Discovery of the acoustic Faraday effect in superfluid ^{3}He-B

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Acoustic waves provide a powerful tool for studying the structure of matter. For example, the speed, attenuation and dispersion of acoustic waves can give useful information on molecular forces and the microscopic mechanisms of absorption and scattering of acoustic energy. In solids, both compression and shear waves occur—longitudinal and transverse sound, respectively. But normal liquids do not support shear forces and consequently transverse waves do not propagate in liquids, with one notable exception. In 1957 Landau predicted[1] that the quantum-liquid phase of helium-3 might support transverse sound waves at sufficiently low temperatures, the restoring forces for shear waves being supplied by the collective quantum behaviour of the particles in the fluid. Such shear waves will involve displacements of the fluid transverse to the direction of propagation, and so define a polarization direction similar to that of electromagnetic waves. Here we confirm experimentally the existence of transverse sound waves in superfluid ^{3}He-B by observing the rotation of the polarization of these waves in the presence of a magnetic field. This phenomenon is the acoustic analogue of the magneto-optic Faraday effect, whereby the polarization direction of an electromagnetic wave is rotated by a magnetic field applied along the propagation direction.

Superfluidity in ^{3}He results from the binding of the ^{3}He particles with nuclear spin $s = 1/2$ into molecules called "Cooper pairs" with binding energy 2Δ (refs 2–5). The pairs undergo a type of Bose–Einstein condensation, which has a close analogy to the Bardeen–Cooper–Schrieffer condensation[6] phenomenon associated with superconductivity in metals. One important difference is that the pairs that form the condensate in ^{3}He have total spin $S = 1$ and an orbital wavefunction with relative angular momentum $L = 1$ ('*p*-wave'). This is in contrast to superconductors which are formed with Cooper pairs of electrons having $S = 0$ and $L = 0$ ('*s*-wave') or, as is the case of high-temperature superconductors, $S = 0$ and $L = 2$ ('*d*-wave'). In superfluid ^{3}He, the spin and orbital angular momentum vectors are locked at a fixed angle to one another. This is called broken relative spin–orbit symmetry[2,3]. The equilibrium superfluid state is described as a condensate of Cooper pairs with a total angular momentum $J = L + S = 0$. In addition, the Cooper pairs can be resonantly excited by sound waves to quantum states with total angular momentum $J = 2$ (ref. 7). This is reminiscent of diatomic molecules which have similar excited states.

The above description applies to the B-phase of superfluid ^{3}He, the most stable phase at low pressure. The acoustic Faraday effect occurs in ^{3}He-B as a consequence of spontaneously broken relative spin–orbit symmetry[8]. An applied field magnetically polarizes the spins of the Cooper pairs which, through coupling to their orbital motion, rotates the polarization of transverse sound. The rotational excitations of Cooper pairs are essential[8] to our observation of the propagation of transverse acoustic waves in ^{3}He-B because they significantly increase the sound velocity making the sound mode much easier to detect; the closer the sound energy is to the energy of the Cooper-pair excited state, the stronger is this effect. Furthermore, the Cooper-pair excited states have a linear Zeeman splitting with magnetic field[9,10]. Of the five $(2J + 1)$ Zeeman substates there is one, $m_J = +1$, which couples to right circularly polarized transverse sound, and a second, $m_J = -1$, couples to left circularly polarized sound. Thus the speeds of these two transverse waves are different in a magnetic field. We call this property acoustic birefringence. It leads to the acoustic Faraday effect where the magnetic field rotates the polarization direction of linearly polarized sound. Our measurements show that the rotation angle can be as large as 1.4×10^7 degrees cm^{-1} T^{-1}, much larger than the usual magneto-optical Faraday effect[11].

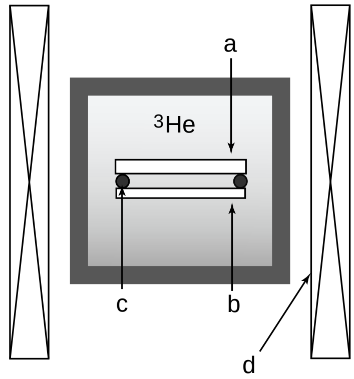

Figure 1 Acoustic cavity for transverse sound. Our short-path-length acoustic cavity consists of two quartz transducers 6.3 mm in diameter, labelled a and b. Their ground planes are face-to-face, and the spacing between the transducers is established by two parallel gold-coated tungsten wires (c). One of the cavity walls is a 12-MHz, AC-cut transducer (b) used for transverse sound excitation and detection. The other face of the cavity is a 17-MHz, X-cut transducer (a) for excitation and detection of longitudinal sound, which we used to determine the cavity length of 31 μm. The acoustic cavity is immersed in liquid ^{3}He at a pressure near 4.5 bar. The experimental cell is cooled by a copper nuclear demagnetization refrigerator (not shown). A superconducting solenoid (d) is placed outside the sample liquid enclosed within a superconducting shield. Temperatures were determined by SQUID-based paramagnetic salt thermometry.

Excitation and detection of transverse sound is provided by a high-Q (~3,000), AC-cut, quartz transducer with a fundamental resonance frequency of 12 MHz. It generates and detects shear waves with a specific linear polarization. The detection method is based on measurement of the electrical impedance of the transducer using a frequency-modulated spectrometer[12]. All measurements were performed at 82.26 MHz, the seventh harmonic of the transducer, with frequency modulation at 400 Hz and a modulation amplitude of 3 kHz. The electrical impedance of the transducer is a direct measure of the acoustic impedance of the surrounding liquid ^{3}He in the acoustic cavity that is shown in Fig. 1. Linearly polarized waves are excited by the transducer, reflected from the opposite surface of the acoustic cell, and detected by the same transducer. Under conditions of high attenuation there is no reflected wave, and the acoustic response is determined by the bulk acoustic impedance, $Z_a = \rho\omega/q$ where ρ is the density of the liquid, ω is the sound frequency, $q = k + i\alpha$ is the complex wavenumber, α is the attenuation, and $2\pi/k$ is the wavelength. A change in either the attenuation or the phase velocity, $C_\phi = \omega/k$, produces a change in the impedance, Z_a.

On cooling into the highly attenuating superfluid region, the acoustic response varies smoothly with temperature (Fig. 2). If the attenuation is low, there is interference between the source and reflected waves which modulates the local acoustic impedance detected by the transducer. Consequently, the acoustic response oscillates as the phase velocity changes with temperature. The oscillations in Fig. 2 at low temperatures correspond to interference between outgoing and reflected waves, and so they indicate the existence of some form of propagating wave. Each period of the oscillations corresponds to a change in velocity sufficient to increase, or decrease, by unity the number of half wavelengths in the cavity. The amplitude of the oscillations increases as the temperature is reduced, indicating that attenuation of the sound mode decreases with decreasing temperature.

The features labelled A and B in Fig. 2 are identified with known physical processes for sound absorption in superfluid ^{3}He-B (refs 3, 13, 14). Feature A corresponds to onset of the dissociation of Cooper pairs by sound where $\hbar\omega = 2\Delta(T_A)$, where $\hbar\omega$ is the energy of the sound quantum and $\hbar$ is Planck's constant. In the temperature range between T_A and the superfluid transition temperature, T_C, the attenuation of the liquid is extremely high owing to this mechanism. The point B corresponds to resonant absorption of sound at $\hbar\omega = 1.5\Delta(T)$ by the excited Cooper pairs with angular momentum $J = 2$ (ref. 8). Transverse sound is extinguished below this temperature. Early attempts to observe transverse sound in the normal phase of ^{3}He were inconclusive[15–17], and furthermore, it was originally expected that the transverse mode would be suppressed in the superfluid phase[18,19]. More recent theoretical work[8] clarified the role of the Cooper-pair excitations, showing that they increase the transverse sound speed which results in a more robust propagating transverse acoustic wave at low temperatures in superfluid ^{3}He-B. The first experimental evidence for this can be found in the acoustic impedance measurements of ref. 20.

The proof that the impedance oscillations correspond to propagating transverse sound is given in Fig. 3. In Fig. 3a we show data sets obtained at a pressure of 4.42 bar in magnetic fields of 52 G, 101 G and 152 G. The principal feature is that the magnetic field modulates the zero-field oscillations shown in Fig. 2. Our detector is only sensitive to linearly polarized transverse sound having a specific direction. Application of a field of 52 G in the direction of wave propagation suppresses the oscillations near $T = 0.465\ T_C$ that were present in zero field. This corresponds to a 90° rotation of the polarization of the first reflected transverse sound wave making the polarization orthogonal to the detection direction. Doubling the magnetic field restores the transverse sound oscillations at this temperature. The oscillations are suppressed once again by tripling the field to 152 G. We also note that near the points labelled 90° and 270° in Fig. 3, there are smaller-amplitude impedance oscillations with shorter period than the primary oscillations. These come from interference of doubly reflected waves within the cavity. We demonstrate this fact with a simple, but powerful, simulation of the acoustic impedance oscillations (Fig. 3b; the results of the simulation are discussed later).

In zero field, superfluid ^{3}He-B is non-magnetic and non-birefringent. Linearly polarized transverse sound is the superposition of two circularly polarized waves having the same velocity and attenuation. Application of a magnetic field gives rise to acoustic circular birefringence through the Zeeman splitting of the excited states of the Cooper pairs that couple to the transverse sound modes: thus right- and left-circularly polarized waves propagate with different speeds, $C_\pm = C_\phi \pm \delta C_\phi$. For magnetic fields well below 1 kG, the difference in propagation speeds is linear in the magnetic field, $\delta C_\phi \propto H$. This implies that a linearly polarized wave generated by the transducer undergoes Faraday rotation of its polarization as it propagates. On reflection from the opposite wall of the cavity, the linearly polarized wave with $\mathbf{q}\|\mathbf{H}$ reverses direction. The reflected wave propagates with the polarization rotating with the same handedness relative to the direction of the field; that is, the rotation of the polarization accumulates after reflection from a surface. The spatial period for rotation of the polarization by 360° is given by:

$$\Lambda = 4\pi\left(\frac{C_\phi}{\omega}\right)\left|\frac{C_\phi}{\delta C_\phi}\right| \quad (1)$$

The Faraday effect produces a sinusoidal modulation of the impedance oscillations as a function of magnetic field with a period that is inversely proportional to the field: that is, $\Lambda \propto 1/H$. The constant of proportionality in magneto-optics is called the Verdet constant, $V = 2\pi/H\Lambda$.

In Fig. 3b we show the result of our numerical calculation of the sound wave amplitude in the direction detected by the transducer. The oscillations shown in the figure come from interference between the source wave and multiply reflected waves. The calculation uses the attenuation and phase velocity measured in zero field.

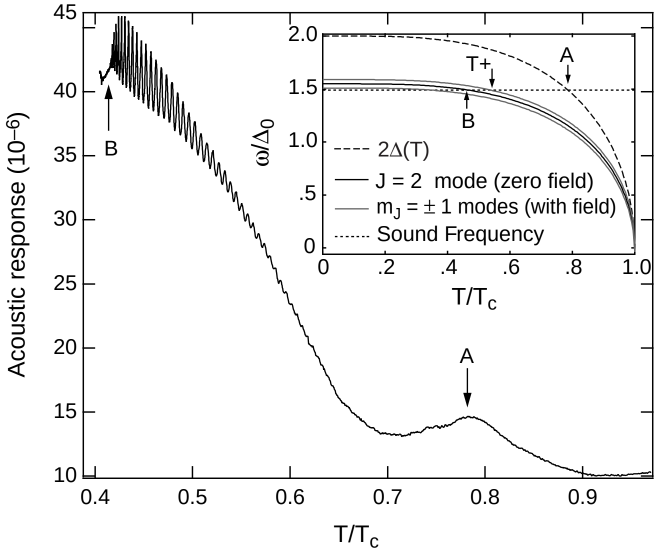

Figure 2 Temperature dependence of the acoustic cavity response. Measurements of the transverse acoustic impedance are shown at a pressure of 4.31 bar in zero magnetic field. The feature labelled A corresponds to acoustic pair-breaking, $\hbar\omega = 2\Delta(T_A)$. Impedance oscillations develop for temperatures $T < 0.6T_C$, and grow in amplitude at lower temperatures as the attenuation of transverse waves decreases. Near $T \approx T_B = 0.41T_C$ the propagating mode is extinguished by absorption of transverse sound via resonant excitation of Cooper pairs with total angular momentum $J = 2$ at $\hbar\omega = 1.5\Delta(T_B)$. The pair-breaking and resonance conditions of $J = 2$ Cooper-pair excitations for fixed frequency are shown in the inset; Δ_0 is the energy gap at zero temperature.

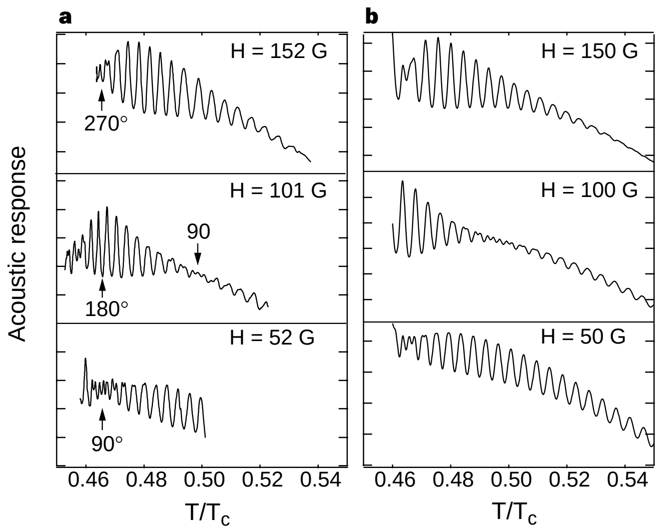

Figure 3 Magnetic field dependence of the acoustic cavity response. **a**, The sound amplitude measured at 4.42 bar. The angles are indicated for rotation of the polarization of transverse sound by 90°, 180° and 270°. **b**, Simulation of the acoustic impedance for the same path length cell and same temperature, pressure and magnetic fields. The wavelength and attenuation of transverse sound in zero field were used as the input data for the calculation. The Verdet constant for the simulation was determined by the position of the minimum in the impedance oscillations at $H = 52$ G shown in **a**.

The Verdet constant is obtained from the measurement at 52 G. The simulation reproduces all the observed features of the impedance as a function of temperature, including the maximum in the modulation at $T/T_C = 0.415$, $H = 101$ G, and the minimum at $T/T_C = 0.415$, $H = 152$ G, which confirms that the Faraday period is proportional to $1/H$. The simulation also produces the fine-structure oscillations in the impedance near the points labelled 90° and 270°. The fine structure is observed when the polarization rotates by an odd multiple of 90° upon a single round trip in the cell. Then waves that traverse the cell twice are 180° out of phase relative to the source wave, and consequently the period of the impedance oscillations is halved. The amplitude of the oscillations is substantially reduced because of attenuation over the longer path-length. This structure provides proof that impedance oscillations are modulated by the Faraday effect for propagating transverse waves. The impedance data from our experiments were analysed to obtain the spatial period for rotation of the polarization, and were found to be in agreement with the theoretical prediction[8] for the Faraday rotation period. The theoretical results for the period can be expressed in the form

$$\Lambda = K\frac{\sqrt{T/T_{+} - 1}}{gH} \tag{2}$$

for fields $H \ll 1$ kG and temperatures above and near the extinction point B. The temperature T_+ corresponds to the extinction of transverse sound by resonant excitation of Cooper pairs with $J = 2$, $m_J = +1$, at a slightly higher temperature than the B extinction point in zero field as shown in Fig. 2 (for example, at $H = 100$ G, $T_+ - T_B \approx 1\ \mu$K). The magnitude of the Faraday rotation period depends on accurately known superfluid properties, contained in the parameter K; it also depends on one parameter that is not well-established, the Landé g-factor, g, for the Zeeman splitting of the Cooper-pair excited state with $J = 2$.

Movshovich *et al.*[21] analysed the splitting of the $J = 2$ multiplet in the absorption spectrum of longitudinal sound, finding a value of $g = 0.042$. In that experiment it was not possible to resolve the splitting except for fields above 2 kG. At these high fields, the nonlinear field dependence due to the Paschen–Back effect[22,23] becomes comparable to the linear Zeeman splitting[24,25], which makes it difficult to determine the Landé g-factor accurately. We have analysed our measurements of the acoustic Faraday effect to determine the g-factor with high accuracy at low fields, which eliminates the complication of the high-field Paschen–Back effect. We find $g = 0.020 \pm 0.002$. We interpret our significantly smaller value of the Landé g-factor as meaning that there are important $L = 3$ ('*f*-wave') pairing correlations in the superfluid condensate, about 7% of the dominant *p*-wave interactions[26]. □

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Similarities between the growth dynamics of university research and of competitive economic activities

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Quantifying the dynamics of research activities is of considerable current interest, not least because of recent changes in research and development (R&D) funding[1–9]. Here we quantify and analyse university research activities, and compare their growth dynamics with those of business firms[10–14]. Our study involves the analysis of five distinct databases, the largest of which is a National Science Foundation database of the R&D expenditures in science and engineering for a 17-year period (1979–95) in 719 United States universities. We find that the distribution of growth rates displays a 'universal' form that does not depend on the size of the university or on the measure of size used; and the width of this distribution decays with size as a power law. These findings are quantitatively similar to those of business firms[10–14], and so are consistent with the hypothesis that the growth dynamics of complex organizations are governed by universal mechanisms. One possible explanation for these similarities is that the combination of peer review and government direction leads to an outcome similar to that induced by market forces (where the analogues of peer review and government direction are, respectively, consumer evaluation and product regulation).

In the study of physical systems, the scaling properties of fluctuations in the output of a system often yield information regarding the underlying processes responsible for the observed macroscopic behaviour[15,16]. Here we analyse the fluctuations in the growth rates of university research activities, using five different measures of research activity. The first measure of the size of a university's research activities that we consider is R&D expenditure.