The Left Hand of the Electron in Superfluid ^{3}He

J. A. Sauls

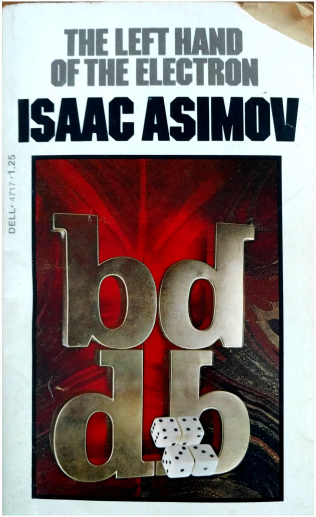
Northwestern University

- Oleksii Shevtsov

- Parity violation
- Superfluid ^{3}He
- Edge States & Currents
- Electron Bubbles in ^{3}He
- Anomalous Hall Effect
- Electron Transport in ^{3}He

▶ NSF Grant DMR-1508730

- An Essay on the Discovery of Parity Violation by the Weak Interaction



Parity Violation in Beta Decay of ^{60}Co - Physical Review 105, 1413 (1957)

Experimental Test of Parity Conservation in Beta Decay*

C. S. Wu, *Columbia University, New York, New York*

AND

E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson,
National Bureau of Standards, Washington, D. C.

(Received January 15, 1957)

- T. D. Lee and C. N. Yang, Phys Rev 104, 204 (1956)

$$^{60}Co \rightarrow \, ^{60}Ni + e^- + \bar{\nu}$$

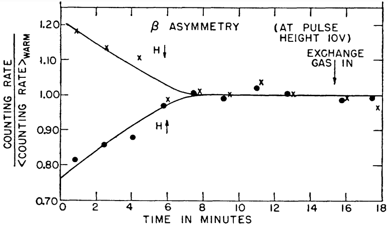
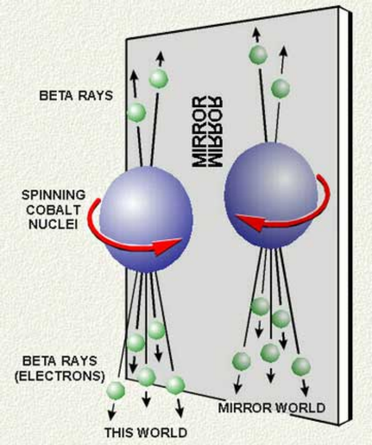


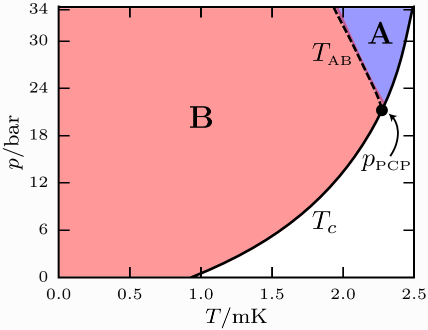
Fig. 2. Gamma anisotropy and beta asymmetry for polarizing field pointing up and pointing down.



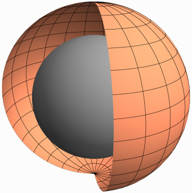
- Current of Beta electrons is (anti) correlated with the Spin of the ^{60}Co nucleus.
  $\langle \vec{S} \cdot \vec{p} \rangle \neq 0 \rightsquigarrow$ Parity violation

BCS Condensate Amplitude: $\Psi_{\alpha\beta}(p) = \langle \psi_\alpha(p)\psi_\beta(-p) \rangle$

J. Wiman & J. A. Sauls, PRB 92, 144515 (2015)



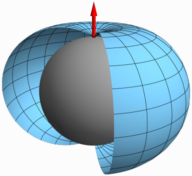
"Isotropic" BW State



$J = 0, \; J_z = 0$

$H = \mathtt{SO(3)_J} \times \mathtt{T}$

Chiral AM State $\vec{l} = \hat{\mathbf{z}}$



$L_z = 1, \; S_z = 0$

$H = \mathtt{U(1)_S} \times \mathtt{U(1)_{L_z\text{-}N}} \times \mathtt{Z_2}$

$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix}_{BW} = \begin{pmatrix} p_x - ip_y \sim e^{-i\phi} & p_z \\ p_z & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix}_{AM} = \begin{pmatrix} p_x + ip_y \sim e^{+i\phi} & 0 \\ 0 & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

Signatures of Broken T and P Symmetry in ^{3}He-A

What is the Signature & Evidence for Chirality of Superfluid ^{3}He-A?

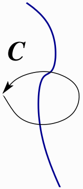
Spontaneous Symmetry Breaking $\rightsquigarrow$ Emergent Topology of ^{3}He-A

Chirality + Topology $\rightsquigarrow$ Edge States & Chiral Edge Currents

Broken T and P $\rightsquigarrow$ Anomalous Hall Effect for electrons in ^{3}He-A

Topology in Real Space

$\Psi(\mathbf{r}) = |\Psi(r)|\, e^{i\vartheta(\mathbf{r})}$



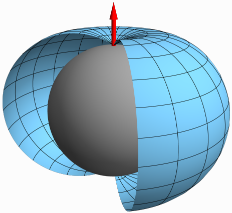
Phase Winding

$$N_C = \frac{1}{2\pi}\oint_C d\mathbf{l}\cdot\frac{1}{|\Psi|}\mathrm{Im}[\boldsymbol{\nabla}\Psi] \in \{0, \pm1, \pm2, \ldots$$

- Massless Fermions confined in the Vortex Core

Chiral Symmetry $\rightsquigarrow$
Topology in Momentum Space

$\Psi(\mathbf{p}) = \Delta(p_x \pm i p_y) \sim e^{\pm i\varphi_{\mathbf{p}}}$



Topological Quantum Number: $L_z = \pm1$

$$N_{2D} = \frac{1}{2\pi}\oint d\mathbf{p}\cdot\frac{1}{|\Psi(\mathbf{p})|}\mathrm{Im}[\boldsymbol{\nabla}_{\mathbf{p}}\Psi(\mathbf{p})] = L_z$$

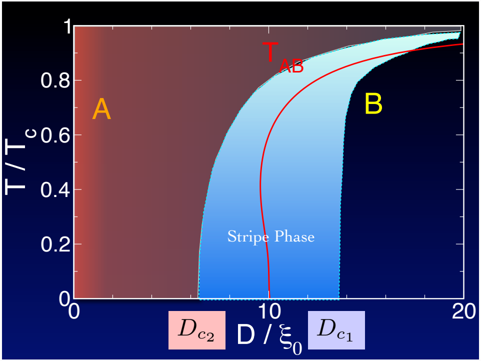
- Massless Chiral Fermions
  - Nodal Fermions in 3D
  - Edge Fermions in 2D

Symmetry or Normal Liquid ^{3}*He*: $G = SO(3)_S \times SO(2)_L \times U(1)_N \times P \times T$

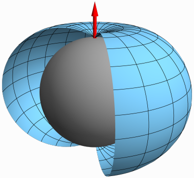
- Length Scale for Strong Confinement:

$$\xi_0 = \hbar v_f / 2\pi k_B T_c \approx 20 - 80\,\text{nm}$$

A. Vorontsov & JAS, PRL, 2007

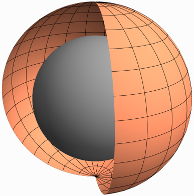


Chiral AM State $\vec{l} = \hat{\mathbf{z}}$



$L_z = 1,\ S_z = 0$

"Isotropic" BW State

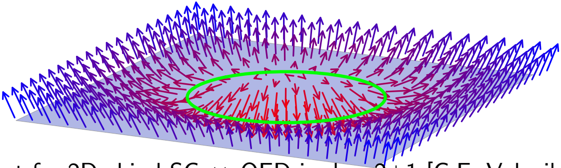


$J = 0,\ J_z = 0$

Hamiltonian for quasi-2D Chiral Superconductor (Sr_2RuO_4 & ^{3}He-A Film):

$$\widehat{H} = \begin{pmatrix} (|\mathbf{p}|^2/2m^* - \mu) & c(p_x + ip_y) \\ c(p_x - ip_y) & -(|\mathbf{p}|^2/2m^* - \mu) \end{pmatrix} = \vec{\mathbf{m}}(\mathbf{p}) \cdot \widehat{\vec{\tau}}$$

$\vec{\mathbf{m}} = (\, cp_x\,,\, \mp cp_y\,, \xi(\mathbf{p}))$ with $|\vec{\mathbf{m}}(\mathbf{p})|^2 = \left(|\mathbf{p}|^2/2m - \mu\right)^2 + c^2|\mathbf{p}|^2 > 0\;, \mu \neq 0$



Topological Invariant for 2D chiral SC ↔ QED in d = 2+1 [G.E. Volovik, JETP 1988]:

$$N_{2\text{D}} = \pi \int \frac{d^2p}{(2\pi)^2}\, \hat{\mathbf{m}}(\mathbf{p}) \cdot \left(\frac{\partial \hat{\mathbf{m}}}{\partial p_x} \times \frac{\partial \hat{\mathbf{m}}}{\partial p_y}\right) = \begin{cases} \pm 1\,; & \mu > 0 \text{ and } \Delta \neq 0 \\ 0\,; & \mu < 0 \text{ or } \Delta = 0 \end{cases}$$

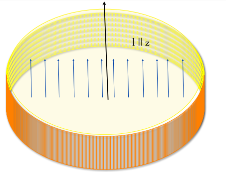
"Vacuum" $(\Delta = 0)$ with $N_{2D} = 0$ | ^{3}He-A $(\Delta \neq 0)$ with $N_{2D} = 1$

Zero Energy Fermions ↑ Confined on the Edge

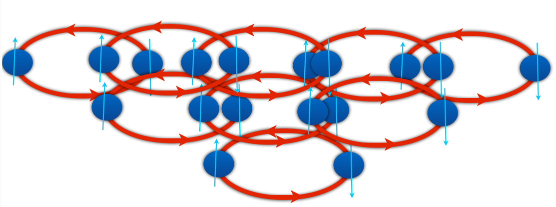
Chiral P-wave BEC Molecules or BCS Pairs (N Fermions):

$$|\,\Phi_N\,\rangle = \left[\iint d\mathbf{r}_1 d\mathbf{r}_2\, \varphi_{s_1 s_2}(\mathbf{r}_1 - \mathbf{r}_2)\, \psi^\dagger_{s_1}(\mathbf{r}_1)\psi^\dagger_{s_2}(\mathbf{r}_2)\right]^{N/2} |\,\text{vac}\,\rangle$$

- $\varphi_{s_1 s_2}(\mathbf{r}) = f(|\mathbf{r}|/\xi)\;(x+iy)\;\chi_{s_1 s_2}(S=1, M_S=0)$
    - BEC ($\xi < a$) vs. BCS ($\xi > a$)



$L_z = (N/2)\hbar$



$L_z = (N/2)\hbar\,(a/\xi)^2 \ll (N/2)\hbar$? (P.W. Anderson & P. Morel, 1960, A. Leggett, 1975)

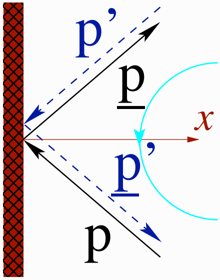
- $L_z|\,\Phi_N\,\rangle = (N/2)\hbar\,|\,\Phi_N\,\rangle$ independent of (a/ξ)! - McClure-Takagi (PRL, 1979)

BCS Limit: Currents are confined on the Edge

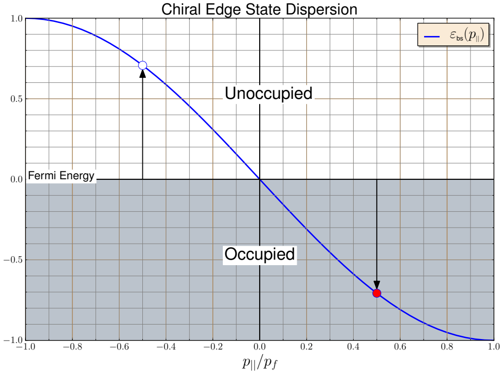
Edge Fermions: $G^{R}_{\text{edge}}(\mathbf{p},\varepsilon;x) = \frac{\pi\Delta|\mathbf{p}_x|}{\varepsilon + i\gamma - \varepsilon_{\text{bs}}(\mathbf{p}_{||})}\, e^{-x/\xi_\Delta}$

Confinement: $\xi_\Delta = \hbar v_f/2\Delta \approx 10^2 - 10^3\, \mathring{A} \gg \hbar/p_f$

- $\varepsilon_{\text{bs}} = -c\,p_{||}$ with $c = \Delta/p_f \ll v_f$



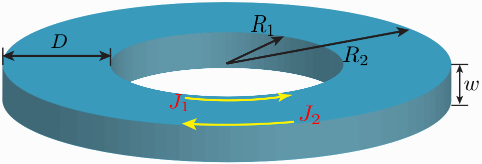
- Broken P & T $\rightsquigarrow$ Edge Current



▶ M. Stone, R. Roy, PRB 69, 184511 (2004)

▶ J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

^{3}He-A confined in a toroidal cavity



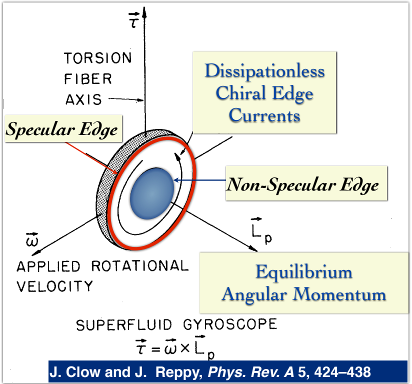
- $R_1, R_2, R_1 - R_2 \gg \xi_0$

- Sheet Current: $J = \frac{1}{4}\, n\, \hbar$ ($n = N/V$ = ^{3}He density)
- Counter-propagating Edge Currents: $J_1 = -J_2 = \frac{1}{4}\, n\, \hbar$
- Angular Momentum:

$$L_z = 2\pi\, h\, (R_1^2 - R_2^2) \times \frac{1}{4}\, n\, \hbar = (N/2)\, \hbar$$

McClure-Takagi Result

Possible Gyroscopic Experiment to Measure of $L_z(T)$
H. Choi (KAIST) [sub-micron mechanical gyroscope @ 200 μK]



Thermal Signature of Chiral Edge States

- Power Law for $T \lesssim 0.5T_c$

$$L_z = (N/2)\hbar\,(1 - c\,(T/\Delta)^2)$$

Toroidal Geometry with Engineered Surfaces

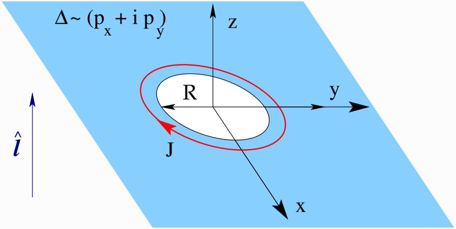
- Incomplete Screening

$$L_z > (N/2)\hbar$$

Direct Signature of Edge Currents

- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

Unbounded Film of ^{3}He-A perforated by a Hole

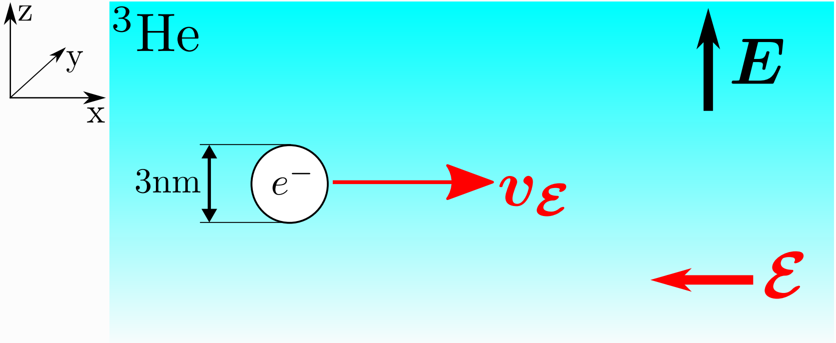


- $R \gg \xi_0 \approx 100\,\text{nm}$

- Magnitude of the Sheet Current: $\frac{1}{4}\, n\, \hbar$ ($n = N/V = {}^3$He density)
- Edge Current *Counter*-Circulates: $J = -\frac{1}{4}\, n\, \hbar$ w.r.t. Chirality: $\hat{\mathbf{l}} = +\mathbf{z}$
- Angular Momentum: $L_z = 2\pi\, h\, R^2 \times (-\frac{1}{4}\, n\, \hbar) = -(N_{\text{hole}}/2)\, \hbar$

N_{hole} = Number of ^{3}He atoms excluded from the Hole

∴ An object in ^{3}He-A *inherits* angular momentum from the Condensate of Chiral Pairs!



- Bubble with $R \simeq 1.5$ nm, $0.1\,\text{nm} \simeq \lambda_f \ll R \ll \xi_0 \simeq 80\,\text{nm}$
- Effective mass $M \simeq 100 m_3$ (m_3 – atomic mass of ^{3}He)
- QPs mean free path $l \gg R$, Knudsen limit
- Mobility of ^{3}He is *independent of T* for $T_c < T < 50$ mK

B. Josephson and J. Leckner, PRL 23, 111 (1969)

Determination of the Electron Bubble Radius

(i) Energy required to create a bubble:

$$E(R,P) = E_0(U_0,R) + 4\pi R^2\gamma + \frac{4\pi}{3}R^3 P, \quad P \text{ – pressure}$$

(ii) For $U_0 \to \infty$:

$E_0 = -U_0 + \pi^2\hbar^2/2m_e R^2$ – ground state energy

(iii) Surface Energy: hydrostatic surface tension $\rightsquigarrow \gamma = 0.15\,\mathrm{erg/cm^2}$

(iv) Minimizing E w.r.t. $R \rightsquigarrow P = \pi\hbar^2/4m_e R^5 - 2\gamma/R$

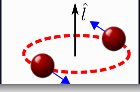
(v) For zero pressure, $P = 0$:

$$R = \left(\frac{\pi\hbar^2}{8m_e\gamma}\right)^{1/4} \approx 2.38\,\mathrm{nm} \quad \rightsquigarrow \quad k_f R = 18.67$$

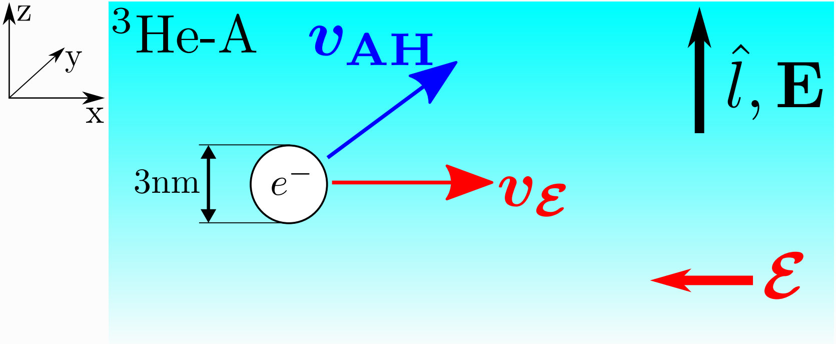
Transport $\rightsquigarrow k_f R = 11.17$

- A. Ahonen et al., J. Low Temp. Phys., 30(1):205228, 1978

Electron bubbles in chiral superfluid ^{3}He-A

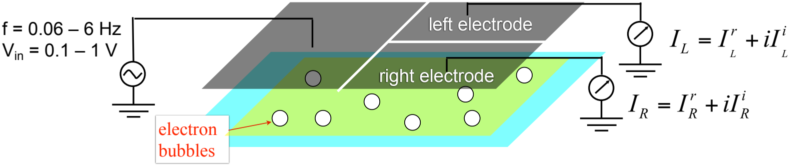


$$\Delta_A(\hat{\mathbf{k}}) = \Delta \frac{k_x + ik_y}{k_f} = \Delta\, e^{i\phi_{\mathbf{k}}}$$



- Electric current: $\mathbf{v} = \overbrace{\mu_\perp \boldsymbol{\mathcal{E}}}^{\mathbf{v}_{\mathcal{E}}} + \overbrace{\mu_{\mathsf{AH}} \boldsymbol{\mathcal{E}} \times \hat{\mathbf{l}}}^{\mathbf{v}_{\mathsf{AH}}}$ Salmelin et al. PRL **63**, 868 (1989)
- Hall ratio: $\tan\alpha = v_{\mathsf{AH}}/v_{\mathcal{E}} = |\mu_{\mathsf{AH}}/\mu_\perp|$

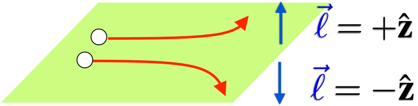
Measurement of the Transverse e$^-$ mobility in Superfluid ^{3}He Films



Transverse Force from *Skew* Scattering

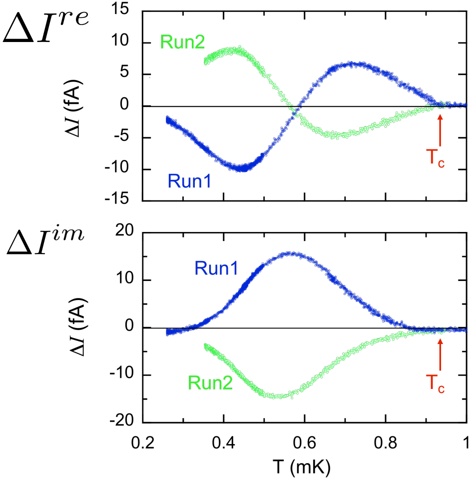
$$\rightsquigarrow \quad \Delta I = I_R - I_L \neq 0$$

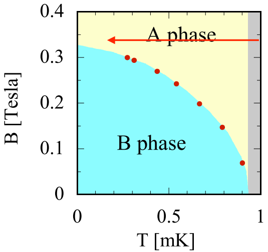
$$\vec{v} = \left[\mu_\perp \vec{E} + \mu_{xy} \hat{\ell} \times \vec{E} \right]$$



H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

Transverse e⁻ bubble current in ^{3}He-A $\Delta I = I_R - I_L$





Single Domains:

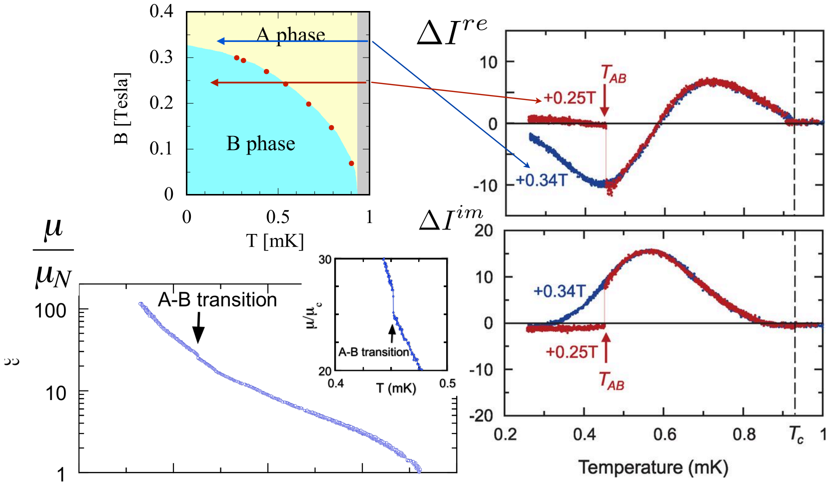
Run 1 $\vec{\ell} = +\hat{\mathbf{z}}$

Run 2 $\vec{\ell} = -\hat{\mathbf{z}}$

$$\frac{|I_R - I_L|}{I_R + I_L} \approx 6\,\%$$

H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

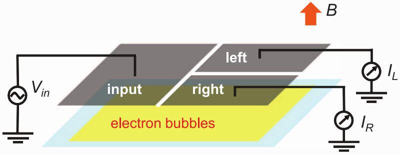
Zero Transverse e⁻ current in ^{3}He-B (*T - symmetric phase*)

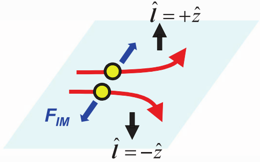


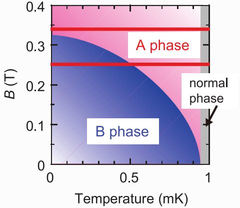
H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

Mobility of Electron Bubbles in ^{3}He-A

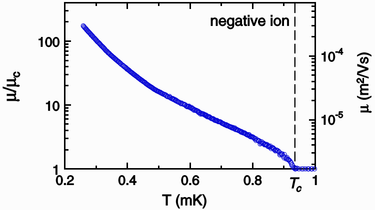
▶ H. Ikegami et al., Science **341**, 59 (2013); JPSJ **82**, 124607 (2013); JPSJ **84**, 044602 (2015)

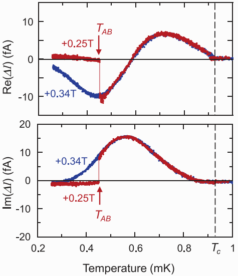


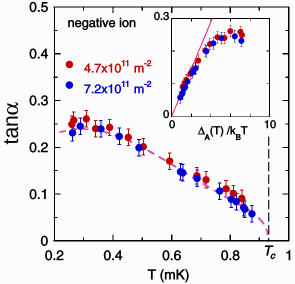




Electric current: $\mathbf{v} = \overbrace{\mu_\perp \boldsymbol{\mathcal{E}}}^{\mathbf{v}_{\mathcal{E}}} + \overbrace{\mu_{\text{AH}} \boldsymbol{\mathcal{E}} \times \hat{\mathbf{l}}}^{\mathbf{v}_{\text{AH}}}$ Hall ratio: $\tan\alpha = v_{\text{AH}}/v_{\mathcal{E}} = |\mu_{\text{AH}}/\mu_\perp|$







Forces on the Electron bubble in ^{3}He-A:

(i) $M\frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} + \mathbf{F}_{\text{QP}}$, $\mathbf{F}_{QP}$ – force from quasiparticle collisions

(ii) $\mathbf{F}_{QP} = -\overleftrightarrow{\eta}\cdot\mathbf{v}$, $\overleftrightarrow{\eta}$ – generalized Stokes tensor

(iii) $\overleftrightarrow{\eta} = \begin{pmatrix} \eta_{\perp} & \eta_{\text{AH}} & 0 \\ -\eta_{\text{AH}} & \eta_{\perp} & 0 \\ 0 & 0 & \eta_{\parallel} \end{pmatrix}$ for chiral symmetry with $\hat{\mathbf{l}} \parallel \mathbf{e}_z$

(iv) $M\frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} - \eta_{\perp}\mathbf{v} + \frac{e}{c}\mathbf{v}\times\mathbf{B}_{\text{eff}}$, for $\boldsymbol{\mathcal{E}} \perp \hat{\mathbf{l}}$

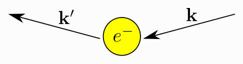
(v) $\mathbf{B}_{\text{eff}} = -\frac{c}{e}\eta_{\text{AH}}\hat{\mathbf{l}}$ $\quad B_{\text{eff}} \simeq 10^3 - 10^4\ \text{T}$!!!

(vi) $\frac{d\mathbf{v}}{dt} = 0 \rightsquigarrow \mathbf{v} = \overleftrightarrow{\mu}\boldsymbol{\mathcal{E}}$, where $\overleftrightarrow{\mu} = e\overleftrightarrow{\eta}^{-1}$

$$\mu_{\parallel} = \frac{e}{\eta_{\parallel}}\,, \quad \mu_{\perp} = e\,\frac{\eta_{\perp}}{\eta_{\perp}^2 + \eta_{\text{AH}}^2}, \quad \mu_{\text{AH}} = -e\,\frac{\eta_{\text{AH}}}{\eta_{\perp}^2 + \eta_{\text{AH}}^2}$$

▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

T-matrix description of Quasiparticle-Ion scattering



(i) Lippmann-Schwinger equation for the T-matrix ($\varepsilon = E + i\eta\,;\ \eta \to 0^+$):

$$\hat{T}^R_S(\mathbf{k}',\mathbf{k},E) = \hat{T}^R_N(\mathbf{k}',\mathbf{k}) + \int \frac{d^3k''}{(2\pi)^3} \hat{T}^R_N(\mathbf{k}',\mathbf{k}'') \Big[\hat{G}^R_S(\mathbf{k}'',E) - \hat{G}^R_N(\mathbf{k}'',E)\Big] \hat{T}^R_S(\mathbf{k}'',\mathbf{k},E)$$

$$\hat{G}^R_S(\mathbf{k},E) = \frac{1}{\varepsilon^2 - E^2_{\mathbf{k}}} \begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad E_{\mathbf{k}} = \sqrt{\xi_k^2 + |\Delta(\hat{\mathbf{k}})|^2}, \ \ \xi_k = \frac{\hbar^2 k^2}{2m^*} - \mu$$

(ii) Normal-state T-matrix:

$$\hat{T}^R_N(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = \begin{pmatrix} t^R_N(\hat{\mathbf{k}}',\hat{\mathbf{k}}) & 0 \\ 0 & -[t^R_N(-\hat{\mathbf{k}}',-\hat{\mathbf{k}})]^\dagger \end{pmatrix}$$ in p-h (Nambu) space, where

$$t^R_N(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = -\frac{1}{\pi N_f} \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin\delta_l P_l(\hat{\mathbf{k}}' \cdot \hat{\mathbf{k}}),$$ $P_l(x)$ – Legendre function

- Hard-sphere potential $\rightsquigarrow \tan\delta_l = j_l(k_f R)/n_l(k_f R)$ – spherical Bessel functions

$k_f R$ – the only parameter to be determined by experiment!

$$\hat{\mathcal{G}}^R_S(\mathbf{r}',\mathbf{r},E) = \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} e^{i\mathbf{k}'\mathbf{r}'} e^{-i\mathbf{k}\mathbf{r}} \hat{\mathcal{G}}^R_S(\mathbf{k}',\mathbf{k},E)$$

$$\hat{\mathcal{G}}^R_S(\mathbf{k}',\mathbf{k},E) = (2\pi)^3 \hat{G}^R_S(\mathbf{k},E)\delta_{\mathbf{k}',\mathbf{k}} + \hat{G}^R_S(\mathbf{k}',E)\hat{T}_S(\mathbf{k}',\mathbf{k},E)\hat{G}^R_S(\mathbf{k},E)$$

$$\hat{G}^R_S(\mathbf{k},E) = \frac{1}{\varepsilon^2 - E^2_{\mathbf{k}}} \begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad \varepsilon = E + i\eta, \quad \eta \to 0^+$$

$$N(\mathbf{r},E) = -\frac{1}{2\pi} \mathrm{Im}\left\{ \mathrm{Tr}\left[\hat{\mathcal{G}}^R_S(\mathbf{r},\mathbf{r},E) \right] \right\}$$

$$\mathbf{j}(\mathbf{r}) = \frac{\hbar}{4mi} k_B T \sum_{n=-\infty}^{\infty} \lim_{\mathbf{r}\to\mathbf{r}'} \mathrm{Tr}\left[(\boldsymbol{\nabla}_{\mathbf{r}'} - \boldsymbol{\nabla}_{\mathbf{r}}) \hat{\mathcal{G}}^M(\mathbf{r}',\mathbf{r},\epsilon_n) \right]$$

$$\hat{\mathcal{G}}^R_S(\mathbf{r}',\mathbf{r},E) = \hat{\mathcal{G}}^M_S(\mathbf{r}',\mathbf{r},\epsilon_n)\Big|_{i\epsilon_n \to \varepsilon}, \text{ for } n \geq 0$$

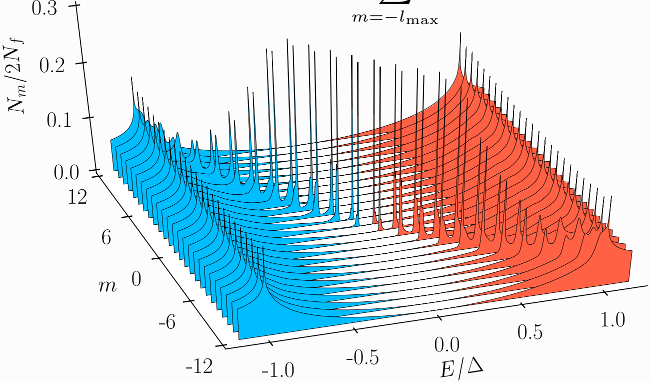
$$\hat{\mathcal{G}}^M_S(\mathbf{k},\mathbf{k}',-\epsilon_n) = \left[\hat{\mathcal{G}}^M_S(\mathbf{k}',\mathbf{k},\epsilon_n) \right]^\dagger$$

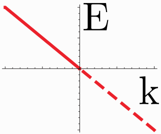
Weyl Fermion Spectrum bound to the Electron Bubble

$$\mu_N = \frac{e}{n_3 p_f \sigma_N^{tr}} \quad \Leftarrow \quad \mu_N^{exp} = 1.7 \times 10^{-6} \, \frac{m^2}{V\, s}$$

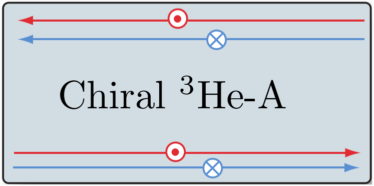
$$\tan\delta_l = j_l(k_f R)/n_l(k_f R) \quad \Rightarrow \quad \sigma_N^{tr} = \frac{4\pi}{k_f^2}\sum_{l=0}^{\infty}(l+1)\sin^2(\delta_{l+1}-\delta_l) \quad \rightsquigarrow \quad k_f R = 11.17$$

$$N(\mathbf{r}, E) = \sum_{m=-l_{max}}^{l_{max}} N_m(\mathbf{r}, E), \quad l_{max} \simeq k_f R$$

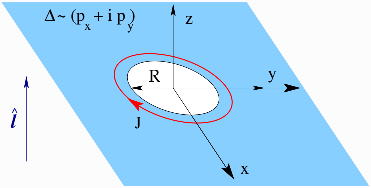


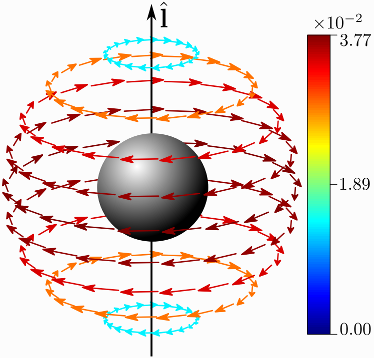


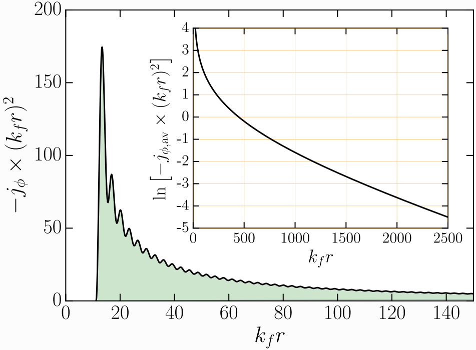
Current density bound to an electron bubble ($k_f R = 11.17$)



$\Longrightarrow$

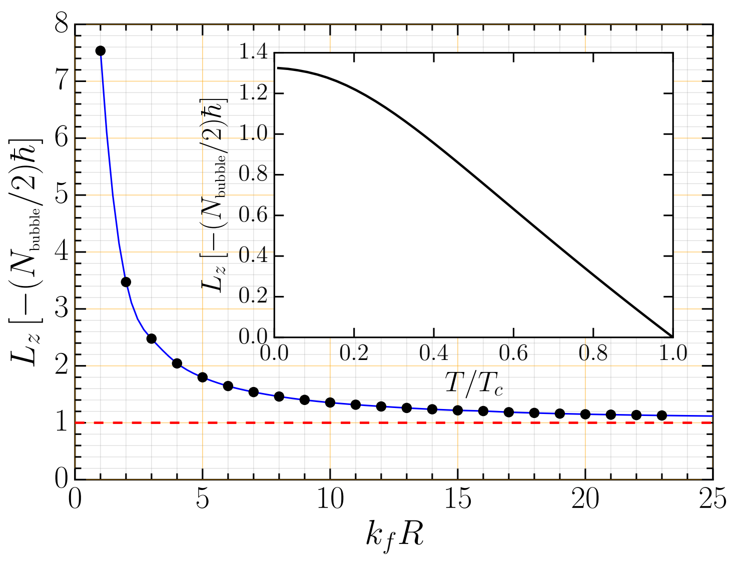






$$\mathbf{j}(\mathbf{r})/v_f N_f k_B T_c = j_\phi(\mathbf{r})\hat{\mathbf{e}}_\phi \quad \Longrightarrow \quad \mathbf{L}(T \to 0) \approx -\hbar N_{\mathrm{bubble}}\hat{\mathbf{l}}/2$$

$$\mathbf{L}(T \rightarrow 0) \approx -\hbar N_{\text{bubble}} \hat{\mathbf{l}}/2 \,; \qquad N_{\text{bubble}} = n_3 \, \frac{4\pi}{3} R^3 \approx 200 \; ^3\text{He atoms}$$



(i) Fermi's golden rule and the QP scattering rate:

$$\Gamma(\mathbf{k}',\mathbf{k}) = \frac{2\pi}{\hbar} W(\hat{\mathbf{k}}',\hat{\mathbf{k}})\delta(E_{\mathbf{k}'} - E_{\mathbf{k}}), \;\; W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = \frac{1}{2}\sum_{\tau'\sigma';\tau\sigma} |\overbrace{\langle\,\mathbf{k}',\sigma',\tau'\,|}^{\text{outgoing}}\hat{T}_S\overbrace{|\,\mathbf{k},\sigma,\tau\,\rangle}^{\text{incoming}}|^2$$

(ii) Drag force from QP-ion collisions (linear in $\mathbf{v}$): ▶ Baym et al. PRL **22**, 20 (1969)

$$\mathbf{F}_{\text{QP}} = -\sum_{\mathbf{k},\mathbf{k}'} \hbar(\mathbf{k}'-\mathbf{k})\left[\hbar\mathbf{k}'\mathbf{v} f_{\mathbf{k}}\left(-\frac{\partial f_{\mathbf{k}'}}{\partial E}\right) - \hbar\mathbf{k}\mathbf{v}(1-f_{\mathbf{k}'})\left(-\frac{\partial f_{\mathbf{k}}}{\partial E}\right)\right]\Gamma(\mathbf{k}',\mathbf{k})$$

(iii) Broken microscopic reversibility:

Broken TR and mirror symmetries in ^{3}He-A $\Rightarrow$ fixed $\hat{\mathbf{l}}$ $\rightsquigarrow$ $W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) \neq W(\hat{\mathbf{k}},\hat{\mathbf{k}}')$

(iv) Generalized Stokes tensor:

$$\mathbf{F}_{\text{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v} \;\; \rightsquigarrow \;\; \eta_{ij} = n_3 p_f \int_0^{\infty} dE\left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E), \quad \overleftrightarrow{\eta} = \begin{pmatrix} \eta_{\perp} & \eta_{\text{AH}} & 0 \\ -\eta_{\text{AH}} & \eta_{\perp} & 0 \\ 0 & 0 & \eta_{\parallel} \end{pmatrix}$$

$n_3 = \frac{k_f^3}{3\pi^2}$ – ^{3}He particle density, $\sigma_{ij}(E)$ – transport scattering cross section,

$f(E) = [\exp(E/k_B T) + 1]^{-1}$ – Fermi Distribution

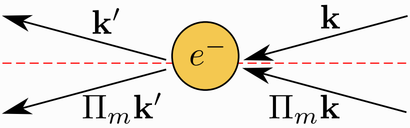
Mirror-symmetric scattering ⇒ longitudinal drag force

$$\mathbf{F}_{\text{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^\infty dE\left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E)$$

Subdivide by mirror symmetry:

$$W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}),$$

$$\sigma_{ij}(E) = \sigma_{ij}^{(+)}(E) + \sigma_{ij}^{(-)}(E),$$



$$\sigma_{ij}^{(+)}(E) = \frac{3}{4}\int_{E\geq|\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'}\int_{E\geq|\Delta(\hat{\mathbf{k}})|}\frac{d\Omega_{\mathbf{k}}}{4\pi}\left[(\hat{\mathbf{k}}'_i-\hat{\mathbf{k}}_i)(\hat{\mathbf{k}}'_j-\hat{\mathbf{k}}_j)\right]\frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E)$$

Mirror-symmetric cross section: $W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W(\hat{\mathbf{k}},\hat{\mathbf{k}}')]/2$

$$\frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2\frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}}')|^2}}W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}})\frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}})|^2}}$$

$\rightsquigarrow$ Stokes Drag $\eta_{xx}^{(+)} = \eta_{yy}^{(+)} \equiv \eta_\perp,\ \eta_{zz}^{(+)} \equiv \eta_\parallel$, No transverse force $\left[\eta_{ij}^{(+)}\right]_{i\neq j} = 0$

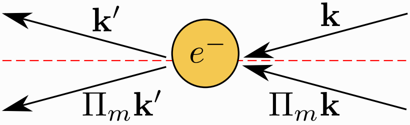
Mirror-antisymmetric scattering ⇒ transverse force

$$\mathbf{F}_{\text{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^\infty dE \left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E)$$

Subdivide by mirror symmetry:

$$W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}),$$

$$\sigma_{ij}(E) = \sigma^{(+)}_{ij}(E) + \sigma^{(-)}_{ij}(E),$$



$$\sigma^{(-)}_{ij}(E) = \frac{3}{4}\int_{E\geq|\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'} \int_{E\geq|\Delta(\hat{\mathbf{k}})|} \frac{d\Omega_{\mathbf{k}}}{4\pi}\left[\epsilon_{ijk}(\hat{\mathbf{k}}'\times\hat{\mathbf{k}})_k\right]\frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E)\left[f(E)-\frac{1}{2}\right]$$

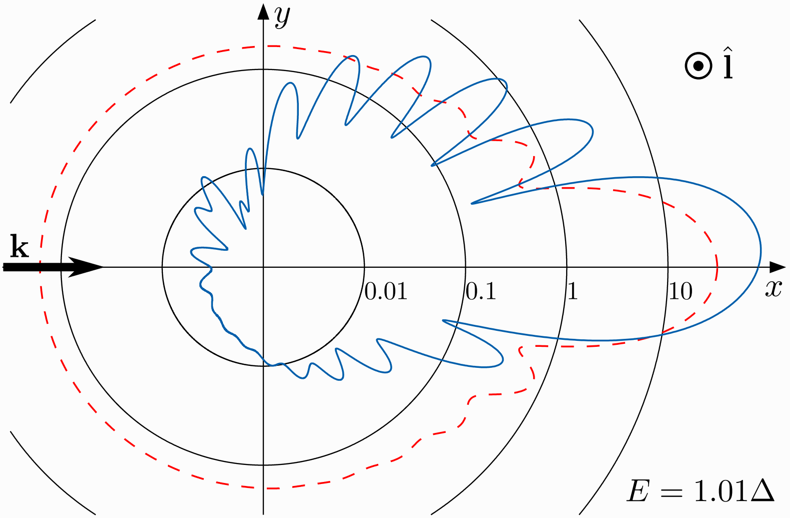
Mirror-antisymmetric cross section: $W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) - W(\hat{\mathbf{k}},\hat{\mathbf{k}}')]/2$

$$\frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2 \frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}}')|^2}} W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) \frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}})|^2}}$$

Transverse force $\quad \eta^{(-)}_{xy} = -\eta^{(-)}_{yx} \equiv \eta_{\text{AH}} \quad \Rightarrow \quad$ anomalous Hall effect

▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

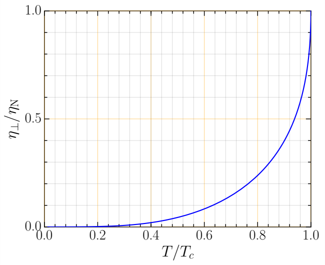
Differential cross section for Bogoliubov QP-Ion Scattering

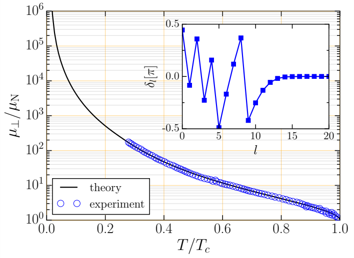


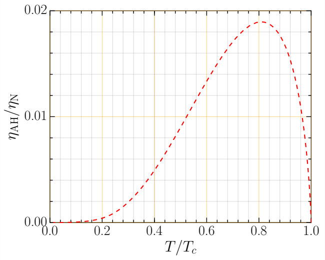
▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

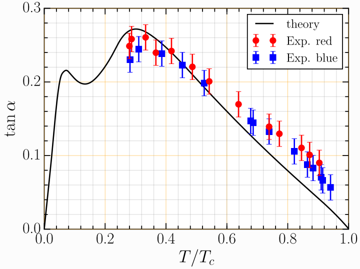
Theoretical and Experimental Comparison for the Electron Mobility in ^{3}He-A

$$\mu_\perp = e\,\frac{\eta_\perp}{\eta_\perp^2+\eta_{\mathrm{AH}}^2}, \quad \mu_{\mathrm{AH}} = -e\,\frac{\eta_{\mathrm{AH}}}{\eta_\perp^2+\eta_{\mathrm{AH}}^2}, \quad \tan\alpha = \left|\frac{\mu_{\mathrm{AH}}}{\mu_\perp}\right| = \frac{\eta_{\mathrm{AH}}}{\eta_\perp}, \quad k_f R = 11.17$$









▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Summary

- Electrons in ^{3}He-A are "dressed" by a spectrum of Weyl Fermions
- Electrons in ^{3}He-A are "Left handed" in a Right-handed Chiral Vacuum
  $\rightsquigarrow L_z \approx -(N_{bubble}/2)\hbar \approx -100\,\hbar$
- Experiment: RIKEN mobility experiments $\rightsquigarrow$ Observation an AHE in ^{3}He-A
- Scattering of Bogoliubov QPs by the dressed Ion
  $\rightsquigarrow$ Drag Force $(-\eta_\perp \mathbf{v})$ and Transverse Force $(\frac{e}{c}\mathbf{v} \times \mathbf{B}_{eff})$ on the Ion
- *Anomalous Hall Field:* $\mathbf{B}_{\text{eff}} \approx \frac{\Phi_0}{3\pi^2}\, k_f^2\, (k_f R)^2 \left(\frac{\eta_{\text{AH}}}{\eta_{\text{N}}}\right) \mathbf{l} \simeq 10^3 - 10^4\,\text{T}\,\mathbf{l}$
- Mechanism: Skew/Andreev Scattering of Bogoliubov QPs by the dressed Ion
- Origin: Broken Mirror & Time-Reversal Symmetry $\rightsquigarrow W(\mathbf{k},\mathbf{k}') \neq W(\mathbf{k}',\mathbf{k})$
- Theory: $\rightsquigarrow$ Quantitative account of RIKEN mobility experiments
- Ongoing: New directions for Novel Transport in ^{3}He-A & Chiral Superconductors