Condensed Matter Physics Seminar, UCSD, June 2, 2021

The Left Hand of the Electron *in Chiral Superconductors*

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- Oleksii Shevtsov (Boeing) • Wave Ngampruetikorn (ITS, CUNY)

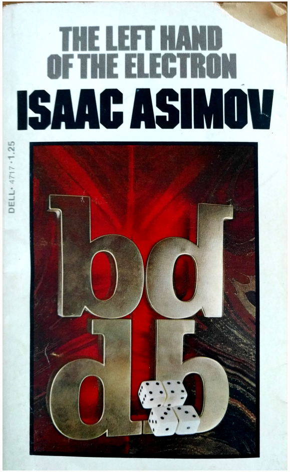
- Left-Handed Electrons in a Chiral Vacuum
- P and T violation
- Anomalous Hall Effect in ^{3}He-A
- Anomalous Thermal Hall Effect in Chiral SCs

▶NSF Grant DMR-1508730

▶H. Ikegami, Y. Tsutsumi, K. Kono, Science **341**, 59 (2013)
▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)
▶V. Ngampruetikorn and JAS, PRL 124, 157002 (2020)

The Left Hand of the Electron, Issac Asimov, circa 1971

- An Essay on the Discovery of Parity Violation by the Weak Interaction



- ... And Reflections on Mirror Symmetry in Nature

Parity Violation in Beta Decay of ^{60}Co - Physical Review 105, 1413 (1957)

Experimental Test of Parity Conservation in Beta Decay*

C. S. Wu, *Columbia University, New York, New York*

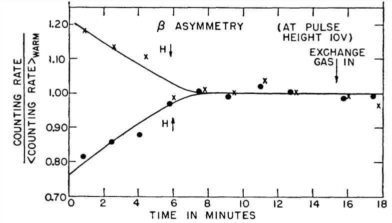
AND

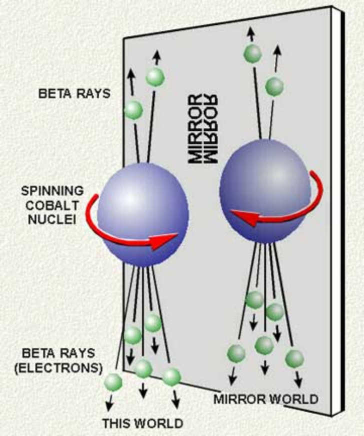
E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, *National Bureau of Standards, Washington, D. C.*

(Received January 15, 1957)

- T. D. Lee and C. N. Yang, Phys Rev 104, 204 (1956)

$$^{60}Co \to {}^{60}Ni + e^- + \bar{\nu}$$



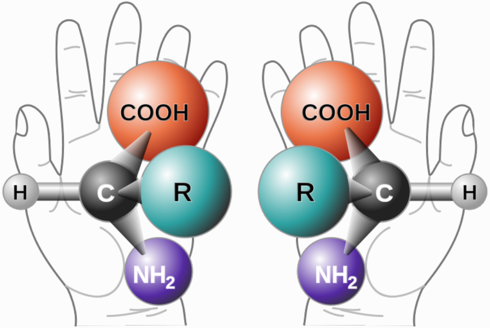


- Current of Beta electrons is (anti) correlated with the Spin of the ^{60}Co nucleus.

$\langle \vec{S} \cdot \vec{p} \rangle \neq 0 \rightsquigarrow$ Parity violation

Chiral Quantum Matter

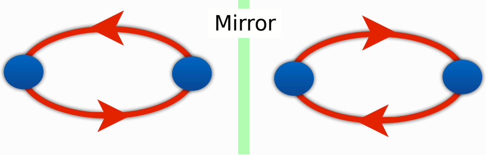
Molecular Chiral Enantiomers



Handedness: Broken Mirror Symmetry

Chiral Diatomic Molecules

$$\Psi(\mathbf{r}) = f(r)\,(x + iy)$$



Broken Mirror Symmetries

$$\Pi_{zx}\,\Psi(\mathbf{r}) = f(r)\,(x - iy)$$

Broken Time-Reversal Symmetry

$$\mathrm{T}\,\Psi(\mathbf{r}) = f(r)\,(x - iy)$$

Realized in Superfluid ^{3}He-A & possibly the ground states in unconventional superconductors

Signatures: Chiral, Edge Fermions $\rightsquigarrow$ Anomalous Hall Transport

Chiral Superconductors

Ground states exhibiting:

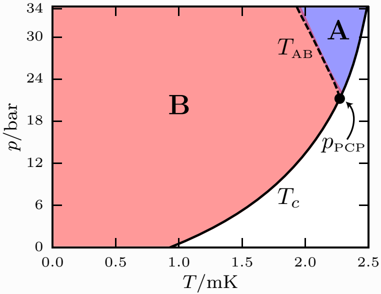
- Emergent Topology of a Broken-Symmetry Ground State of Cooper Pairs
- Weyl-Majorana excitations of the Ground State
- Ground-State Edge Currents and Angular Momemtum
- Broken P and T $\rightsquigarrow$ Anomalous Hall-Type Transport

Where are They?

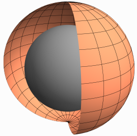
- ^{3}He-A: definitive chiral p-wave condensate; quantitative theory-experimental confirmation
- UPt_3: electronic analog to ^{3}He: Multiple Superconducting Phases; evidence of chirality
- Sr_2RuO_4: proposed as the electronic analog of ^{3}He-A; evidence of chirality, but ... d-wave?
- Other candidates: URu_2Si_2; SrPtAs, doped graphene ...

The ^{3}He Paradigm: Maximal Symmetry: $\mathtt{G} = \mathtt{SO(3)_S} \times \mathtt{SO(3)_L} \times \mathtt{U(1)_N} \times \mathtt{P} \times \mathtt{T} \rightarrow$ Superfluid Phases of ^{3}He

J. Wiman & J. A. Sauls, PRB 92, 144515 (2015)



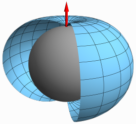
"Isotropic" BW State



$J = 0,\ J_z = 0$

$H = \mathtt{SO(3)_J} \times \mathtt{T}$

Chiral AM State $\vec{l} = \hat{\mathbf{z}}$



$L_z = 1,\ S_{z'} = 0$

$H = \mathtt{U(1)_S} \times \mathtt{U(1)_{L_z\text{-}N}} \times \mathtt{Z_2}$

Spin-Triplet BCS Condensate Amplitude: $\widehat{\Psi} = \begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix} \leftarrow \Psi_{\alpha\beta}(p) = \langle \psi_\alpha(p)\psi_\beta(-p)\rangle$

$$\widehat{\Psi}_{BW} = \begin{pmatrix} p_x - ip_y \sim e^{-i\phi} & p_z \\ p_z & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

$$\widehat{\Psi}_{AM} = \begin{pmatrix} p_x + ip_y \sim e^{+i\phi} & 0 \\ 0 & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

Fully Gapped: $\widehat{\Psi}^{\dagger}_{BW}\widehat{\Psi}_{BW} = |\Delta|^2$

Nodal Points: $\widehat{\Psi}^{\dagger}_{AM}\widehat{\Psi}_{AM} = |\Delta|^2 \sin^2\theta$

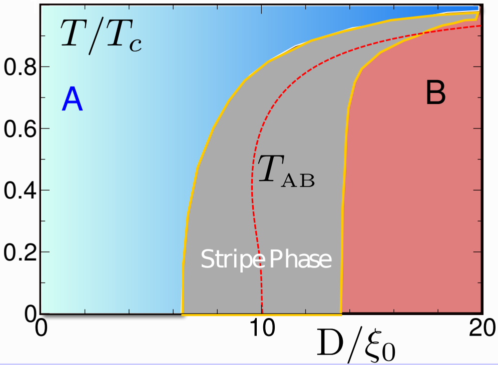
Realization of Broken Time-Reversal and Mirror Symmetry by the Ground State of ^{3}He Films

$$\mathrm{SO(3)_S \times SO(3)_L \times U(1)_N \times \boxed{T} \times \boxed{P}}$$
$$\Downarrow$$
$$\mathrm{SO(2)_S \times U(1)_{N\text{-}L_z} \times \boxed{Z_2}}$$

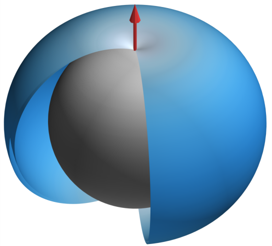
- Length Scale for Strong Confinement:

$$\xi_0 = \hbar v_f / 2\pi k_B T_c \approx 20 - 80\,\mathrm{nm}$$

- L. Levitov et al., Science 340, 6134 (2013)
- A. Vorontsov & J. A. Sauls, PRL 98, 045301 (2007)



Chiral ABM State $\vec{l} = \hat{\mathbf{z}}$



$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix}_{ABM} = \begin{pmatrix} p_x + ip_y \sim \boxed{e^{+i\phi}} & 0 \\ 0 & p_x + ip_y \sim \boxed{e^{+i\phi}} \end{pmatrix}$$

Realization of Broken Time-Reversal and Mirror Symmetry by the Ground State of ^{3}He Films

$$SO(3)_S \times SO(3)_L \times U(1)_N \times \boxed{T} \times \boxed{P}$$
$$\Downarrow$$
$$SO(2)_S \times U(1)_{N-L_z} \times \boxed{Z_2}$$

- Length Scale for Strong Confinement:

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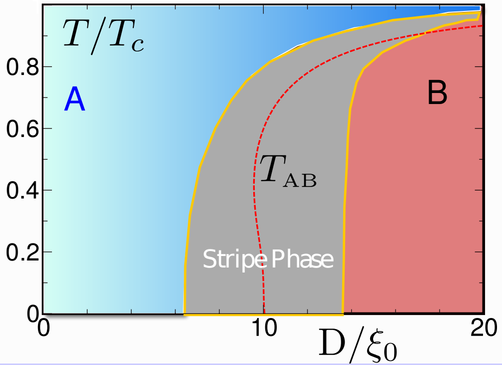
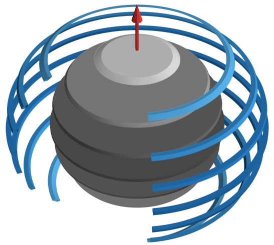


Chiral ABM State $\vec{l} = \hat{\mathbf{z}}$



$$L_z = 1,\; S_z = 0$$

Ground-State Angular Momentum

$$\langle \widehat{L}_z \rangle = \frac{N}{2}\hbar \;\; ?$$

Open Question

- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix}_{ABM} = \begin{pmatrix} p_x + ip_y \sim \boxed{e^{+i\phi}} & 0 \\ 0 & p_x + ip_y \sim \boxed{e^{+i\phi}} \end{pmatrix}$$

Signatures of Broken T and P Symmetry in ^{3}He-A

Evidence for the Chirality of Superfluid ^{3}He-A

$\Downarrow$

Broken T and P $\rightsquigarrow$ Anomalous Hall Effect for Electrons in ^{3}He-A

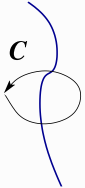
Broken Symmetries $\rightsquigarrow$ Topology of ^{3}He-A

Chirality $+$ Topology $\rightsquigarrow$ Chiral Edge States

Real-Space vs. Momentum-Space Topology

Topology in Real Space

$\Psi(\mathbf{r}) = |\Psi(r)|\, e^{i\vartheta(\mathbf{r})}$



Phase Winding

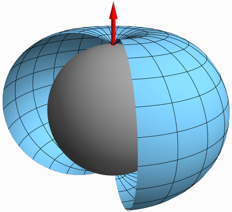
$$N_C = \frac{1}{2\pi} \oint_C d\vec{l} \cdot \frac{1}{|\Psi|} \mathrm{Im}[\boldsymbol{\nabla}\Psi] \in \{0, \pm 1, \pm 2, \dots\}$$

▶ Massless Fermions confined in the Vortex Core

Chiral Symmetry ⇝

Topology in Momentum Space

$\Psi(\mathbf{p}) = \Delta(p_x \pm i p_y) \sim \boxed{e^{\pm i\varphi_{\mathbf{p}}}}$



Topological Quantum Number: $L_z = \pm 1$

$$N_{2D} = \frac{1}{2\pi} \oint d\mathbf{p} \cdot \frac{1}{|\Psi(\mathbf{p})|} \mathrm{Im}[\boldsymbol{\nabla}_{\mathbf{p}}\Psi(\mathbf{p})] = L_z$$

▶ Massless Chiral Fermions

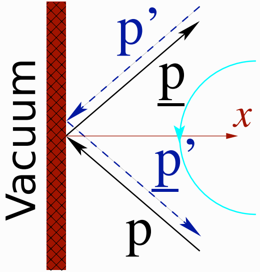
▶ Nodal Fermions in 3D

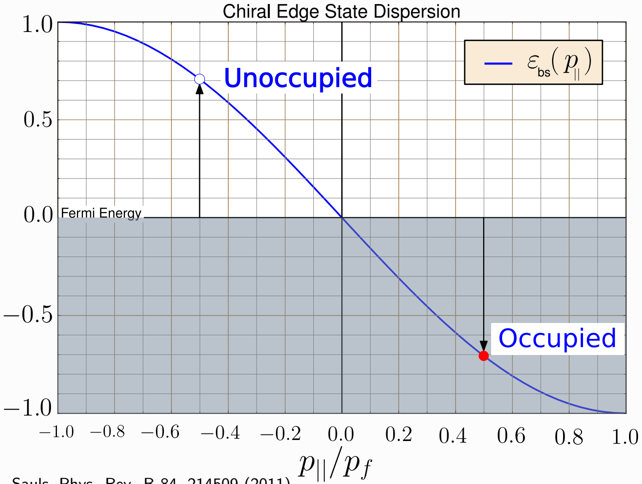
▶ Edge Fermions in 2D

Massless Chiral Fermions in the 2D ^{3}He-A Films

Edge Fermions: $G^{\mathrm{R}}_{\text{edge}}(\mathbf{p},\varepsilon;x) = \dfrac{\pi\Delta|\mathbf{p}_x|}{\varepsilon + i\gamma - \varepsilon_{\text{bs}}(\mathbf{p}_{||})}$ e^{-x/ξ_Δ} $\xi_\Delta = \hbar v_f/2\Delta \approx 10^2\,\mathring{A} \gg \hbar/p_f$

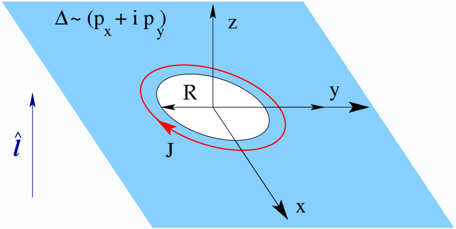
- $\varepsilon_{\text{bs}} = -c\,p_{||}$ with $c = \Delta/p_f \ll v_f$
- Broken P & T $\rightsquigarrow$ Edge Current





- M. Stone, R. Roy, PRB 69, 184511 (2004)
- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

Chiral Edge Current Circulating a Hole or Defect in a Chiral Superfluid



- $R \gg \xi_0 \approx 100\,\text{nm}$
- Sheet Current :

$$J \equiv \int dx\, J_\varphi(x)$$

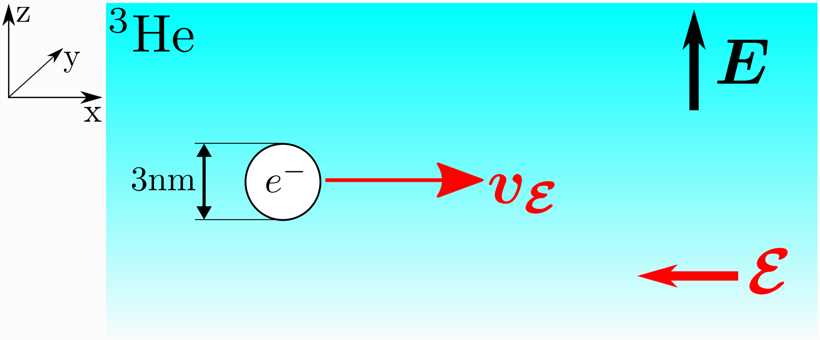
- Quantized Sheet Current: $\frac{1}{4}\, n\, \hbar$ $(n = N/V = {}^3\text{He density})$
- Edge Current *Counter*-Circulates: $J = -\frac{1}{4}\, n\, \hbar$ w.r.t. Chirality: $\hat{\mathbf{l}} = +\mathbf{z}$
- Angular Momentum: $L_z = 2\pi\, h\, R^2 \times (-\frac{1}{4}\, n\, \hbar) = -(N_{\text{hole}}/2)\, \hbar$

$N_{\text{hole}}/2 =$ Number of ^{3}He Cooper Pairs excluded from the Hole

$\therefore$ An object in ^{3}He-A *inherits* angular momentum from the Condensate of Chiral Pairs!

- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

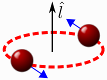
Electron bubbles in the Normal Fermi liquid phase of ^{3}He



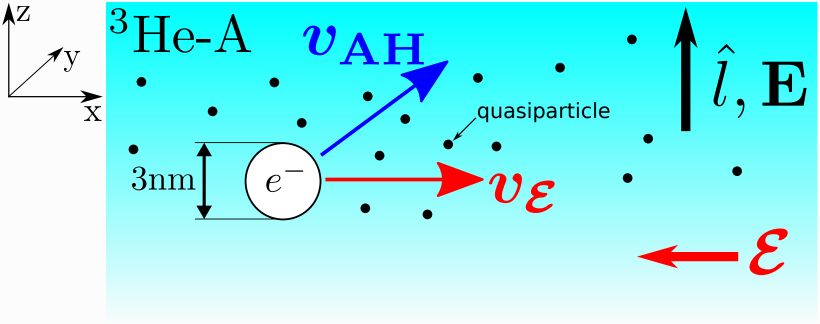
- Bubble with $R \simeq 1.5$ nm, $0.1\,\text{nm} \simeq \lambda_f \ll R \ll \xi_0 \simeq 80\,\text{nm}$
- Effective mass $M \simeq 100 m_3$ (m_3 – atomic mass of ^{3}He)
- QPs mean free path $l \gg R$
- Mobility of ^{3}He is *independent of* T for $T_c < T < 50$ mK

B. Josephson and J. Leckner, PRL 23, 111 (1969)

Electron bubbles in chiral superfluid ^{3}He-A

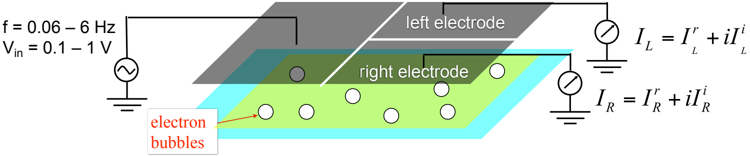


$\Delta(\hat{k}) = \Delta(\hat{k}_x + i\hat{k}_y) = \Delta\, e^{i\phi_{\mathbf{k}}}$



- Current: $\mathbf{v} = \overbrace{\mu_\perp \boldsymbol{\mathcal{E}}}^{\mathbf{v}_{\mathcal{E}}} + \overbrace{\mu_{\text{AH}} \boldsymbol{\mathcal{E}} \times \hat{\mathbf{l}}}^{\mathbf{v}_{\text{AH}}}$ R. Salmelin, M. Salomaa & V. Mineev, PRL **63**, 868 (1989)
- Hall ratio: $\boxed{\tan\alpha = v_{\text{AH}}/v_{\mathcal{E}} = |\mu_{\text{AH}}/\mu_\perp|}$

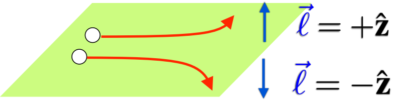
Measurement of the Transverse e$^-$ mobility in Superfluid ^{3}He Films



Transverse Force from ***Skew* Scattering**

$$\rightsquigarrow \quad \Delta I = I_R - I_L \neq 0$$

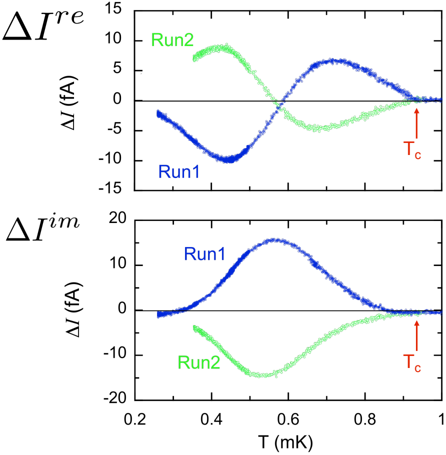
$$\vec{v} = \left[\mu_\perp \vec{E} + \boxed{\mu_{xy}\,\hat{\ell} \times \vec{E}}\right]$$

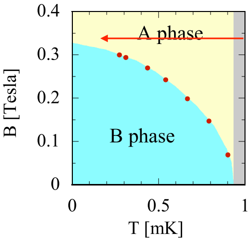


H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

Detection of Broken Time-Reversal & Mirror Symmetry in ^{3}He-A

Transverse e$^-$ bubble current in ^{3}He-A $\quad \Delta I = I_R - I_L$





Single Domains:

Run 1 $\quad \vec{\ell} = +\hat{\mathbf{z}}$

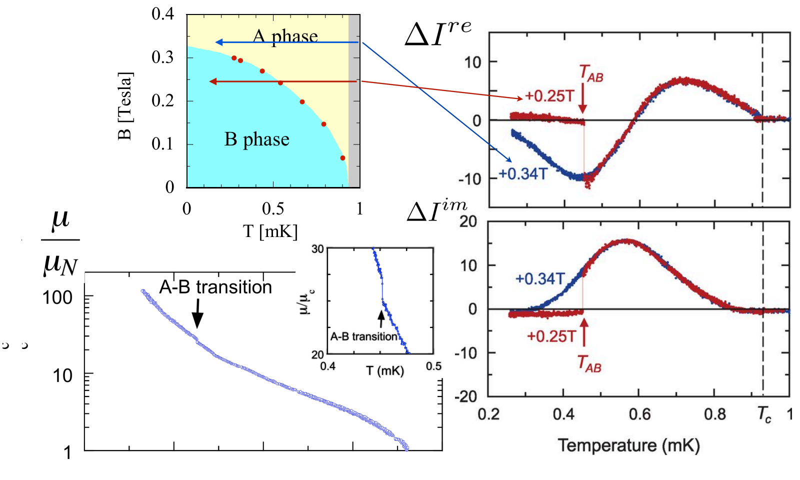
Run 2 $\quad \vec{\ell} = -\hat{\mathbf{z}}$

$$\frac{|I_R - I_L|}{I_R + I_L} \approx 6\,\%$$

H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

Detection of Broken Time-Reversal & Mirror Symmetry in ^{3}He-A

Zero Transverse e$^-$ current in ^{3}He-B (*T - symmetric phase*)



H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

- Structure of Electrons in Superfluid ^{3}He-A
- Forces of Moving Electrons in Superfluid ^{3}He-A

$$\Downarrow$$

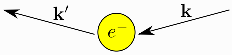
- Scattering Theory of ^{3}He Quasiparticles by Electron Bubbles

Forces on the Electron bubble in ^{3}He-A:

- $M\frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} + \mathbf{F}_{\mathrm{QP}}$, $\mathbf{F}_{QP}$ – force from quasiparticle collisions
- $\mathbf{F}_{QP} = -\overleftrightarrow{\eta} \cdot \mathbf{v}$, $\overleftrightarrow{\eta}$ – generalized Stokes tensor
- $\overleftrightarrow{\eta} = \begin{pmatrix} \eta_\perp & \eta_{\mathsf{AH}} & 0 \\ -\eta_{\mathsf{AH}} & \eta_\perp & 0 \\ 0 & 0 & \eta_\parallel \end{pmatrix}$ for broken PT symmetry with $\hat{\mathbf{l}} \parallel \mathbf{e}_z$
- $M\frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} - \eta_\perp \mathbf{v} + \frac{e}{c}\mathbf{v} \times \mathbf{B}_{\mathrm{eff}}$, for $\boldsymbol{\mathcal{E}} \perp \hat{\mathbf{l}}$
- $\mathbf{B}_{\mathrm{eff}} = -\frac{c}{e}\eta_{\mathsf{AH}}\hat{\mathbf{l}}$ $\quad B_{\mathrm{eff}} \simeq 10^3 - 10^4\ \mathrm{T}$!!!
- Mobility: $\frac{d\mathbf{v}}{dt} = 0 \rightsquigarrow \mathbf{v} = \overleftrightarrow{\mu}\boldsymbol{\mathcal{E}}$, where $\overleftrightarrow{\mu} = e\overleftrightarrow{\eta}^{-1}$

▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

T-matrix description of Quasiparticle-Ion scattering



- Lippmann-Schwinger equation for the T-matrix ($\varepsilon = E + i\eta\,;\ \eta \to 0^+$):

$$\hat{T}_S^R(\mathbf{k}', \mathbf{k}, E) = \hat{T}_N^R(\mathbf{k}', \mathbf{k}) + \int \frac{d^3k''}{(2\pi)^3} \hat{T}_N^R(\mathbf{k}', \mathbf{k}'') \Big[\hat{G}_S^R(\mathbf{k}'', E) - \hat{G}_N^R(\mathbf{k}'', E)\Big] \hat{T}_S^R(\mathbf{k}'', \mathbf{k}, E)$$

$$\hat{G}_S^R(\mathbf{k}, E) = \frac{1}{\varepsilon^2 - E_{\mathbf{k}}^2} \begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad E_{\mathbf{k}} = \sqrt{\xi_k^2 + |\Delta(\hat{\mathbf{k}})|^2}, \ \ \xi_k = \frac{\hbar^2 k^2}{2m^*} - \mu$$

- Normal-state T-matrix:

$$\hat{T}_N^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = \begin{pmatrix} t_N^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}) & 0 \\ 0 & -[t_N^R(-\hat{\mathbf{k}}', -\hat{\mathbf{k}})]^\dagger \end{pmatrix}$$ in p-h (Nambu) space, where

$$t_N^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = -\frac{1}{\pi N_f} \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin\delta_l P_l(\hat{\mathbf{k}}' \cdot \hat{\mathbf{k}}),$$ $P_l(x)$ – Legendre function

- Hard-sphere potential $\rightsquigarrow \tan\delta_l = j_l(k_f R)/n_l(k_f R)$ – spherical Bessel functions

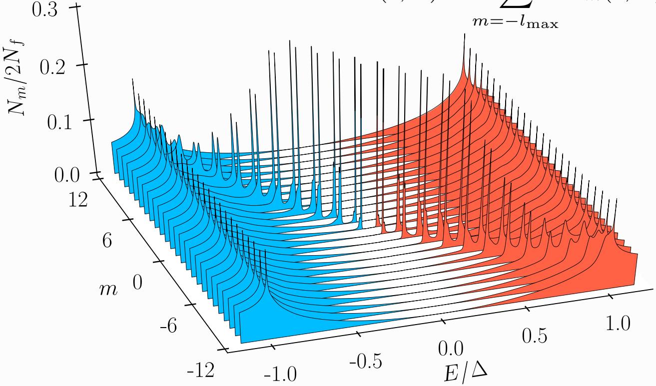
  - $k_f R$ – determined by the Normal-State Mobility $\rightsquigarrow$ $k_f R = 11.17\ (R = 1.42\,\mathrm{nm})$

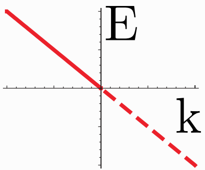
Weyl Fermion Spectrum bound to the Electron Bubble

$$\mu_{\mathsf{N}} = \frac{e}{n_3 p_f \sigma_{\mathsf{N}}^{\mathsf{tr}}} \quad \Leftarrow \quad \mu_{\mathsf{N}}^{\mathsf{exp}} = 1.7 \times 10^{-6}\,\frac{m^2}{V\,s}$$

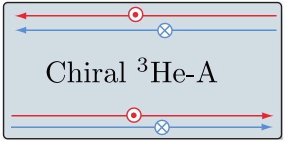
$$\tan\delta_l = j_l(k_f R)/n_l(k_f R) \quad \Rightarrow \quad \sigma_{\mathsf{N}}^{\mathsf{tr}} = \frac{4\pi}{k_f^2}\sum_{l=0}^{\infty}(l+1)\sin^2(\delta_{l+1}-\delta_l) \quad \rightsquigarrow \quad \boxed{k_f R = 11.17}$$

$$N(\mathbf{r},E) = \sum_{m=-l_{\max}}^{l_{\max}} N_m(\mathbf{r},E), \quad l_{\max} \simeq k_f R$$





Current bound to an electron bubble ($k_f R = 11.17$)



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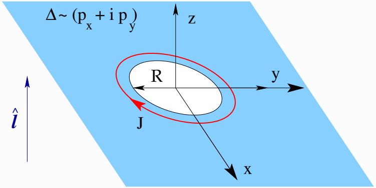
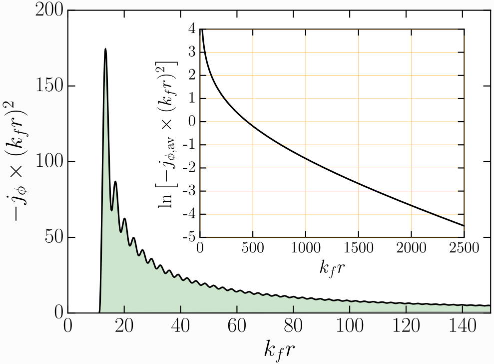
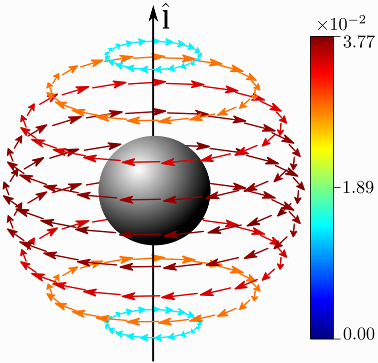




$\Longrightarrow$



$\mathbf{j}(\mathbf{r})/v_f N_f k_B T_c = j_\phi(\mathbf{r})\hat{\mathbf{e}}_\phi$

$\mathbf{L}(T \to 0) \approx -\hbar N_{\mathrm{bubble}}/2\,\hat{\mathbf{l}} \approx -100\,\hbar\,\hat{\mathbf{l}}$

▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Determination of the Stokes Tensor from the QP-Ion T-matrix

(i) Fermi's golden rule and the QP scattering rate:

$$\Gamma(\mathbf{k}',\mathbf{k}) = \frac{2\pi}{\hbar} W(\hat{\mathbf{k}}',\hat{\mathbf{k}})\delta(E_{\mathbf{k}'} - E_{\mathbf{k}}), \;\; W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = \frac{1}{2}\sum_{\tau'\sigma';\tau\sigma} |\overbrace{\langle\,\mathbf{k}',\sigma',\tau'\,|}^{\text{outgoing}}\hat{T}_S\overbrace{|\,\mathbf{k},\sigma,\tau\,\rangle}^{\text{incoming}}|^2$$

(ii) Drag force from QP-ion collisions (linear in $\mathbf{v}$): ▶Baym et al. PRL **22**, 20 (1969)

$$\mathbf{F}_{\mathsf{QP}} = -\sum_{\mathbf{k},\mathbf{k}'}\hbar(\mathbf{k}'-\mathbf{k})\left[\hbar\mathbf{k}'\mathbf{v}f_{\mathbf{k}}\left(-\frac{\partial f_{\mathbf{k}'}}{\partial E}\right) - \hbar\mathbf{k}\mathbf{v}(1-f_{\mathbf{k}'})\left(-\frac{\partial f_{\mathbf{k}}}{\partial E}\right)\right]\Gamma(\mathbf{k}',\mathbf{k})$$

(iii) Microscopic reversibility condition: $W(\hat{\mathbf{k}}',\hat{\mathbf{k}}:+\mathbf{l}) = W(\hat{\mathbf{k}},\hat{\mathbf{k}}':-\mathbf{l})$

Broken T and mirror symmetries in ^{3}He-A $\Rightarrow$ fixed $\hat{\mathbf{l}}$ $\rightsquigarrow$ $\boxed{W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) \neq W(\hat{\mathbf{k}},\hat{\mathbf{k}}')}$

(iv) Generalized Stokes tensor:

$$\mathbf{F}_{\mathsf{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v} \;\;\rightsquigarrow\;\; \boxed{\eta_{ij} = n_3 p_f \int_0^\infty dE\left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E)}\,, \quad \overleftrightarrow{\eta} = \begin{pmatrix} \eta_\perp & \eta_{\mathsf{AH}} & 0 \\ -\eta_{\mathsf{AH}} & \eta_\perp & 0 \\ 0 & 0 & \eta_\parallel \end{pmatrix}$$

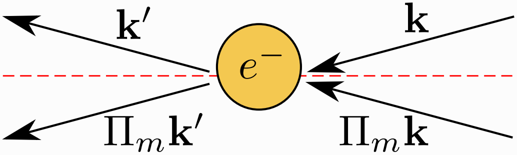
$n_3 = \dfrac{k_f^3}{3\pi^2}$ – ^{3}He particle density, $\sigma_{ij}(E)$ – transport scattering cross section,

$f(E) = [\exp(E/k_BT)+1]^{-1}$ – Fermi Distribution

Mirror-symmetric scattering $\Rightarrow$ longitudinal drag force

$$\mathbf{F}_{\mathsf{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^\infty dE\left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E)$$

Subdivide by mirror symmetry:



$$W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = \boxed{W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}})} + W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}),$$

$$\sigma_{ij}(E) = \boxed{\sigma_{ij}^{(+)}(E)} + \sigma_{ij}^{(-)}(E),$$

$$\sigma_{ij}^{(+)}(E) = \frac{3}{4}\int_{E\geq|\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'}\int_{E\geq|\Delta(\hat{\mathbf{k}})|}\frac{d\Omega_{\mathbf{k}}}{4\pi}\boxed{[(\hat{\mathbf{k}}_i'-\hat{\mathbf{k}}_i)(\hat{\mathbf{k}}_j'-\hat{\mathbf{k}}_j)]}\frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E)$$

Mirror-symmetric cross section: $\boxed{W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W(\hat{\mathbf{k}},\hat{\mathbf{k}}')]/2}$

$$\frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2\frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}}')|^2}}W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}})\frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}})|^2}}$$

$\rightsquigarrow$ Stokes Drag $\eta_{xx}^{(+)} = \eta_{yy}^{(+)} \equiv \eta_\perp,\ \eta_{zz}^{(+)} \equiv \eta_\parallel$, No transverse force $\boxed{\left[\eta_{ij}^{(+)}\right]_{i\neq j} = 0}$

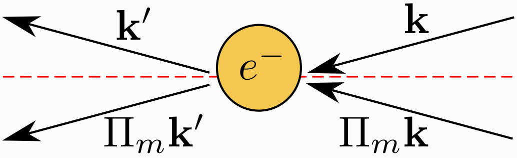
Mirror-antisymmetric scattering $\Rightarrow$ transverse force

$$\mathbf{F}_{\mathsf{QP}} = -\overleftrightarrow{\eta} \cdot \mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^\infty dE \left(-2\frac{\partial f}{\partial E}\right) \sigma_{ij}(E)$$

Subdivide by mirror symmetry:

$$W(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = W^{(+)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}) + W^{(-)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}),$$

$$\sigma_{ij}(E) = \sigma_{ij}^{(+)}(E) + \sigma_{ij}^{(-)}(E),$$



$$\sigma_{ij}^{(-)}(E) = \frac{3}{4} \int_{E \geq |\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'} \int_{E \geq |\Delta(\hat{\mathbf{k}})|} \frac{d\Omega_{\mathbf{k}}}{4\pi} \left[\epsilon_{ijk}(\hat{\mathbf{k}}' \times \hat{\mathbf{k}})_k\right] \frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E) \left[f(E) - \frac{1}{2}\right]$$

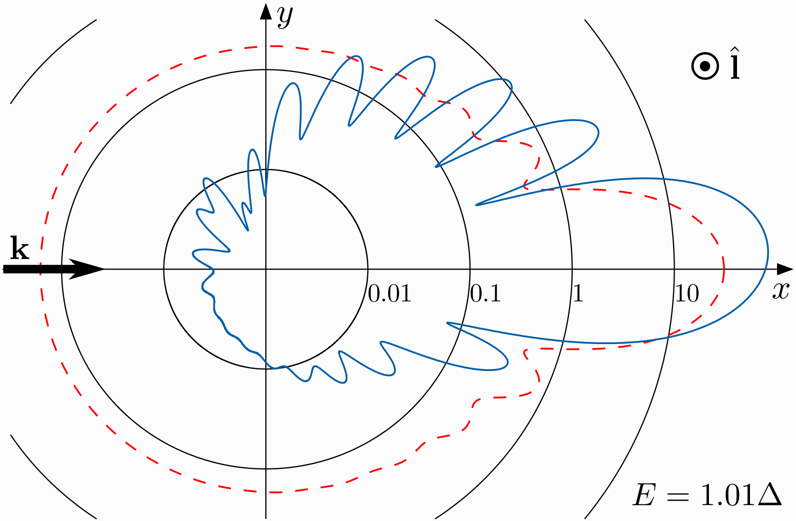
Mirror-antisymmetric cross section: $W^{(-)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}', \hat{\mathbf{k}}) - W(\hat{\mathbf{k}}, \hat{\mathbf{k}}')]/2$

$$\frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2 \frac{E}{\sqrt{E^2 - |\Delta(\hat{\mathbf{k}}')|^2}} W^{(-)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}) \frac{E}{\sqrt{E^2 - |\Delta(\hat{\mathbf{k}})|^2}}$$

Transverse force $\quad \eta_{xy}^{(-)} = -\eta_{yx}^{(-)} \equiv \eta_{\mathsf{AH}} \quad \Rightarrow \quad$ anomalous Hall effect

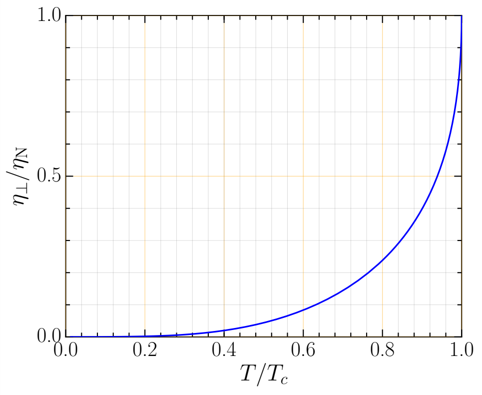
▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

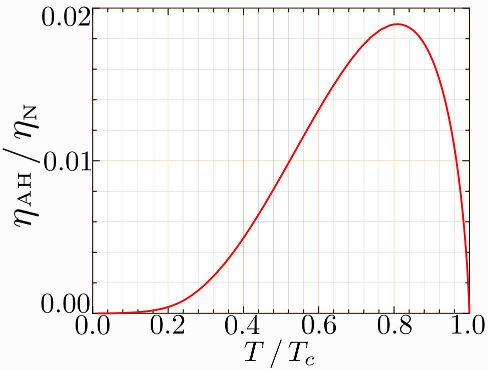
Differential cross section for Bogoliubov QP-Ion Scattering $k_f R = 11.17$



▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Theoretical Results for the Drag and Transverse Forces





- $\Delta p_x \approx p_f \qquad \sigma^{\text{tr}}_{xx} \approx \sigma^{\text{tr}}_{\text{N}} \approx \pi R^2$
- $F_x \approx n\, v_x\, \Delta p_x\, \sigma^{\text{tr}}_{xx}$
  $\approx n\, v_x\, p_f\, \sigma^{\text{tr}}_{\text{N}}$

- $\Delta p_y \approx \boxed{\hbar/R} \quad \sigma^{\text{tr}}_{xy} \approx (\Delta(T)/k_{\text{B}}T_c)^2 \sigma^{\text{tr}}_{\text{N}}$
- $F_y \approx n\, v_x\, \Delta p_y\, \sigma^{\text{tr}}_{xy}$
  $\approx n\, v_x \; \boxed{(\hbar/R)\, \sigma^{\text{tr}}_{\text{N}} (\Delta(T)/k_{\text{B}}T_c)^2}$

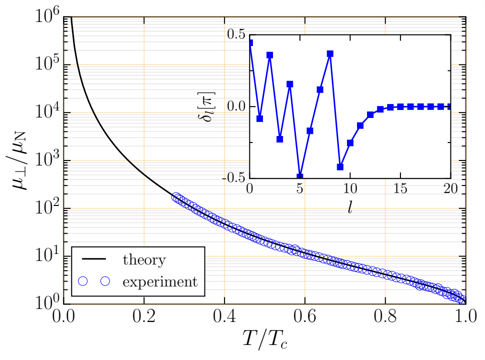
$$\boxed{|F_y/F_x| \approx \frac{\hbar}{p_f R}\,(\Delta(T)/k_B T_c)^2}$$

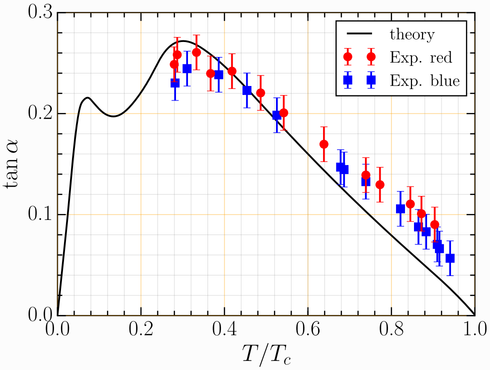
$k_f R = 11.17$

Branch Conversion Scattering in a Chiral Condensate

▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Comparison between Theory and Experiment for the Drag and Transverse Forces





- $$\mu_\perp = e\,\frac{\eta_\perp}{\eta_\perp^2 + \eta_{\mathrm{AH}}^2}$$
- $$\mu_{\mathrm{AH}} = -e\,\frac{\eta_{\mathrm{AH}}}{\eta_\perp^2 + \eta_{\mathrm{AH}}^2}$$

- $$\tan\alpha = \left|\frac{\mu_{\mathrm{AH}}}{\mu_\perp}\right| = \frac{\eta_{\mathrm{AH}}}{\eta_\perp}$$
- Electron Bubble Radius: $k_f R = 11.17$

▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

▶O. Shevtsov and JAS, JLTP 187, 340–353 (2017)

Summary

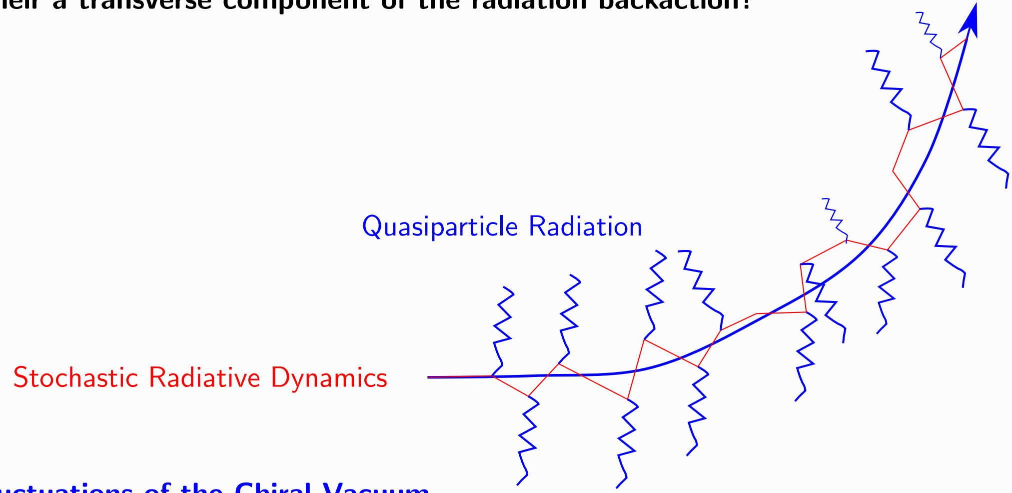
- Electrons in ^{3}He-A are "dressed" by a spectrum of Chiral Fermions
- Electrons are "Left handed" in a Right-handed Chiral Vacuum $\rightsquigarrow L_z \approx -100\,\hbar$
- Experiment: RIKEN mobility experiments $\rightsquigarrow$ Observation an AHE in ^{3}He-A
- Origin: Broken Mirror & Time-Reversal Symmetry
- Theory: Scattering of Bogoliubov QPs by the dressed Ion $\rightsquigarrow$
  - Drag Force $(-\eta_\perp \mathbf{v})$ • Transverse Force $(\frac{e}{c}\mathbf{v} \times \mathbf{B}_{\text{eff}})$
- *Anomalous Hall Field:* $\boxed{\mathbf{B}_{\text{eff}} \approx \frac{\Phi_0}{3\pi^2}\, k_f^2\, (k_f R)^2 \left(\frac{\eta_{\text{AH}}}{\eta_{\text{N}}}\right) \mathbf{l} \simeq 10^3 - 10^4\,\text{T}\,\mathbf{l}}$

N.B. This theory fails as $T \to 0$ ... but also suggests

Bulk Signature of BTRS in UPt_3,Sr_2RuO_4 $\rightsquigarrow$ Thermal Hall Effects?

Anomalous Thermal Hall Transport in ^{3}He-A & Chiral Superconductors

Is their a transverse component of the radiation backaction?



Fluctuations of the Chiral Vacuum

- Mesoscopic Ion coupled and driven through a Chiral “Bath”

Chiral superconductors

★ Majorana edge states, edge currents, exotic vortices etc.

★ Where are they?

✓✓✓ ^{3}**He-A** — Definitively a chiral p-wave superfluid

→ quantitative theory of the bulk signature of broken P & T

✓✓? **UPt$_3$** — Strong evidence of chirality (Polar Kerr effect) and recent SANS field hysteresis (Avers et al. Nat. Phys 2020)

✤ What is the precise the nature of the chiral order parameter?

✓?? **Sr$_2$RuO$_4$ & UTe$_2$** — Evidence from Polar Kerr

Zero-Field Thermal Hall Effect for studying Chiral SC

- Hall effect requires
  1. **Broken time-reversal symmetry** (TRS)
  2. **Broken mirror symmetry**

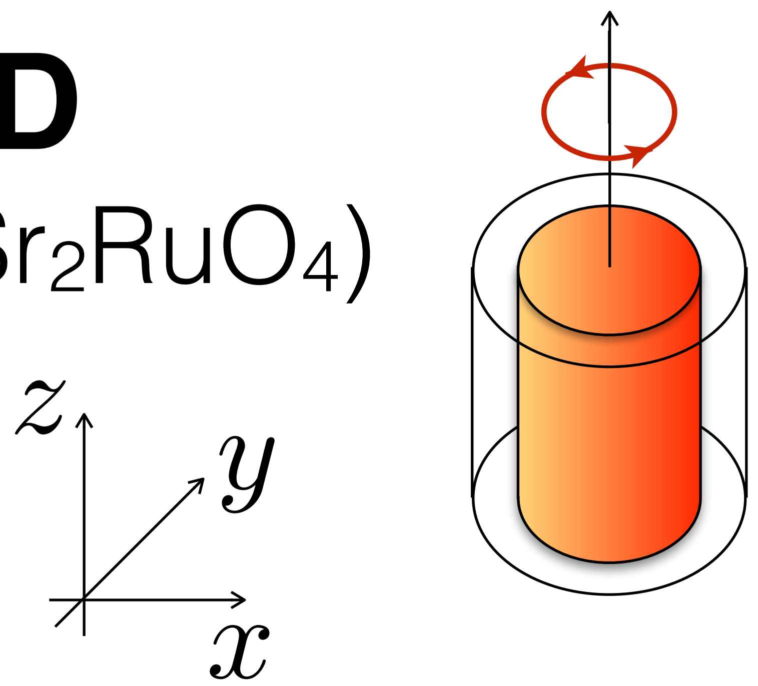
Broken in Chiral SC

- Hall experiments could …

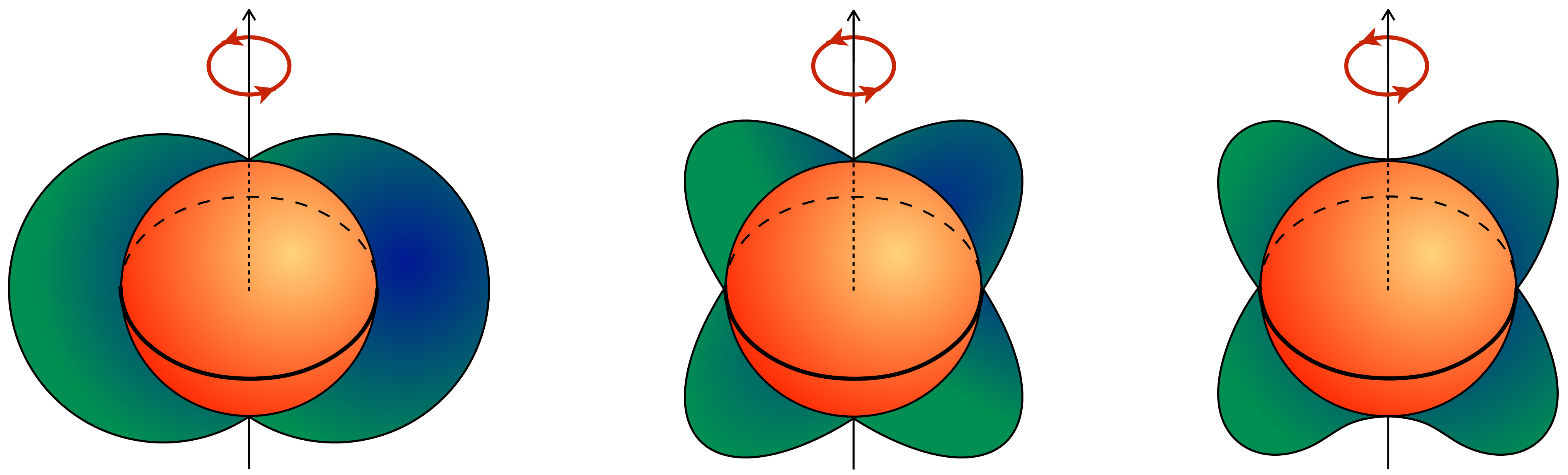
★ **Verify Chiral pairing**

★ **Identify Order parameter** (winding number & gap structure)

2D
(Sr_2RuO_4)



3D
(UPt_3)



Anomalous Thermal Hall effects from edge and bulk

Edge Hall Effect

✤ For Chiral *p*-wave ground states

[Read & Green, PRB (2000)]

sensitive

✤ **Edge Mode Spectrum is** to surface disorder

Bulk Hall Effect

★ Induced by *impurity scattering* in the bulk

★ Often dominant when present

This work

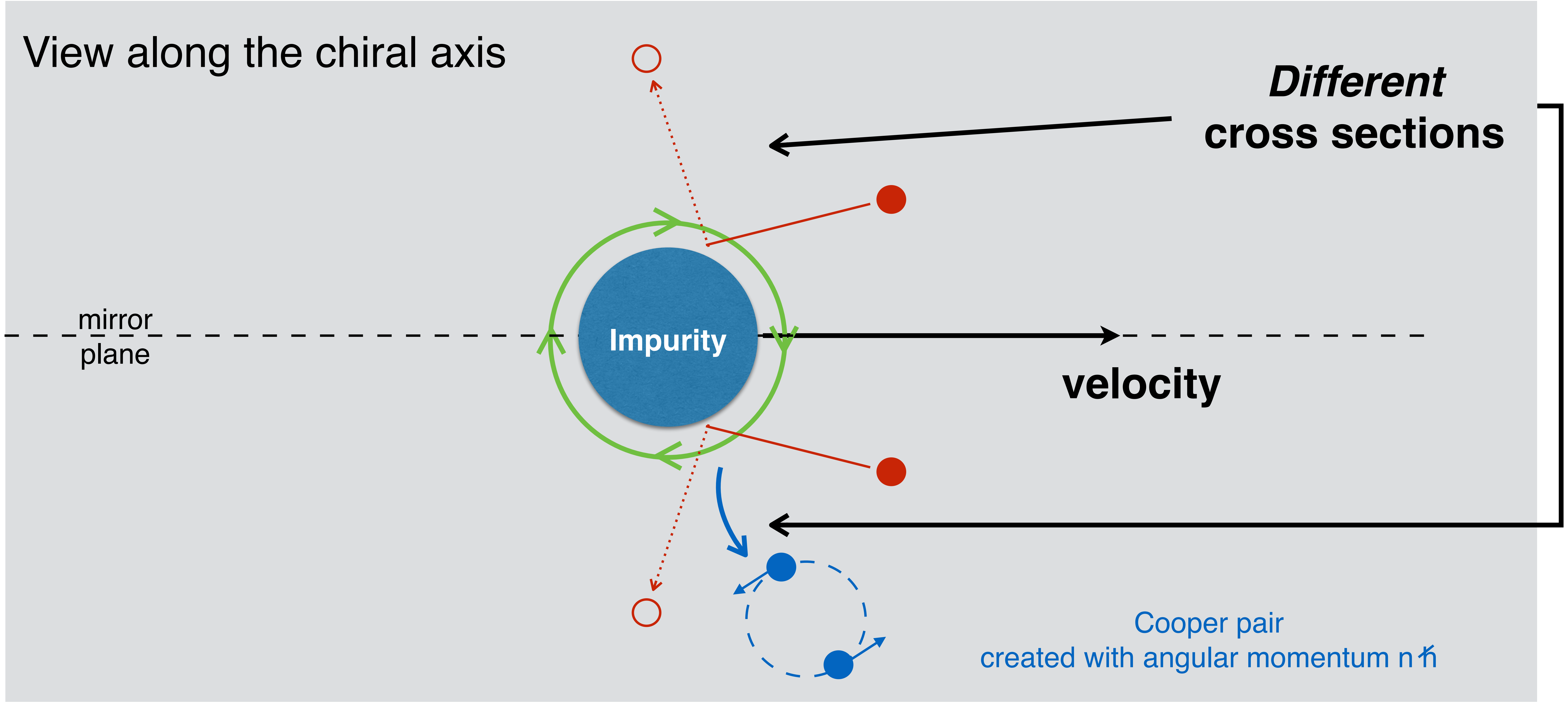
★ Both indicate **Broken Time-reversal & Mirror Symmetries** ★

Impurity ***Edge currents*** reflect broken mirror and T symmetries

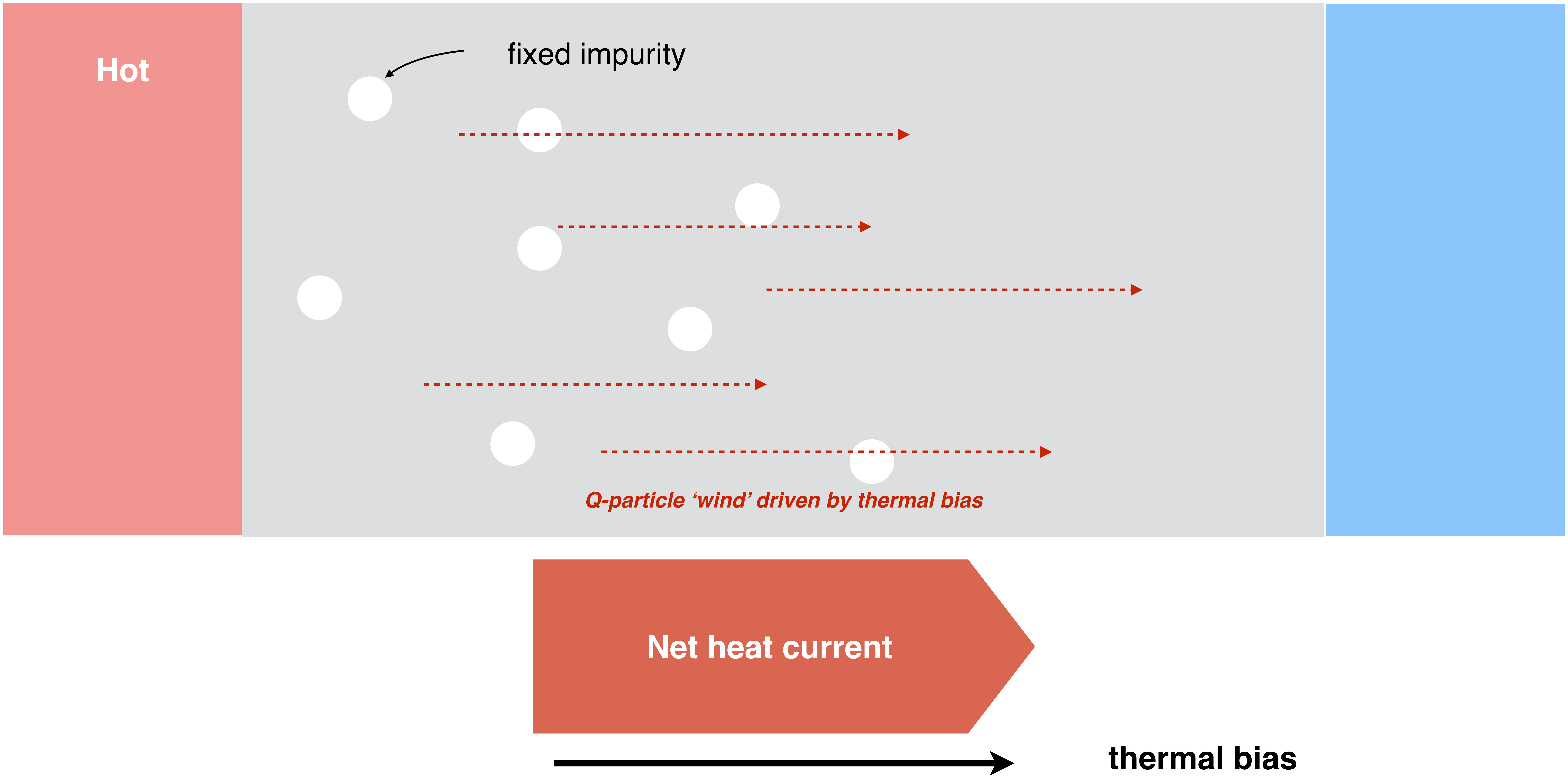
Branch conversion scattering

$e_m \longrightarrow h_{m-n}$

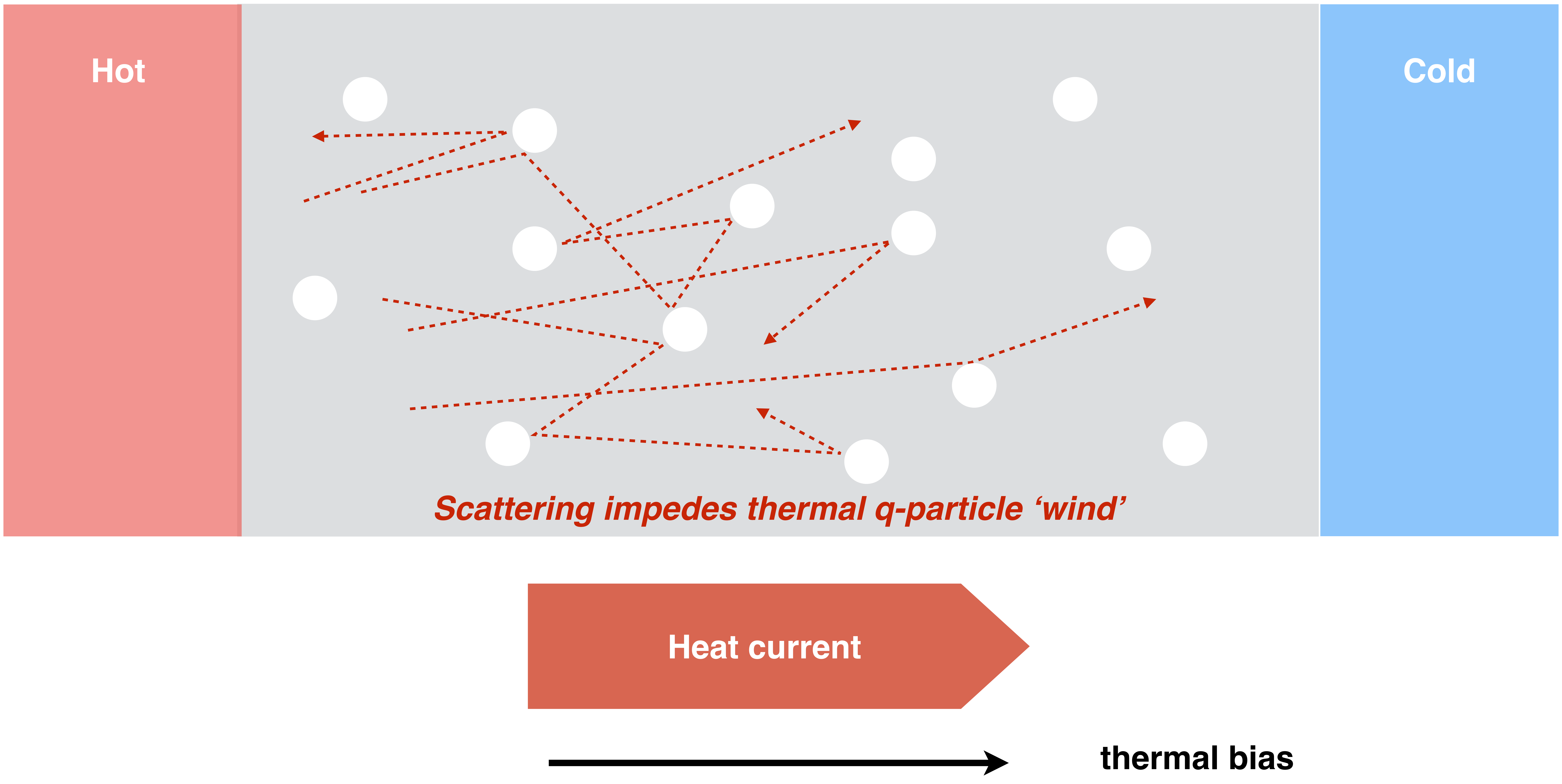
Origin of the ATHE



What happens when a q-particle ‘wind’ blows past fixed impurities?

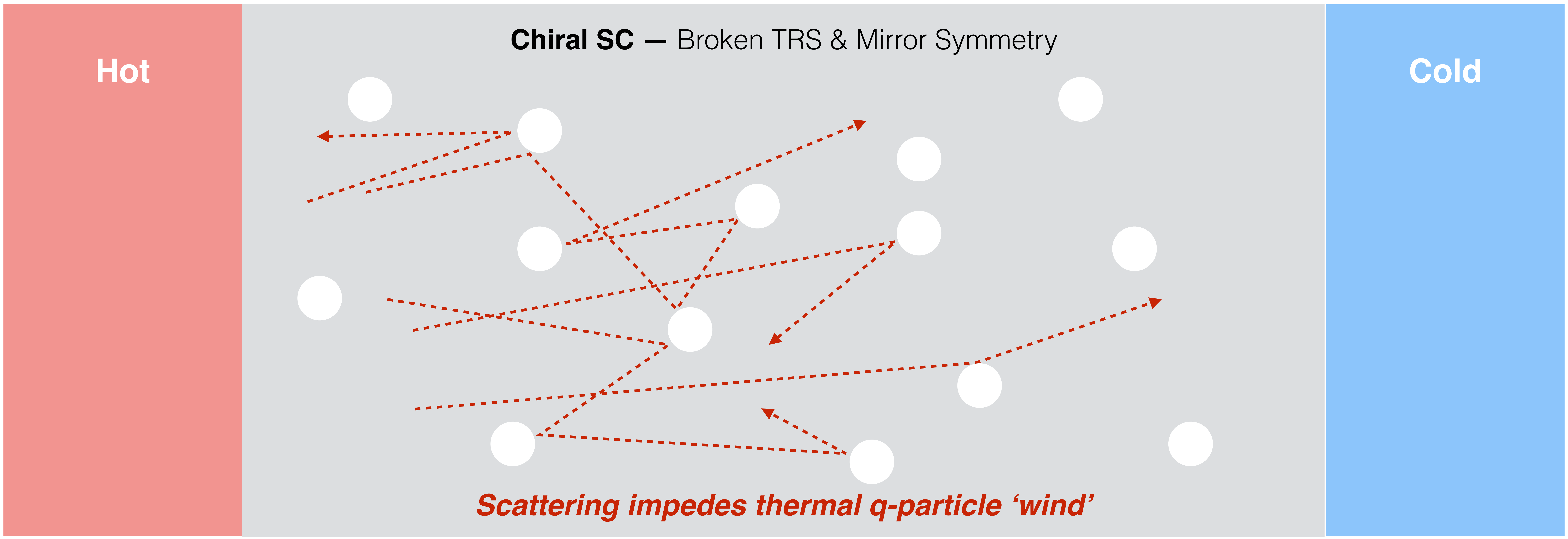


Generally scattering lowers transport conductivity

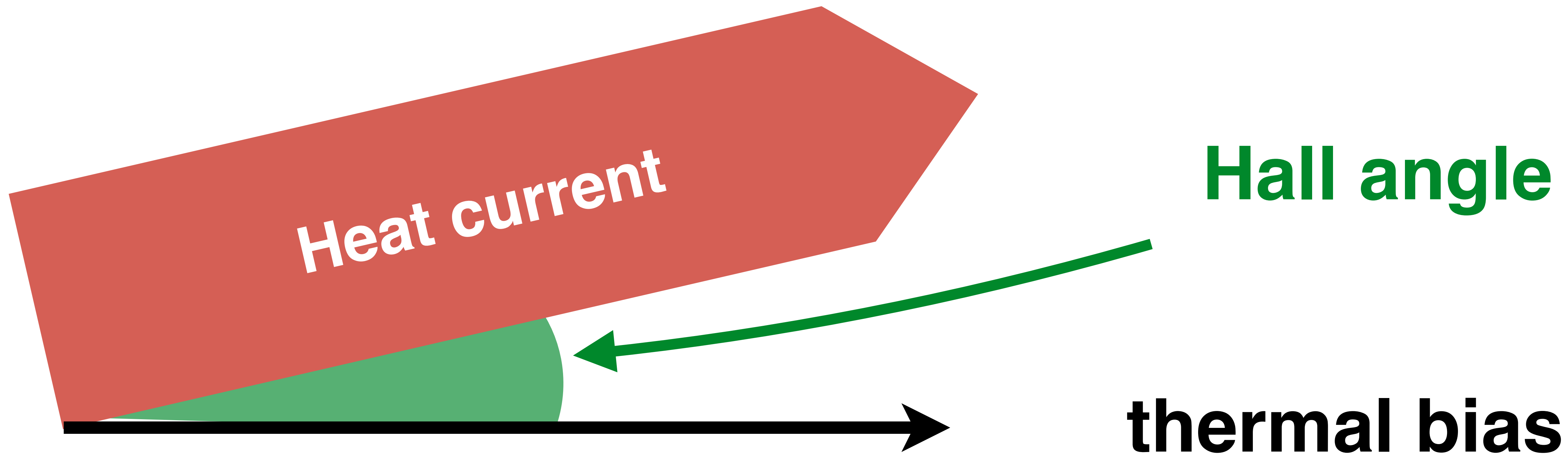


Cross section asymmetry in chiral SC ‘kicks’ q-particle sideway

The reverse of e^- bubble ^{3}He-A



Impurity scattering induces Anomalous Thermal Hall Effect!



Impurities in Chiral SCs

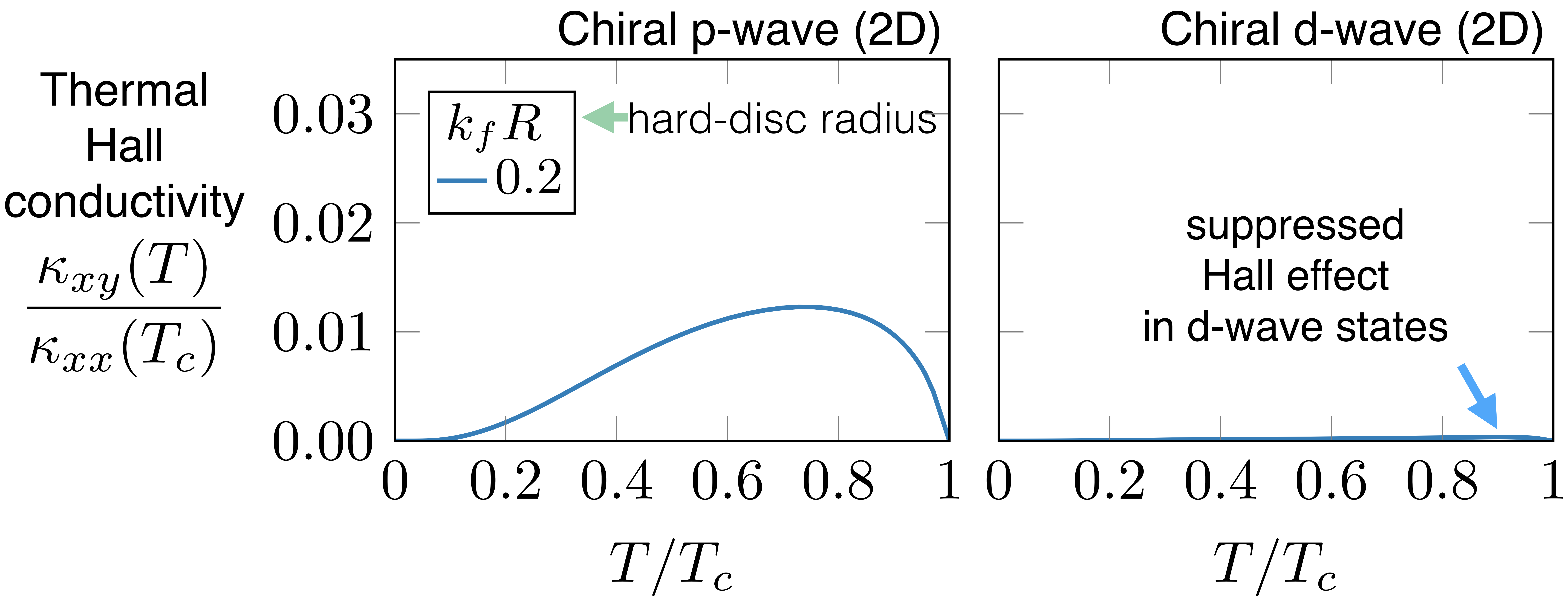
- **Standard model for impurity effects - Point-like impurities**
  — **scattering in the *s*-wave channel only**
  — ***Hall Effect ONLY in Chiral p-wave states***

★ **Point-like means $k_f R \ll 1$**

It's never true!

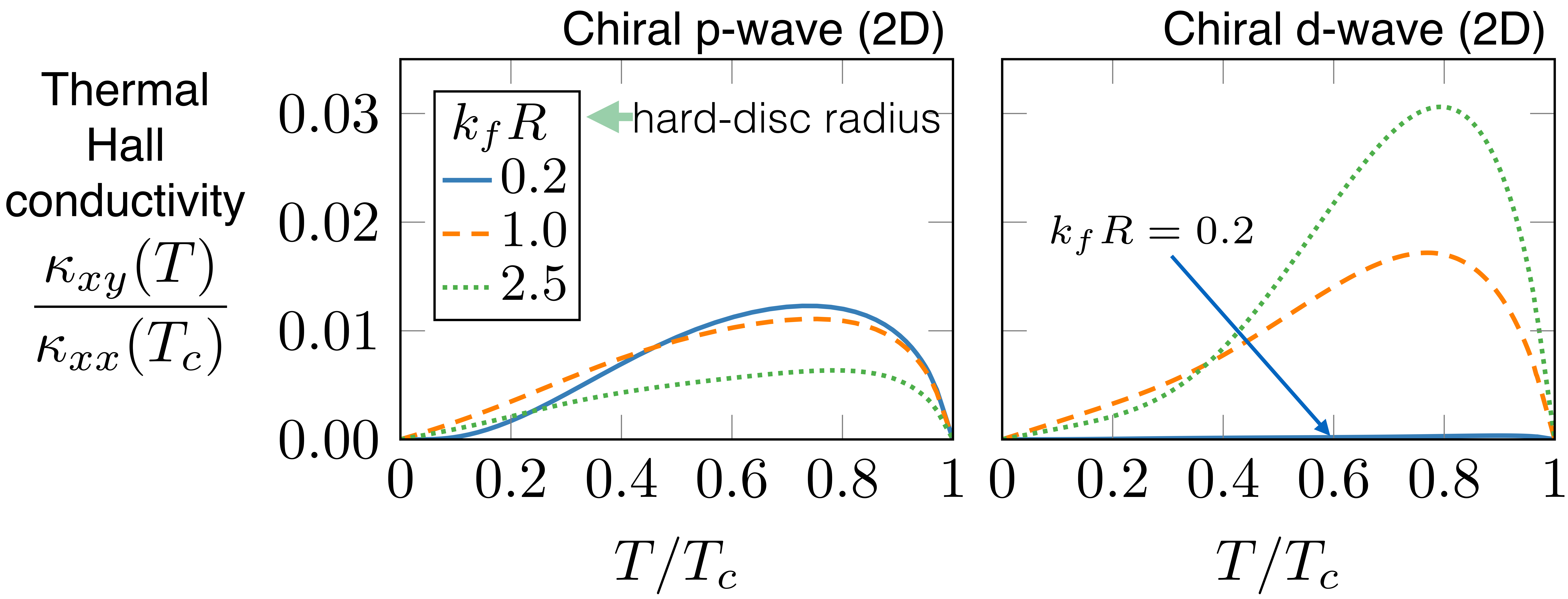
★ ATHE in Chiral SCs depends on **Finite-size Impurities** ★

Point-like impurity — Hall effect in *p+ip* only



★ T-dependence - Branch conversion (Andreev) Scattering

Finite size impurity — Hall effect in *p+ip*, *d+id*, *f+if*



★ Hall current is **sensitive to chiral winding number**

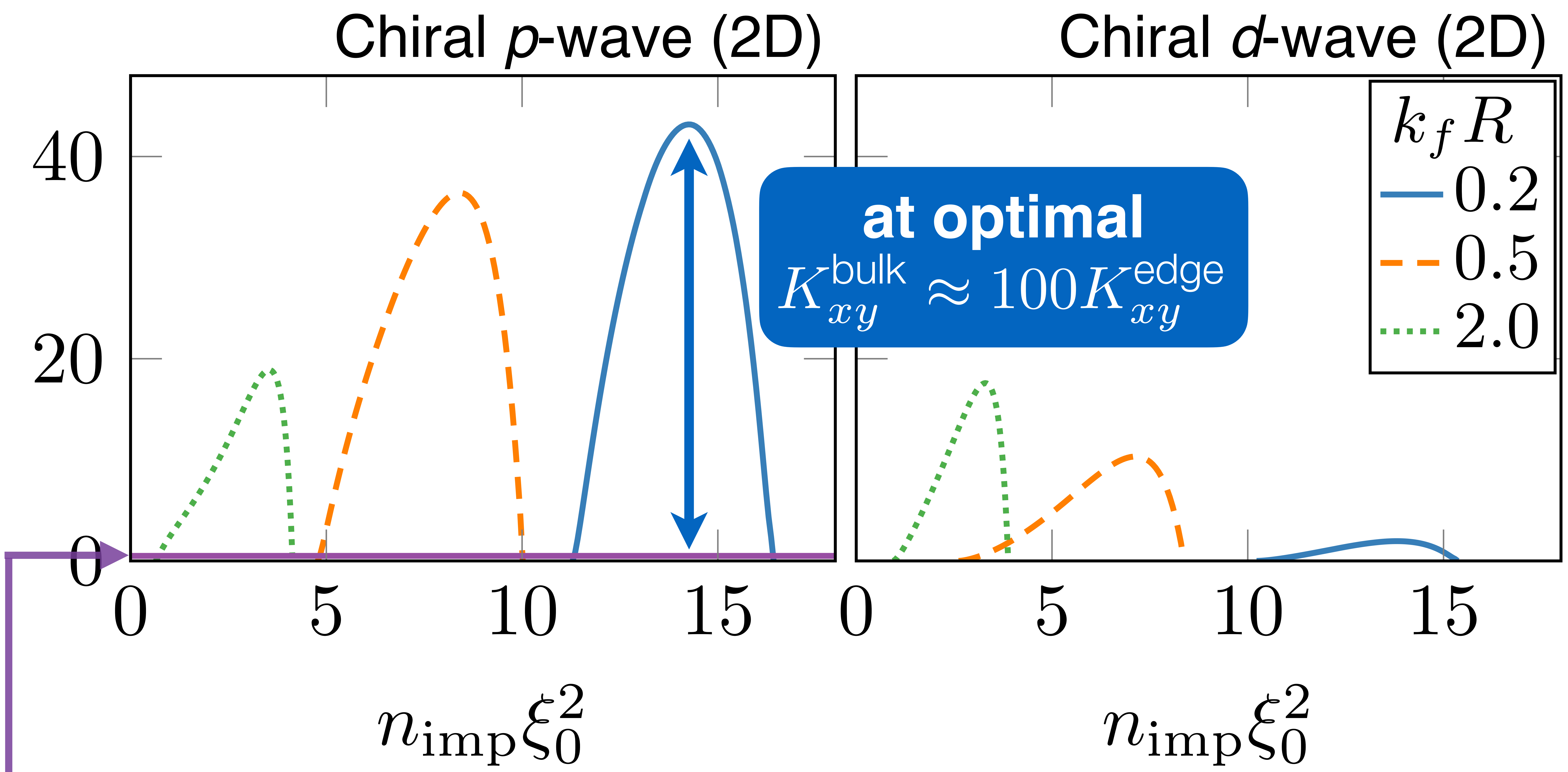
✓**Good probe for chirality**

Impurity density chosen so that normal-state conductivity is fixed

Impurity-Induced Thermal Hall transport at zero T (N(0) finite)
Bulk ATHE effect dominates Edge ATHE

Zero-T Thermal Hall Conductance

$$\frac{K_{xy}^{\text{bulk}}}{\pi^2 k_B^2 T/6\pi\hbar}$$



conservative estimate

$k_f \xi_0 = 100$

$\xi_0 = \dfrac{\hbar v_f}{2\pi k_B T_{c0}}$

$(K_{xy}^{\text{bulk}} \propto k_f \xi_0)$

Edge Hall conductance $(T \to 0)$

$K_{xy}^{\text{edge}} = \pi^2 k_B^2 T/6\pi\hbar$ (*p*-wave)

★ **Bulk effect dominates** ★

(when present)

Conclusions

★ **Thermal Hall Effect** can be used for **verifying and studying Chiral SC**

★ Point-like impurity model [$k_f R \ll 1$] no! spuriously rules out Hall Effect in all but chiral *p*-wave states ~~— **Finite**~~**-size impurity needed**

★ **Bulk Hall Effect** is **often dominant** but requires low-energy states

★ If no low-energy states exist — Edge Effect dominates even in dirty samples

✦ We have **full set of results** for **3D gaps** proposed for **UPt_3**

Thank you!

NSF DMR-1508730

CAPST

Fermilab

Northwestern

Center for **Applied Physics and Superconducting Technologies**

Thank You!

The End

Determination of the Electron Bubble Radius

(i) Energy required to create a bubble:

$$E(R,P) = E_0(U_0,R) + 4\pi R^2\gamma + \frac{4\pi}{3}R^3 P, \quad P \text{ – pressure}$$

(ii) For $U_0 \to \infty$: $E_0 = -U_0 + \pi^2\hbar^2/2m_e R^2$ – ground state energy

(iii) Surface Energy: hydrostatic surface tension $\rightsquigarrow \gamma = 0.15\,\mathrm{erg/cm^2}$

(iv) Minimizing E w.r.t. $R \rightsquigarrow P = \pi\hbar^2/4m_e R^5 - 2\gamma/R$

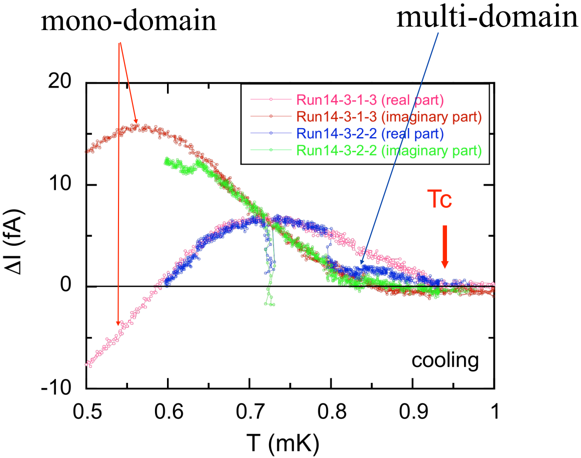
(v) For zero pressure, $P = 0$:

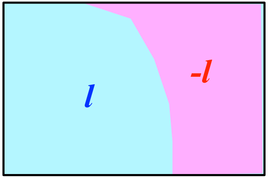
$$R = \left(\frac{\pi\hbar^2}{8m_e\gamma}\right)^{1/4} \approx 2.38\,\mathrm{nm} \quad \rightsquigarrow \quad k_f R = 18.67$$

Transport $\rightsquigarrow k_f R = 11.17$

▶A. Ahonen et al., J. Low Temp. Phys., 30(1):205–228, 1978

Chiral Domains & Domain Switching

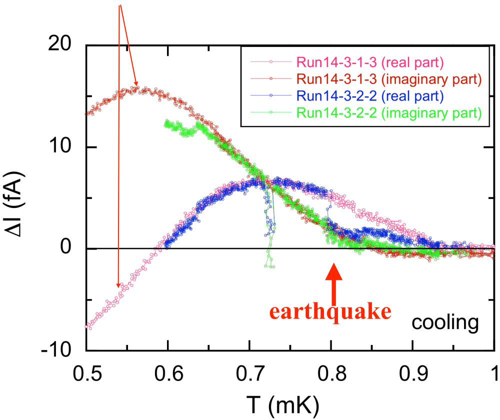


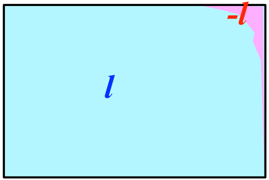


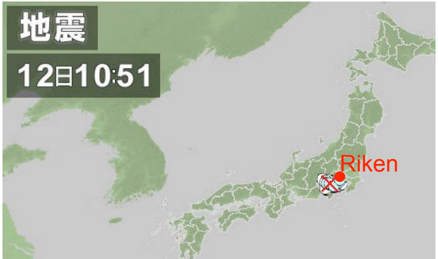
Chiral Domains & Domain Switching

mono-domain

multi-domain



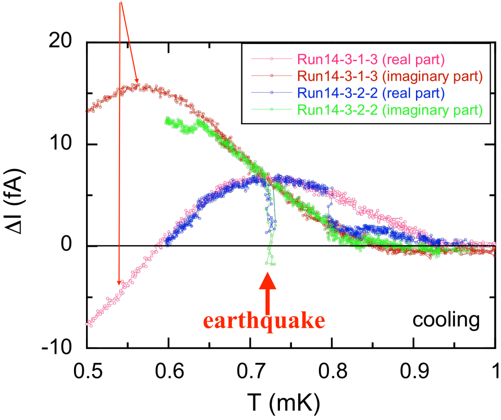


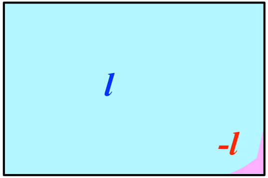


Chiral Domains & Domain Switching

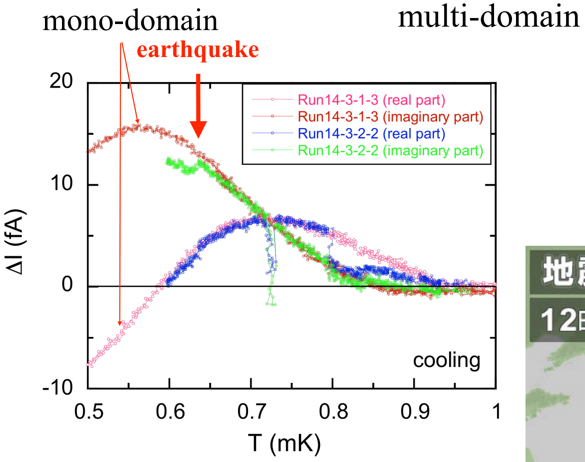
mono-domain

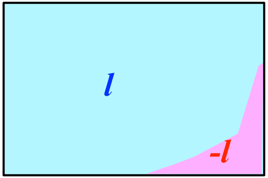
multi-domain



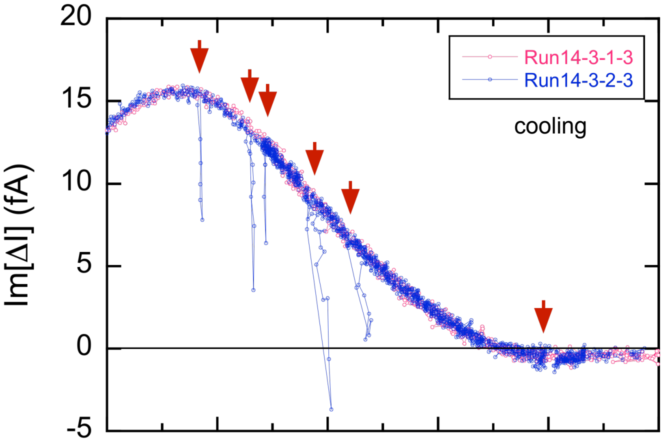


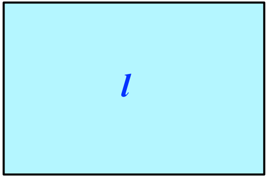
Chiral Domains & Domain Switching





Single Chiral Domain





small earthquakes

mono-domain stability

Two Fluid Motion for a moving electron bubble as $T \to 0$

- An ion moving through a fluid experiences a force originating from the scattering of excitations off the ion.
- ^{3}He-A at $T \neq 0$ ^{3}He-A: a condensate of chiral Cooper pairs & a fluid of "normal" chiral Fermions.

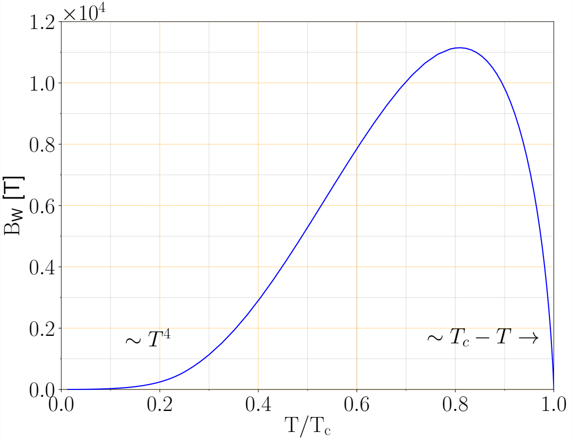
$$\boxed{M\frac{d\mathbf{V}}{dt} = e\mathbf{E} + e\mathbf{V}\times\mathbf{B}_W - \eta\,\mathbf{V}}$$

  - Dynamical Effective Mass of the Ion: M $\leftarrow$ Backflow & Virtual Excitations
  - Stokes Drag Force on the Ion: $\mathbf{F}_{\text{drag}} = -\eta\mathbf{V}$ $\leftarrow$ Dynamic Viscosity
  - Chiral Effective Magnetic Field: $\mathbf{B}_W = -\frac{c}{e}\eta_{xy}\hat{\mathbf{l}}$ $\leftarrow$ Anomalous Hall Response

- Stokes' drag for a sphere of radius R: $\eta = 6\pi\,\nu\,R \rightsquigarrow$ Reynold's Number: $\text{Re} \equiv \dfrac{2\rho V R}{\nu}$
- Normal ^{3}He: $\rho = 0.0819\,\text{g/cm}^3$ $\quad \mu_{\text{N}} = \dfrac{e}{n_3 p_f \sigma_{\text{N}}^{\text{tr}}} \simeq 1.7\times 10^{-6}\,\text{m}^2/\text{V-s}$ $\quad \rightsquigarrow R = 1.42\,\text{nm}$ $\quad k_f R = 11.17$
- Derived Parameters: $\nu_{\text{N}} = \dfrac{\eta_{\text{N}}}{6\pi R} = 3.5\times 10^{-6}\,\text{Pa-s}$ $\quad \text{Re}_{\text{N}} = 6.7\times 10^{-6}$ $\quad \text{B}_{\text{N}} \equiv \dfrac{c}{e}\eta_{\text{N}} = 5.9\times 10^5\,\text{T}$
- Reynold's Number for flow past an electron bubble in ^{3}He-A: $\boxed{\text{Re} = \text{Re}_{\text{N}}\left(\frac{\eta_{\text{N}}}{\eta}\right)^{3/2} \xrightarrow[T\to 0]{} \sim \left(\frac{T_c}{T}\right)^{9/2} !}$

Vanishing of the Effective Magnetic Field for $T \to 0$

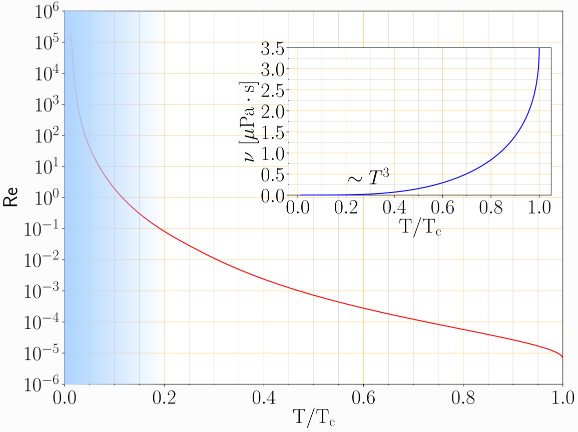
$$B_{\mathsf{W}} = \boxed{5.9 \times 10^5\ \mathsf{T}} \left(\frac{\eta_{xy}}{\eta_{\mathsf{N}}}\right)$$



$$\eta_{xy}/\eta_{\mathsf{N}}|_{T=0.8\,T_c} \approx \frac{\hbar}{p_f R}$$

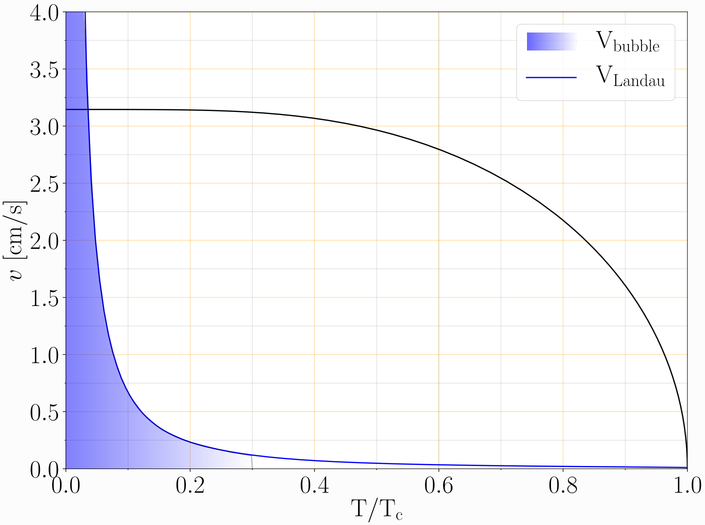
Breakdown of Laminar Flow

$$\mathsf{Re} = \mathsf{Re}_{\mathsf{N}} \left(\frac{\eta_{\mathsf{N}}}{\eta}\right)^{3/2} \xrightarrow[T\to 0]{} \sim \left(\frac{T_c}{T}\right)^{9/2}$$



$$\mathsf{Re}_{\mathsf{N}} = 6.7 \times 10^{-6}$$

Breakdown of Scattering Theory for $T \to 0$



Electron Bubble Velocity

- $V_{\mathsf{N}} = \mu_{\mathsf{N}} E_{\mathsf{N}} = 1.01 \times 10^{-4}\,\mathsf{m/s}$
- $V = \mu_{\mathsf{N}} E_{\mathsf{N}} \sqrt{\dfrac{\eta_{\mathsf{N}}}{\eta}}$

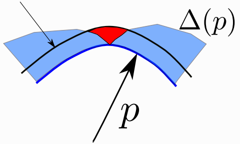
Maximum Landau critical velocity

- $V_c^{\mathsf{max}} \approx 155 \times 10^{-4}\,\mathsf{m/s}\,\dfrac{\Delta_{\mathsf{A}}(T)}{k_b T_c}$

Nodal Superfluids:

- $V_c = \Delta(p)/p_f \to 0$ for $p \to p_{\mathsf{node}}$

$$\varepsilon_{\mathrm{Doppler}} = p_f V$$



- Radiation Dominated Damping for $T \lesssim 0.1 T_c$

Momentum-Space Topology of Nambu-Bogoliubov Hamiltonian

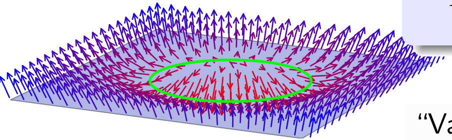
Hamiltonian for 2D Chiral Superfluid (^{3}He-A Thin Film & Sr_2RuO_4):

$$\widehat{H} = \begin{pmatrix} (|\mathbf{p}|^2/2m^* - \mu) & c(p_x + ip_y) \\ c(p_x - ip_y) & -(|\mathbf{p}|^2/2m^* - \mu) \end{pmatrix} = \vec{\mathbf{m}}(\mathbf{p}) \cdot \widehat{\vec{\boldsymbol{\tau}}}$$

$$\vec{\mathbf{m}} = (\, cp_x \,,\, \mp cp_y \,, \xi(\mathbf{p})) \text{ with } |\vec{\mathbf{m}}(\mathbf{p})|^2 = \left(|\mathbf{p}|^2/2m - \mu\right)^2 + c^2|\mathbf{p}|^2 > 0 \,, \mu \neq 0$$

▶ Topological Invariant for 2D chiral SC $\leftrightarrow$ QED in d = 2+1 [G.E. Volovik, JETP 1988]:

$$N_{\text{C}} = \int \frac{d^2p}{4\pi}\, \hat{\mathbf{m}}(\mathbf{p}) \cdot \left(\frac{\partial \hat{\mathbf{m}}}{\partial p_x} \times \frac{\partial \hat{\mathbf{m}}}{\partial p_y}\right) = \begin{cases} \pm 1 \,; & \mu > 0 \text{ and } \Delta \neq 0 \\ 0 \,; & \mu < 0 \text{ or } \Delta = 0 \end{cases}$$

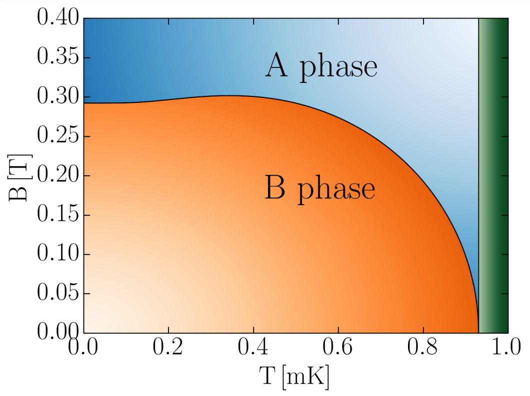


"Vacuum" ($\Delta = 0$) & $N_{\text{c}} = 0$ | ^{3}He-A ($\Delta \neq 0$) with $N_{\text{c}} = 1$

Zero Energy Fermions $\uparrow$ Confined on the Edge

Superfluid Phases of ^{3}He in a Magnetic Field for $P < P_{\text{PCP}}$

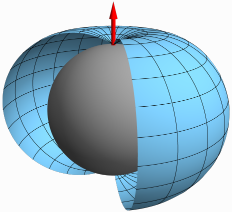
Zeeman Energy for Spin-Triplet Pairing



Spin-Triplet, P-wave Order Parameter

$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix} = \begin{pmatrix} -\mathbf{d}_x + i\mathbf{d}_y & \mathbf{d}_z \\ \mathbf{d}_z & \mathbf{d}_x + i\mathbf{d}_y \end{pmatrix}$$

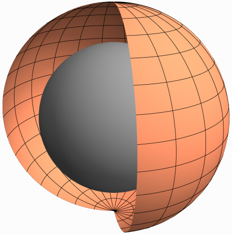
ABM State $\vec{l} = \hat{\mathbf{z}}$



$L_z = 1,\ S_y = 0$

$\Psi_{\uparrow\uparrow} = \Psi_{\downarrow\downarrow} = \hat{p}_x + i\hat{p}_y$

"Isotropic" BW State



$J = 0,\ J_z = 0$

$\Psi_{\uparrow\uparrow} = \hat{p}_x - i\hat{p}_y,\ \Psi_{\downarrow\downarrow} = \hat{p}_x + i\hat{p}_y,\ \Psi_{\uparrow\downarrow} = \hat{p}_z$

Mobility of an electron bubble in the Normal Fermi Liquid

- $t_{\mathsf{N}}^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E) = \sum_{l=0}^{\infty} (2l+1) t_l^R(E) P_l(\hat{\mathbf{k}}' \cdot \hat{\mathbf{k}})$
- $t_l^R(E) = -\frac{1}{\pi N_f} e^{i\delta_l(E)} \sin \delta_l(E)$
- $\frac{d\sigma}{d\Omega_{\mathbf{k}'}} = \left(\frac{m^*}{2\pi\hbar^2} \right)^2 |t_{\mathsf{N}}^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E)|^2$
- Non-resonant scattering at $T \ll E_f/k_{\mathsf{B}} \approx 3\,\mathsf{K} \rightsquigarrow \delta_l(E \approx E_f)$
- $\sigma_{\mathsf{N}}^{\mathsf{tr}} = \int \frac{d\Omega_{\mathbf{k}'}}{4\pi} (1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}') \frac{d\sigma}{d\Omega_{\mathbf{k}'}} = \boxed{\frac{4\pi}{k_f^2} \sum_{l=0}^{\infty} (l+1) \sin^2(\delta_{l+1} - \delta_l)}$
- $\boxed{\mu_{\mathsf{N}} = \frac{e}{n_3 p_f \sigma_{\mathsf{N}}^{\mathsf{tr}}}}, \quad p_f = \hbar k_f, \quad n_3 = \frac{k_f^3}{3\pi^2}$

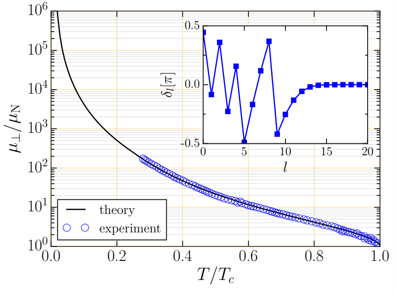
Theoretical Models for the QP-ion potential

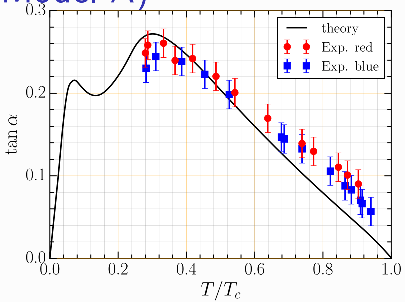
- $$U(r) = \begin{cases} U_0, & r < R, \\ -U_1, & R < r < R', \\ 0, & r > R'. \end{cases}$$
- $\rightsquigarrow$ Hard-Sphere Potential: $U_1 = 0$, $R' = R$, $U_0 \to \infty$
- $U(x) = U_0\,[1 - \tanh[(x-b)/c]]$, $\quad x = k_f r$
- $U(x) = U_0/\cosh^2[\alpha x^n]$, $\quad x = k_f r \quad$ (Pöschl-Teller-like potential)
- Random phase shifts: $\{\delta_l \,|\, l = 1 \ldots l_{\max}\}$ are generated with δ_0 is an adjustable parameter
- Parameters for all models are chosen to fit the experimental value of the normal-state mobility, $\mu_N^{\exp} = 1.7 \times 10^{-6}\, m^2/V \cdot s$

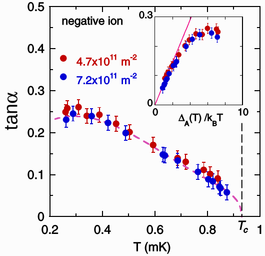
Theoretical Models for the QP-ion potential

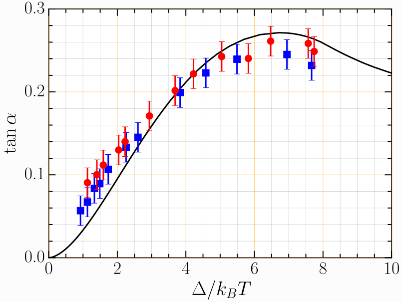
Label	Potential	Parameters
Model A	hard sphere	$k_f R = 11.17$
Model B	repulsive core & attractive well	$U_0 = 100E_f, U_1 = 10E_f, k_f R' = 11, R/R' = 0.36$
Model C	random phase shifts model 1	$l_{\max} = 11$
Model D	random phase shifts model 2	$l_{\max} = 11$
Model E	Pöschl-Teller-like	$U_0 = 1.01E_f, k_f R = 22.15, \alpha = 3 \times 10^{-5}, n = 4$
Model F	Pöschl-Teller-like	$U_0 = 2E_f, k_f R = 19.28, \alpha = 6 \times 10^{-5}, n = 4$
Model G	hyperbolic tangent	$U_0 = 1.01E_f, k_f R = 14.93, b = 12.47, c = 0.246$
Model H	hyperbolic tangent	$U_0 = 2E_f, k_f R = 14.18, b = 11.92, c = 0.226$
Model I	soft sphere 1	$U_0 = 1.01E_f, k_f R = 12.48$
Model J	soft sphere 2	$U_0 = 2E_f, k_f R = 11.95$

Hard-sphere model with $k_f R = 11.17$ (Model A)

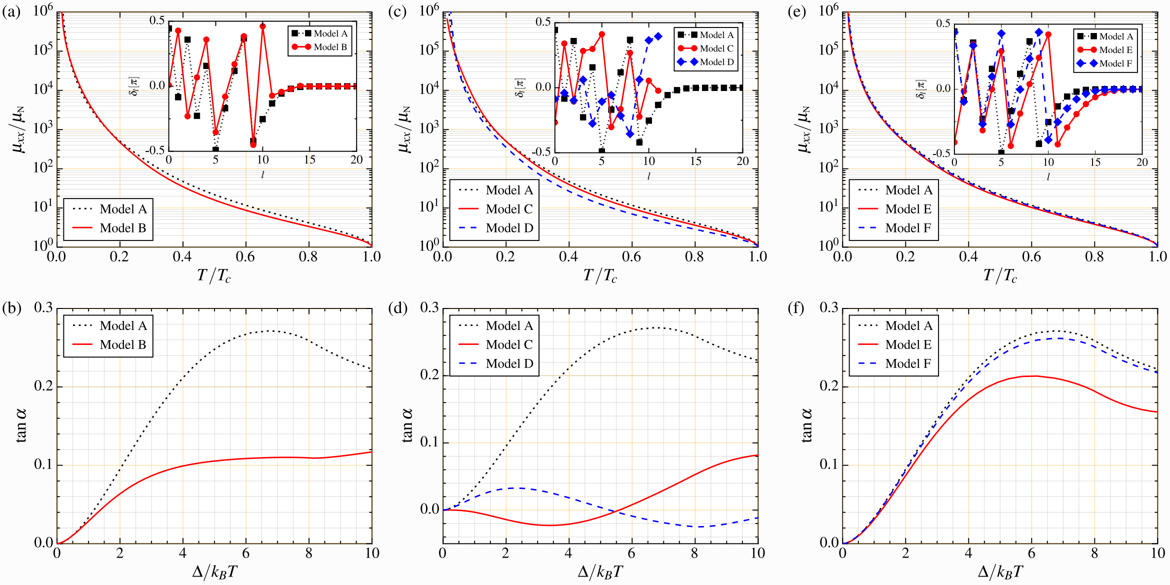




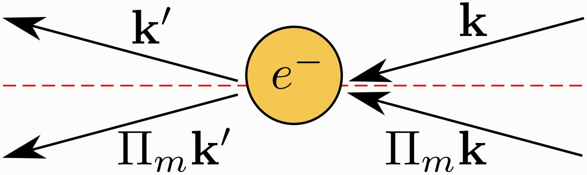




Comparison with Experiment for Models for the QP-ion potential



Broken Time-Reversal ($\mathtt{T}$) & mirror (Π_{m}) symmetries in Chiral Superfluids



- Broken TRS: $\mathtt{T} \cdot (\hat{p}_x + i\hat{p}_y) = (\hat{p}_x - i\hat{p}_y)$
- Broken mirror symmetry: $\Pi_{\mathrm{m}} \cdot (\hat{p}_x + i\hat{p}_y) = (\hat{p}_x - i\hat{p}_y)$
- Chiral symmetry: $\mathtt{C} = \mathtt{T} \times \Pi_{\mathrm{m}} \quad \rightsquigarrow \quad \mathtt{C} \cdot (\hat{p}_x + i\hat{p}_y) = (\hat{p}_x + i\hat{p}_y)$
- Microscopic reversibility for chiral superfluids: $W(\hat{\mathbf{k}}', \hat{\mathbf{k}}; +\hat{\mathbf{l}}) = W(\hat{\mathbf{k}}, \hat{\mathbf{k}}'; -\hat{\mathbf{l}})$
- $\therefore$ For BTRS: the chiral axis $\hat{\mathbf{l}}$ is fixed $\rightsquigarrow$ $\boxed{W(\hat{\mathbf{k}}', \hat{\mathbf{k}}; \hat{\mathbf{l}}) \neq W(\hat{\mathbf{k}}, \hat{\mathbf{k}}'; \hat{\mathbf{l}})}$

Calculation of LDOS and Current Density

$$\hat{\mathcal{G}}_S^R(\mathbf{r}', \mathbf{r}, E) = \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} e^{i\mathbf{k}'\mathbf{r}'} e^{-i\mathbf{k}\mathbf{r}} \hat{\mathcal{G}}_S^R(\mathbf{k}', \mathbf{k}, E)$$

$$\hat{\mathcal{G}}_S^R(\mathbf{k}', \mathbf{k}, E) = (2\pi)^3 \hat{G}_S^R(\mathbf{k}, E)\delta_{\mathbf{k}',\mathbf{k}} + \hat{G}_S^R(\mathbf{k}', E)\hat{T}_S(\mathbf{k}', \mathbf{k}, E)\hat{G}_S^R(\mathbf{k}, E)$$

$$\hat{G}_S^R(\mathbf{k}, E) = \frac{1}{\varepsilon^2 - E_{\mathbf{k}}^2} \begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad \varepsilon = E + i\eta, \quad \eta \to 0^+$$

$$N(\mathbf{r}, E) = -\frac{1}{2\pi} \mathrm{Im} \left\{ \mathrm{Tr} \left[\hat{\mathcal{G}}_S^R(\mathbf{r}, \mathbf{r}, E) \right] \right\}$$

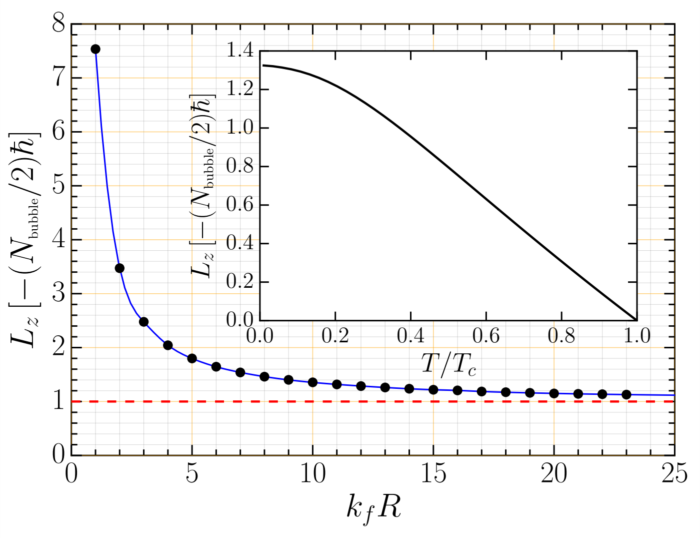
$$\mathbf{j}(\mathbf{r}) = \frac{\hbar}{4mi} k_B T \sum_{n=-\infty}^{\infty} \lim_{\mathbf{r}\to\mathbf{r}'} \mathrm{Tr} \left[(\boldsymbol{\nabla}_{\mathbf{r}'} - \boldsymbol{\nabla}_{\mathbf{r}}) \hat{\mathcal{G}}^M(\mathbf{r}', \mathbf{r}, \epsilon_n) \right]$$

$$\hat{\mathcal{G}}_S^R(\mathbf{r}', \mathbf{r}, E) = \hat{\mathcal{G}}_S^M(\mathbf{r}', \mathbf{r}, \epsilon_n) \Big|_{i\epsilon_n \to \varepsilon}, \text{ for } n \geq 0$$

$$\hat{\mathcal{G}}_S^M(\mathbf{k}, \mathbf{k}', -\epsilon_n) = \left[\hat{\mathcal{G}}_S^M(\mathbf{k}', \mathbf{k}, \epsilon_n) \right]^\dagger$$

Angular momentum of an electron bubble in ^{3}He-A ($k_f R = 11.17$)

$$\mathbf{L}(T \to 0) \approx -\hbar N_{\text{bubble}} \hat{\mathbf{l}}/2\,; \qquad N_{\text{bubble}} = n_3 \frac{4\pi}{3} R^3 \approx 200\ ^3\text{He atoms}$$



Temperature scaling of the Stokes tensor components

- For $1 - \dfrac{T}{T_c} \to 0^+$:

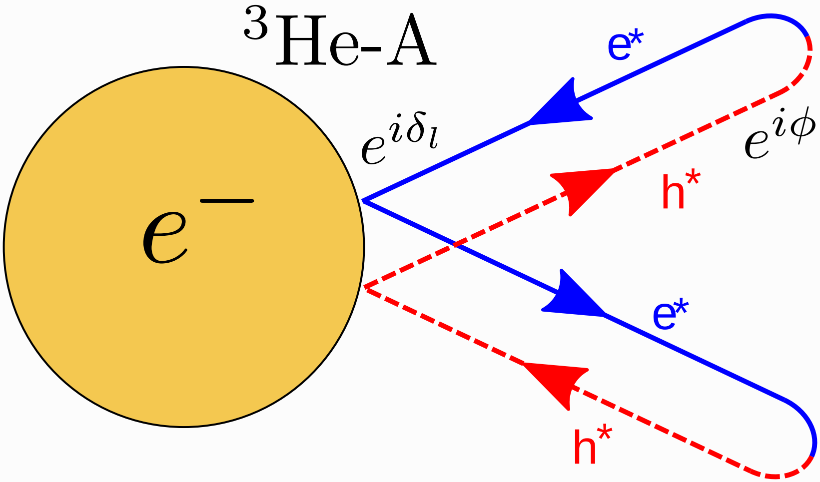
$$\frac{\eta_\perp}{\eta_\mathsf{N}} - 1 \propto -\Delta(T) \propto \sqrt{1 - \frac{T}{T_c}}$$

$$\frac{\eta_\mathsf{AH}}{\eta_\mathsf{N}} \propto \Delta^2(T) \propto 1 - \frac{T}{T_c}$$

- For $\dfrac{T}{T_c} \to 0^+$:

$$\frac{\eta_\perp}{\eta_\mathsf{N}} \propto \left(\frac{T}{T_c}\right)^2$$

$$\frac{\eta_\mathsf{AH}}{\eta_\mathsf{N}} \propto \left(\frac{T}{T_c}\right)^3$$



$$\Delta(\hat{\mathbf{k}}) = \Delta \sin\theta e^{i\phi}$$

Obtaining Thermal Hall currents from Quasiclassical Linear Response Theory

propagators (encode spectrum)

Quasiclassical Transport Equations

+

order parameter

Gap Equation

+

effects of impurities

T-matrix Equations

Solve

Self-consistent Equilibrium states

Input

Linearize & Solve

Keldysh response

diagonal piece

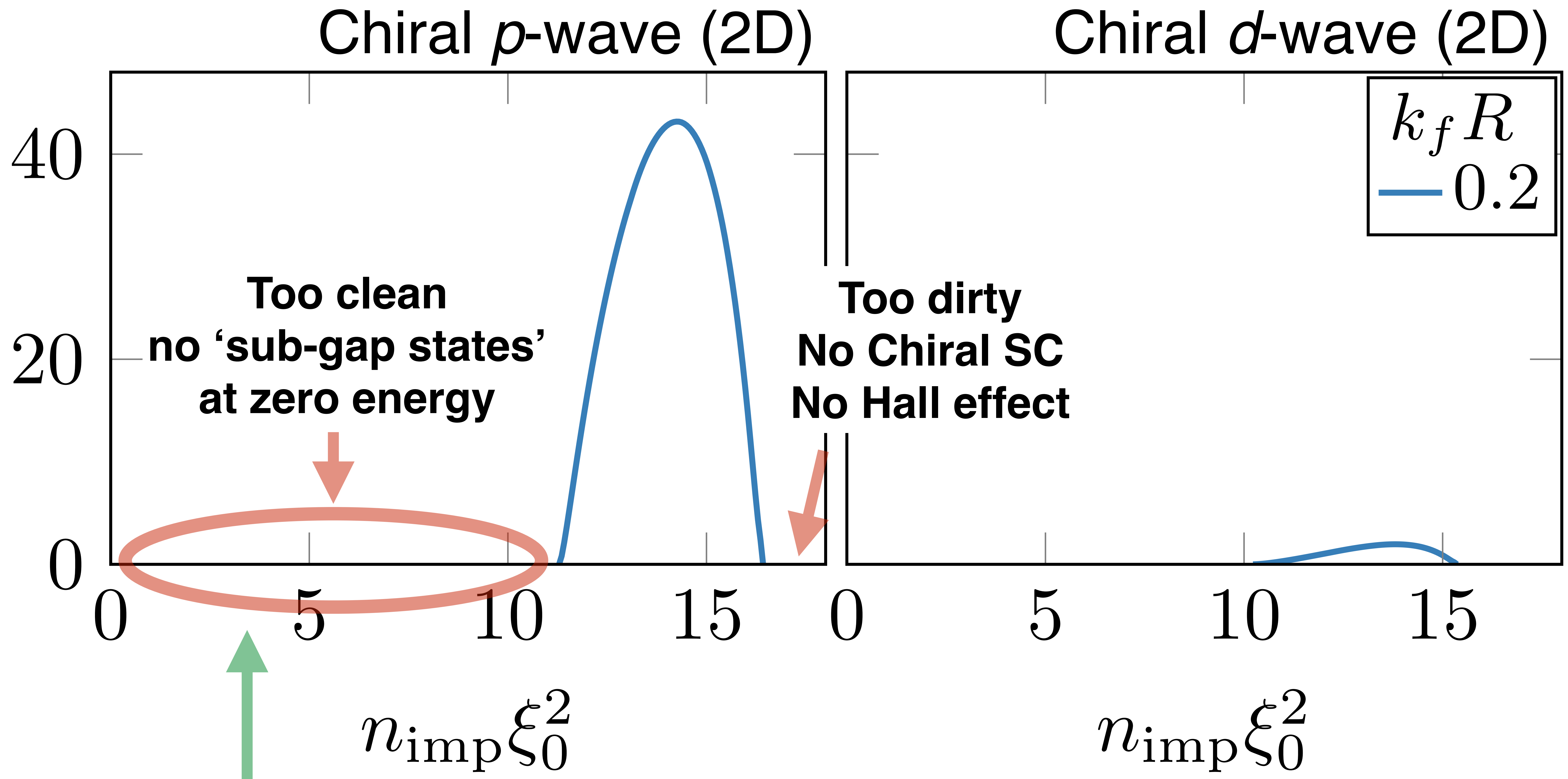
"occupied spectral function"

Heat Currents: $\mathbf{j}^{(\varepsilon)} = 2N_f \int \frac{d\varepsilon}{4\pi i} \left\langle \varepsilon \mathbf{v}_{\hat{\mathbf{p}}} \delta g^K(\hat{\mathbf{p}};\varepsilon) \right\rangle_{\hat{\mathbf{p}}}$

Hall effect vs impurity density at zero T
There is density threshold for Hall effect

Zero-T Thermal Hall Conductance

$$\frac{K_{xy}^{\text{bulk}}}{\pi^2 k_B^2 T/6\pi\hbar}$$



conservative estimate

$$k_f \xi_0 = 100$$

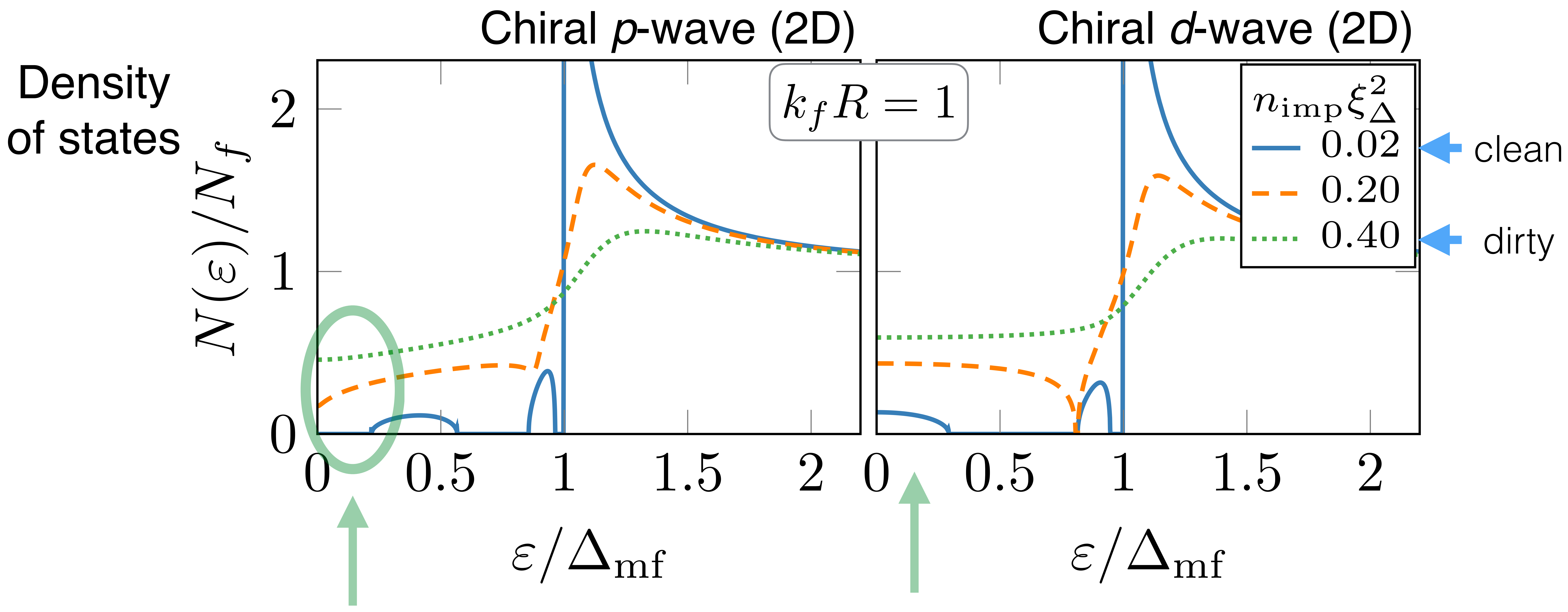
$$\xi_0 = \frac{\hbar v_f}{2\pi k_B T_{c0}}$$

$$(K_{xy}^{\text{bulk}} \propto k_f \xi_0)$$

★ **Edge effect dominates** even in dirty systems ★

(no bulk signal)

Band broadening makes low-T transport possible

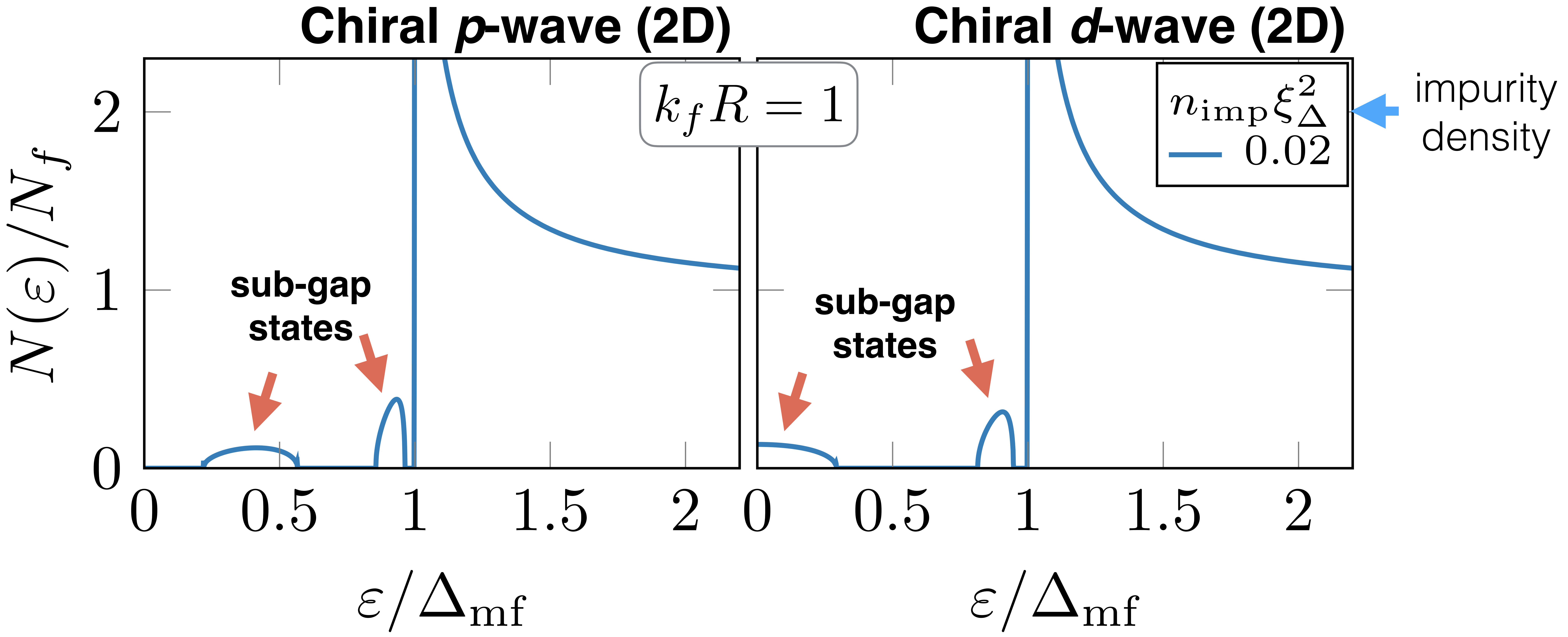


Impurity-induced states for low-T transport

Impurity-induced states depend on chiral order

Hall currents carried by *sub-gap* states

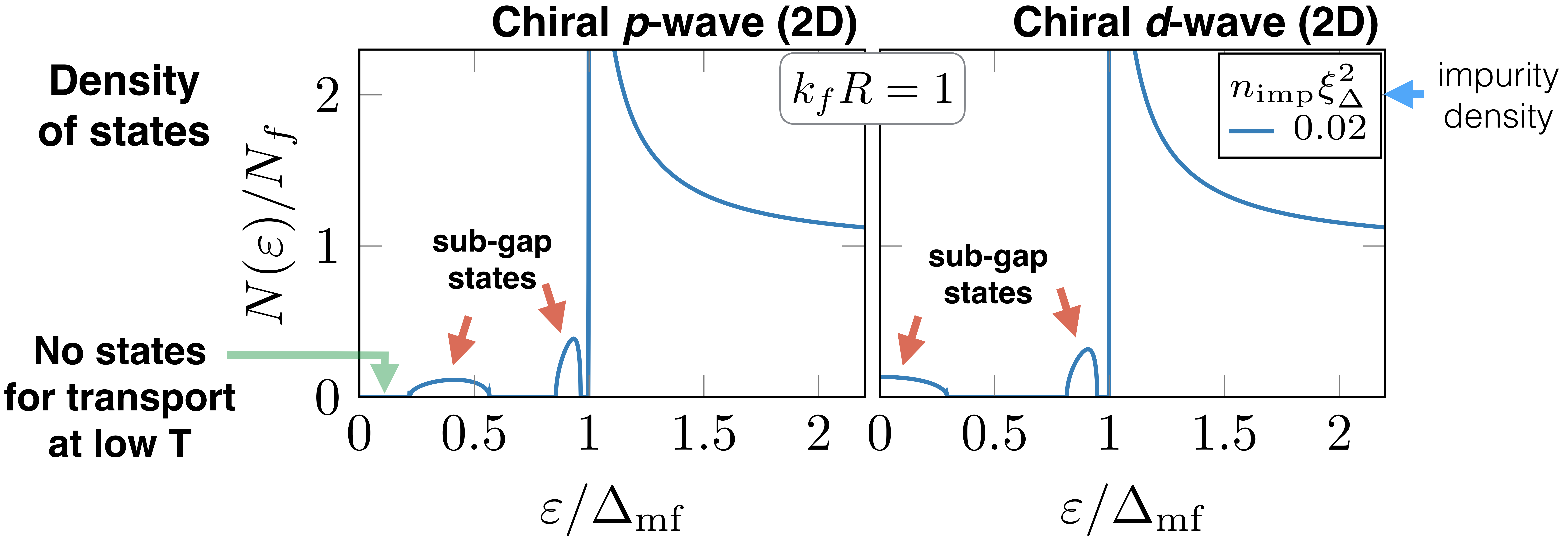
Density of states



★ Repeated Andreev scattering from order parameter variation (chiral winding) ⟶ **Bound states with energies depending on winding #**

★ **Hall currents also depend on winding #**

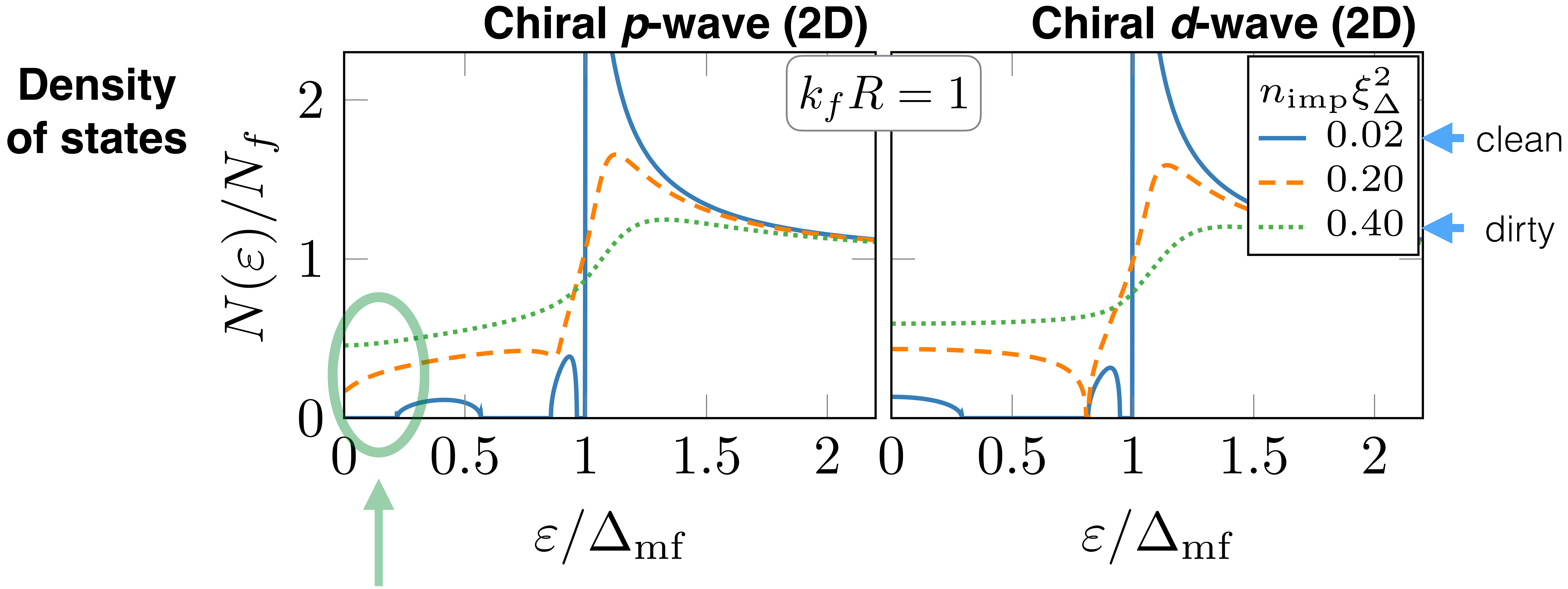
Hall currents carried by *sub-gap* states



★ Repeated Andreev scattering from order parameter variation (chiral winding) → **Bound states with energies depending on winding #**

★ Carried by these states, **Hall currents also depend on winding #**

Band broadening makes low-T transport possible

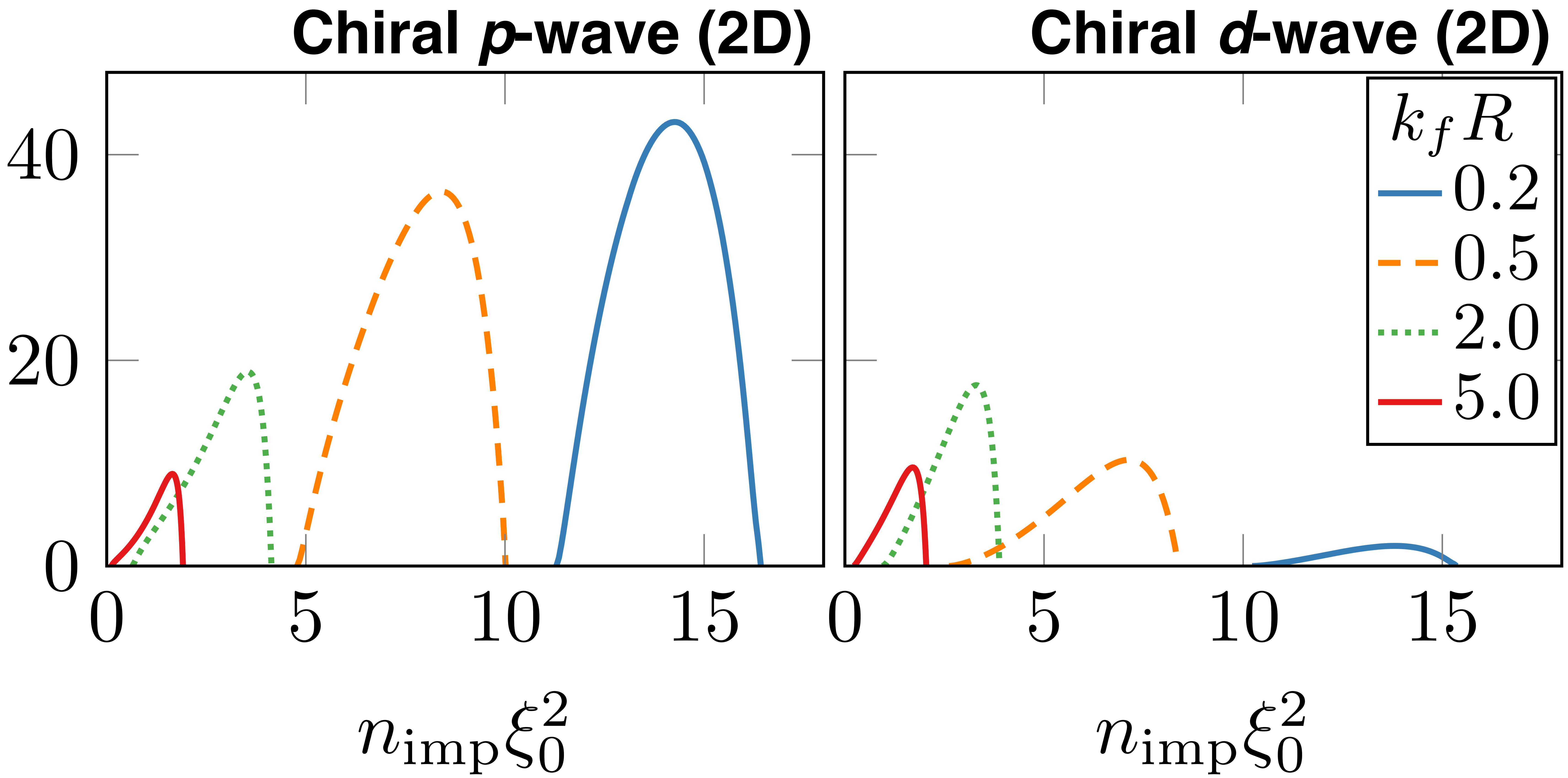


States available for transport at low T from broadened sub-gap states

Zero-T thermal Hall transport
Non-monotonic dependence on impurity radius

Zero-T Thermal Hall Conductance

$$\frac{K_{xy}^{\text{bulk}}}{\pi^2 k_B^2 T / 6\pi\hbar}$$



conservative estimate

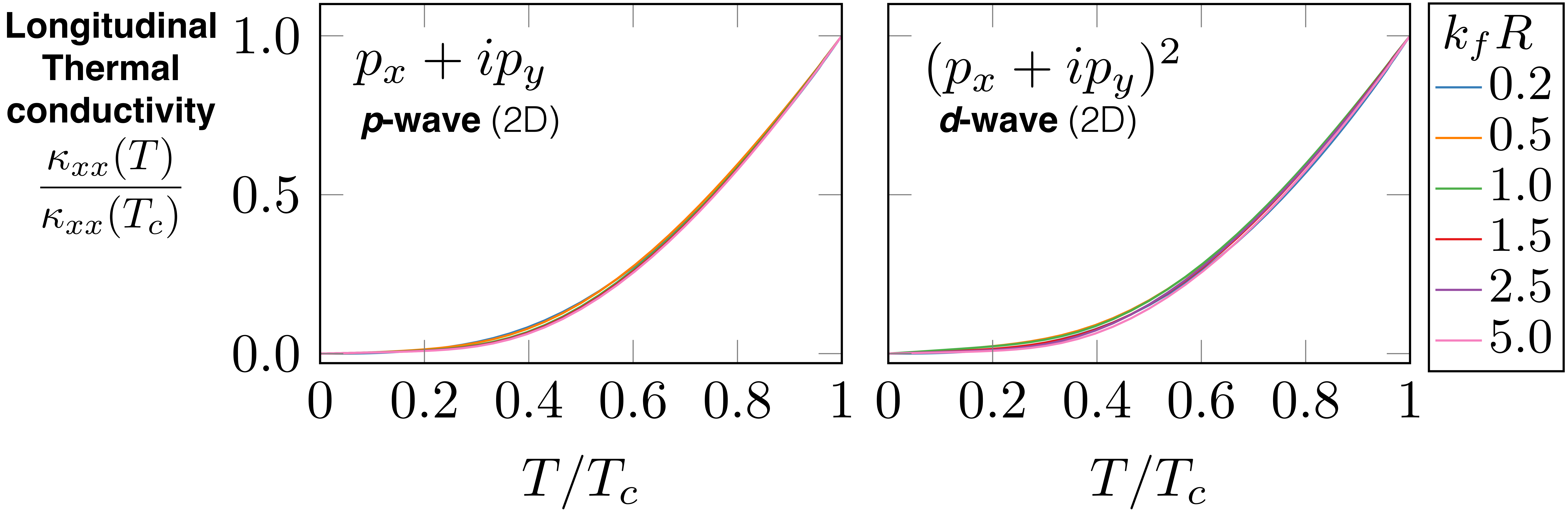
$k_f \xi_0 = 100$

$\xi_0 = \frac{\hbar v_f}{2\pi k_B T_{c0}}$

$(K_{xy}^{\text{bulk}} \propto k_f \xi_0)$

(hard-disc)

Finite-size impurity — Longitudinal thermal currents hardly affected by impurity size



★ Characterized by scattering rate

★ Insensitive to gap symmetry / impurity size

Impurity density chosen so that normal-state conductivity is fixed