International Conference on Ultra-Low Temperature Physics, Heidelberg, Germany

Spontaneous Symmetry Breaking & Topology of the Superfluid Phases of ^{3}He

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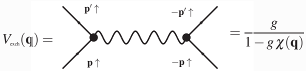
August 18, 2017

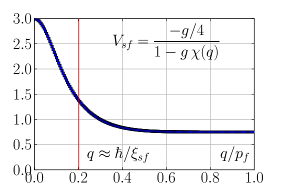
- Symmetry & Broken Symmetry of ^{3}He
- Dynamical Consequences: Bosonic Spectrum
- Topology of the Bulk Phases
- Signatures: Weyl & Majorana Fermions

• Research supported by US National Science Foundation Grant DMR-1508730

Spin-Fluctuation Mediated Pairing ⇝ Odd-Parity, Spin-Triplet Pairing for ^{3}He

- A. Layzer and D. Fay, Int. J. Magn. 1, 135 (1971)





$$-g_l = (2l+1)\int \frac{d\Omega_{\hat{p}}}{4\pi} \int \frac{d\Omega_{\hat{p}'}}{4\pi} V_{\text{sf}}(\mathbf{p},\mathbf{p}')\,P_l(\hat{p}\cdot\hat{p}')$$

- $-g_l$ is a function of $g \approx 0.75$ & $\xi_{\text{sf}} \approx 5\hbar/p_f$
- $l = 1$ (p-wave) is dominant pairing channel
  - $\hat{p}_x + i\hat{p}_y \sim \sin\theta_{\hat{p}}\; e^{+i\phi_{\hat{p}}} \rightsquigarrow l_z = +1$
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- $S = 1$, $S_z = 0$, ± 1 pairing fluctuations

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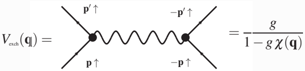

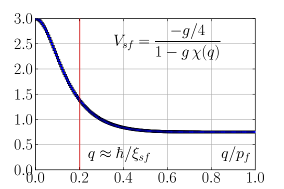

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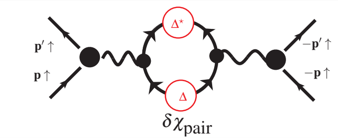
- Feedback on V_{sf} ⇝ Multiple Stable Superfluid Phases



$\chi_A \approx \chi_N > \chi_B \rightsquigarrow \frac{1}{3}\chi_N \rightsquigarrow$ Superfluid A-phase

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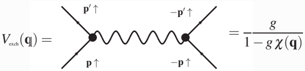

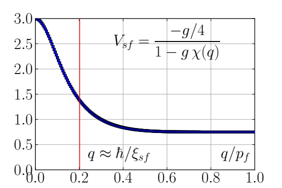

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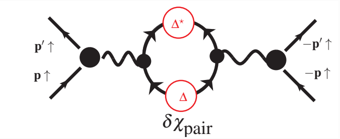

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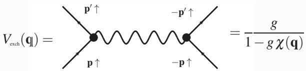

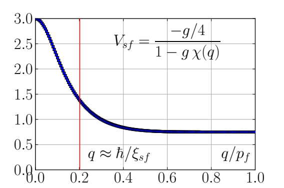

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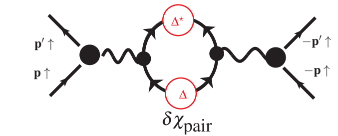

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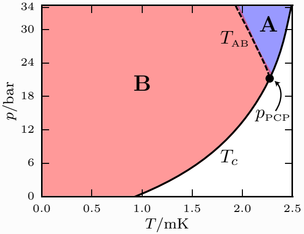
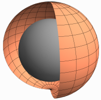
Poster Fri-038, Joshua Wiman

Maximal Symmetry $\mathtt{G} = \mathtt{SO(3)}_\mathsf{S} \times \mathtt{SO(3)}_\mathsf{L} \times \mathtt{U(1)}_\mathsf{N} \times \mathtt{P} \times \mathtt{T} \times \mathtt{C} \rightarrow$ Superfluid Phases of ^{3}He

J. Wiman & J. A. Sauls, PRB 92, 144515 (2015)



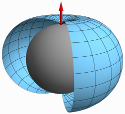
"Isotropic" BW State



$J = 0,\ J_z = 0$

$H = \mathtt{SO(3)}_\mathsf{J} \times \mathtt{T}$

Chiral AM State $\vec{l} = \hat{\mathbf{z}}$



$L_z = 1,\ S_{z'} = 0$

$H = \mathtt{U(1)}_\mathsf{S} \times \mathtt{U(1)}_{\mathsf{L}_z\text{-}\mathsf{N}} \times \mathtt{Z}_2$

Spin-Triplet Condensate Amplitudes: $\widehat{\Psi} = \begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix} \leftarrow \Psi_{\alpha\beta}(p) = \langle \psi_\alpha(p)\psi_\beta(-p)\rangle$

$$\widehat{\Psi}_{BW} = \Delta \begin{pmatrix} p_x - ip_y \sim e^{-i\phi} & p_z \\ p_z & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

$$\widehat{\Psi}_{AM} = \Delta \begin{pmatrix} p_x + ip_y \sim e^{+i\phi} & 0 \\ 0 & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

Fully Gapped: $\widehat{\Psi}^\dagger_{BW}\widehat{\Psi}_{BW} = |\Delta|^2$

Nodal Points: $\widehat{\Psi}^\dagger_{AM}\widehat{\Psi}_{AM} = |\Delta|^2 \sin^2\theta$

New Bosonic Excitations

New Bosonic Excitations

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN)

CERN-PH-EP/2012-220
2013/01/29

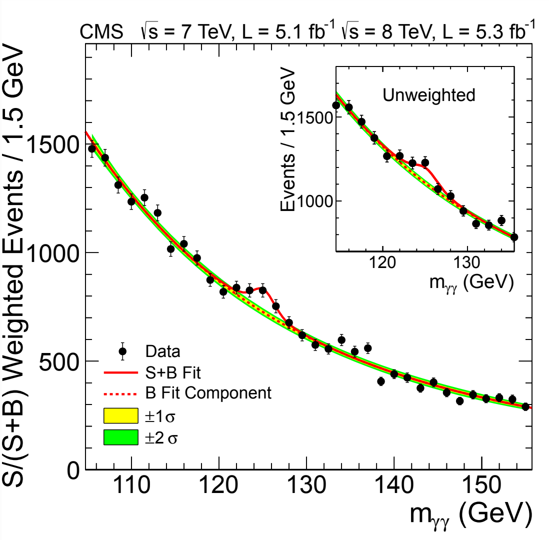
CMS-HIG-12-028

Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC

The CMS Collaboration

2013

Higgs Boson with mass $M = 125$ GeV

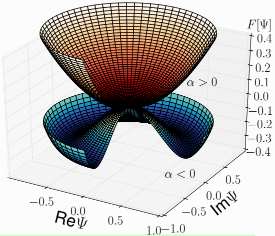


Dynamical Consequences of Spontaneous Symmetry Breaking

Scalar Higgs Boson (spin $J=0$) [P. Higgs, PRL 13, 508 1964]

Energy Functional for the Higgs Field

$$\mathscr{U}[\Delta] = \int dV \left\{ \alpha\, |\Delta|^2 + \beta\, |\Delta|^4 + \frac{1}{2}c^2\, |\nabla\Delta|^2 \right\}$$



- Broken Symmetry State: $\Delta = \sqrt{|\alpha|/2\beta}$

Space-Time Fluctuations about the Broken Symmetry Vacuum State

$\Delta(\mathbf{r},t) = \Delta + D(\mathbf{r},t)$ ▶ Eigenmodes: $D^{(\pm)} = D \pm D^*$ (Conjugation Parity)

$$\mathscr{L} = \int d^3r \left\{ \frac{1}{2}[(\dot{D}^{(+)})^2 + (\dot{D}^{(-)})^2] - 2\Delta^2\,(D^{(+)})^2 - \frac{1}{2}[c^2(\nabla D^{(+)})^2 + c^2(\nabla D^{(-)})^2] \right\}$$

- $\partial_t^2 D^{(-)} - c^2\nabla^2 D^{(-)} = 0$

  Massless Nambu-Goldstone Mode

- $\partial_t^2 D^{(+)} - c^2\nabla^2 D^{(+)} + 4\Delta^2\, D^{(+)} = 0$

  Massive Higgs Mode: $M = 2\Delta$

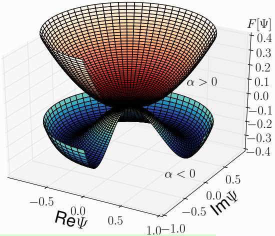
Dynamical Consequences of Spontaneous Symmetry Breaking

BCS Condensation of Spin-Singlet ($S = 0$), S-wave ($L = 0$) "Scalar" Cooper Pairs

Ginzburg-Landau Functional

$$F[\Delta] = \int dV \left\{ \alpha\, |\Delta|^2 + \beta\, |\Delta|^4 + \kappa\, |\nabla\Delta|^2 \right\}$$

- Order Parameter: $\Delta = \sqrt{|\alpha|/2\beta}$



Space-Time Fluctuations of the Condensate Order Parameter

$\Delta(\mathbf{r},t) = \Delta + D(\mathbf{r},t)$ ▶ Eigenmodes: $D^{(\pm)} = D \pm D^*$ (Fermion "Charge" Parity)

$$\mathscr{L} = \int d^3r \left\{ \frac{1}{2}[(\dot{D}^{(+)})^2 + (\dot{D}^{(-)})^2] - 2\Delta^2\,(D^{(+)})^2 - \frac{1}{2}[v^2(\nabla D^{(+)})^2 + v^2(\nabla D^{(-)})^2] \right\}$$

- $\partial_t^2 D^{(-)} - v^2\nabla^2 D^{(-)} = 0$

  Anderson-Bogoliubov Mode

- $\partial_t^2 D^{(+)} - v^2\nabla^2 D^{(+)} + 4\Delta^2\, D^{(+)} = 0$

  Amplitude Higgs Mode: $M = 2\Delta$

Ginzburg-Landau Functional for Superfluid ^{3}He

- Maximal Symmetry of ^{3}He: $\texttt{G} = \texttt{SO(3)}_\texttt{L} \times \texttt{SO(3)}_\texttt{S} \times \texttt{U(1)}_\texttt{N} \times \texttt{P} \times \texttt{T} \times \texttt{C}$

- Order Parameter for P-wave $(L = 1)$, Spin-Triplet $(S = 1)$ Pairing

$$\widehat{\Psi}(\hat{p}) = \overbrace{\begin{pmatrix} S_\mathbf{x} & S_\mathbf{y} & S_\mathbf{z} \end{pmatrix}}^{\text{Spin Basis}} \times \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix} \times \overbrace{\begin{pmatrix} \hat{p}_x \\ \hat{p}_y \\ \hat{p}_z \end{pmatrix}}^{\text{Orbital Basis}}$$

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- GL Functional: $A_{\alpha i}$ $\rightsquigarrow$ vector under both $\mathtt{SO(3)}_{\mathsf{S}}$ $[\alpha]$ and $\mathtt{SO(3)}_{\mathsf{L}}$ $[i]$

$$\begin{aligned} \mathscr{U}[A] &= \int d^3r \Big[\alpha(T) \mathrm{Tr}\left\{AA^\dagger\right\} + \beta_1 \left|\mathrm{Tr}\left\{AA^{\mathrm{tr}}\right\}\right|^2 + \beta_2 \left(\mathrm{Tr}\left\{AA^\dagger\right\}\right)^2 \\ &+ \beta_3 \mathrm{Tr}\left\{AA^{\mathrm{tr}}(AA^{\mathrm{tr}})^*\right\} + \beta_4 \mathrm{Tr}\left\{(AA^\dagger)^2\right\} + \beta_5 \mathrm{Tr}\left\{AA^\dagger(AA^\dagger)^*\right\} \\ &+ \kappa_1 \partial_i A_{\alpha j} \partial_i A^*_{\alpha j} + \kappa_2 \partial_i A_{\alpha i} \partial_j A^*_{\alpha j} + \kappa_3 \partial_i A_{\alpha j} \partial_j A^*_{\alpha i} \Big] \end{aligned}$$

- Mermin, Ambegaokar, Brinkman, Anderson, circa 1974

Lagrangian Field Theory for Bosonic Excitations of Superfluid ^{3}He-B

$$^3\text{He-B:}\quad B_{\alpha i} = \frac{1}{\sqrt{3}}\Delta\,\delta_{\alpha i} \quad L=1,\ S=1 \rightsquigarrow J=0 \quad C=+1$$

- Symmetry of ^{3}He-B: $\mathtt{H} = \mathtt{SO(3)}_{\mathtt{J}} \times \mathtt{T}$
- Fluctuations: $\mathscr{D}_{\alpha i}(\mathbf{r},t) = A_{\alpha i}(\mathbf{r},t) - B_{\alpha i} = \sum_{J,m} D_{J,m}(\mathbf{r},t)\, t^{(J,m)}_{\alpha i}$
- Lagrangian:

$$\mathscr{L} = \int d^3r \left\{ \tau\, \mathrm{Tr}\left\{\dot{\mathscr{D}}\dot{\mathscr{D}}^\dagger\right\} - \alpha\, \mathrm{Tr}\left\{\mathscr{D}\mathscr{D}^\dagger\right\} - \sum_{p=1}^{5} \beta_p u_p(\mathscr{D}) - \sum_{l=1}^{3} K_l v_l(\partial\mathscr{D}) \right\}$$

$$\partial_t^2 D^{(\mathtt{C})}_{J,m} + E^{(\mathtt{C})}_{J,m}(\mathbf{q})^2 D^{(\mathtt{C})}_{J,m} = \frac{1}{\tau}\eta^{(\mathtt{C})}_{J,m}$$

with $\quad J = \{0,1,2\}\,, m = -J \ldots +J\,, \mathtt{C} = \pm 1$

- *Time-Dependent GL Theory for Bosonic Excitations of Superfluid ^{3}He-B*: JAS & T. Mizushima, PRB 95, 094515 (2017)

Spectrum of Bosonic Modes of Superfluid ^{3}He-B : Condensate is $J^{\mathtt{C}} = 0^+$

▶ 4 Nambu-Goldstone Modes & 14 Higgs modes

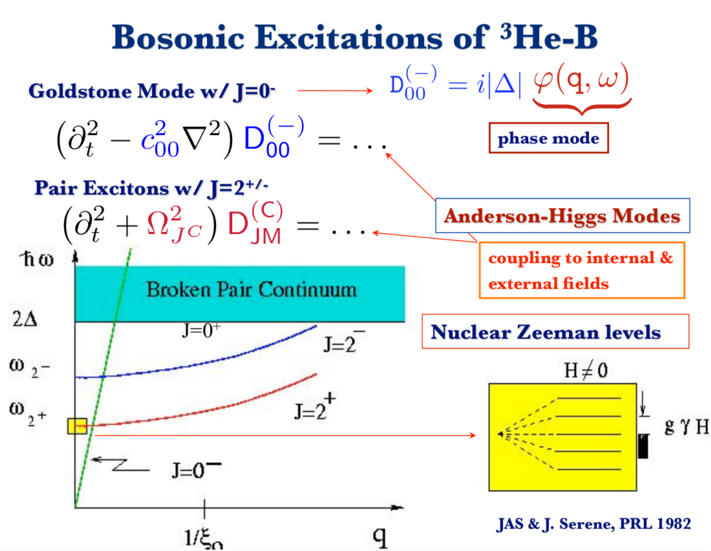
$$E^{(\mathtt{C})}_{J,m}(\mathbf{q}) = \sqrt{M^2_{J,\mathtt{C}} + \left(c^{(\mathtt{C})}_{J,|m|}|\mathbf{q}|\right)^2}$$

Mode	Symmetry	Mass	Name
$D^{(+)}_{0,m}$	$J=0,\ \mathtt{C}=+1$	2Δ	Amplitude Higgs
$D^{(-)}_{0,m}$	$J=0,\ \mathtt{C}=-1$	0	NG Phase Mode
$D^{(+)}_{1,m}$	$J=1,\ \mathtt{C}=+1$	0	NG Spin-Orbit Modes
$D^{(-)}_{1,m}$	$J=1,\ \mathtt{C}=-1$	2Δ	AH Spin-Orbit Modes
$D^{(+)}_{2,m}$	$J=2,\ \mathtt{C}=+1$	$\sqrt{\frac{8}{5}}\Delta$	2^+ AH Modes
$D^{(-)}_{2,m}$	$J=2,\ \mathtt{C}=-1$	$\sqrt{\frac{12}{5}}\Delta$	2^- AH Modes

▶ Vdovin, Maki, Wölfle, Serene, Nagai, Volovik, Schopohl, McKenzie, JAS ...

▶ *Broken Symmetry & Nonequilibrium Superfluid ^{3}He*, Les Houches Lectures, arXiv:cond-mat/9910260 (1999), J.A. Sauls

Collective Mode Spectrum for ^{3}He-B



- *Broken Symmetry & Nonequilibrium Superfluid ^{3}He*, Les Houches Lectures, arXiv:cond-mat/9910260 (1999), J.A. Sauls

First Observations of Higgs Bosons in a BCS Condensate - Superfluid ^{3}He-B

Observation of a New Sound-Attenuation Peak in Superfluid ^{3}He-*B*

R. W. Giannetta,[a] A. Ahonen,[b] E. Polturak, J. Saunders,
E. K. Zeise, R. C. Richardson, and D. M. Lee
Laboratory of Atomic and Solid State Physics and Materials Science Center, Cornell University, Ithaca, New York 14853
(Received 25 March 1980)

Results of zero-sound attenuation measurements in ^{3}He-*B*, at frequencies up to 60 MHz and pressures between 0 and 20 bars, are reported. At frequencies of 30 MHz and above, a new attenuation feature is observed which bears the signature of a collective mode of the superfluid.

VOLUME 45, NUMBER 4 PHYSICAL REVIEW LETTERS 28 JULY 1980

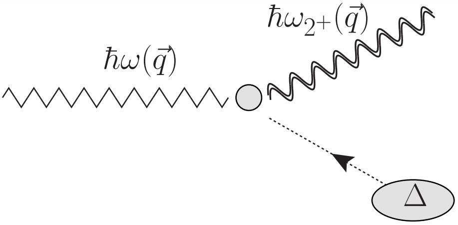
Measurements of High-Frequency Sound Propagation in ^{3}He-*B*

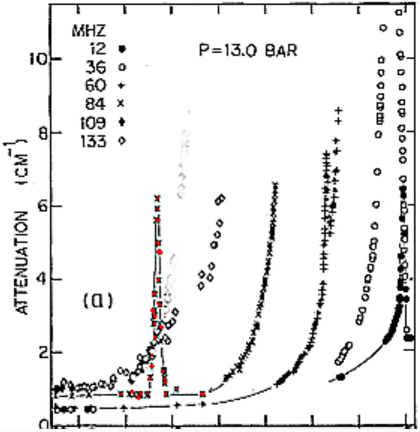
D. B. Mast, Bimal K. Sarma, J. R. Owers-Bradley, I. D. Calder,
J. B. Ketterson, and W. P. Halperin
Department of Physics and Astronomy and Materials Research Center, Northwestern University, Evanston, Illinois 60201
(Received 10 April 1980)

Measurements of the attenuation and velocity of pulsed high-frequency sound have been performed up to 133 MHz in superfluid ^{3}He-*B*. A new collective mode of the order parameter was discovered at a frequency extrapolated to T_c of $\omega = (1.165 \pm 0.05)\Delta_{BCS}(T_c)$, where $\Delta_{BCS}(T)$ is the energy gap in the weak-coupling BCS theory. The group velocity has been observed to decrease by as much as $\frac{2}{3}$ of the zero-sound velocity.

Excitation of the $J^C = 2^+$, $m_J = 0$ Higgs Mode by Phonon Absorption

Higgs Mode with mass: $M = 500$ neV and spin $J^C = 2^+$ at ULT-Northwestern





T/T_c

- D. Mast et al. Phys. Rev. Lett. 45, 266 (1980).

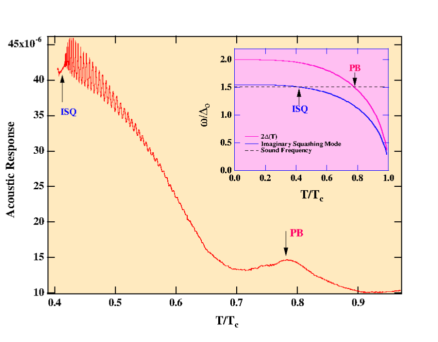
$J = 2^{-}$, $m = \pm 1$ Higgs Modes Transport Mass and Spin

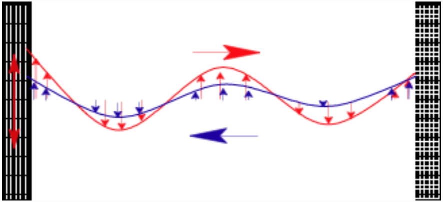
$J=2^-$, $m=\pm1$ Higgs Modes Transport Mass and Spin

- "Transverse Waves in Superfluid ^{3}He-B", G. Moores and JAS, JLTP 91, 13 (1993)

$$C_t(\omega)=\sqrt{\frac{F_1^s}{15}}v_f\left[\rho_n(\omega)+\frac{2}{5}\rho_s(\omega)\underbrace{\left\{\frac{\omega^2}{(\omega+i\Gamma)^2-\frac{12}{5}\Delta^2-\frac{2}{5}(q^2v_f^2)}\right\}}_{D^{(-)}_{2,\pm1}}\right]^{\frac{1}{2}}$$

Transverse Zero Sound Propagation in Superfluid ^{3}He-B: *Cavity Oscillations of TZS*

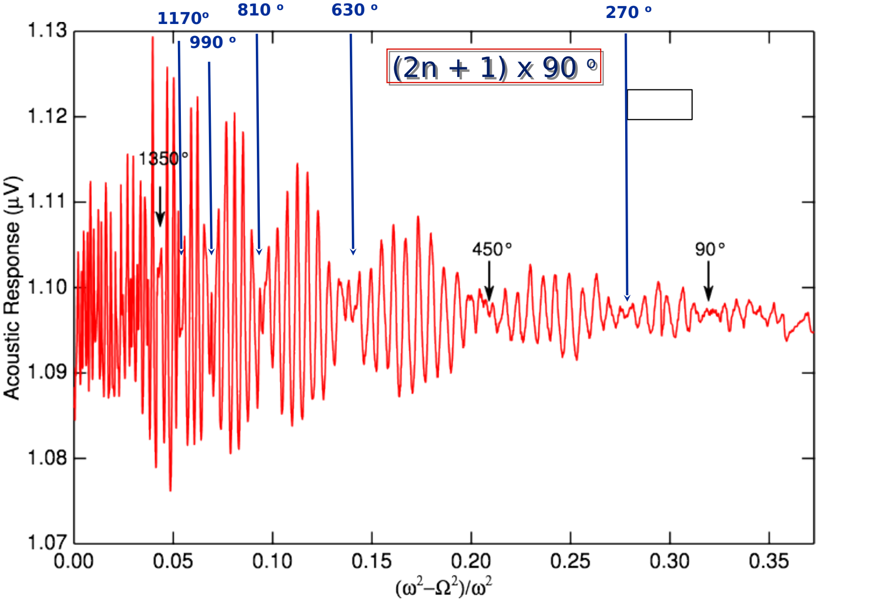




- Y. Lee et al. Nature 400 (1999)

B ⟶

Large Faraday Rotations vs. ``Blue Tuning'' B = 1097 G



C. Collett et al., Phys. Rev. B 87, 024502 (2013)

$J = 1^+$, $m = 0, \pm 1$ NG Modes $\rightsquigarrow$ Pseudo-NG Modes

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ARTICLE

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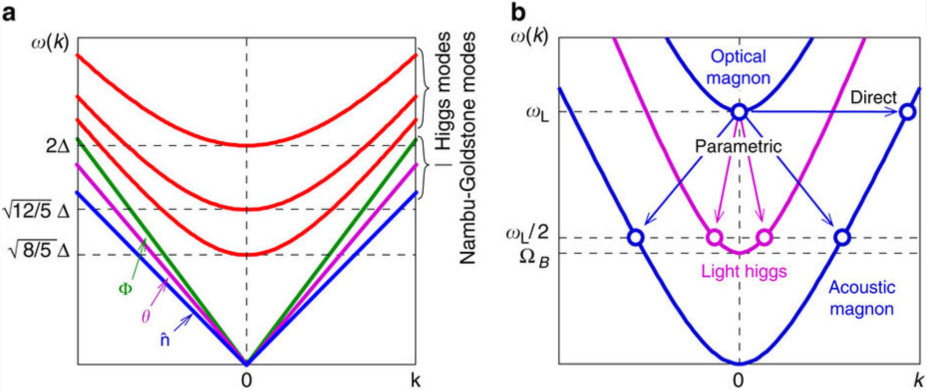
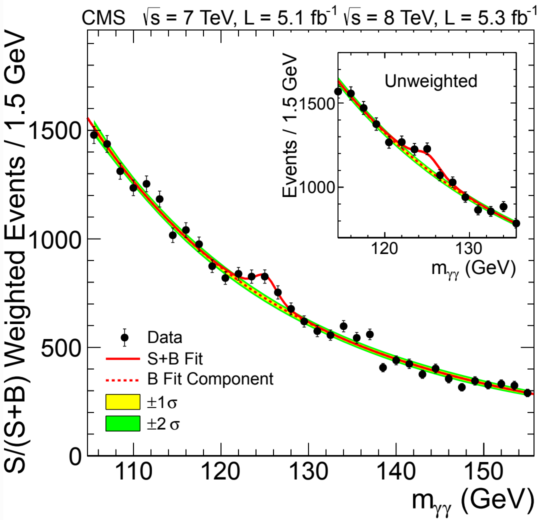
Light Higgs channel of the resonant decay of magnon condensate in superfluid ^{3}He-B

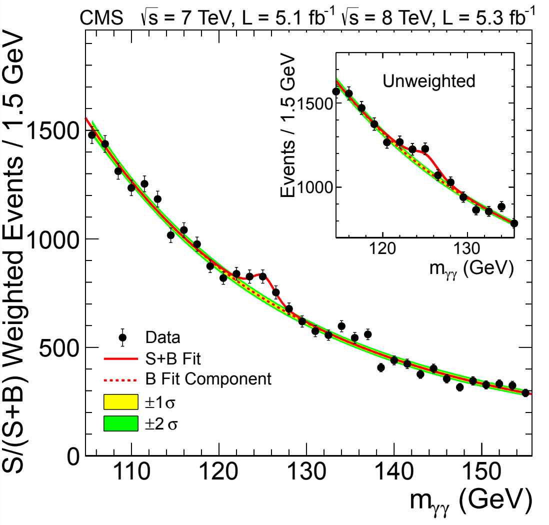
V.V. Zavjalov[1], S. Autti[1], V.B. Eltsov[1], P.J. Heikkinen[1] & G.E. Volovik[1,2]



Higgs Boson with mass $M = 125$ GeV



Higgs Boson with mass $M = 125$ GeV



Is this all there is?

Higgs Boson with mass $M = 125$ GeV - *Is this all there is?*

- *Higgs Bosons in Particle Physics and in Condensed Matter*
  G.E. Volovik & M. Zubkov, PRD 87, 075016 (2013)

- GEV & MZ: $m_{\text{top}} \approx 175\,\text{GeV}$, $M_{\text{H},-} = 125\,\text{GeV}$, $\therefore$ NSR $\rightsquigarrow$ $M_{\text{H},+} \approx 270\,\text{GeV}$

  - *Boson-Fermion Relations in BCS type Theories*
    Y. Nambu, Physica D, 15, 147 (1985)

- Broken Symmetry State: $\rightsquigarrow$ Fermion mass: $m_{\text{F}} = \Delta$
- Nambu's Sum Rule ("empirical observation"): $\sum_C M_{J,C}^2 = (2m_{\text{F}})^2$

Mode	Symmetry	Mass	Name
$D_{0,m}^{(+)}$	$J=0$, $\mathtt{C}=+1$	2Δ	Amplitude Higgs
$D_{0,m}^{(-)}$	$J=0$, $\mathtt{C}=-1$	0	NG Phase Mode
$D_{1,m}^{(+)}$	$J=1$, $\mathtt{C}=+1$	0	NG Spin-Orbit Modes
$D_{1,m}^{(-)}$	$J=1$, $\mathtt{C}=-1$	2Δ	AH Spin-Orbit Modes
$D_{2,m}^{(+)}$	$J=2$, $\mathtt{C}=+1$	$\sqrt{\frac{8}{5}}\Delta$	2^+ AH Modes
$D_{2,m}^{(-)}$	$J=2$, $\mathtt{C}=-1$	$\sqrt{\frac{12}{5}}\Delta$	2^- AH Modes

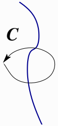
Superfluid ^{3}He as Topological Quantum Matter

Confinement, Excitations & New Phases

Real-Space & Momentum-Space Topology of Superfluid ^{3}He

Topology in Real Space

$\Psi(\mathbf{r}) = |\Psi(r)|\, e^{i\vartheta(\mathbf{r})}$



Phase Winding

$$N_C = \frac{1}{2\pi}\oint_C d\vec{l}\cdot \frac{1}{|\Psi|}\mathrm{Im}[\nabla\Psi] \in \{0, \pm 1, \pm 2, \dots\}$$

- Massless Fermions confined in the Vortex Core

- Point Defects & Domain Walls
- Quantized Spin-Current Vortices
- **"Half-Quantum" Mass-Spin Vortices**

Real-Space & Momentum-Space Topology of Superfluid ^{3}He

Phase Winding

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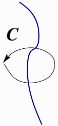
$$\Psi(\mathbf{r}) = |\Psi(r)|\, e^{i\vartheta(\mathbf{r})}$$



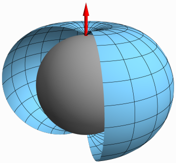
$$N_C = \frac{1}{2\pi}\oint_C d\vec{l}\cdot\frac{1}{|\Psi|}\mathrm{Im}[\nabla\Psi] \in \{0, \pm1, \pm2, \ldots\}$$

- Massless Fermions confined in the Vortex Core

- Point Defects & Domain Walls
- Quantized Spin-Current Vortices
- **“Half-Quantum” Mass-Spin Vortices**

Chiral Symmetry ⇝ Topology in Momentum Space

$$\Psi(\mathbf{p}) = \Delta(p_x \pm i p_y) \sim e^{\pm i\varphi_{\mathbf{p}}}$$



Topological Quantum Number: $L_z = \pm 1$

$$N_{2D} = \frac{1}{2\pi}\oint d\mathbf{p}\cdot\frac{1}{|\Psi(\mathbf{p})|}\mathrm{Im}[\nabla_{\mathbf{p}}\Psi(\mathbf{p})] = L_z$$

- Massless Chiral Fermions
  - Nodal Fermions in 3D
  - Edge Fermions in 2D

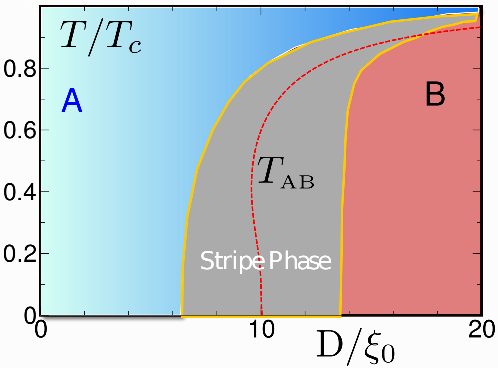
- G. Volovik and V. Mineev, Line and Point Singularities in Superfluid ^{3}He, JETP Letters, 24, 561 (1976)

Broken Time-Reversal and Mirror Symmetry by the Vacuum State of ^{3}He Films

- Length Scale for Strong Confinement:

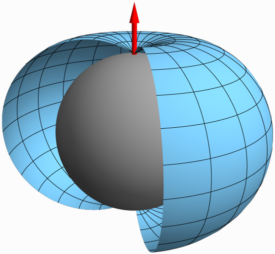
$$\xi_0 = \hbar v_f / 2\pi k_B T_c \approx 20 - 80\,\text{nm}$$

A. Vorontsov & J. A. Sauls, PRL 98, 045301 (2007)



$$\mathrm{SO}(3)_{\mathrm{S}} \times \mathrm{SO}(3)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{N}} \times \mathrm{T} \times \mathrm{P}$$
$$\Downarrow$$
$$\mathrm{SO}(2)_{\mathrm{S}} \times \mathrm{U}(1)_{\mathrm{N\text{-}L}_z} \times \mathrm{Z}_2$$

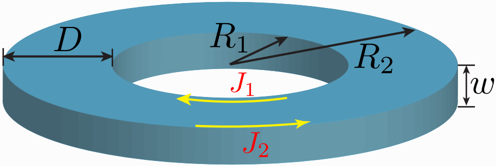
Chiral AM State $\vec{l} = \hat{\mathbf{z}}$



$L_z = 1,\ S_z = 0$

$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix}_{AM} = \begin{pmatrix} p_x + ip_y \sim e^{+i\phi} & 0 \\ 0 & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

^{3}He-A confined in a toroidal cavity



- $R_1, R_2, R_1 - R_2 \gg \xi_0$

- Sheet Current: $J = \frac{1}{4} n\hbar$ ($n = N/V = {}^3$He density)
- Counter-propagating Edge Currents: $J_1 = -J_2 = \frac{1}{4} n\hbar$
- Angular Momentum:

$$L_z = 2\pi h(R_1^2 - R_2^2) \times \frac{1}{4} n\hbar = (N/2)\hbar$$

McClure-Takagi's Global Symmetry Result
PRL 43, 596 (1979)

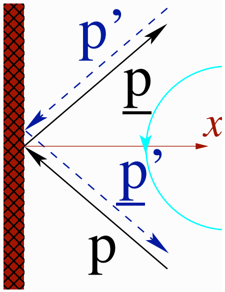
- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

Weyl Fermions in the 2D Chiral Sr_2RuO_4 and ^{3}He-A Films

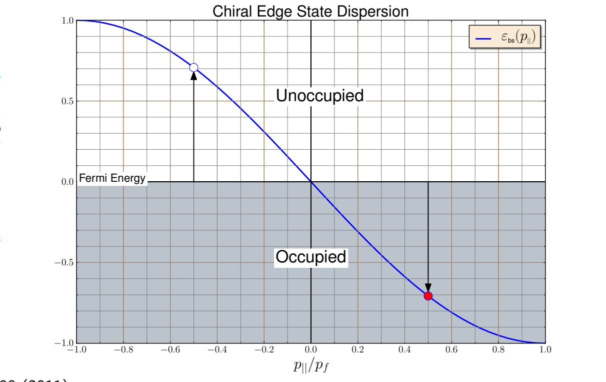
Edge Fermions: $G^{R}_{\text{edge}}(\mathbf{p},\varepsilon;x) = \dfrac{\pi\Delta|\mathbf{p}_x|}{\varepsilon + i\gamma - \varepsilon_{\text{bs}}(\mathbf{p}_{||})}\, e^{-x/\xi_\Delta}$

Confinement: $\xi_\Delta = \hbar v_f/2\Delta \approx 10^2 - 10^3\,\AA \gg \hbar/p_f$

- $\varepsilon_{\text{bs}} = -c\,p_{||}$ with $c = \Delta/p_f \ll v_f$



- Broken P & T $\rightsquigarrow$ Edge Current

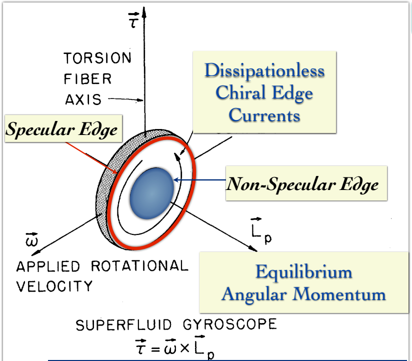


- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

Long-Standing Challenge: Detect the Ground-State Angular Momentum of ^{3}He-A

Possible Gyroscopic Experiment to Measure of $L_z(T)$

- Hyoungsoon Choi (KAIST) [micro-mechanical gyroscope @ 200 μK]



J. Clow and J. Reppy, *Phys. Rev. A* 5, 424–438

Thermal Signature of Massless Chiral Fermions

- Power Law for $T \lesssim 0.5 T_c$

$$L_z = (N/2)\hbar \,(1 - c\,(T/\Delta)^2\,)$$

Toroidal Geometry with Engineered Surfaces

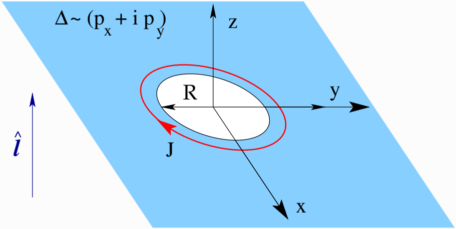
- Incomplete Screening

$$L_z > (N/2)\hbar$$

Direct Signature of Edge Currents

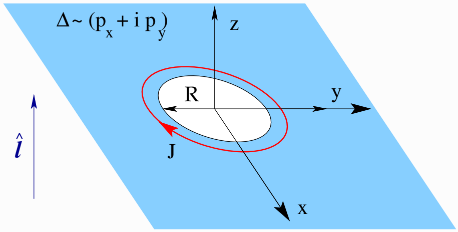
- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)
- Y. Tsutsumi, K. Machida, JPSJ 81, 074607 (2012)

Unbounded Film of ^{3}He-A perforated by a Hole



- $R \gg \xi_0 \approx 100\,\text{nm}$

Unbounded Film of ^{3}He-A perforated by a Hole

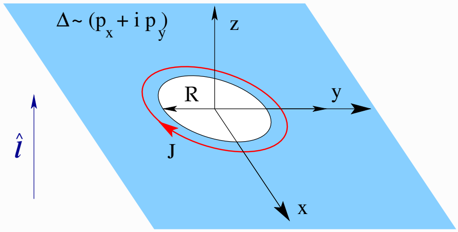


- $R \gg \xi_0 \approx 100\,\mathrm{nm}$

- Magnitude of the Sheet Current: $\frac{1}{4}n\hbar$ ($n = N/V = {}^3$He density)
- Edge Current *Counter*-Circulates: $J = -\frac{1}{4}n\hbar$ w.r.t. Chirality: $\hat{\mathbf{l}} = +\mathbf{z}$

Chiral Edge Current Circulating a Hole or Defect in a Chiral Superfluid

Unbounded Film of ^{3}He-A perforated by a Hole



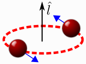
- $R \gg \xi_0 \approx 100\,\text{nm}$

- Magnitude of the Sheet Current: $\frac{1}{4} n \hbar$ $(n = N/V = {}^3\text{He density})$
- Edge Current *Counter*-Circulates: $J = -\frac{1}{4} n \hbar$ w.r.t. Chirality: $\hat{\mathbf{l}} = +\mathbf{z}$
- Angular Momentum: $L_z = 2\pi h R^2 \times (-\frac{1}{4} n \hbar) = -(N_{\text{hole}}/2)\,\hbar$

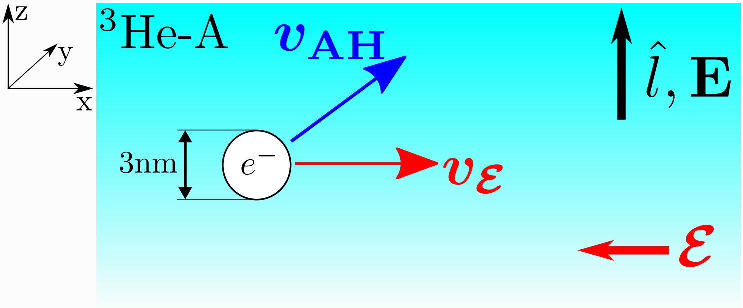
N_{hole} = Number of ^{3}He atoms excluded from the Hole

$\therefore$ An object in ^{3}He-A *inherits* angular momentum from the Condensate of Chiral Pairs!

Electron bubbles in chiral superfluid ^{3}He-A



$$\Delta_A(\hat{\mathbf{k}}) = \Delta \frac{k_x + ik_y}{k_f} = \Delta\, e^{i\phi_{\mathbf{k}}}$$



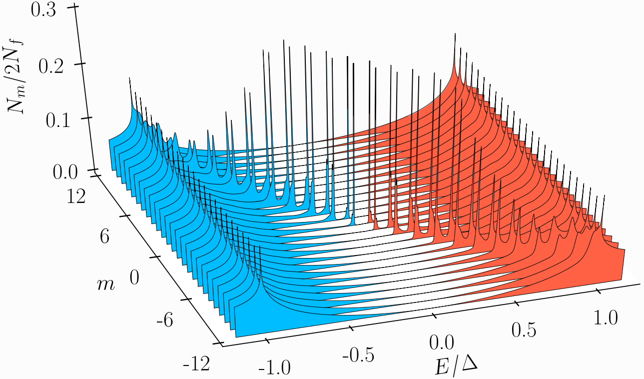
- Current: $\mathbf{v} = \overbrace{\mu_\perp \mathscr{E}}^{\mathbf{v}_{\mathscr{E}}} + \overbrace{\mu_{\text{AH}} \mathscr{E} \times \hat{\mathbf{l}}}^{\mathbf{v}_{\text{AH}}}$ R. Salmelin, M. Salomaa & V. Mineev, PRL **63**, 868 (1989)
- Hall ratio: $\tan\alpha = v_{\text{AH}}/v_{\mathscr{E}} = |\mu_{\text{AH}}/\mu_\perp|$

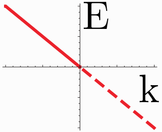
Weyl Fermion Spectrum bound to the Electron Bubble

$$\mu_{\mathsf{N}} = \frac{e}{n_3 p_f \sigma_{\mathsf{N}}^{\mathsf{tr}}} \quad \Leftarrow \quad \mu_{\mathsf{N}}^{\mathsf{exp}} = 1.7 \times 10^{-6}\,\frac{m^2}{V\,s}$$

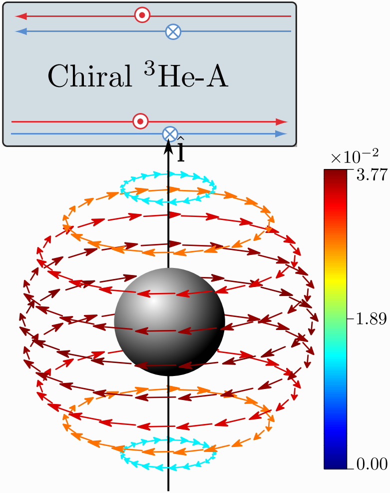
$$\tan\delta_l = j_l(k_f R)/n_l(k_f R) \;\Rightarrow\; \sigma_{\mathsf{N}}^{\mathsf{tr}} = \frac{4\pi}{k_f^2}\sum_{l=0}^{\infty}(l+1)\sin^2(\delta_{l+1}-\delta_l) \;\rightsquigarrow\; k_f R = 11.17$$

$$N(\mathbf{r},E) = \sum_{m=-l_{\max}}^{l_{\max}} N_m(\mathbf{r},E), \quad l_{\max} \simeq k_f R$$

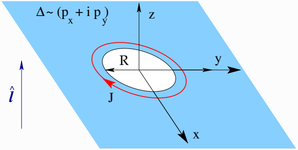


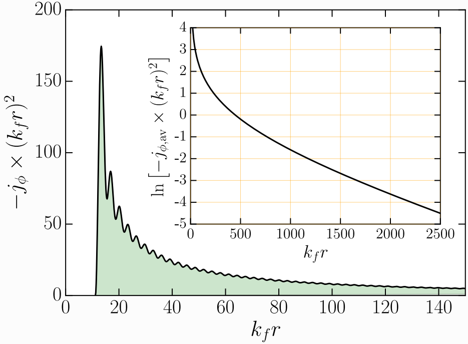


Current density bound to an electron bubble ($k_f R = 11.17$)



$\Longrightarrow$

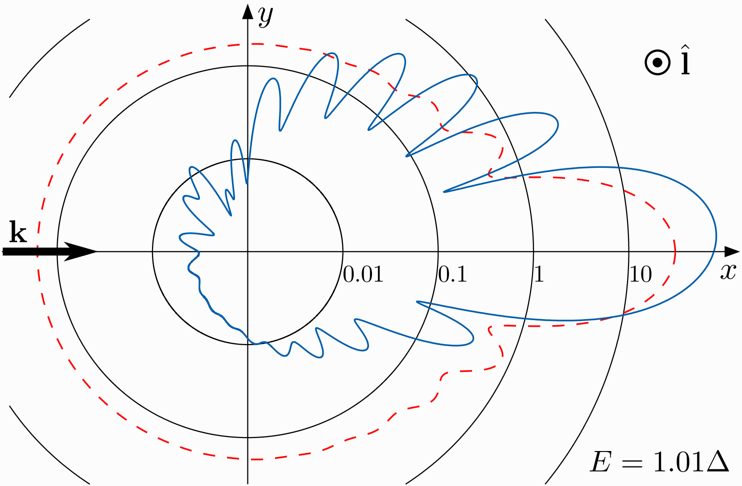




$\mathbf{j}(\mathbf{r})/v_f N_f k_B T_c = j_\phi(\mathbf{r})\hat{\mathbf{e}}_\phi \quad \Longrightarrow \quad \mathbf{L}(T \to 0) \approx -\hbar N_{\text{bubble}} \hat{\mathbf{l}}/2$

- O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Differential cross section for Bogoliubov QP-Ion Scattering $k_f R = 11.17$



- O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Forces on the Electron bubble in ^{3}He-A:

- $M\dfrac{d\mathbf{v}}{dt} = e\mathscr{E} + \mathbf{F}_{\text{QP}}$, $\mathbf{F}_{QP}$ – force from quasiparticle collisions
- $\mathbf{F}_{QP} = -\overleftrightarrow{\eta}\cdot\mathbf{v}$, $\overleftrightarrow{\eta}$ – generalized Stokes tensor
- $\overleftrightarrow{\eta} = \begin{pmatrix} \eta_\perp & \eta_{\mathsf{AH}} & 0 \\ -\eta_{\mathsf{AH}} & \eta_\perp & 0 \\ 0 & 0 & \eta_\parallel \end{pmatrix}$ for chiral symmetry with $\hat{\mathbf{l}} \parallel \mathbf{e}_z$
- $M\dfrac{d\mathbf{v}}{dt} = e\mathscr{E} - \eta_\perp\mathbf{v} + \dfrac{e}{c}\mathbf{v}\times\mathbf{B}_{\text{eff}}$, for $\mathscr{E} \perp \hat{\mathbf{l}}$

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- $\mathbf{B}_{\text{eff}} = -\frac{c}{e}\eta_{\text{AH}}\hat{\mathbf{l}}$ $\quad B_{\text{eff}} \simeq 10^3 - 10^4$ T !!!

Forces on the Electron bubble in ^{3}He-A:

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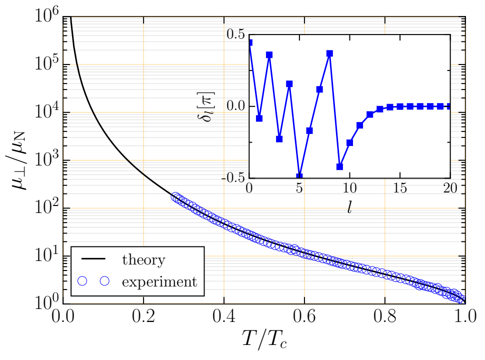
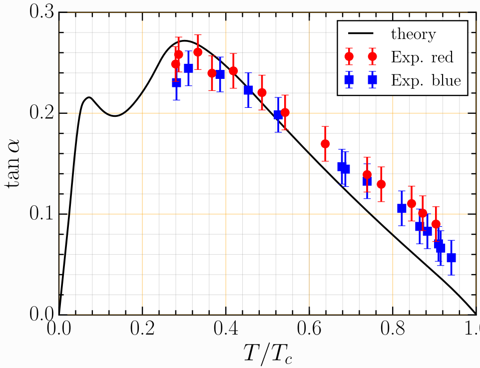
- $M\frac{d\mathbf{v}}{dt} = e\mathscr{E} - \eta_{\perp}\mathbf{v} + \frac{e}{c}\mathbf{v}\times\mathbf{B}_{\text{eff}}$, for $\mathscr{E} \perp \hat{\mathbf{l}}$
- $\mathbf{B}_{\text{eff}} = -\frac{c}{e}\eta_{\text{AH}}\hat{\mathbf{l}}$ $\quad B_{\text{eff}} \simeq 10^3 - 10^4\ \text{T}$!!!
- $\frac{d\mathbf{v}}{dt} = 0 \rightsquigarrow \mathbf{v} = \overleftrightarrow{\mu}\mathscr{E}$, where $\overleftrightarrow{\mu} = e\overleftrightarrow{\eta}^{-1}$

- O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Comparison between Theory and Experiment for the Drag and Transverse Forces





- $$\mu_\perp = e\,\frac{\eta_\perp}{\eta_\perp^2+\eta_{\mathsf{AH}}^2}$$
- $$\mu_{\mathsf{AH}} = -e\,\frac{\eta_{\mathsf{AH}}}{\eta_\perp^2+\eta_{\mathsf{AH}}^2}$$

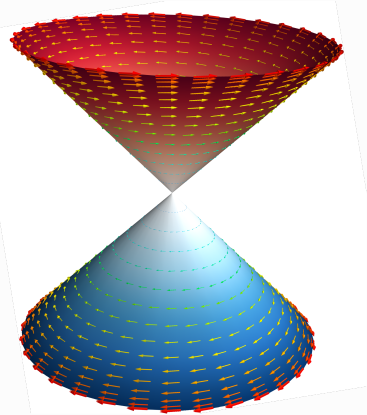
- $$\tan\alpha = \left|\frac{\mu_{\mathsf{AH}}}{\mu_\perp}\right| = \frac{\eta_{\mathsf{AH}}}{\eta_\perp}$$
- Hard-Sphere Model: $k_f R = 11.17$

- O. Shevtsov and JAS, PRB 96, 064511 (2016)
- O. Shevtsov and JAS, JLTP 187, 340353 (2017)

Spontaneously Broken Relative Spin-Orbit Symmetry in ^{3}He-B

Symmetry Protected Topology in Momentum Space

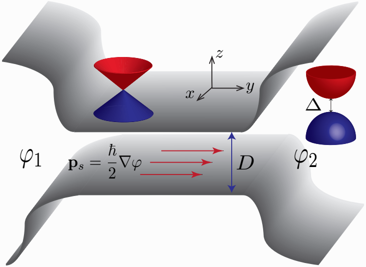
- Massless Helical Majorana Fermions
- Vacuum Spin-Currents
- Ising Magnetic Response
- Ground-State Helical Spin-Currents
- Signatures of Helical Majorana Fermions
- Topological Phase Transitions



- G. Volovik, The Universe in a Helium Droplet, Oxford Press (2003)
- T. Mizushima, Y. Tsutsumi, M. Sato & K. Machida, Symmetry protected topological ^{3}He-B, JPCM 27 113203 (2015)
- Y. Nagato, S. Higashitani & K. Nagai, Strong Anisotropy in Spin Suceptibility of ^{3}He-B Films, JPSJ 78, 123603 (2009)
- Hao Wu & J. Sauls, Majorana excitations, spin & mass currents in topological superfluid ^{3}He-B, PRB 88, 184506 (2013)

Condensate Flow and Backflow from Majorana Excitations

Condensate Flow: $\mathbf{p}_s \equiv m\mathbf{v}_s = \frac{\hbar}{2}\nabla\varphi$



- Flow Field Breaks T-symmetry, but not Topological Protection
- Doppler Shifted Majorana Spectrum: $\varepsilon \rightarrow \varepsilon = c|\mathbf{p}_{||}| + \mathbf{p}_{||} \cdot \mathbf{v}_s$
- Thermal Signature: $\vec{J} = n\,\mathbf{p}_s \times \left(1 - \frac{27\pi\zeta(3)}{2}\frac{\xi_\Delta}{D}\frac{\Delta_\perp}{\Delta_{||}}\frac{m^*}{m_3}\left(\frac{T}{\Delta_{||}}\right)^3\right)$

- T. Mizushima et al., PRL 109, 165301, (2012)
- Hao Wu, JAS, PRB 88, 18 184506 (2013)

Topology of Two-dimensional Chiral Superfluid ^{3}He-A- Lateral Confinement

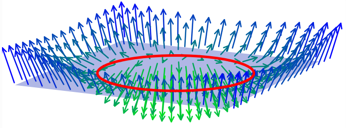
Bogoliubov Equations - Fermion Excitations

$\widehat{\mathscr{H}_B} = \vec{\mathbf{m}}(\mathbf{p}) \cdot \widehat{\vec{\tau}}$ with

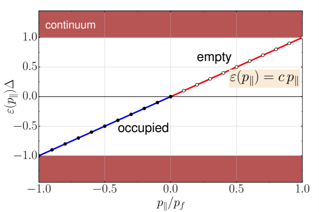
$$\vec{\mathbf{m}}(\mathbf{p}) = (\Delta p_x/p_f, \Delta p_y/p_f, \xi(\mathbf{p}))$$

Topological superfluid - Winding number

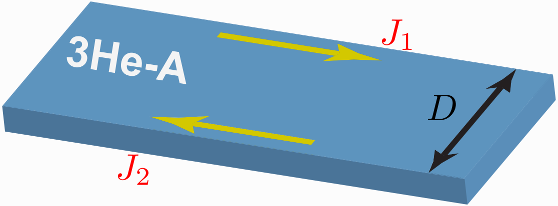
$$N_{\mathsf{C}} = \frac{1}{4\pi}\int d^2p\, \hat{\mathbf{m}} \cdot \left(\partial_{p_x}\hat{\mathbf{m}} \times \partial_{p_y}\hat{\mathbf{m}}\right) = \pm 1$$



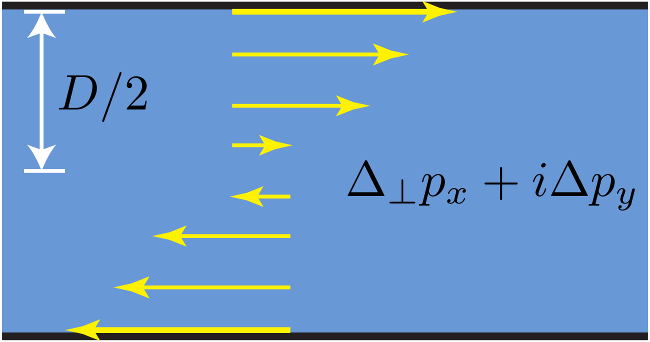
Edge states: **gapless** Weyl fermions



Edge Currents: $\mathbf{J} = \mathbf{n}\hbar/4$

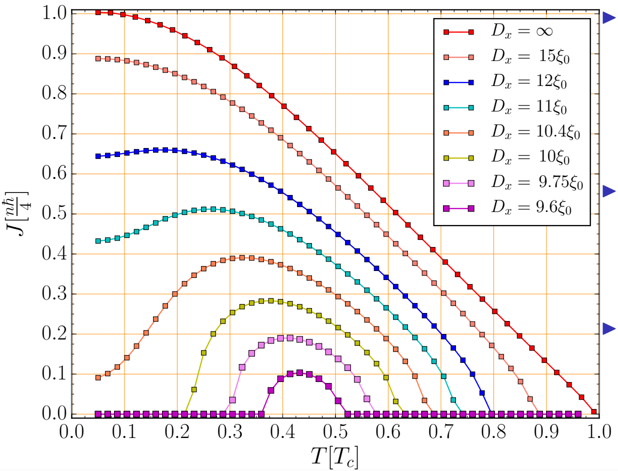


Edge current in Laterally Confined ^{3}He-A



$$J(T) = \int_0^{D/2} dx\; j_y(x;T) \xrightarrow[T=0]{D\to\infty} \frac{n\hbar}{4}$$

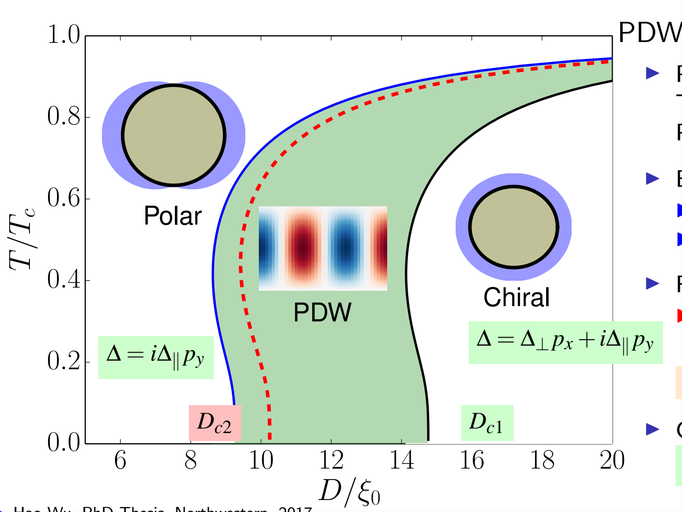
Edge Current in Laterally Confined ^{3}He-A



- **Hybridization:** Suppression of the Edge Current for $D < \infty$
- **Non-Chiral Phase** with $J = 0$ for $T_{c_1} < T \leq T_c$
- **Re-entrance:** Polar $\rightarrow$ Chiral $\rightarrow$ Polar Phase Transitions

- Hao Wu, PhD Thesis, Northwestern, 2017

Phase Diagram Polar → Pair Density Wave (PDW) → Chiral Phases



PDW Phase

- Periodic Lattice of Time-Reversed Pairs of Chiral Domains
- Broken Symmetries:
  - Time reversal
  - Translations
- Relative Symmetry:
  - Time reversal

    ×

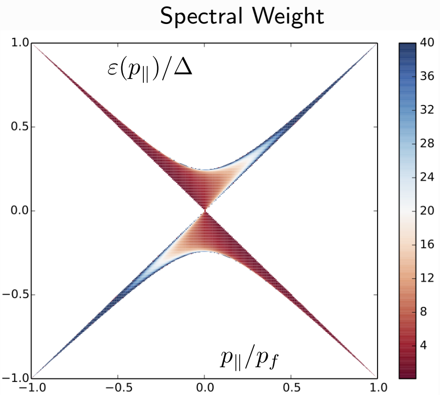
    Half-Lattice Translation
- Confinement Range:

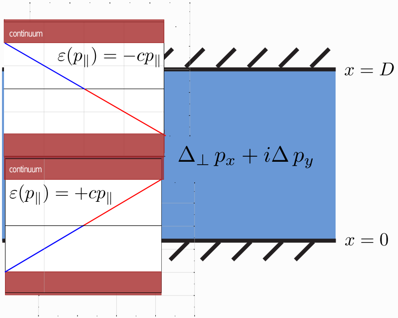
  $D_{c_2} - D_{c_1} \approx 6\xi_0 \approx 0.5\,\mu\text{m}$

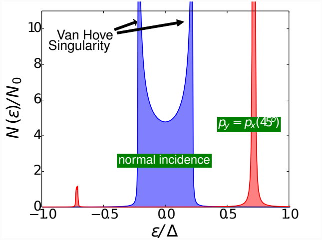
- Hao Wu, PhD Thesis, Northwestern, 2017

The End

Sub-Gap Spectrum: Hybridized Weyl Branches under confinement at $x = 0$





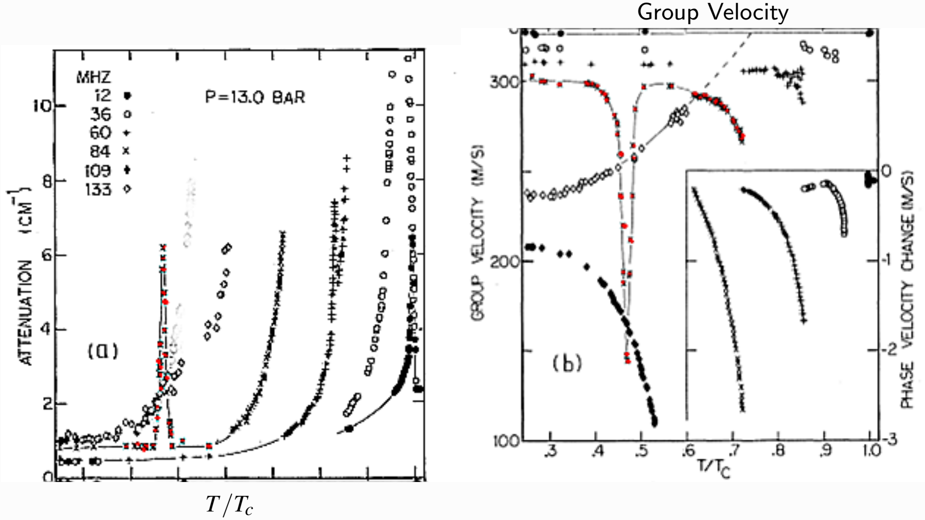


- Energy band at each $p_{\|}$
- Van Hove singularities

- Hao Wu, PhD Thesis, Northwestern, 2017

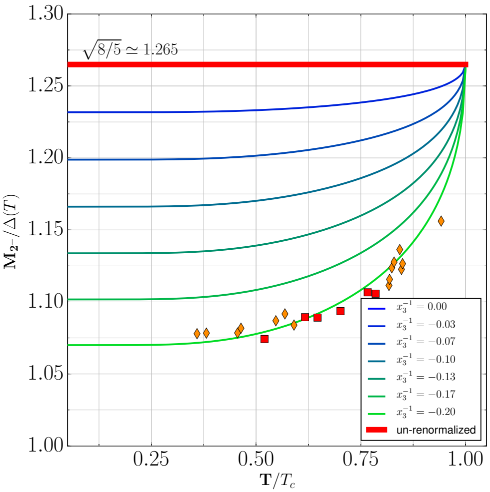
Excitation of the $J^{\mathrm{C}} = 2^+$, $m_J = 0$ Higgs Mode by Phonon Absorption

Higgs Mode with mass: $M = 500$ neV and spin $J^{\mathrm{C}} = 2^+$ at ULT-Northwestern



- D. Mast et al. Phys. Rev. Lett. 505, 266 (1980).

Vacuum Polarization $\rightsquigarrow$ Mass shift of the $J^{\mathrm{C}} = 2^+$ Higgs Mode in ^{3}He-B



- Measurements: D. Mast et al. PRL 45, 266 (1980)
- exchange p-h channel: $F_2^a = -0.88$ (from Magnetic susceptibility of ^{3}He-B)
  - *attractive* f-wave interaction $\rightsquigarrow$
    Higgs Modes with $J = 4^{\pm}$ with $M \lesssim 2\Delta$!

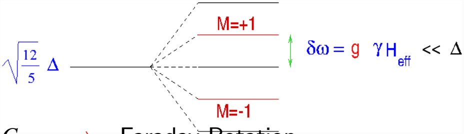
- JAS & J. Serene, Coupling of Order-Parameter Modes with L>1 to Zero Sound in ^{3}He-B, Phys. Rev. B 23, 4798 (1982)
- JAS and T. Mizushima, On Nambu's Boson-Fermion Mass Relations, Phys. Rev. B 95, 094515 (2017)

Faraday Rotation: Magneto-Acoustic Birefringence of Transverse Currents

- "Magneto-Acoustic Rotation of Transverse Waves in ^{3}He-B", J. A. Sauls et al., Physica B, 284,267 (2000)

$$C_{\substack{\text{RCP}\\\text{LCP}}}(\omega) = v_f \left[\frac{F_1^s}{15}\rho_n(\omega) + \frac{2F_1^s}{75}\rho_s(\omega) \underbrace{\left\{ \frac{\omega^2}{(\omega+i\Gamma)^2 - \Omega_{2,\pm}^{(-)}(\mathbf{q})} \right\}}_{D_{2,\pm 1}^{(-)}} \right]^{\frac{1}{2}}$$

$$\Omega_{2,\pm}^{(-)}(\mathbf{q}) = \sqrt{\frac{12}{5}}\Delta \pm g_{2^-}\,\gamma H_{\text{eff}}$$



- Circular Birefringence $\Longrightarrow$ $C_{\text{RCP}} \neq C_{\text{LCP}}$ $\Longrightarrow$ Faraday Rotation

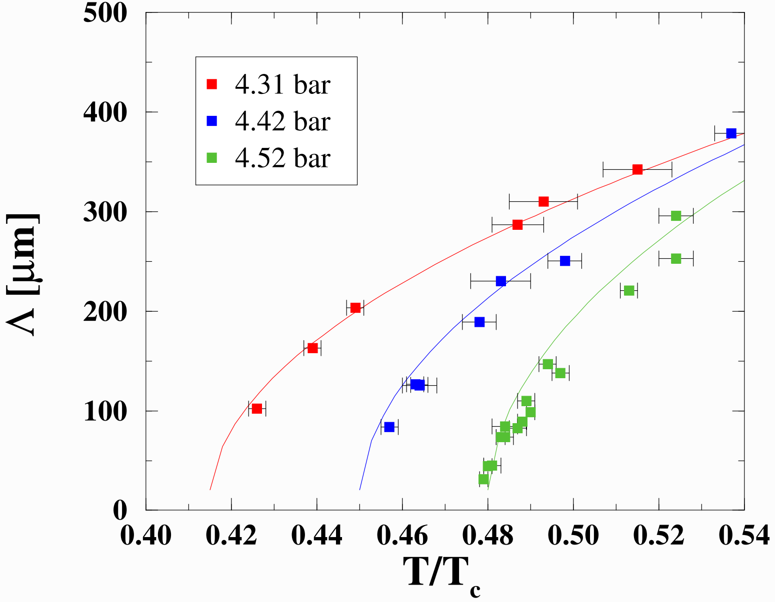
$$\left(\frac{C_{\text{RCP}} - C_{\text{LCP}}}{C_t}\right) \simeq g_{2^-} \left(\frac{\gamma H_{\text{eff}}}{\omega}\right)$$

- Faraday Rotation Period $(\gamma H_{\text{eff}} \ll (\omega - \Omega_2^{(-)}))$:

$$\Lambda \simeq \frac{4\pi C_t}{g_{2^-}\,\gamma H} \simeq 500\,\mu m\,, \quad H = 200\,G$$

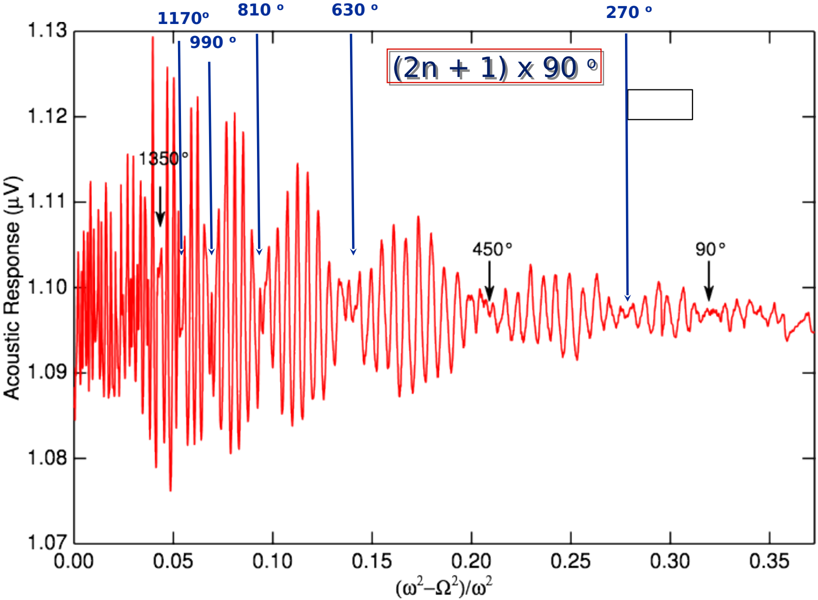
- *Discovery of the acoustic Faraday effect in superfluid ^{3}He-B*, Y. Lee, et al. Nature 400, 431 (1999)

Faraday Rotation: Magneto-Acoustic Birefringence of Transverse Currents



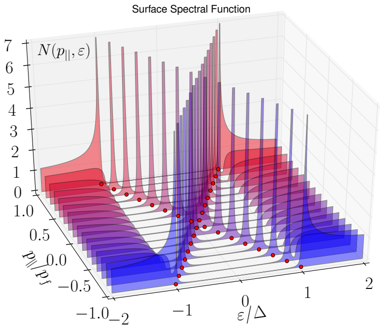
- “Broken Symmetry & Non-Equilibrium Superfluid ^{3}He”, J. A. Sauls
  Lecture Notes - Les Houches 1999, Eds. H. Godfrin & Y. Bunkov, Elsievier (2000)

Large Faraday Rotations vs. ``Blue Tuning'' B = 1097 G



C. Collett et al., Phys. Rev. B 87, 024502 (2013)

Fermionic Spectrum confined on the Surface of ^{3}He-B

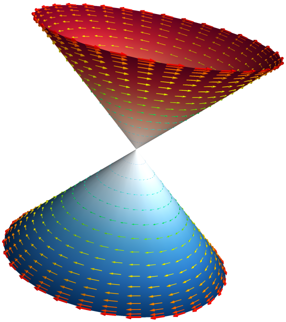


- Surface Spectrum:

$$N_b(\mathbf{p}, z; \varepsilon) = \frac{\pi}{2} \Delta_\perp \hat{p}_z e^{-2\Delta_\perp z / v_f} \times [\delta(\varepsilon - c|\mathbf{p}_{||}|) + \delta(\varepsilon + c|\mathbf{p}_{||}|)]$$

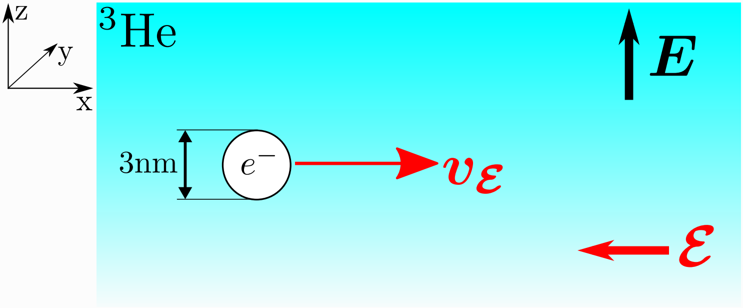
- Y. Tsutsumi, K. Machida, JPSJ 81, 074607 (2012)
- Hao Wu, JAS, Phys. Rev. B 88, 18 184506 (2013)

- **Surface Majorana Modes**



- $\varepsilon_b^\pm = \pm c|\mathbf{p}_{||}|, \quad c = \Delta_{||}/p_f \ll v_f$
- Helical Spin-Orbit Locking: $\vec{s} \perp \mathbf{p}$
- $\varepsilon_b^- < 0 \rightsquigarrow$ Helical Spin Current at $T = 0$
- $K_{xy} = \frac{1}{6} \, n_{2D} v_f \frac{\hbar}{2} \times (1 - a\, T^3)$

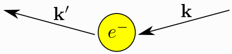
Electron bubbles in the Normal Fermi liquid phase of ^{3}He



- Bubble with $R \simeq 1.5$ nm, $0.1\,\text{nm} \simeq \lambda_f \ll R \ll \xi_0 \simeq 80\,\text{nm}$
- Effective mass $M \simeq 100 m_3$ (m_3 – atomic mass of ^{3}He)
- QPs mean free path $l \gg R$
- Mobility of ^{3}He is *independent of T* for $T_c < T < 50$ mK

  B. Josephson and J. Leckner, PRL 23, 111 (1969)

T-matrix description of Quasiparticle-Ion scattering



- Lippmann-Schwinger equation for the T-matrix ($\varepsilon = E + i\eta\,;\ \eta \to 0^+$):

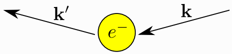
$$\hat{T}^R_S(\mathbf{k}',\mathbf{k},E) = \hat{T}^R_N(\mathbf{k}',\mathbf{k}) + \int \frac{d^3k''}{(2\pi)^3}\hat{T}^R_N(\mathbf{k}',\mathbf{k}'')\Big[\hat{G}^R_S(\mathbf{k}'',E) - \hat{G}^R_N(\mathbf{k}'',E)\Big]\hat{T}^R_S(\mathbf{k}'',\mathbf{k},E)$$

$$\hat{G}^R_S(\mathbf{k},E) = \frac{1}{\varepsilon^2 - E^2_{\mathbf{k}}}\begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad E_{\mathbf{k}} = \sqrt{\xi_k^2 + |\Delta(\hat{\mathbf{k}})|^2}, \ \ \xi_k = \frac{\hbar^2 k^2}{2m^*} - \mu$$

- Normal-state T-matrix:

$$\hat{T}^R_N(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = \begin{pmatrix} t^R_N(\hat{\mathbf{k}}',\hat{\mathbf{k}}) & 0 \\ 0 & -[t^R_N(-\hat{\mathbf{k}}',-\hat{\mathbf{k}})]^\dagger \end{pmatrix} \quad \text{in p-h (Nambu) space}$$

T-matrix description of Quasiparticle-Ion scattering



- Lippmann-Schwinger equation for the T-matrix ($\varepsilon = E + i\eta\,;\ \eta \to 0^+$):

$$\hat{T}_S^R(\mathbf{k}',\mathbf{k},E) = \hat{T}_N^R(\mathbf{k}',\mathbf{k}) + \int \frac{d^3k''}{(2\pi)^3}\hat{T}_N^R(\mathbf{k}',\mathbf{k}'')\left[\hat{G}_S^R(\mathbf{k}'',E) - \hat{G}_N^R(\mathbf{k}'',E)\right]\hat{T}_S^R(\mathbf{k}'',\mathbf{k},E)$$

$$\hat{G}_S^R(\mathbf{k},E) = \frac{1}{\varepsilon^2 - E_\mathbf{k}^2}\begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad E_\mathbf{k} = \sqrt{\xi_k^2 + |\Delta(\hat{\mathbf{k}})|^2}, \ \ \xi_k = \frac{\hbar^2k^2}{2m^*} - \mu$$

- Normal-state T-matrix:

$$\hat{T}_N^R(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = \begin{pmatrix} t_N^R(\hat{\mathbf{k}}',\hat{\mathbf{k}}) & 0 \\ 0 & -[t_N^R(-\hat{\mathbf{k}}',-\hat{\mathbf{k}})]^\dagger \end{pmatrix}$$ in p-h (Nambu) space, where

$$t_N^R(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = -\frac{1}{\pi N_f}\sum_{l=0}^{\infty}(2l+1)e^{i\delta_l}\sin\delta_l P_l(\hat{\mathbf{k}}'\cdot\hat{\mathbf{k}}), \quad P_l(x)$$ – Legendre function

- Hard-sphere potential $\rightsquigarrow \tan\delta_l = j_l(k_fR)/n_l(k_fR)$ – spherical Bessel functions

  - k_fR – determined by the Normal-State Mobility

Calculation of LDOS and Current Density

$$\hat{\mathscr{G}}_S^R(\mathbf{r}',\mathbf{r},E) = \int \frac{d^3k}{(2\pi)^3}\int \frac{d^3k'}{(2\pi)^3} e^{i\mathbf{k}'\mathbf{r}'} e^{-i\mathbf{k}\mathbf{r}} \hat{\mathscr{G}}_S^R(\mathbf{k}',\mathbf{k},E)$$

$$\hat{\mathscr{G}}_S^R(\mathbf{k}',\mathbf{k},E) = (2\pi)^3 \hat{G}_S^R(\mathbf{k},E)\delta_{\mathbf{k}',\mathbf{k}} + \hat{G}_S^R(\mathbf{k}',E)\hat{T}_S(\mathbf{k}',\mathbf{k},E)\hat{G}_S^R(\mathbf{k},E)$$

$$\hat{G}_S^R(\mathbf{k},E) = \frac{1}{\varepsilon^2 - E_{\mathbf{k}}^2}\begin{pmatrix} \varepsilon+\xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon-\xi_k \end{pmatrix}, \quad \varepsilon = E + i\eta, \;\; \eta \to 0^+$$

$$N(\mathbf{r},E) = -\frac{1}{2\pi}\mathrm{Im}\left\{\mathrm{Tr}\left[\hat{\mathscr{G}}_S^R(\mathbf{r},\mathbf{r},E)\right]\right\}$$

$$\mathbf{j}(\mathbf{r}) = \frac{\hbar}{4mi}k_BT\sum_{n=-\infty}^{\infty}\lim_{\mathbf{r}\to\mathbf{r}'}\mathrm{Tr}\left[(\nabla_{\mathbf{r}'}-\nabla_{\mathbf{r}})\hat{\mathscr{G}}^M(\mathbf{r}',\mathbf{r},\varepsilon_n)\right]$$

$$\hat{\mathscr{G}}_S^R(\mathbf{r}',\mathbf{r},E) = \hat{\mathscr{G}}_S^M(\mathbf{r}',\mathbf{r},\varepsilon_n)\Big|_{i\varepsilon_n\to\varepsilon}, \text{ for } n \geq 0$$

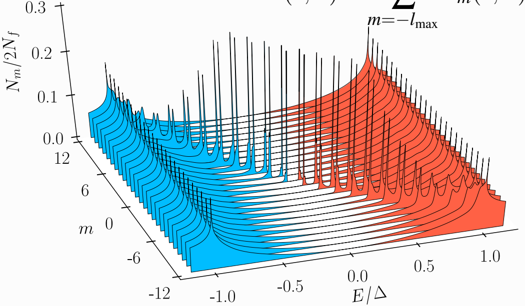
$$\hat{\mathscr{G}}_S^M(\mathbf{k},\mathbf{k}',-\varepsilon_n) = \left[\hat{\mathscr{G}}_S^M(\mathbf{k}',\mathbf{k},\varepsilon_n)\right]^\dagger$$

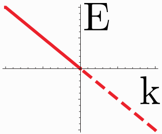
Weyl Fermion Spectrum bound to the Electron Bubble

$$\mu_{\mathsf{N}} = \frac{e}{n_3 p_f \sigma_{\mathsf{N}}^{\mathsf{tr}}} \quad \Leftarrow \quad \mu_{\mathsf{N}}^{\mathsf{exp}} = 1.7 \times 10^{-6}\,\frac{m^2}{V\,s}$$

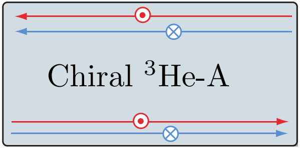
$$\tan\delta_l = j_l(k_f R)/n_l(k_f R) \;\Rightarrow\; \sigma_{\mathsf{N}}^{\mathsf{tr}} = \frac{4\pi}{k_f^2}\sum_{l=0}^{\infty}(l+1)\sin^2(\delta_{l+1}-\delta_l) \;\rightsquigarrow\; k_f R = 11.17$$

$$N(\mathbf{r},E) = \sum_{m=-l_{\max}}^{l_{\max}} N_m(\mathbf{r},E), \quad l_{\max} \simeq k_f R$$

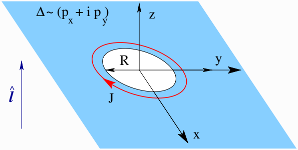




Current density bound to an electron bubble ($k_f R = 11.17$)



$\Longrightarrow$



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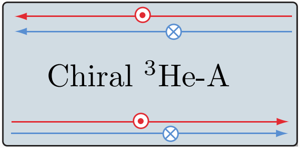
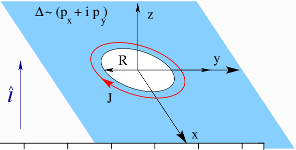
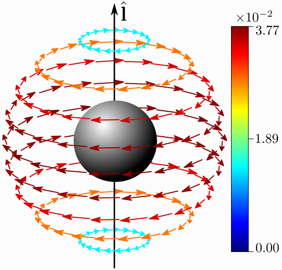
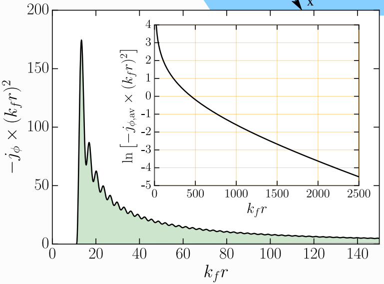


$\Longrightarrow$





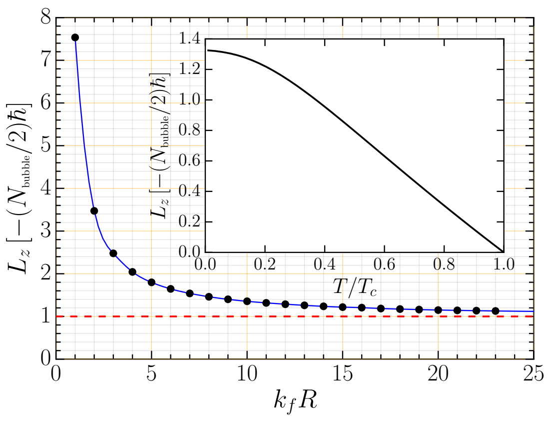


$\mathbf{j}(\mathbf{r})/v_f N_f k_B T_c = j_\phi(\mathbf{r})\hat{\mathbf{e}}_\phi \quad \Longrightarrow \quad \mathbf{L}(T \to 0) \approx -\hbar N_{\mathrm{bubble}} \hat{\mathbf{l}}/2$

- O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Angular momentum of an electron bubble in ^{3}He-A ($k_f R = 11.17$)

$$\mathbf{L}(T \to 0) \approx -\hbar N_{\text{bubble}} \hat{\mathbf{l}}/2\,; \qquad N_{\text{bubble}} = n_3 \frac{4\pi}{3} R^3 \approx 200\ ^3\text{He atoms}$$



Determination of the Stokes Tensor from the QP-Ion T-matrix

(i) Fermi's golden rule and the QP scattering rate:

$$\Gamma(\mathbf{k}',\mathbf{k}) = \frac{2\pi}{\hbar} W(\hat{\mathbf{k}}',\hat{\mathbf{k}})\,\delta(E_{\mathbf{k}'} - E_{\mathbf{k}}),$$

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(ii) Drag force from QP-ion collisions (linear in $\mathbf{v}$): ▶ Baym et al. PRL **22**, 20 (1969)

$$\mathbf{F}_{\text{QP}} = -\sum_{\mathbf{k},\mathbf{k}'} \hbar(\mathbf{k}'-\mathbf{k})\left[\hbar\mathbf{k}'\mathbf{v} f_{\mathbf{k}}\left(-\frac{\partial f_{\mathbf{k}'}}{\partial E}\right) - \hbar\mathbf{k}\mathbf{v}(1-f_{\mathbf{k}'})\left(-\frac{\partial f_{\mathbf{k}}}{\partial E}\right)\right]\Gamma(\mathbf{k}',\mathbf{k})$$

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(iii) Microscopic reversibility condition: $W(\hat{\mathbf{k}}',\hat{\mathbf{k}}:+\mathbf{l}) = W(\hat{\mathbf{k}},\hat{\mathbf{k}}':-\mathbf{l})$

Broken T and mirror symmetries in ^{3}He-A $\Rightarrow$ fixed $\hat{\mathbf{l}}$ $\rightsquigarrow$ $W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) \neq W(\hat{\mathbf{k}},\hat{\mathbf{k}}')$

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(iv) Generalized Stokes tensor:

$$\mathbf{F}_{\text{QP}} = -\overset{\leftrightarrow}{\eta}\cdot\mathbf{v}$$

Determination of the Stokes Tensor from the QP-Ion T-matrix

(i) Fermi's golden rule and the QP scattering rate:

$$\Gamma(\mathbf{k}',\mathbf{k}) = \frac{2\pi}{\hbar} W(\hat{\mathbf{k}}',\hat{\mathbf{k}})\,\delta(E_{\mathbf{k}'} - E_{\mathbf{k}}), \quad W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = \frac{1}{2}\sum_{\tau'\sigma';\tau\sigma} |\overbrace{\langle \mathbf{k}',\sigma',\tau'|}^{\text{outgoing}}\hat{T}_S\overbrace{|\mathbf{k},\sigma,\tau\rangle}^{\text{incoming}}|^2$$

(ii) Drag force from QP-ion collisions (linear in $\mathbf{v}$): ▶ Baym et al. PRL **22**, 20 (1969)

$$\mathbf{F}_{\text{QP}} = -\sum_{\mathbf{k},\mathbf{k}'} \hbar(\mathbf{k}'-\mathbf{k})\left[\hbar\mathbf{k}'\mathbf{v} f_{\mathbf{k}}\left(-\frac{\partial f_{\mathbf{k}'}}{\partial E}\right) - \hbar\mathbf{k}\mathbf{v}(1-f_{\mathbf{k}'})\left(-\frac{\partial f_{\mathbf{k}}}{\partial E}\right)\right]\Gamma(\mathbf{k}',\mathbf{k})$$

(iii) Microscopic reversibility condition: $W(\hat{\mathbf{k}}',\hat{\mathbf{k}}:+\mathbf{l}) = W(\hat{\mathbf{k}},\hat{\mathbf{k}}':-\mathbf{l})$

Broken T and mirror symmetries in ^{3}He-A $\Rightarrow$ fixed $\hat{\mathbf{l}}$ $\rightsquigarrow$ $W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) \neq W(\hat{\mathbf{k}},\hat{\mathbf{k}}')$

(iv) Generalized Stokes tensor:

$$\mathbf{F}_{\text{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v} \;\rightsquigarrow\; \eta_{ij} = n_3 p_f \int_0^\infty dE\left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E), \quad \overleftrightarrow{\eta} = \begin{pmatrix} \eta_\perp & \eta_{\text{AH}} & 0 \\ -\eta_{\text{AH}} & \eta_\perp & 0 \\ 0 & 0 & \eta_\parallel \end{pmatrix}$$

$n_3 = \dfrac{k_f^3}{3\pi^2}$ – ^{3}He particle density, $\sigma_{ij}(E)$ – transport scattering cross section,

$f(E) = [\exp(E/k_B T) + 1]^{-1}$ – Fermi Distribution

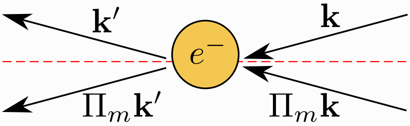
Mirror-symmetric scattering ⇒ longitudinal drag force

$$\mathbf{F}_{\mathsf{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^\infty dE\left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E)$$

Subdivide by mirror symmetry:

$$W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}),$$

$$\sigma_{ij}(E) = \sigma_{ij}^{(+)}(E) + \sigma_{ij}^{(-)}(E),$$



$$\sigma_{ij}^{(+)}(E) = \frac{3}{4}\int_{E\geq|\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'}\int_{E\geq|\Delta(\hat{\mathbf{k}})|}\frac{d\Omega_{\mathbf{k}}}{4\pi}\left[(\hat{\mathbf{k}}'_i-\hat{\mathbf{k}}_i)(\hat{\mathbf{k}}'_j-\hat{\mathbf{k}}_j)\right]\frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E)$$

Mirror-symmetric cross section: $W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W(\hat{\mathbf{k}},\hat{\mathbf{k}}')]/2$

$$\frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2\frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}}')|^2}}W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}})\frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}})|^2}}$$

⇝ Stokes Drag $\eta_{xx}^{(+)} = \eta_{yy}^{(+)} \equiv \eta_\perp,\ \eta_{zz}^{(+)} \equiv \eta_\parallel$, No transverse force $\left[\eta_{ij}^{(+)}\right]_{i\neq j} = 0$

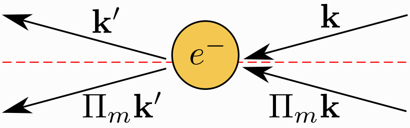
Mirror-antisymmetric scattering ⇒ transverse force

$$\mathbf{F}_{\text{QP}} = -\overleftrightarrow{\eta} \cdot \mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^\infty dE \left(-2\frac{\partial f}{\partial E}\right) \sigma_{ij}(E)$$

Subdivide by mirror symmetry:

$$W(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = W^{(+)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}) + W^{(-)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}),$$

$$\sigma_{ij}(E) = \sigma_{ij}^{(+)}(E) + \sigma_{ij}^{(-)}(E),$$



$$\sigma_{ij}^{(-)}(E) = \frac{3}{4}\int_{E \geq |\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'} \int_{E \geq |\Delta(\hat{\mathbf{k}})|} \frac{d\Omega_{\mathbf{k}}}{4\pi} \left[\varepsilon_{ijk}(\hat{\mathbf{k}}' \times \hat{\mathbf{k}})_k\right] \frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E)\left[f(E) - \frac{1}{2}\right]$$

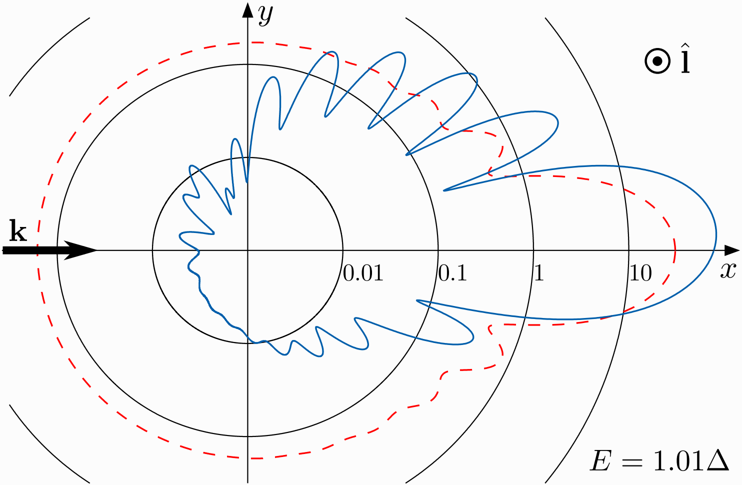
Mirror-antisymmetric cross section: $W^{(-)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}', \hat{\mathbf{k}}) - W(\hat{\mathbf{k}}, \hat{\mathbf{k}}')]/2$

$$\frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2 \frac{E}{\sqrt{E^2 - |\Delta(\hat{\mathbf{k}}')|^2}} W^{(-)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}) \frac{E}{\sqrt{E^2 - |\Delta(\hat{\mathbf{k}})|^2}}$$

Transverse force $\eta_{xy}^{(-)} = -\eta_{yx}^{(-)} \equiv \eta_{\text{AH}}$ ⇒ anomalous Hall effect

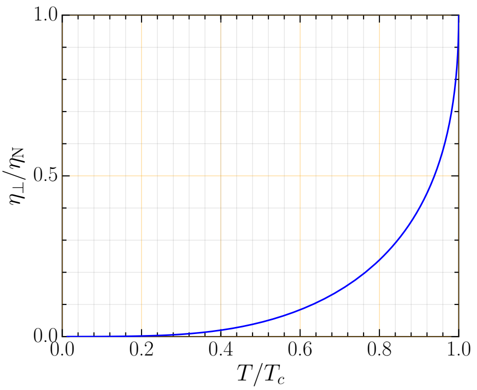
▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

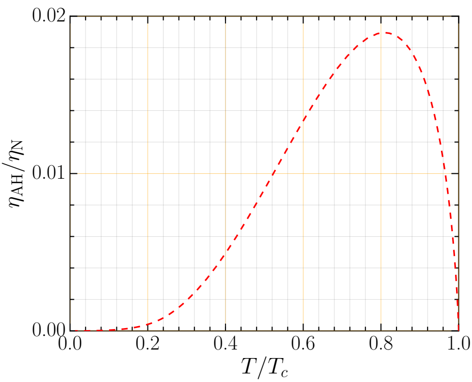
Differential cross section for Bogoliubov QP-Ion Scattering $k_f R = 11.17$



▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Theoretical Results for the Drag and Transverse Forces



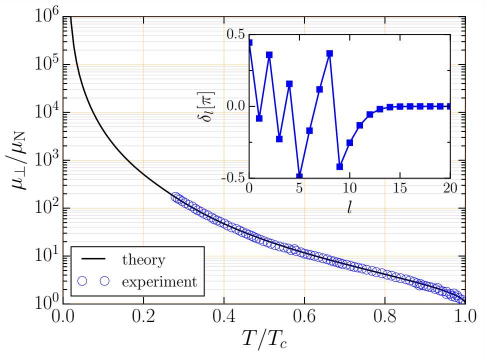


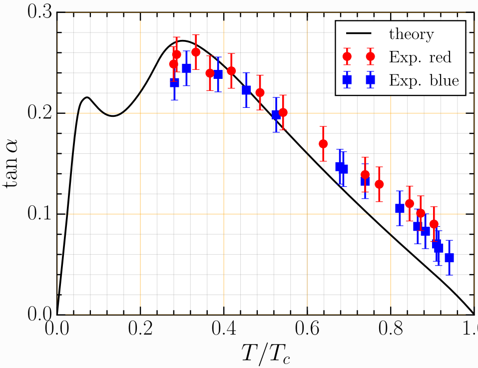
- $\Delta p_x \approx p_f \qquad \sigma^{\text{tr}}_{xx} \approx \sigma^{\text{tr}}_{\text{N}} \approx \pi R^2$
- $F_x \approx n\, v_x\, \Delta p_x\, \sigma^{\text{tr}}_{xx}$
  $\approx n\, v_x\, p_f\, \sigma^{\text{tr}}_{\text{N}}$
- $\Delta p_y \approx \hbar/R\ \sigma^{\text{tr}}_{xy} \approx (\Delta(T)/k_{\text{B}}T_c)^2 \sigma^{\text{tr}}_{\text{N}}$
- $F_y \approx n\, v_x\, \Delta p_y\, \sigma^{\text{tr}}_{xy}$
  $\approx n\, v_x\, (\hbar/R)\, \sigma^{\text{tr}}_{\text{N}} (\Delta(T)/k_{\text{B}}T_c)^2$

$$|F_y/F_x| \approx \frac{\hbar}{p_f R}\,(\Delta(T)/k_B T_c)^2 \qquad k_f R = 11.17$$

- O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Comparison between Theory and Experiment for the Drag and Transverse Forces





- $\mu_\perp = e\,\dfrac{\eta_\perp}{\eta_\perp^2 + \eta_{\mathrm{AH}}^2}$
- $\mu_{\mathrm{AH}} = -e\,\dfrac{\eta_{\mathrm{AH}}}{\eta_\perp^2 + \eta_{\mathrm{AH}}^2}$

- $\tan\alpha = \left|\dfrac{\mu_{\mathrm{AH}}}{\mu_\perp}\right| = \dfrac{\eta_{\mathrm{AH}}}{\eta_\perp}$
- Hard-Sphere Model: $k_f R = 11.17$

- O. Shevtsov and JAS, PRB 96, 064511 (2016)
- O. Shevtsov and JAS, JLTP 187, 340353 (2017)

Theoretical Models for the QP-ion potential

- $U(r) = \begin{cases} U_0, & r < R, \\ -U_1, & R < r < R', \\ 0, & r > R'. \end{cases}$

- $\rightsquigarrow$ Hard-Sphere Potential: $U_1 = 0$, $R' = R$, $U_0 \to \infty$

- $U(x) = U_0 [1 - \tanh[(x-b)/c]], \quad x = k_f r$

- $U(x) = U_0 / \cosh^2[\alpha x^n], \quad x = k_f r$ (Pöschl-Teller-like potential)

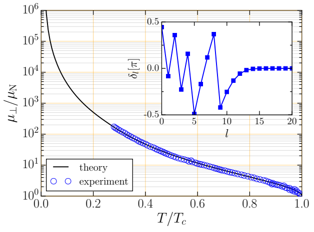
- Random phase shifts: $\{\delta_l | l = 1 \dots l_{\max}\}$ are generated with δ_0 is an adjustable parameter

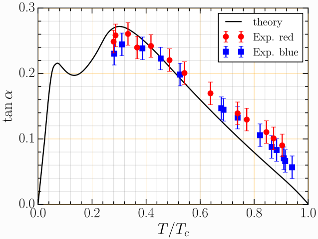
- Parameters for all models are chosen to fit the experimental value of the normal-state mobility, $\mu_N^{\exp} = 1.7 \times 10^{-6} m^2/V \cdot s$

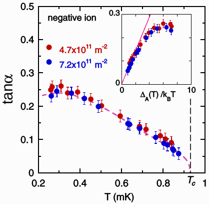
Theoretical Models for the QP-ion potential

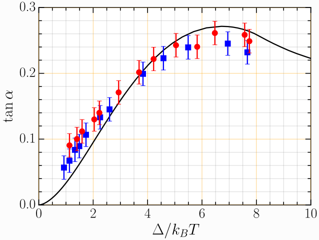
Label	Potential	Parameters
Model A	hard sphere	$k_f R = 11.17$
Model B	repulsive core & attractive well	$U_0 = 100E_f, U_1 = 10E_f, k_f R' = 11, R/R' = 0.36$
Model C	random phase shifts model 1	$l_{\max} = 11$
Model D	random phase shifts model 2	$l_{\max} = 11$
Model E	Pöschl-Teller-like	$U_0 = 1.01E_f, k_f R = 22.15, \alpha = 3 \times 10^{-5}, n = 4$
Model F	Pöschl-Teller-like	$U_0 = 2E_f, k_f R = 19.28, \alpha = 6 \times 10^{-5}, n = 4$
Model G	hyperbolic tangent	$U_0 = 1.01E_f, k_f R = 14.93, b = 12.47, c = 0.246$
Model H	hyperbolic tangent	$U_0 = 2E_f, k_f R = 14.18, b = 11.92, c = 0.226$
Model I	soft sphere 1	$U_0 = 1.01E_f, k_f R = 12.48$
Model J	soft sphere 2	$U_0 = 2E_f, k_f R = 11.95$

Hard-sphere model with $k_f R = 11.17$ (Model A)









Comparison with Experiment for Models for the QP-ion potential

Label	Potential	Parameters
Model A	hard sphere	$k_f R = 11.17$
Model B	attractive well with a repulsive core	$U_0 = 100E_f, U_1 = 10E_f, k_f R' = 11, R/R' = 0.36$
Model C	random phase shifts model 1	$l_{\max} = 11$
Model D	random phase shifts model 2	$l_{\max} = 11$
Model E	Pöschl-Teller-like	$U_0 = 1.01E_f, k_f R = 22.15, \alpha = 3 \times 10^{-5}, n = 4$
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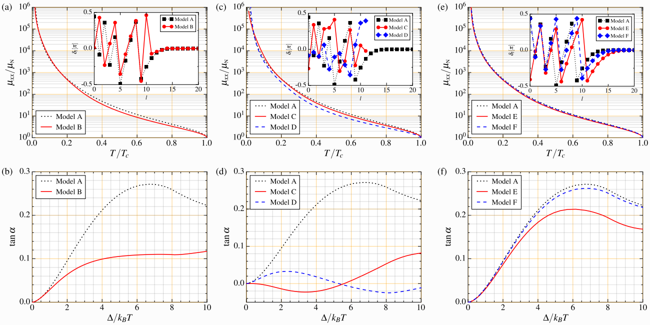
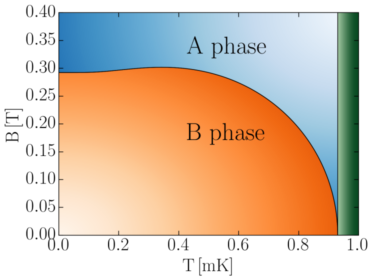


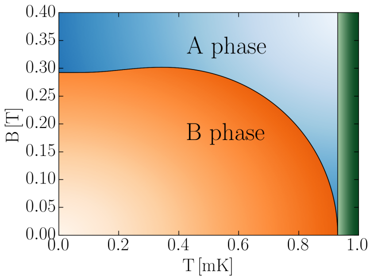
Stabilizing the A-phase at Low Temperatures



Magnetic field $\mathbf{B}$:

- suppresses $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ Cooper pairs: $\rightsquigarrow$ disfavors the B-phase
- favors the chiral, $p_x + ip_y$, A-phase with: $((1+\eta B)|\uparrow\uparrow\rangle + (1-\eta B)|\downarrow\downarrow\rangle)$
- critical field: $B_c(0) \approx 0.3\,\mathrm{T}$

Stabilizing the A-phase at Low Temperatures



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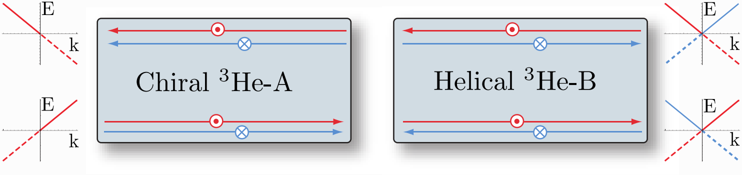
- critical field: $B_c(0) \approx 0.3\,\mathrm{T}$

Topological Edge states:



Calculation of LDOS and Current Density

$$\hat{\mathscr{G}}_S^R(\mathbf{r}',\mathbf{r},E) = \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} e^{i\mathbf{k}'\mathbf{r}'} e^{-i\mathbf{k}\mathbf{r}} \hat{\mathscr{G}}_S^R(\mathbf{k}',\mathbf{k},E)$$

$$\hat{\mathscr{G}}_S^R(\mathbf{k}',\mathbf{k},E) = (2\pi)^3 \hat{G}_S^R(\mathbf{k},E)\delta_{\mathbf{k}',\mathbf{k}} + \hat{G}_S^R(\mathbf{k}',E)\hat{T}_S(\mathbf{k}',\mathbf{k},E)\hat{G}_S^R(\mathbf{k},E)$$

$$\hat{G}_S^R(\mathbf{k},E) = \frac{1}{\varepsilon^2 - E_{\mathbf{k}}^2} \begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad \varepsilon = E + i\eta, \;\; \eta \to 0^+$$

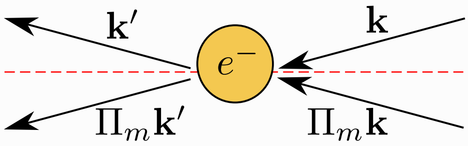
$$N(\mathbf{r},E) = -\frac{1}{2\pi}\mathrm{Im}\left\{\mathrm{Tr}\left[\hat{\mathscr{G}}_S^R(\mathbf{r},\mathbf{r},E)\right]\right\}$$

$$\mathbf{j}(\mathbf{r}) = \frac{\hbar}{4mi} k_B T \sum_{n=-\infty}^{\infty} \lim_{\mathbf{r}\to\mathbf{r}'} \mathrm{Tr}\left[(\boldsymbol{\nabla}_{\mathbf{r}'} - \boldsymbol{\nabla}_{\mathbf{r}})\hat{\mathscr{G}}^M(\mathbf{r}',\mathbf{r},\varepsilon_n)\right]$$

$$\hat{\mathscr{G}}_S^R(\mathbf{r}',\mathbf{r},E) = \hat{\mathscr{G}}_S^M(\mathbf{r}',\mathbf{r},\varepsilon_n)\Big|_{i\varepsilon_n\to\varepsilon}, \text{ for } n \geq 0$$

$$\hat{\mathscr{G}}_S^M(\mathbf{k},\mathbf{k}',-\varepsilon_n) = \left[\hat{\mathscr{G}}_S^M(\mathbf{k}',\mathbf{k},\varepsilon_n)\right]^\dagger$$

Broken time-reversal (T) & mirror (Π_m) symmetries for Chiral Superfluids



(1) Broken TRS: $\mathrm{T}\hat{\mathbf{l}} = -\hat{\mathbf{l}}$

(2) Broken mirror symmetry: $\Pi_m\hat{\mathbf{l}} = -\hat{\mathbf{l}}$

(3) Chiral symmetry: $\mathrm{C} = \mathrm{T} \times \Pi_m$

(4) Microscopic reversibility for chiral superfluids: $W(\hat{\mathbf{k}}',\hat{\mathbf{k}};\hat{\mathbf{l}}) = W(\hat{\mathbf{k}},\hat{\mathbf{k}}';-\hat{\mathbf{l}})$

(5) $\therefore$ For BTRS: the chiral axis $\hat{\mathbf{l}}$ is fixed $\rightsquigarrow W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) \neq W(\hat{\mathbf{k}},\hat{\mathbf{k}}')$

Determination of the Electron Bubble Radius

(i) Energy required to create a bubble:

$$E(R,P) = E_0(U_0,R) + 4\pi R^2\gamma + \frac{4\pi}{3}R^3P, \quad P \text{ – pressure}$$

(ii) For $U_0 \to \infty$: $E_0 = -U_0 + \pi^2\hbar^2/2m_eR^2$ – ground state energy

(iii) Surface Energy: hydrostatic surface tension $\rightsquigarrow \gamma = 0.15\,\mathrm{erg/cm^2}$

(iv) Minimizing E w.r.t. $R \rightsquigarrow P = \pi\hbar^2/4m_eR^5 - 2\gamma/R$

(v) For zero pressure, $P = 0$:

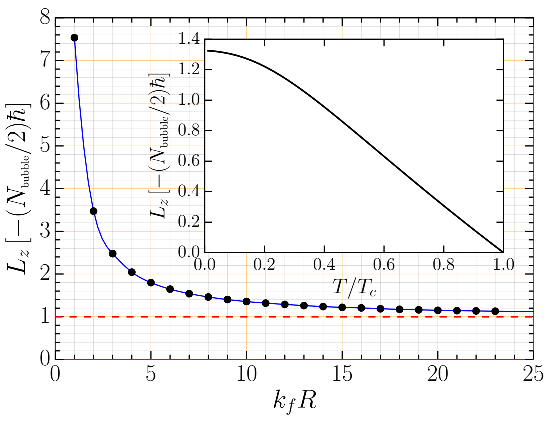
$$R = \left(\frac{\pi\hbar^2}{8m_e\gamma}\right)^{1/4} \approx 2.38\,\mathrm{nm} \;\rightsquigarrow\; k_fR = 18.67$$

Transport $\rightsquigarrow k_fR = 11.17$

- A. Ahonen et al., J. Low Temp. Phys., 30(1):205228, 1978

Angular momentum of an electron bubble in ^{3}He-A ($k_f R = 11.17$)

$$\mathbf{L}(T \to 0) \approx -\hbar N_{\text{bubble}} \hat{\mathbf{l}}/2\,; \qquad N_{\text{bubble}} = n_3 \frac{4\pi}{3} R^3 \approx 200\ {}^3\text{He atoms}$$



Mobility of an electron bubble in the Normal Fermi Liquid

(i) $t^R_{\mathsf{N}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E) = \sum_{l=0}^{\infty}(2l+1)t^R_l(E)P_l(\hat{\mathbf{k}}'\cdot\hat{\mathbf{k}})$

Mobility of an electron bubble in the Normal Fermi Liquid

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(ii) $t_l^R(E) = -\frac{1}{\pi N_f} e^{i\delta_l} \sin \delta_l$

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(iii) $\frac{d\sigma}{d\Omega_{\mathbf{k}'}} = \left(\frac{m^*}{2\pi\hbar^2} \right)^2 |t^R_{\mathsf{N}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E)|^2$

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(iv) $\sigma_{\mathsf{N}}^{\mathsf{tr}} = \int\frac{d\Omega_{\mathbf{k}'}}{4\pi}(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')\frac{d\sigma}{d\Omega_{\mathbf{k}'}} = \frac{4\pi}{k_f^2}\sum_{l=0}^{\infty}(l+1)\sin^2(\delta_{l+1}-\delta_l)$

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(v) $\mu_{\mathsf{N}} = \frac{e}{n_3 p_f \sigma_{\mathsf{N}}^{\mathsf{tr}}}, \quad p_f = \hbar k_f, \quad n_3 = \frac{k_f^3}{3\pi^2}$

Calculation of LDOS and Current Density

$$\hat{\mathscr{G}}_S^R(\mathbf{r}',\mathbf{r},E) = \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} e^{i\mathbf{k}'\mathbf{r}'} e^{-i\mathbf{k}\mathbf{r}} \hat{\mathscr{G}}_S^R(\mathbf{k}',\mathbf{k},E)$$

$$\hat{\mathscr{G}}_S^R(\mathbf{k}',\mathbf{k},E) = (2\pi)^3 \hat{G}_S^R(\mathbf{k},E)\delta_{\mathbf{k}',\mathbf{k}} + \hat{G}_S^R(\mathbf{k}',E)\hat{T}_S(\mathbf{k}',\mathbf{k},E)\hat{G}_S^R(\mathbf{k},E)$$

$$\hat{G}_S^R(\mathbf{k},E) = \frac{1}{\varepsilon^2 - E_{\mathbf{k}}^2} \begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad \varepsilon = E + i\eta, \;\; \eta \to 0^+$$

$$N(\mathbf{r},E) = -\frac{1}{2\pi}\mathrm{Im}\left\{\mathrm{Tr}\left[\hat{\mathscr{G}}_S^R(\mathbf{r},\mathbf{r},E)\right]\right\}$$

$$\mathbf{j}(\mathbf{r}) = \frac{\hbar}{4mi} k_B T \sum_{n=-\infty}^{\infty} \lim_{\mathbf{r}\to\mathbf{r}'} \mathrm{Tr}\left[(\boldsymbol{\nabla}_{\mathbf{r}'} - \boldsymbol{\nabla}_{\mathbf{r}})\hat{\mathscr{G}}^M(\mathbf{r}',\mathbf{r},\varepsilon_n)\right]$$

$$\hat{\mathscr{G}}_S^R(\mathbf{r}',\mathbf{r},E) = \hat{\mathscr{G}}_S^M(\mathbf{r}',\mathbf{r},\varepsilon_n)\Big|_{i\varepsilon_n\to\varepsilon}, \text{ for } n \geq 0$$

$$\hat{\mathscr{G}}_S^M(\mathbf{k},\mathbf{k}',-\varepsilon_n) = \left[\hat{\mathscr{G}}_S^M(\mathbf{k}',\mathbf{k},\varepsilon_n)\right]^\dagger$$

Temperature scaling of the Stokes tensor components

- For $1 - \frac{T}{T_c} \to 0^+$:

$$\frac{\eta_\perp}{\eta_{\mathsf{N}}} - 1 \propto -\Delta(T) \propto \sqrt{1 - \frac{T}{T_c}}$$

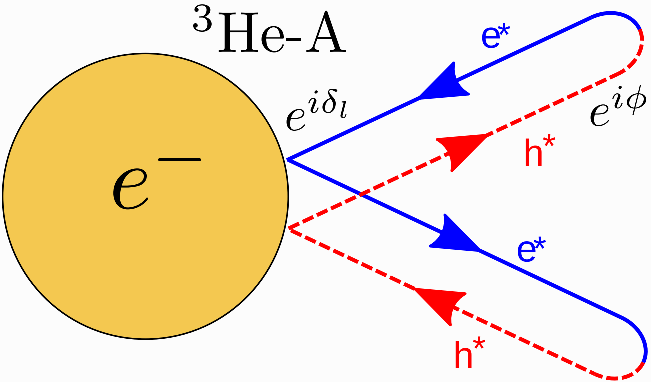
$$\frac{\eta_{\mathsf{AH}}}{\eta_{\mathsf{N}}} \propto \Delta^2(T) \propto 1 - \frac{T}{T_c}$$

- For $\frac{T}{T_c} \to 0^+$:

$$\frac{\eta_\perp}{\eta_{\mathsf{N}}} \propto \left(\frac{T}{T_c}\right)^2$$

$$\frac{\eta_{\mathsf{AH}}}{\eta_{\mathsf{N}}} \propto \left(\frac{T}{T_c}\right)^3$$

Multiple Andreev Scattering $\rightsquigarrow$ Formation of Weyl fermions on e-bubbles



$$\Delta(\hat{\mathbf{k}}) = \Delta \sin\theta e^{i\phi}$$