uparrow$ vs. $\downarrow$ spin quasiparticles by the polarized surface leads to an additional contribution to the splitting of the $\uparrow\uparrow$ vs. $\downarrow\downarrow$ transitions, $\lambda_{\mathrm{J}} \propto J$, which is determined by the non-magnetic, u_0, and exchange, J, interactions and the density of ^{3}He coating the aerogel.[22] Below we extend the analysis of Ref. 22 and examine the role of the exchange coupling on the possible $A_1 - A_2$ splitting of the superfluid phases of ^{3}He in aerogel.

The suppression of the AB transition in both pure and disordered ^{3}He is quadratic in field on a scale set by $g_{\mathrm{AB}} \sim$ mK/kG^2.[3] Thus, for fields, $B \gg B^* = \lambda^{\mathrm{A1}}/g_{\mathrm{AB}} \sim 1\,\mathrm{G}$, and temperatures, $T \approx T_c$, the $\uparrow\downarrow$ pairs are suppressed. In the field and temperature range of interest we can restrict the full p-wave, spin-triplet order parameter to two components, d_+ for $\uparrow\uparrow$ pairs and d_- for $\downarrow\downarrow$ pairs. We assume that the orbital state is the same for both spin-components and of the axial (ABM) form, $\chi(\hat{\mathbf{p}}) = \hat{\mathbf{p}} \cdot (\hat{\mathbf{m}} + i\hat{\mathbf{n}})/\sqrt{2}$. Thus, in pure ^{3}He the full Ginzburg-Landau (GL) free energy functional (c.f. Ref. 23) reduces to[11]

$$\begin{aligned} \Omega[\mathsf{d}_+, \mathsf{d}_-] &= \alpha\left(|\mathsf{d}_+|^2 + |\mathsf{d}_-|^2\right) - \eta\, B\left(|\mathsf{d}_+|^2 - |\mathsf{d}_-|^2\right) \\ &+ \beta_{24}\left(|\mathsf{d}_+|^2 + |\mathsf{d}_-|^2\right)^2 + 4\beta_5\,|\mathsf{d}_+|^2\,|\mathsf{d}_-|^2, \quad (2) \end{aligned}$$

where α, η and β_{i} are the known material parameters for superfluid ^{3}He; $\alpha(T) = \frac{1}{3}N_f \ln(T/T_c)$ determines the zero-field transition and $\eta = \frac{1}{3}N_f\lambda^{\mathrm{A1}}/T_c$ determines the A_1 ($\uparrow\uparrow$) transition, where N_f is the single-spin density of states at the Fermi level. The fourth-order coefficients determine the relative stability of the possible phases. In particular, $\beta_{24} > 0$, and $\beta_5 < 0$ favors an equal-spin-pairing (ESP) phase with $|\mathsf{d}_+| = |\mathsf{d}_-|$. The linear field term is symmetry breaking and competes with the fourth-order terms. The latter wins at lower temperatures and gives rise to the A_2 transition where $\downarrow\downarrow$ spins condense, with $T_c^{A_2} = T_c - \lambda^{\mathrm{A2}}\,B$ and

$$\lambda^{\mathrm{A2}} = \left(\frac{\beta_{245}}{-\beta_5}\right)\lambda^{\mathrm{A1}}. \quad (3)$$

Within the homogeneous, isotropic scattering model (HSM) the rotational symmetry of ^{3}He in aerogel is preserved on the coherence length scale, and the GL free energy has the same form as in pure ^{3}He, but with material parameters, $\bar{\alpha}$, $\bar{\eta}$, etc., that are modified by the effects of scattering by the aerogel (we use a 'bar' to denote the material parameters in the presence of aerogel scattering). These effects were calculated within the quasiclassical theory to leading order in T_c/E_f (weak-coupling), and one finds[13]

$$\bar{\alpha} = \frac{1}{3}N_f\left[\ln(T/T_{c0}) - 2\,S_1(x)\right], \quad (4)$$

where $x = v_f/2\pi T\ell$, and ℓ is the mean free path of quasiparticles scattering off the aerogel. In Eq. (4), and hereafter, we denote the transition temperature for pure ^{3}He by T_{c0}. The superfluid transition in aerogel is determined by the condition $\bar{\alpha}(T_c) = 0$, and the function, S_1, is

$$S_1(x) = \sum_{n=0}^{\infty}\left(\frac{1}{2n+1+x} - \frac{1}{2n+1}\right). \quad (5)$$

The parameter η is directly proportional to the high-energy vertex, Λ, and so is un-renormalized by impurities to leading order in $\hbar/p_f\ell$. Thus, $\bar{\eta} = \eta = \frac{1}{3}N_f\lambda^{\mathrm{A1}}/T_{c0}$, where λ^{A1} is the rate for the splitting of the A_1-A transition in pure ^{3}He. Although η is un-renormalized, the splitting parameters for the A_1 and A_2 transitions are renormalized by the impurity corrections to transition temperature and, in general, the β parameters.

The weak-coupling results for the fourth order coefficients are,[13]

$$\begin{aligned} \bar{\beta}_{24} &= -2\bar{\beta}_5 = 4(\bar{\beta}^{\mathrm{wc}} + \bar{\beta}^{\bar{\sigma}}) \quad (6) \\ \bar{\beta}^{\mathrm{wc}} &= \frac{N_f}{30\pi^2T^2}\,S_3(x) \\ \bar{\beta}^{\bar{\sigma}} &= \frac{N_f}{9\pi^2T^2}\,(\bar{\sigma}_0 - \tfrac{1}{2})\,x\,S_4(x), \quad (7) \end{aligned}$$

where $\bar{\sigma}_0$ is the dimensionless, non-magnetic, s-wave scattering cross-section, $0 < \bar{\sigma}_0 < 1$ (see below), and

$$S_p(x) = \sum_{n=0}^{\infty}\left(\frac{1}{2n+1+x}\right)^p, \quad p > 1. \quad (8)$$

Note that the ratio, $\bar{r}_\beta = \bar{\beta}_{245}/(-\bar{\beta}_5) \equiv 1$ in the weak-coupling limit, even with (isotropic) impurity scattering. However, this ratio deviates substantially from 1 in pure ^{3}He, particularly at high pressures, e.g. $r_\beta \approx 0.47$ at $p = 33.4\,\mathrm{bar}$. Thus, the asymmetry of the $A_1 - A$ vs. the $A_2 - A$ transitions is a measure of the strong-coupling corrections to the β parameters, $\delta\beta^{\mathrm{sc}} = \beta - \beta^{\mathrm{wc}}$, which are of order

$$\frac{\delta\beta^{\mathrm{sc}}}{\beta^{\mathrm{wc}}} \sim \frac{T_c}{E_f}\,\langle|\Gamma_{\mathrm{N}}|^2\rangle_{\mathrm{FS}}, \quad (9)$$

compared to the weak-coupling values, where $\langle\ldots\rangle_{\mathrm{FS}}$ is a Fermi-surface average of the normal-state quasiparticle-quasiparticle collision rate, $\propto |\Gamma_{\mathrm{N}}|^2$.

Corrections to the weak-coupling β parameters from quasiparticle scattering off the aerogel strands are of order $\delta\bar{\beta}^{\mathrm{wc}}/\beta^{\mathrm{wc}} \sim x_c = v_f/2\pi\ell T_c$, which is small for high-porosity aerogels, but comparable to the strong-coupling corrections from quasiparticle-quasiparticle collisions. Based on the suppression of T_c and the aerogel mean free path we estimate $x_c \simeq 0.12$ at high pressures.

If we neglect aerogel scattering corrections to the intermediate quasiparticle states that enter the strong-coupling self-energies,[24] then the relative strong-coupling corrections for ^{3}He in aerogel are scaled relative to their bulk ratios by the ratio of transition temperatures,

$$\frac{\delta\bar{\beta}^{\mathrm{sc}}}{\bar{\beta}^{\mathrm{wc}}} = \frac{\delta\beta^{\mathrm{sc}}}{\beta^{\mathrm{wc}}}\left(\frac{T_c}{T_{c0}}\right). \quad (10)$$

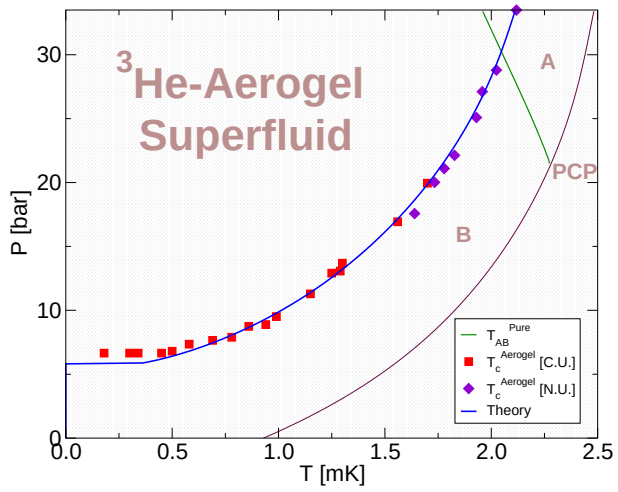

FIG. 1: The phase diagram for ^{3}He in 98% aerogel. The data are from Refs. 4 and 25. The theoretical curve is calculated from $\bar{\alpha}(T_c) = 0$ using Eq. (4) in zero field with the effective pair-breaking parameter $\tilde{x}$ evaluated with $\xi_a = 502\,\text{Å}$ and $\ell = 1400\,\text{Å}$. The phase boundaries for pure ^{3}He are shown for comparison.

This approximation gives a good qualitative description of the suppression of strong-coupling parameters for ^{3}He in aerogel as measured by the field-dependence of the AB transition.[3]

A theoretical calculation of impurity scattering corrections to the strong-coupling β parameters within the spin-fluctuation feedback theory of Brinkman and Anderson was carried out by Baramidze and Kharadze.[16] Their theory predicts a suppression of strong-coupling effects with increased disorder, but at a rate that is slower than that predicted just on the basis of the suppression of T_c. We can use the results of Ref. 16 to estimate the strong-coupling correction to the predicted A_2 transition for ^{3}He in aerogel. The results of Ref. 16 depend on a high-energy vertex, which we determine by comparison with the magnitude of strong-coupling ratio, $r_\beta = \beta_{245}/(-\beta_s)$ for pure ^{3}He. In particular, we fix the ratio, $\delta\beta^{\rm sc}/\beta^{\rm wc}$ for pure ^{3}He, using the measured value of r_β: $\frac{1}{2}\delta\beta^{\rm sc}/\beta^{\rm wc} = (1-r_\beta)/(1+r_\beta)$. Then, the impurity corrections to the strong-coupling β-parameters calculated in Ref. 16, give $\bar{r}_\beta = (1-\frac{1}{2}\delta\bar{\beta}^{\rm sc}/\bar{\beta}^{\rm wc})/(1+\frac{1}{2}\delta\bar{\beta}^{\rm sc}/\bar{\beta}^{\rm wc})$ where[30]

$$\frac{\delta\bar{\beta}^{\rm sc}}{\bar{\beta}^{\rm wc}} = \frac{\delta\beta^{\rm sc}}{\beta^{\rm wc}}\left(\frac{T_c}{T_{c0}}\right)\left(\frac{S_2(x_c)/S_2(0)}{S_3(x_c)/S_3(0)}\right). \qquad (11)$$

To leading order in the pair-breaking parameter, $\delta\bar{\beta}^{\rm sc}/\bar{\beta}^{\rm wc} \approx (\delta\beta^{\rm sc}/\beta^{\rm wc})(1-a\,x_c)$. Based on Eq. (10), $a \simeq 2.47$. The rate of suppression is reduced to $a \simeq 1.28$ based on Eq. (11). In what follows we use Eq. (11) to estimate the suppression of the strong-coupling correction for $T_c^{\rm A2}$. This correction turns out to be small, and relatively unimportant on the scale of corrections that are required to explain the lack of an A_1-A_2 splitting for $B \leq 5$kG.

Finally, before discussing the effects of the liquid-solid exchange coupling, we consider a simplified version of an *inhomogeneous* scattering model discussed by Ref. 13 that incorporates correlations of the aerogel. The length scale at which aerogel reveals inhomogeneity, $\xi_a \sim 30-100\,$nm, is typically comparable to the pair correlation length, ξ, and has a substantial effect on the transition temperature of ^{3}He in aerogel, particularly at high pressures. The inhomogeneity of the aerogel on scales $\xi_a \sim \xi$ leads to higher superfluid transition temperatures than predicted by the HSM with the same quasiparticle mean free path. Regions of lower aerogel density, of size of order ξ_a, are available for formation of the condensate. Thus, the qualitative picture is that of a random distribution of low density regions, 'voids', with a typical length scale, ξ_a, in an aerogel with a quasiparticle mean-free path, ℓ. When $\xi \sim \xi_a \ll \ell$, the superfluid transition is determined by the pairbreaking effects of dense regions surrounding the 'voids', and scales as $\delta T_c/T_{c0} \propto -(\xi/\xi_a)^2$. However, when the pair size is much larger than ξ_a the aerogel is effectively homogeneous on the scale of the pairs and pairbreaking results from homogeneous scattering defined by the transport mean free path, which scales as $\delta T_c/T_{c0} \propto -(\xi/\ell)$. This latter limit is achieved at low pressures. We incorporate the correlation effect by introducing an effective pairbreaking parameter in Eq. (4) that interpolates between these two limits, $x \to \tilde{x} = x/(1+\zeta_a^2/x)$, where $\zeta_a \equiv \xi_a/\ell$. This heuristic treatment of aerogel correlations provides a good description of the pressure dependence of T_c in zero field for ^{3}He in aerogel over the whole pressure range, as shown in Fig. 1 for $\ell = 1400\,$Å and $\xi_a = 502\,$Å. Alternatively, we can adjust the mean-free path, ℓ, with pressure in order to simulate the correlation effect on T_c. However, we prefer to identify ℓ with the pressure-independent geometric mean-free path and introduce aerogel correlation effects via the effective pair-breaking parameter, $\tilde{x}$. In either scenario, the GL theory for ^{3}He in aerogel predicts transitions for the A_1 and A_2 phases, which correspond to the condensation of $\uparrow\uparrow$ and $\downarrow\downarrow$ Cooper pairs as in pure ^{3}He; the transition temperatures are of the same form,

$$T_c^{\rm A1} = T_c + \bar{\lambda}^{\rm A1}B\,, \quad T_c^{\rm A2} = T_c - \bar{\lambda}^{\rm A2}B\,, \qquad (12)$$

but with renormalized parameters,

$$\bar{\lambda}^{\rm A1} = \lambda^{\rm A1}\left(\frac{T_c}{T_{c0}}\right)\left(1+2\tilde{x}_c' S_2(\tilde{x}_c)\right), \bar{\lambda}^{\rm A2} = \bar{r}_\beta\bar{\lambda}^{\rm A1}\,, \qquad (13)$$

where $\tilde{x}_c' \equiv T_c d\tilde{x}_c/dT_c$, and $\bar{r}_\beta = \bar{\beta}_{245}/(-\bar{\beta}_5)$ is calculated including both impurity scattering and strong-coupling corrections as described above. These results predict an A_1-A_2 splitting of $\Delta T_c^{A_1-A_2}/B = \left(\bar{\lambda}^{\rm A1}+\bar{\lambda}^{\rm A2}\right) \simeq 6.3\mu$K/kG at $p = 33.4\,$bar, comparable to that of pure ^{3}He. There is currently no experimental evidence of an A_1-A_2 splitting in ^{3}He-aerogel. Since the width of the transition is less than $20\,\mu$K, inhomogeneities within the aerogel cannot account for the absence of the A_1-A_2 splitting.

For ^{3}He in aerogel an additional mechanism contributing to the splitting of the $\uparrow\uparrow$ and $\downarrow\downarrow$ transitions is possible.[22] It originates from an exchange coupling between liquid ^{3}He quasiparticles and the surface ^{3}He spins adsorbed on the silica structure. Such a surface solid of ^{3}He has been observed for ^{3}He impregnated into silica aerogel. The signature is a Curie-like susceptibility, $\chi_s = C/(T-\Theta_s)$, with a Curie temperature, $\Theta_s \approx 0.4\,$mK.[21]

Thus, the model for scattering of quasiparticles by aerogel that we adopt is a modified version of the scattering model described above which includes an exchange coupling between ^{3}He quasiparticles in the liquid and localized ^{3}He atoms bound to the silica aerogel structure. This coupling is described by an exchange term in the quasiparticle-impurity potential,

$$u = u_0 + J\mathbf{S}\cdot\boldsymbol{\sigma}\,, \tag{14}$$

where J is the liquid-solid exchange coupling, $\mathbf{S}$ is the localized ^{3}He spin operator and $\boldsymbol{\sigma}$ is the Pauli spin operator for the ^{3}He quasiparticles. There are no direct measurements of J for ^{3}He on aerogel, and theoretical calculations for ^{3}He on planar substrates give indirect exchange interactions that vary over a wide range of values, $J_{\rm ind}\sim 0.1\,\mu{\rm K}-1.0\,{\rm mK}$, and may be either ferromagnetic or anti-ferromagnetic depending on the specific mechanism and details of the theoretical model (c.f. Ref. 26).

In a magnetic field, $\mathbf{B}=-B\hat{\mathbf{z}}$, the solid ^{3}He spins are polarized, $\mathbf{S}=S(T,B)\hat{\mathbf{z}}$, with $S(T,B)=\mathscr{P}(B,T)\mathsf{s}$, where $0\le\mathscr{P}\le 1$ is the fractional polarization and $\mathsf{s}=\frac{1}{2}$. For sufficiently low fields, and temperatures well above the ordering temperature for the solid ^{3}He spins, the polarization is expected to be linear in field with $\mathscr{P}(B,T)\approx|\mu|B/k_BT$. In this limit the A_1-A_2 splitting is given by Eqs. (12)-(13), but with $\bar{\lambda}^{\rm A1}\to\bar{\lambda}^{\rm A1}+\lambda_{\rm J}$, where $\lambda_{\rm J}$ represents the effect of the surface polarization and exchange coupling on the transition temperatures for $\uparrow\uparrow$ vs. $\downarrow\downarrow$ pairs. The polarization-induced splitting, $\lambda_{\rm J}\propto J$, depends on the sign of the exchange coupling. Thus, this term may either enhance or reduce the intrinsic splitting, $\bar{\lambda}^{\rm A1}$. In what follows we calculate the exchange splitting, $\lambda_{\rm J}$, and discuss the result in relation to the existing data for T_c.

To calculate the liquid-solid exchange contribution to the A_1-A_2 splitting we use the quasiclassical theory of superfluid ^{3}He,[27] with effects of scattering by the aerogel described by the HSM,[13] modified to include the exchange coupling in Eq. (14). The instability temperatures for $\uparrow\uparrow$ and $\downarrow\downarrow$ Cooper pairs are obtained by solving the weak-coupling gap equation for the spin-triplet components of the order parameter,

$$\frac{1}{3}\ln(T/T_{c0})\Delta(\hat{\mathbf{p}}) = T\sum_{\varepsilon_n}\int\frac{d\Omega_{\hat{\mathbf{p}}'}}{4\pi}(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')\times \left(\mathbf{f}(\hat{\mathbf{p}}';\varepsilon_n)-\pi\frac{\Delta(\hat{\mathbf{p}}')}{|\varepsilon_n|}\right), \tag{15}$$

where $\mathbf{f}(\hat{\mathbf{p}};\varepsilon_n)=(f_1,f_2,f_3)$ are the "pair" amplitudes for the three spin-triplet states: $f_{\uparrow\downarrow}=f_3$, $f_{\uparrow\uparrow}=(-f_1+if_2)$ and $f_{\downarrow\downarrow}=(f_1+if_2)$. The pairing interaction and density of states at the Fermi level, as well as the cutoff, have already been adsorbed into T_{c0}.

The scattering of quasiparticles off the aerogel structure is described by a random distribution of scattering centers ("impurities"). The impurity self-energy, to leading order in $\hbar/\tau E_f$, is determined by a t-matrix for multiple scattering by a single impurity and the mean density of impurities,

$$\hat{\Sigma}_{\rm imp}(\hat{\mathbf{p}};\varepsilon_n) = n_{\rm s}\,\hat{\mathsf{t}}(\hat{\mathbf{p}},\hat{\mathbf{p}};\varepsilon_n)\,. \tag{16}$$

The model for scattering of quasiparticles by aerogel that we adopt is described by an isotropic, non-magnetic scattering amplitude, u_0, and an exchange term in the quasiparticle-impurity scattering potential; in 4×4 Nambu representation $\hat{\mathsf{u}}=u_0\hat{1}+JS\hat{\Sigma}_z$, where $\hat{\Sigma}=(\hat{\mathsf{i}}+\hat{\tau}_3)\sigma/2+(\hat{\mathsf{i}}-\hat{\tau}_3)\sigma^{\rm tr}/2$ is the Nambu representation for the quasiparticle spin. For simplicity we also assume J to be isotropic. The t-matrix for repeated scattering of quasiparticles off a random distribution of these polarized scattering centers is

$$\hat{\mathsf{t}} = \hat{\mathsf{u}} + N_f\hat{\mathsf{u}}\langle\hat{\mathsf{g}}\rangle\hat{\mathsf{t}}, \tag{17}$$

where $\langle\hat{\mathsf{g}}\rangle$ is the Fermi-surface-averaged propagator. For normal ^{3}He in aerogel and even in the presence of magnetic fields and magnetic scattering, the propagator reduces to $\hat{\mathsf{g}}_{\rm N}=-i\pi{\rm sgn}(\varepsilon_n)\hat{\tau}_3$. Thus, the solution to the scattering t-matrix is given by

$$\hat{\mathsf{t}} = \frac{1}{\pi N_f}\left(\hat{\mathsf{i}}+is_\varepsilon\hat{\mathsf{u}}\hat{\tau}_3\right)^{-1}\hat{\mathsf{u}}\,, \tag{18}$$

where $s_\varepsilon={\rm sgn}(\varepsilon_n)$, and the dimensionless scattering potential is $\hat{\mathsf{u}}=\mathsf{u}\hat{\mathsf{i}}+\mathsf{v}\hat{\Sigma}_z$, with $\mathsf{u}=\pi N_f u_0$, $\mathsf{v}=\pi N_f JS$.

For non-magnetic scattering ($S=0$) the t-matrix is parameterized by the s-wave scattering phase shift, $\delta_0=\tan^{-1}(\mathsf{u})$,

$$\hat{\mathsf{t}} = \frac{1}{\pi N_f}\sin\delta_0\, e^{-is_\varepsilon\delta_0\hat{\tau}_3}\,. \tag{19}$$

In this minimal model for aerogel scattering, the mean density of impurities and scattering rate for normal quasiparticles are fixed by the mean free path, ℓ, and scattering cross-section, σ,

$$n_{\rm s} = \frac{1}{\sigma\ell}\,,\quad\text{with}\quad \sigma=\frac{4\pi\hbar^2}{p_f^2}\,\bar{\sigma}_0\,, \tag{20}$$

where the normalized cross-section is related to the scattering potential by,

$$\bar{\sigma}_0 = \frac{\mathsf{u}^2}{1+\mathsf{u}^2}\,. \tag{21}$$

Note that $\bar{\sigma}_0\to0$ is the Born scattering limit, while $\bar{\sigma}_0\to1$ is the unitary limit.

When $S\neq0$ there are different phase shifts for the scattering of $\uparrow(+)$ and $\downarrow(-)$ spin quasiparticles, which we parameterize as

$$\delta^\pm = \delta_0\pm\Delta\delta\,. \tag{22}$$

The t-matrix can now be expressed as,

$$\begin{aligned}\hat{\mathsf{t}} &= \tfrac{1}{\pi N_f}\left\{\sin\delta_0\cos(\Delta\delta)\hat{\mathsf{i}}+\cos\delta_0\sin(\Delta\delta)\hat{\Sigma}_z\right\}\\ &\times\ e^{-is_\varepsilon\delta_0\hat{\tau}_3}\,e^{-is_\varepsilon\Delta\delta\hat{\Sigma}_z\hat{\tau}_3}\,.\end{aligned} \tag{23}$$

The quasiparticle-impurity scattering rates for $\uparrow$ and $\downarrow$ quasiparticles are calculated from the retarded self-energy, $\hat{\Sigma}^{\rm R}_{\rm imp}=n_s\hat{\mathsf{t}}^{\rm R}$, obtained from Eq. (23) by setting $s_\varepsilon=+$. Thus, for quasi-particles the self-energy for spin $\sigma_z=\uparrow$ and $\sigma_z=\downarrow$ becomes,

$$\Sigma^{\rm R}_{\uparrow,\downarrow} = \Gamma_{\rm N}\sin\delta_{\uparrow,\downarrow}\left(\cos\delta_{\uparrow,\downarrow}-i\sin\delta_{\uparrow,\downarrow}\right), \tag{24}$$

where $\Gamma_{\rm N} = n_s/\pi N_f$. The scattering rates for $\uparrow$ and $\downarrow$ spin quasiparticles are then,

$$\frac{1}{2\tau_\pm} = \Gamma_{\rm N} \sin^2\delta^\pm = \Gamma_{\rm N}\bar{\sigma}_\pm = \Gamma_{\rm N}\frac{({\rm u}\pm{\rm v})^2}{1+({\rm u}\pm{\rm v})^2}\,, \tag{25}$$

where $\bar{\sigma}_\pm$ is the dimensionless cross-section for scattering of $\uparrow$ vs. $\downarrow$ spin quasiparticles. In both the unitary ($\delta_0 \to \pi/2$) and the Born ($\delta_0 \to 0$) limits, the $\uparrow$ and $\downarrow$ spin scattering rates are equivalent,

$$\frac{1}{2\tau_\pm} \to \begin{cases} \Gamma_{\rm N}\cos^2(\Delta\delta)\,, & \delta_0 = \pi/2 \\ \Gamma_{\rm N}\sin^2(\Delta\delta)\,, & \delta_0 = 0\,. \end{cases} \tag{26}$$

Only when $\delta_0 \neq 0,\pi$ is the scattering rate for $\uparrow$ and $\downarrow$ spin quasiparticles different. In general, we can parameterize the scattering rates as

$$\frac{1}{\tau^\pm} = \frac{1}{\bar{\tau}} \pm \frac{1}{\tau_{\rm s}}\,, \tag{27}$$

or equivalently,

$$\frac{1}{\tau_{\rm s}} = \frac{1}{\bar{\tau}}\left(\frac{\bar{\sigma}_+ - \bar{\sigma}_-}{\bar{\sigma}_+ + \bar{\sigma}_-}\right), \tag{28}$$

where $1/\bar{\tau}$ is the polarization-independent scattering rate. It is convenient to express the normal-state self-energy in terms of base particle-hole matrices,

$$\hat{\Sigma}_{\rm N} = \Sigma_{11}\hat{1} + \Sigma_{13}\hat{\Sigma}_z + \Sigma_{31}\hat{\tau}_3 + \Sigma_{33}\hat{\tau}_3\hat{\Sigma}_z\,, \tag{29}$$

with components

$$\Sigma_{11} = +\frac{1}{2}\Gamma_{\rm N}\sin(2\delta_0)\cos(\Delta\delta)\,, \tag{30}$$

$$\Sigma_{13} = +\frac{1}{2}\Gamma_{\rm N}\cos(2\delta_0)\sin(\Delta\delta)\,, \tag{31}$$

$$\Sigma_{31} = -\frac{i}{2}\Gamma_{\rm N}\, s_\varepsilon[1-\cos(2\delta_0)\cos(2\Delta\delta)]\,, \tag{32}$$

$$\Sigma_{33} = -\frac{i}{2}\Gamma_{\rm N}\, s_\varepsilon \sin(2\delta_0)\sin(2\Delta\delta)\,. \tag{33}$$

To calculate the instability temperatures for $\uparrow\uparrow$ and $\downarrow\downarrow$ pairs we need the off-diagonal propagator to linear order in the pairing self-energy. Thus, we expand the transport equation, self-energies and normalization condition in powers of $\hat{\Delta}$. The zeroth-order terms are the normal-state propagator and self-energy (Eq. (29)). To first-order we obtain,[27]

$$\left[i\varepsilon_n\hat{\tau}_3 - \hat{\Sigma}_{\rm N}\,,\hat{\mathfrak{g}}^{(1)}\right] = [\hat{\Delta}\,,\hat{\mathfrak{g}}_{\rm N}]\,, \tag{34}$$

and $\hat{\tau}_3\hat{\mathfrak{g}}^{(1)} + \hat{\mathfrak{g}}^{(1)}\hat{\tau}_3 = 0$ from the normalization condition. We reduce the equations to 2×2 spin-space by writing $\hat{\mathsf{H}}_{\rm N} = i\varepsilon_n\hat{\tau}_3 - \hat{\Sigma}_{\rm N} = \frac{1}{2}(\hat{1}+\hat{\tau}_3)\mathsf{H}_{\rm N} + \frac{1}{2}(\hat{1}-\hat{\tau}_3)\bar{\mathsf{H}}_{\rm N}$ with $\mathsf{H}_{\rm N} = i\varepsilon_n - \Sigma_{\rm N}$ and $\bar{\mathsf{H}}_{\rm N} = -i\varepsilon_n - \bar{\Sigma}_{\rm N}$. Note that $\hat{\mathfrak{g}}^{(1)}$ is purely off-diagonal, with the upper-right pair amplitude satisfying the equation in spin-space,

$$\mathsf{H}_{\rm N} f^{(1)} - f^{(1)}\bar{\mathsf{H}}_{\rm N} = 2i\pi\,{\rm sgn}(\varepsilon_n)\,\Delta\,. \tag{35}$$

Projecting out the spin-triplet components, we obtain, $(i\varepsilon_n - \Sigma_\pm)\,f_\pm = i\pi\,{\rm sgn}(\varepsilon_n)\,\Delta_\pm$ with $\Sigma_\pm = \Sigma_{31} \pm \Sigma_{33} = -i{\rm sgn}(\varepsilon_n)/2\tau_\pm$, and for $f_{\uparrow\uparrow(\downarrow\downarrow)} \equiv f_{+(-)} = \mp f_1 + if_2$,

$$f_\pm = \frac{\pi\Delta_\pm}{|\varepsilon_n| + 1/2\tau_\pm}\,. \tag{36}$$

The linearized gap equations for $\Delta_\pm$ are given by Eq. (15) with $\mathbf{f} \to f^\pm$, $\Delta \to \Delta^\pm$ and $T \to T_c^\pm$. For non-unitary, axial states,

$$\Delta_\pm(\hat{\mathbf{p}}) = {\rm d}_\pm\,(\hat{p}_x + i\hat{p}_y)/\sqrt{2}\,, \tag{37}$$

the eigenvalue equation for ${\rm d}_\pm$ yields the weak-coupling equation for the instability temperatures, $T_c^\pm$. In the absence of the polarization, the aerogel transition temperature is given by

$$\ln(T_c/T_{c0}) = 2\,S_1(x_c)\,, \tag{38}$$

where $x_c = 1/2\pi\tau T_c$, and the spin-independent rate for quasiparticles scattering off the aerogel is given by

$$\frac{1}{2\tau} = \Gamma_{\rm N}\sin^2\delta_0 \equiv \Gamma_{\rm N}\,\bar{\sigma}_0\,. \tag{39}$$

In the presence of a liquid-solid exchange coupling, and polarization of the solid ^{3}He, the instability temperatures for the $\uparrow\uparrow$ and $\downarrow\downarrow$ condensates are given by

$$\ln\left(T_c^\pm/T_{c0}\right) = 2\,S_1(x^\pm)\,, \tag{40}$$

where $x^\pm \equiv 1/2\pi\tau_\pm T_c^\pm$. For $u_0 \neq 0$, the leading order polarization correction to the scattering cross-sections gives

$$\frac{1}{\bar{\tau}} = \frac{1}{\tau}\,, \quad \frac{1}{2\tau_{\rm s}} = 2n_s JS(T,B)\sqrt{\bar{\sigma}_0}(1-\bar{\sigma}_0)^{3/2}\,. \tag{41}$$

In the low-field region, and above the magnetic ordering temperature, $\mathscr{P} = |\mu|B/k_BT_c$, and we obtain,

$$\lambda_{\rm J} = g_{\rm J}\left(\frac{|\mu|}{k_B}\right)\left(\frac{(1-\bar{\sigma}_0)^{3/2}}{\sqrt{\bar{\sigma}_0}}\right)\left(\frac{-2x_c\,S_2(x_c)}{1-2x_c\,S_2(x_c)}\right), \tag{42}$$

where $x_c = (\xi_0/\ell)(T_{c0}/T_c)$, and the dimensionless exchange coupling is

$$g_{\rm J} = 2\pi N_f J\,{\sf s}\,. \tag{43}$$

Note the impurity-induced exchange splitting vanishes in the unitary limit.[31] Equation (42) is easily generalized to include aerogel correlations within the heuristic 'random void' model described above; the result for $\lambda_{\rm J}$ has the same form as Eq. (42), but with $-x_c\,S_2(x_c) \to \tilde{x}_c'\,S_2(\tilde{x}_c)$, where $\tilde{x}_c = x_c^2/(x_c+\zeta_a^2)$, $x_c = 1/2\pi\tau T_c$, and $\tilde{x}_c' \equiv T_c\,d\tilde{x}_c/dT_c$.

The effects of the liquid-solid exchange coupling, $g_{\rm J}$, and the polarization of the solid ^{3}He coating the aerogel strands on the A_1-A_2 splitting are shown in Fig. 2, and compared with measurements of the superfluid transition in 98% aerogel reported in Ref. 25; these authors found no evidence of an A_1-A_2 splitting for fields up to $B = 5\,$kG. The data for the superfluid transition of ^{3}He in 98% aerogel for fields up to $B = 5\,$kG are

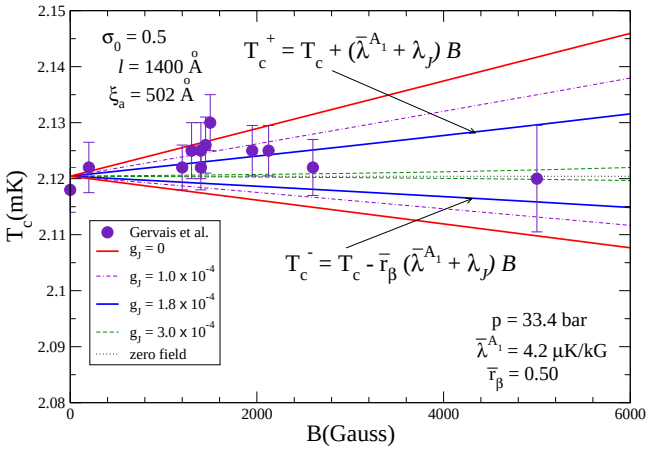

FIG. 2: The low-field (linear) splitting of T_c with magnetic field for ^{3}He in aerogel with a mean free path of $\ell = 140$nm, a correlation length of $\xi_a = 50$nm and a typical cross-section, $\bar{\sigma}_0 = 1/2$. The splitting for ^{3}He-aerogel without liquid-solid exchange is indicated by the solid (red) lines. The data points are taken from Ref. 25.

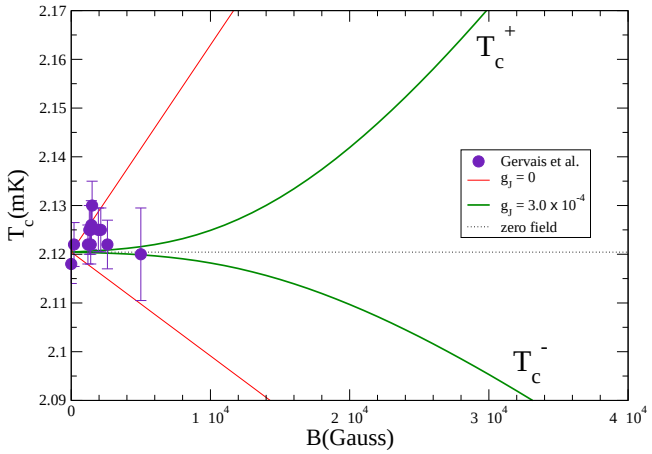

FIG. 3: The field-evolution of the splitting of T_c for ^{3}He in aerogel with the same parameters as those used in Fig. 2. The linear field splitting expected in the absence of polarized solid ^{3}He is indicated by the solid (red) lines. The nonlinear field evolution of the splitting is indicated by the thick (green) lines and corresponds to the value of $g_J = 3.0 \times 10^{-4}$. The Curie temperature is taken from Ref. 19, $\Theta_s \simeq 0.4$mK, and the exchange field is, $B_s = k_B\Theta_s/|\mu| \simeq 5.14$kG. The data points are from Ref. 25.

shown in Fig. 2. The error bars are conservative estimates of the uncertainty in defining T_c; the experiment shows no evidence of a splitting to within the error of determining T_c, and consequently we can assume that the splitting to be less than the error bars for T_c.[32]

The calculation of the A_1-A_2 splitting includes aerogel correlations, which are most important at high pressures. Both the mean-free path, ℓ, and the aerogel correlation length, ξ_a, contribute. The values of $\ell = 1400$Å and $\xi_a = 502$Å correspond to $T_c = 2.12$mK at $p = 33.4$bar and yield close agreement with $T_c(p)$ over the full pressure range. The dimensionless cross-section, $\bar{\sigma}_0$, is not known with any certainty; there is likely a distribution of cross-sections provided by the aerogel. In the absence of detailed knowledge we assume an average value of $\bar{\sigma}_0 = 1/2$. The values of $\bar{\lambda}^{A1}$ and $\bar{\lambda}^{A2}$ for pure ^{3}He, and thus the strong-coupling parameter, r_β, are taken from Ref. 6. The effects of non-magnetic scattering by aerogel lead to small corrections for $\bar{\lambda}^{A1}$ and $\bar{\lambda}^{A2}$; these terms alone (shown in Fig. 2 as $g_J = 0$) generate an A_1-A_2 splitting that is substantially larger ($\approx \times 2$) than the error reported for the superfluid ^{3}He transition in Ref. 5. An anti-ferromagnetic exchange coupling ($g_J > 0$) decreases the A_1-A_2 splitting. The magnitude of the predicted splitting is reduced to lie within the error bars for T_c for $g_J = 1.8 \times 10^{-4}$, which corresponds to an exchange coupling of $J \simeq 0.1$mK per liquid ^{3}He spin.

The existing data, while suggestive that the liquid-solid layer coupling may be playing an important role in suppressing the A_1-A_2 splitting, is not conclusive. If scattering by polarized ^{3}He is responsible for the suppressed A_1-A_2 splitting for pure ^{3}He in aerogel, then heat capacity or acoustic attenuation measurements with ^{4}He added to displace the solid ^{3}He, should exhibit an A_1-A_2 splitting that is comparable to that of pure bulk ^{3}He. Measurements of Sprague et al.[21] at $p = 18.7$bar and at $B = 1.47$kG do show and increase in T_c from 1.69mK without ^{4}He coverage, to 1.76mK with the addition of one monolayer of ^{4}He to remove the solid ^{3}He; thus, $\Delta T_c \simeq 70\,\mu$K. By comparison, if we suppress the polarization component of the scattering rate in our theoretical calculation we obtain and increase in T_c from the conventional component of the A_1-A_2 splitting of $\Delta T_c^{A1-A2} = 3.1\,\mu$K/kG$B \simeq 4.6\mu$K, which is more than an order of magnitude smaller than the change in T_c observed by adding ^{4}He. Thus, the addition of ^{4}He also modifies the non-magnetic contribution to the pair-breaking, and this effect is dominant at these low fields.

Measurements on pure ^{3}He in aerogel at higher fields should not suffer from this problem and should be able to resolve some or all of the uncertainty in the mechanism suppressing the A_1-A_2 splitting at low fields. In particular, if an exchange coupling, $J \approx 0.1 - 0.2$mK, is responsible for the suppressed A_1-A_2 splitting at $B \leq 5$kG, then for higher fields, $B \gg B_s = k_B\Theta_s/|\mu| \approx 5$kG the polarization of the solid ^{3}He should saturate, producing a field-independent shift from scattering off the polarized ^{3}He, and an A_1-A_2 splitting that increases with field, for $B \gg B_s$, at a rate comparable to that for pure ^{3}He.

The Curie temperature for the solid ^{3}He provides the temperature and field scale for the polarization, i.e. $\mathscr{P}(B/B_s, T/\Theta_s)$. In order to estimate the field-dependence of the A_1-A_2 splitting at higher fields we use the mean-field theory for the $s = 1/2$, near-neighbor Heisenberg ferromagnet to calculate the polarization.[28] The result is shown in Fig. 3 for the same parameters used to obtain the low-field suppression of the A_1-A_2 splitting shown in Fig. 2. Thus, even if fully suppressed at low fields, $B \lesssim B_s$, the A_1-A_2 splitting should emerge for fields above $B \approx 20$kG.

We thank Yuriy Bunkov, Henri Godfrin, and Bill Halperin for useful discussions, and acknowledge support from NSF grant DMR-9972087.

* Electronic address: sauls@northwestern.edu

[1] B. I. Barker, Y. Lee, L. Polukhina, D. D.Osheroff, L. W. Hrubesh, and J. F. Poco, Phys. Rev. Lett. **85**, 2148 (2000).

[2] P. Brussaard, S. N. Fisher, A. M. Guénault, A. J. Hale, N. Mulders, and G. R. Pickett, Phys. Rev. Lett. **86**, 4580 (2001).

[3] G. Gervais, T. M. Haard, R. Nomura, N. Mulders, and W. P. Halperin, Phys. Rev. Lett. **87**, 035701 (2001).

[4] K. Matsumoto, J. V. Porto, L. Pollack, E. N. Smith, T. L. Ho, and J. M. Parpia, Phys. Rev. Lett. **79**, 253 (1997).

[5] G. Gervais, K. Yawata, N. Mulders, and W. P. Halperin, Phys. Rev. Lett. **88**, 045505 (2002).

[6] U. E. Israelson, B. C. Crooker, H. M. Bozler, and C. M. Gould, Phys. Rev. Lett. **53**, 1943 (1984).

[7] D. C. Sagan, P. G. N. DeVegvar, E. Polturak, L. Friedman, S. S. Yan, E. L. Ziercher, and D. M. Lee, Phys. Rev. Lett. **53**, 1939 (1984).

[8] V. Ambegaokar and N. D. Mermin, Phys. Rev. Lett. **30**, 81 (1973).

[9] K. Levin and O. T. Valls, Phys. Rev. B **23**, 6154 (1981).

[10] K. S. Bedell and K. F. Quader, Phys. Rev. B **23**, 6154 (1981).

[11] N. D. Mermin and G. Stare, Phys. Rev. Lett. **30**, 1135 (1973).

[12] D. Rainer and J. A. Sauls, J. Low Temp. Phys. **110**, 2861 (1998) [http://xxx.lanl.gov/abs/cond-mat/9710001].

[13] E. V. Thuneberg, S.-K. Yip, M. Fogelström, and J. A. Sauls, Phys. Rev. Lett. **80**, 2861 (1998).

[14] E. V. Thuneberg, cond-mat/**9802044**, 18 (1998).

[15] G. Baramidze, G. Kharadze, and G. Vachnadze, Sov. Phys. JETP **63**, 107 (1996).

[16] G. Baramidze and G. Kharadze, cond-mat/ **0111161**, 10 (2001).

[17] P. Sharma and J. A. Sauls, J. Low Temp. Phys. **125**, 115 (2001).

[18] V. P. Mineev and P. L. Krotkov, Phys. Rev. B **65**, 024501:10pp (2002).

[19] D. Sprague, T. Haard, J. Kycia, M. Rand, Y. Lee, P. Hamot, and W. Halperin, Phys. Rev. Lett. **75**, 661 (1995).

[20] Y. M. Bunkov, A. S. Chen, D. J. Cousins, and H. Godfrin, Phys. Rev. Lett. **85**, 3456 (2000).

[21] D. Sprague, T. Haard, J. Kycia, M. Rand, Y. Lee, P. Hamot, and W. Halperin, Phys. Rev. Lett. **77**, 4568 (1996).

[22] G. Baramidze and G. Kharadze, Physica **284-288**, 305 (2000).

[23] D. Vollhardt and P. Wölfle, *The Superfluid Phases of* ^{3}*He* (Taylor & Francis, New York, 1990).

[24] D. Rainer and J. W. Serene, Phys. Rev. **B13**, 4745 (1976).

[25] G. Gervais, K. Yawata, N. Mulders, and W. P. Halperin, Phys. Rev. B **66**, 054528 (2002).

[26] H. Godfrin and R. E. Rapp, Advan. Phys. **44**, 113 (1995).

[27] J. W. Serene and D. Rainer, Phys. Rep. **101**, 221 (1983).

[28] N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Holt, Rinehart, and Winston, New York, 1975).

[29] The A_1 phase corresponds to pairs of ^{3}He quasiparticles with their magnetic moments aligned along the field. We follow the notation of Ref. 23 and define the quantization axis for the spin to be $\hat{\mathbf{z}}||-\mathbf{B}$ to compensate for the negative gyromagnetic ratio of ^{3}He.

[30] This is the result for $\bar{\sigma}_0 = 1/2$. There is a correction to the third term on the right-side of Eq. 11 for $\bar{\sigma}_0 \neq 1/2$.

[31] The splitting also vanishes in the Born limit; although Eq. (42) is not valid in the Born limit since it is based on a expansion of J/u_0. Nevertheless, in the limit $u_0 = 0$, i.e. with only pure exchange coupling the cross-sections for $\uparrow$ and $\downarrow$ spin scattering are equal and the splitting vanishes.

[32] W. P. Halperin, private communication.