The Left Hand of the Electron in a Chiral Vacuum

J. A. Sauls

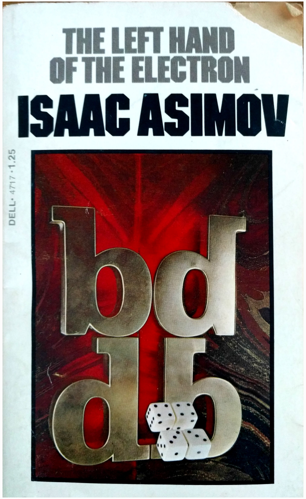
Department of Physics & Astronomy
Hearne Institute of Theoretical Physics
Louisiana State University

- P and T violation ^{3}He
- Low Temperature Physics at NU
- Dynamical Effects of Symmetry Breaking

▶NSF Grant DMR-1508730

The Left Hand of the Electron, Issac Asimov, November 1971

- An Essay on the Discovery of Parity Violation by the Weak Interaction



- ... And Reflections on Mirror Symmetry in Nature

Parity Violation in Beta Decay of ^{60}Co - Physical Review 105, 1413 (1957)

Experimental Test of Parity Conservation in Beta Decay*

C. S. Wu, *Columbia University, New York, New York*

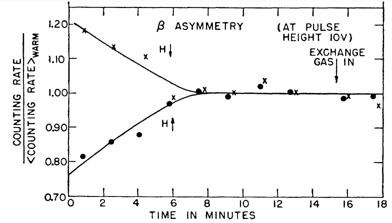
AND

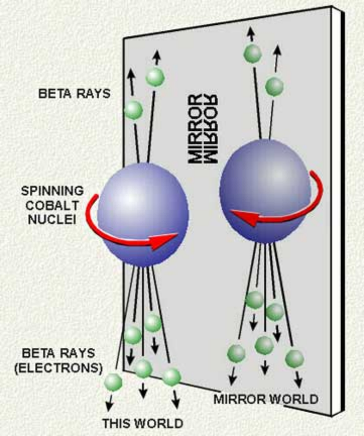
E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson,
National Bureau of Standards, Washington, D. C.

(Received January 15, 1957)

▶ T. D. Lee and C. N. Yang, Phys Rev 104, 204 (1956)

$$^{60}Co \rightarrow {}^{60}Ni + e^- + \bar{\nu}$$



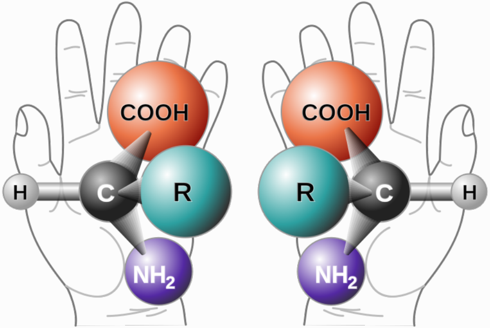


▶ Current of Beta electrons is (anti) correlated with the Spin of the ^{60}Co nucleus.

$\langle \vec{S} \cdot \vec{p} \rangle \neq 0 \rightsquigarrow$ Parity violation

Chiral Quantum Matter

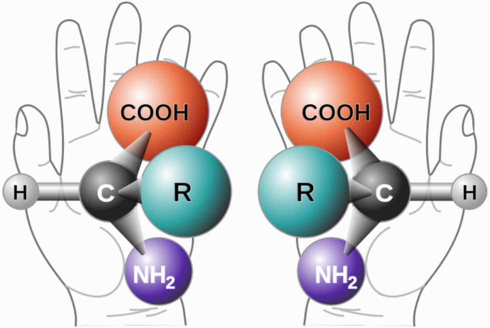
Molecular Chiral Enantiomers



Handedness: Broken Mirror Symmetry

Chiral Quantum Matter

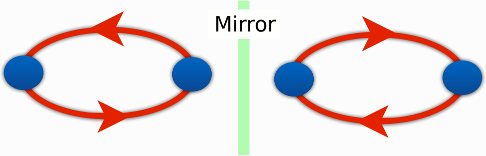
Molecular Chiral Enantiomers



Handedness: Broken Mirror Symmetry

Chiral Diatomic Molecules

$$\Psi(\mathbf{r}) = f(r)\,(x+iy)$$

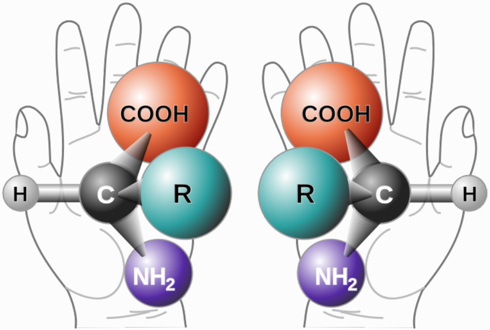


Broken Mirror Symmetries

$$\Pi_{zx}\,\Psi(\mathbf{r}) = f(r)\,(x-iy)$$

Chiral Quantum Matter

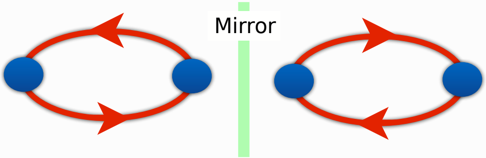
Molecular Chiral Enantiomers



Handedness: Broken Mirror Symmetry

Chiral Diatomic Molecules

$$\Psi(\mathbf{r}) = f(r)\,(x + iy)$$



Broken Mirror Symmetries

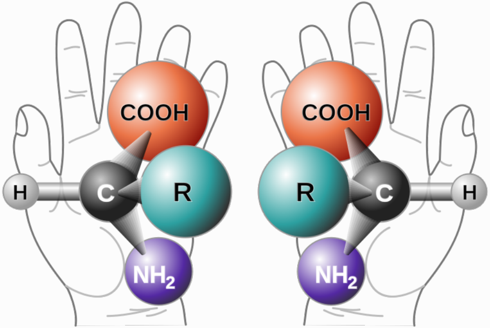
$$\Pi_{zx}\,\Psi(\mathbf{r}) = f(r)\,(x - iy)$$

Broken Time-Reversal Symmetry

$$\mathrm{T}\,\Psi(\mathbf{r}) = f(r)\,(x - iy)$$

Chiral Quantum Matter

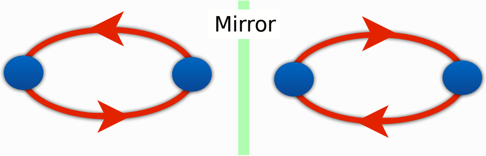
Molecular Chiral Enantiomers



Handedness: Broken Mirror Symmetry

Chiral Diatomic Molecules

$$\Psi(\mathbf{r}) = f(r)\,(x + iy)$$



Broken Mirror Symmetries

$$\Pi_{zx}\,\Psi(\mathbf{r}) = f(r)\,(x - iy)$$

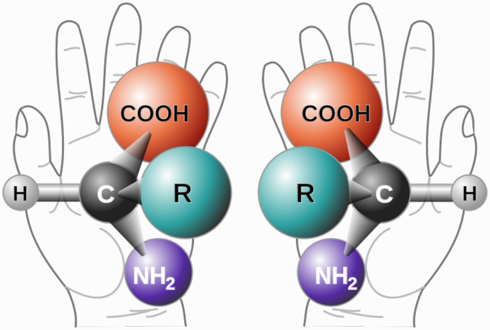
Broken Time-Reversal Symmetry

$$\mathrm{T}\,\Psi(\mathbf{r}) = f(r)\,(x - iy)$$

Signatures: Chiral, Edge Fermions $\rightsquigarrow$ Anomalous Hall Transport

Chiral Quantum Matter

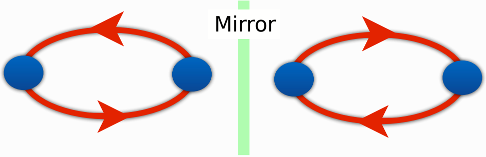
Molecular Chiral Enantiomers



Handedness: Broken Mirror Symmetry

Chiral Diatomic Molecules

$$\Psi(\mathbf{r}) = f(r)\,(x+iy)$$



Broken Mirror Symmetries

$$\Pi_{zx}\,\Psi(\mathbf{r}) = f(r)\,(x-iy)$$

Broken Time-Reversal Symmetry

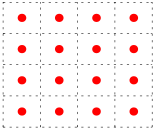
$$\mathrm{T}\,\Psi(\mathbf{r}) = f(r)\,(x-iy)$$

Signatures: Chiral, Edge Fermions $\rightsquigarrow$ Anomalous Hall Transport

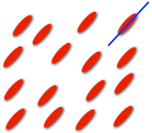
Realized in Superfluid ^{3}He-A & possibly the ground states in unconventional superconductors

▶ P.W. Anderson & P. Morel, Phys. Rev. Lett. 1960

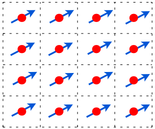
Broken Symmetry, Phase Transitions and Long-Range Order



Solid



Nematic



Ferromagnet

Translations

$\mathsf{G}_{\mathsf{trans}}$

Space Rotations

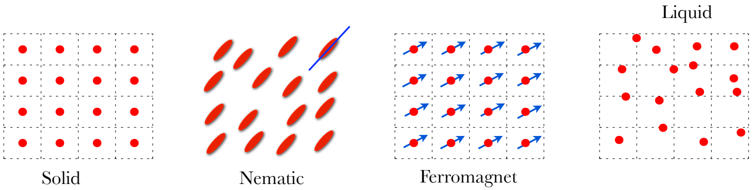
$\mathsf{SO}(3)_{\mathrm{L}}$

Spin Rotation

$\mathsf{SO}(3)_{\mathrm{S}}$

$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r}) \quad Q_{ij} = Q(T)\left(\mathbf{n}_i\mathbf{n}_j - \frac{1}{3}\delta_{ij}\right) \quad \mathbf{M} = \gamma \langle \mathbf{S} \rangle$$

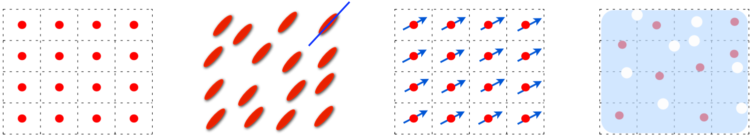
Broken Symmetry, Phase Transitions and Long-Range Order



Translations	Space Rotations	Spin Rotation
G_{trans}	$SO(3)_{\mathrm{L}}$	$SO(3)_{\mathrm{S}}$

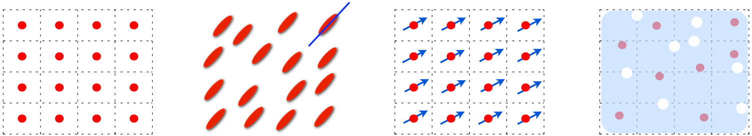
$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r}) \qquad Q_{ij} = Q(T)\left(\mathbf{n}_i\mathbf{n}_j - \frac{1}{3}\delta_{ij}\right) \qquad \mathbf{M} = \gamma\,\langle \mathbf{S} \rangle$$

Broken Symmetry, Phase Transitions and Long-Range Order



Solid	Nematic	Ferromagnet	Super-liquid
Translations	Space Rotations	Spin Rotation	Gauge
$\mathsf{G}_{\mathsf{trans}}$	$\mathsf{SO}(3)_{\mathrm{L}}$	$\mathsf{SO}(3)_{\mathrm{S}}$	$\mathsf{U}(1)_{\mathrm{N}}$
$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r})$	$Q_{ij} = Q(T)\left(\mathbf{n}_i\mathbf{n}_j - \frac{1}{3}\delta_{ij}\right)$	$\mathbf{M} = \gamma \langle \mathbf{S} \rangle$	$\Psi = \langle \psi(\mathbf{r}) \rangle \simeq \sqrt{N/V}\, \boxed{e^{i\vartheta}}$

Broken Symmetry, Phase Transitions and Long-Range Order

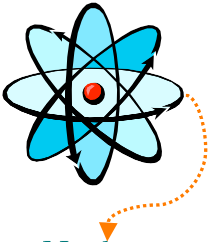


Solid | Nematic | Ferromagnet | Super-liquid

Translations	Space Rotations	Spin Rotation	Gauge
$\mathsf{G}_{\text{trans}}$	$\mathsf{SO}(3)_{\mathrm{L}}$	$\mathsf{SO}(3)_{\mathrm{S}}$	$\mathsf{U}(1)_{\mathrm{N}}$
$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r})$	$Q_{ij} = Q(T)\left(\mathbf{n}_i\mathbf{n}_j - \frac{1}{3}\delta_{ij}\right)$	$\mathbf{M} = \gamma\,\langle\mathbf{S}\rangle$	$\Psi = \langle\psi(\mathbf{r})\rangle \simeq \sqrt{N/V}\,\boxed{e^{i\vartheta}}$

▶ The Superfluid Phases of Liquid Helium Exhibit *all* of these Broken Symmetries!

Helium



$(1s)^2$ - closed electronic shell

- chemically inert
- $S_{electronic} = 0$

^4He	2 protons 2 neutrons	→	$S_{nuclear} = 0$	**Boson**
^3He	2 protons 1 neutron	→	$S_{nuclear} = \frac{1}{2}$	**Fermion**

Quantum Statistics Important for **T < T * ~ 1 K**

Helium Liquids

- Indistinguishability of identical particles becomes important …

$$\lambda = \frac{\hbar}{p} \approx \frac{\hbar}{\sqrt{2\,\mathrm{m}\,k_B\,T}} \qquad > \qquad \mathbf{a} = \sqrt[3]{\frac{V}{N}} \approx \text{Å}$$

$$T < \mathbf{T}^* = \frac{\hbar^2}{2\,\mathrm{m}\,k_B\,\mathbf{a}^2} \approx 3\,\mathrm{K}$$

^{3}He

Fermi Liquid

BCS Superfluid

$T < T_c = 2 \times 10^{-3}$ K

^{4}He

Bose Liquid

Superfluid

$T < T_\lambda = 2.2$ K

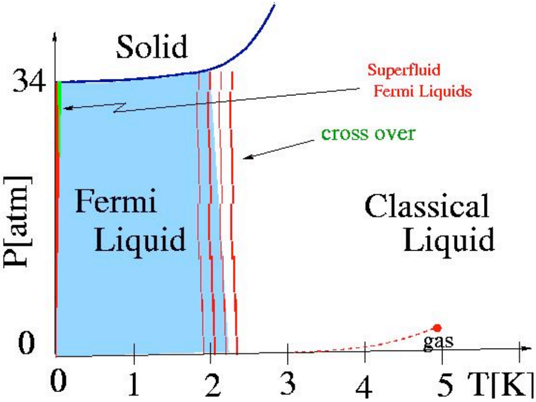
Phase Diagram for ^3He

- Permanent liquid at $P < 34$ atm
- Smooth crossover near $T^* = E_f \sim 2$ K
- ... superfluidity below
- $T_c \sim$ **2 x 10^{-3} K**

D. Osheroff, R. Richardson, D. Lee (1972)

A. J. Leggett (1973)

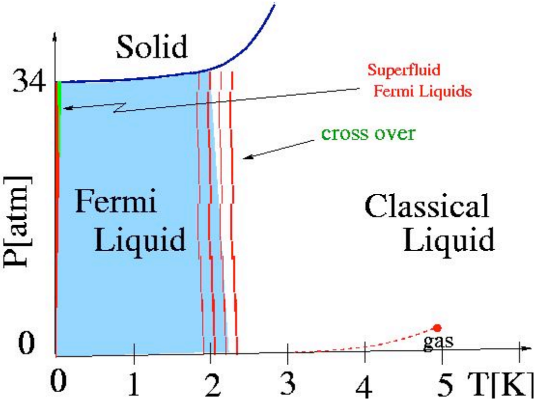
2 Nobel Prizes: 1996 & 2006



Phase Diagram for ^{3}He

- Permanent liquid at $P < 34$ atm
- Smooth crossover near $T^* = E_f \sim 2$ K
- ... superfluidity below
- $T_c \sim$ **2 x 10^{-3} K**

D. Osheroff, R. Richardson, D. Lee (1972)
A. J. Leggett (1973)
2 Nobel Prizes: 1996 & 2006



- **Macroscopic occupation of a *2-particle* quantum state**

$$|\,\Phi_N\,\rangle = \left[\iint d\mathbf{r}_1 d\mathbf{r}_2\, \varphi_{s_1 s_2}(\mathbf{r}_1,\mathbf{r}_2)\, \psi^\dagger_{s_1}(\mathbf{r}_1)\psi^\dagger_{s_2}(\mathbf{r}_2)\right]^{N/2} |\,\mathrm{vac}\,\rangle$$

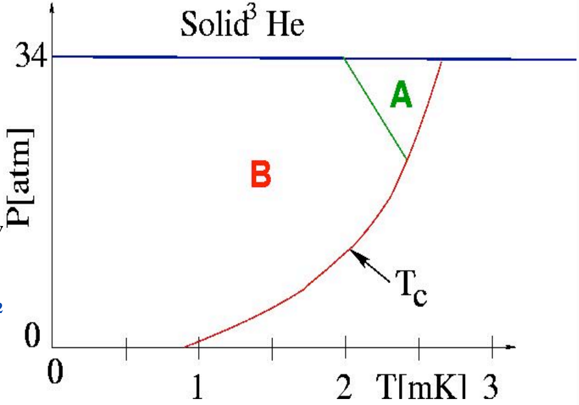
Phase Diagram for ^{3}He

- Permanent liquid at P < 34 atm
- Smooth crossover near T* = E_f ~ 2 K
- ... superfluidity below
- T_c ~ **2 x 10^{-3} K**

D. Osheroff, R. Richardson, D. Lee (1972

A. J. Leggett (1973)

2 Nobel Prizes: 1996 & 2006



- **Macroscopic occupation of a *2-particle* quantum state**

$$|\,\Phi_N\,\rangle = \left[\iint d\mathbf{r}_1 d\mathbf{r}_2\ \varphi_{s_1 s_2}(\mathbf{r}_1,\mathbf{r}_2)\ \psi^\dagger_{s_1}(\mathbf{r}_1)\psi^\dagger_{s_2}(\mathbf{r}_2)\right]^{N/2} |\,\mathrm{vac}\,\rangle$$

$\mathsf{L} = 1$ **“p-wave”** $\mathsf{S} = 1$ **“spin triplet”**

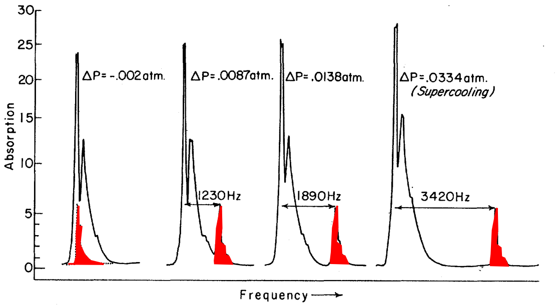
Discovery - NMR Shift in Liquid ^{3}He-A

VOLUME 29, NUMBER 14 PHYSICAL REVIEW LETTERS 2 OCTOBER 1972

New Magnetic Phenomena in Liquid He^3 below 3 mK*

D. D. Osheroff,† W. J. Gully, R. C. Richardson, and D. M. Lee
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14850
(Received 7 July 1972)

Magnetic measurements have been made on a sample of He^3 in a Pomeranchuk cell. Below about 2.7 mK, the NMR line apparently associated with the liquid portion of the sample shifts continuously to higher frequencies during cooling. Below about 2 mK the frequency shift vanishes, and the magnitude of the liquid absorption drops abruptly to approximately $\frac{1}{2}$ its previous value. These measurements are related to the pressure phenomena reported by Osheroff, Richardson, and Lee.



VOLUME 29, NUMBER 18 PHYSICAL REVIEW LETTERS 30 OCTOBER 1972

Interpretation of Recent Results on He^3 below 3 mK: A New Liquid Phase?

A. J. Leggett
School of Mathematical and Physical Sciences, University of Sussex, England
(Received 5 September 1972)

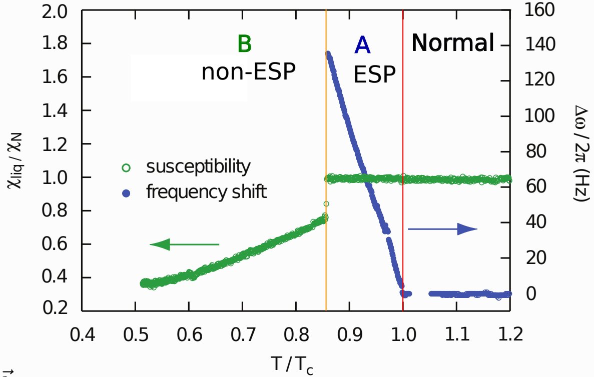
It is demonstrated that recent NMR results in ^{3}He indicate that at 2.65 mK, the liquid makes a second-order transition to a phase in which the "spin-orbit" symmetry is spontaneously broken. The hypothesis that this phase is a BCS-type phase in which pairs form with l odd, $S=1$, $S_z=\pm 1$ leads to reasonable agreement with both NMR and thermodynamic data, but involves some difficulties as to stability.

$$\omega^2 = (\gamma H)^2 + \Omega^2(T) \quad \longrightarrow \quad \omega \simeq \gamma H + \frac{\Omega^2(T)}{2\gamma H} \propto (1 - T/T_c)$$

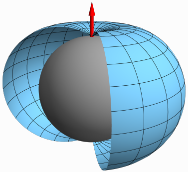
$$\Omega^2 = -\frac{2\gamma^2}{\chi} \langle \Psi | \mathcal{H}_D | \Psi \rangle \qquad \Omega \neq 0 \implies \text{Broken } \textit{relative} \text{ Spin-Orbit Rotation Symmetry}$$

NMR frequency shift and Magnetic Susceptibility

J. Pollanen et al. PRL 107, 195301 (2011)



Chiral AM State $\vec{l}$



$$|\Psi_A\rangle = \Delta \left\{ \overbrace{(p_y + ip_z)}^{L_x=+1} \overbrace{(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)}^{S_z=\pm 1} \right\}$$

orbital FM spin AFM

▶N.B. NMR is not a test for broken mirror or time-reversal symmetry

Maximal Symmetry of ^{3}He: $\mathtt{G} = \mathtt{SO}(3)_{\textsf{S}} \times \mathtt{SO}(3)_{\textsf{L}} \times \mathtt{U}(1)_{\textsf{N}} \times \mathtt{P} \times \mathtt{T}$

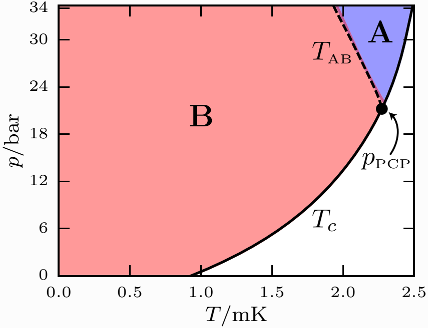
Maximal Symmetry of ^{3}He: $\mathtt{G} = \mathtt{SO(3)_S} \times \mathtt{SO(3)_L} \times \mathtt{U(1)_N} \times \mathtt{P} \times \mathtt{T}$

BCS Condensate of Bound Spin $1/2$ Fermions

$\Downarrow$

Cooper Pairs with Total Spin, $S = 1$ and Orbital Angular Momentum, $L = 1$

J. Wiman & J. A. Sauls, PRB 92, 144515 (2015)



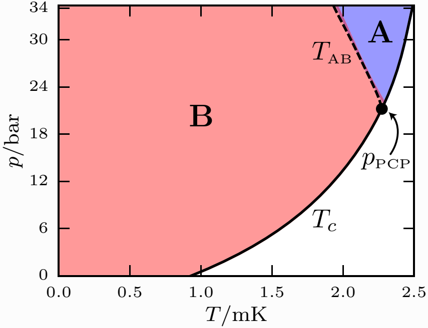
Maximal Symmetry of ^{3}He: $\mathtt{G} = \mathtt{SO(3)_S} \times \mathtt{SO(3)_L} \times \mathtt{U(1)_N} \times \mathtt{P} \times \mathtt{T}$

BCS Condensate of Bound Spin $1/2$ Fermions

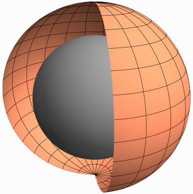
$\Downarrow$

Cooper Pairs with Total Spin, $S = 1$ and Orbital Angular Momentum, $L = 1$

J. Wiman & J. A. Sauls, PRB 92, 144515 (2015)



"Isotropic" BW State



$J = 0,\ J_z = 0$

$H = \mathtt{SO(3)_J} \times \mathtt{T}$

$$|\Psi_B\rangle = \Delta \left\{ \overbrace{\tfrac{1}{\sqrt{2}}(p_x - ip_y)}^{L_z=-1} \underbrace{|\uparrow\uparrow\rangle}_{S_z=+1} + \overbrace{\tfrac{1}{\sqrt{2}}(p_x + ip_y)}^{L_z=+1} \underbrace{|\downarrow\downarrow\rangle}_{S_z=-1} + \overbrace{p_z}^{L_z=0} \underbrace{\tfrac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)}_{S_z=0} \right\}$$

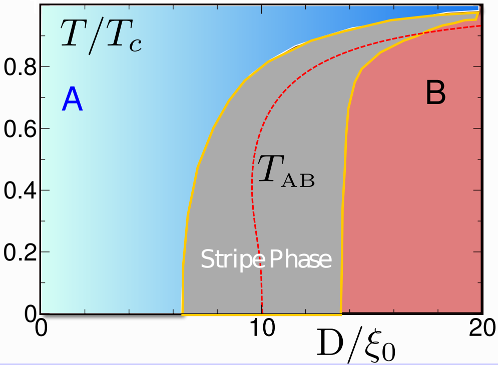
Realization of Broken Time-Reversal and Mirror Symmetry by the Ground State of ^{3}He Films

- Length Scale for Strong Confinement:

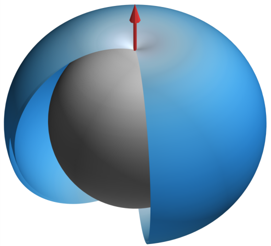
$$\xi_0 = \hbar v_f / 2\pi k_B T_c \approx 20 - 80\,\text{nm}$$

- L. Levitov et al., Science 340, 6134 (2013)
- A. Vorontsov & J. A. Sauls, PRL 98, 045301 (2007)

$$\mathrm{SO(3)_S} \times \mathrm{SO(3)_L} \times \mathrm{U(1)_N} \times \mathrm{T} \times \mathrm{P}$$
$$\Downarrow$$
$$\mathrm{SO(2)_S} \times \mathrm{U(1)_{N\text{-}L_z}} \times \mathrm{Z_2}$$



Chiral ABM State $\vec{l} = \hat{\mathbf{z}}$



$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix}_{ABM} = \begin{pmatrix} p_x + ip_y \sim e^{+i\phi} & 0 \\ 0 & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

Realization of Broken Time-Reversal and Mirror Symmetry by the Ground State of ^{3}He Films

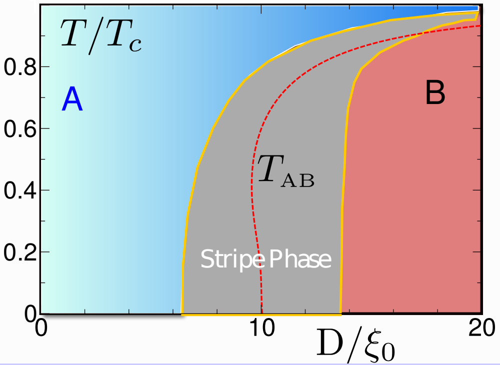
▶ Length Scale for Strong Confinement:

$$\xi_0 = \hbar v_f / 2\pi k_B T_c \approx 20 - 80\,\text{nm}$$

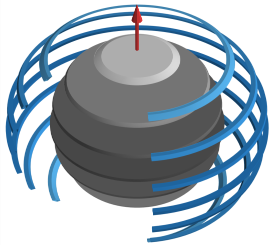
▶ L. Levitov et al., Science 340, 6134 (2013)

▶ A. Vorontsov & J. A. Sauls, PRL 98, 045301 (2007)

$$\mathrm{SO(3)_S} \times \mathrm{SO(3)_L} \times \mathrm{U(1)_N} \times \mathrm{T} \times \mathrm{P}$$
$$\Downarrow$$
$$\mathrm{SO(2)_S} \times \mathrm{U(1)_{N\text{-}L_z}} \times \mathrm{Z_2}$$



Chiral ABM State $\vec{l} = \hat{\mathbf{z}}$



$$L_z = 1,\ S_z = 0$$

Ground-State Angular Momentum

$$\langle \widehat{L}_z \rangle = \frac{N}{2}\hbar \ ?$$

Open Question

▶ J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix}_{ABM} = \begin{pmatrix} p_x + ip_y \sim e^{+i\phi} & 0 \\ 0 & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

Signatures of Broken T and P Symmetry in ^{3}He-A

Evidence for the Chirality of Superfluid ^{3}He-A

$\Downarrow$

Broken T and P $\rightsquigarrow$ Zero-Field Hall Effect for Electrons Moving in ^{3}He-A

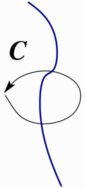
Broken Symmetries $\rightsquigarrow$ Topology of ^{3}He-A

Chirality $+$ Topology $\rightsquigarrow$ Chiral Edge States

Real-Space vs. Momentum-Space Topology

Topology in Real Space

$\Psi(\mathbf{r}) = |\Psi(r)|\, e^{i\vartheta(\mathbf{r})}$



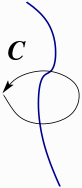
Phase Winding

$$N_C = \frac{1}{2\pi} \oint_C d\vec{l} \cdot \frac{1}{|\Psi|} \mathrm{Im}[\boldsymbol{\nabla}\Psi] \in \{0, \pm 1, \pm 2, \ldots\}$$

▶ Massless Fermions confined in the Vortex Core

Real-Space vs. Momentum-Space Topology

Topology in Real Space
$\Psi(\mathbf{r}) = |\Psi(r)|\, e^{i\vartheta(\mathbf{r})}$

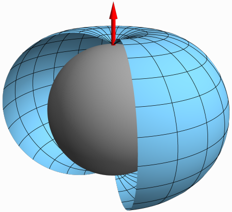


Phase Winding

$$N_C = \frac{1}{2\pi}\oint_C d\vec{l}\cdot \frac{1}{|\Psi|}\mathrm{Im}[\boldsymbol{\nabla}\Psi] \in \{0, \pm 1, \pm 2, \ldots\}$$

▶ Massless Fermions confined in the Vortex Core

Chiral Symmetry $\rightsquigarrow$
Topology in Momentum Space
$\Psi(\mathbf{p}) = \Delta(p_x \pm i p_y) \sim e^{\pm i\varphi_{\mathbf{p}}}$



Topological Quantum Number: $L_z = \pm 1$

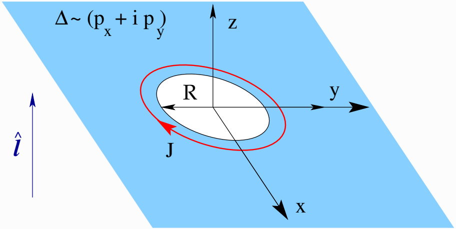
$$N_{\mathrm{2D}} = \frac{1}{2\pi}\oint d\mathbf{p}\cdot \frac{1}{|\Psi(\mathbf{p})|}\mathrm{Im}[\boldsymbol{\nabla}_{\mathbf{p}}\Psi(\mathbf{p})] = L_z$$

▶ Massless Chiral Fermions

▶ Nodal Fermions in 3D

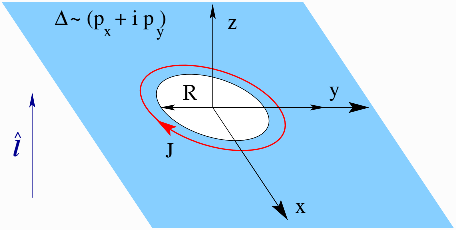
▶ Edge Fermions in 2D

Chiral Edge Current Circulating a Hole or Defect in a Chiral Superfluid



- $R \gg \xi_0 \approx 100\,\text{nm}$
- Sheet Current :
  $J \equiv \int dx\, J_{\varphi}(x)$

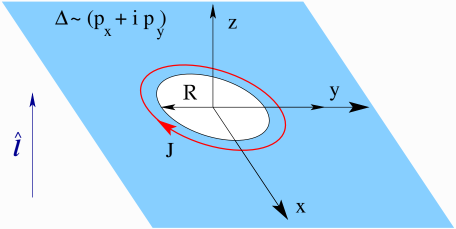
Chiral Edge Current Circulating a Hole or Defect in a Chiral Superfluid



- $R \gg \xi_0 \approx 100\,\text{nm}$
- Sheet Current: $J \equiv \int dx\, J_\varphi(x)$

- Quantized Sheet Current: $\frac{1}{4}\, n\, \hbar$ ($n = N/V = {}^3$He density)
- Edge Current *Counter*-Circulates: $J = -\frac{1}{4}\, n\, \hbar$ w.r.t. Chirality: $\hat{\mathbf{l}} = +\mathbf{z}$

Chiral Edge Current Circulating a Hole or Defect in a Chiral Superfluid



- $R \gg \xi_0 \approx 100\,\mathrm{nm}$
- Sheet Current :
  $J \equiv \int dx\, J_\varphi(x)$

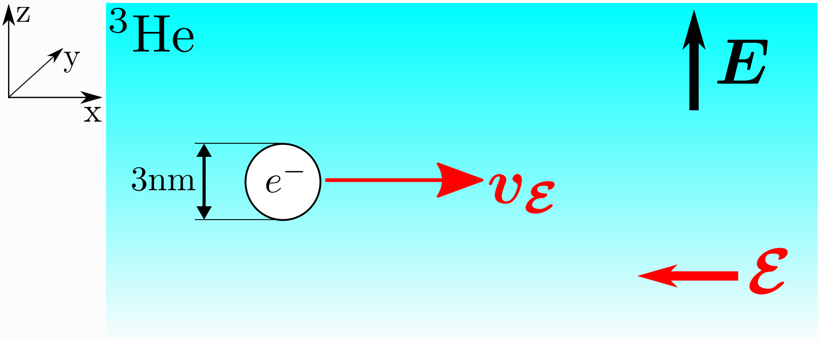
- Quantized Sheet Current: $\frac{1}{4}\, n\, \hbar$ $(n = N/V = {}^3$He density$)$
- Edge Current *Counter*-Circulates: $J = -\frac{1}{4}\, n\, \hbar$ w.r.t. Chirality: $\hat{\mathbf{l}} = +\mathbf{z}$
- Angular Momentum: $L_z = 2\pi\, h\, R^2 \times (-\frac{1}{4}\, n\, \hbar) = -(N_{\text{hole}}/2)\,\hbar$

$N_{\text{hole}}/2 =$ Number of ^{3}He Cooper Pairs excluded from the Hole

∴ An object in ^{3}He-A *inherits* angular momentum from the Condensate of Chiral Pairs!

- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

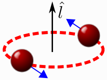
Electron bubbles in the Normal Fermi liquid phase of ^{3}He



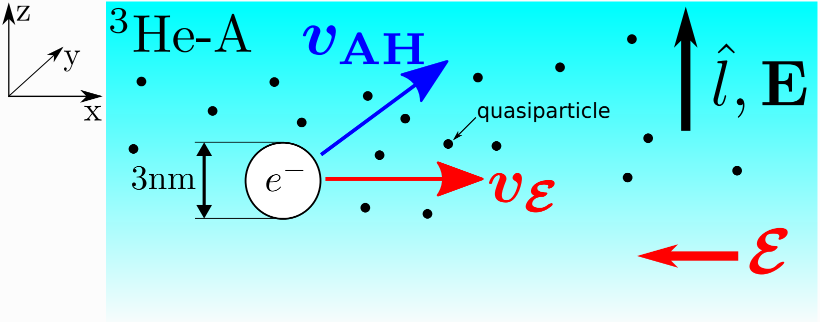
- Bubble with $R \simeq 1.5$ nm, $0.1\,\text{nm} \simeq \lambda_f \ll R \ll \xi_0 \simeq 80\,\text{nm}$
- Effective mass $M \simeq 100 m_3$ (m_3 – atomic mass of ^{3}He)
- QPs mean free path $l \gg R$
- Mobility of ^{3}He is *independent of* T for $T_c < T < 50$ mK

B. Josephson and J. Leckner, PRL 23, 111 (1969)

Electron bubbles in chiral superfluid ^{3}He-A

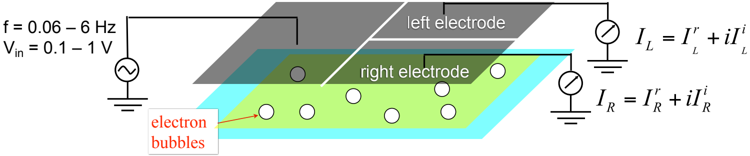


$\Delta(\hat{k}) = \Delta(\hat{k}_x + i\hat{k}_y) = \Delta\, e^{i\phi_{\mathbf{k}}}$



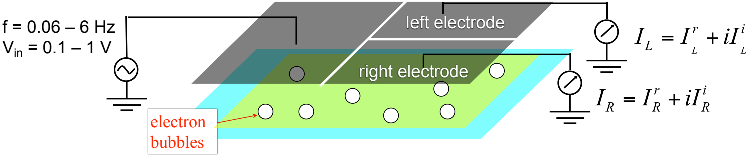
- Current: $\mathbf{v} = \overbrace{\mu_\perp \boldsymbol{\mathcal{E}}}^{\mathbf{v}_{\mathcal{E}}} + \overbrace{\mu_{\mathrm{AH}} \boldsymbol{\mathcal{E}} \times \hat{\mathbf{l}}}^{\mathbf{v}_{\mathrm{AH}}}$ R. Salmelin, M. Salomaa & V. Mineev, PRL **63**, 868 (1989)
- Hall ratio: • $\tan\alpha = v_{\mathrm{AH}}/v_{\mathcal{E}} = |\mu_{\mathrm{AH}}/\mu_\perp|$

Measurement of the Transverse e⁻ mobility in Superfluid ^{3}He Films



H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

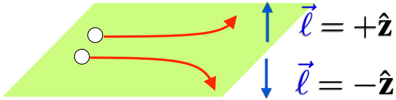
Measurement of the Transverse e⁻ mobility in Superfluid ^{3}He Films



Transverse Force from ***Skew*** **Scattering**

$$\rightsquigarrow \quad \Delta I = I_R - I_L \neq 0$$

$$\vec{v} = \left[\mu_\perp \vec{E} + \mu_{xy}\, \hat{\ell} \times \vec{E}\right]$$

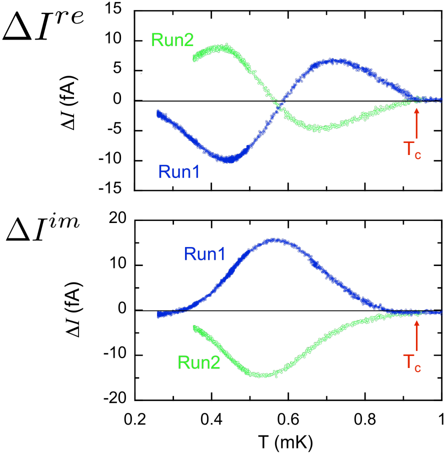


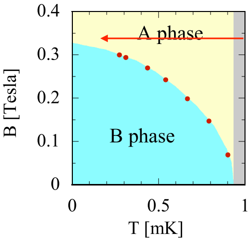
H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

Detection of Broken Time-Reversal & Mirror Symmetry in ^{3}He-A

Transverse e$^-$ bubble current in ^{3}He-A

$$\Delta I = I_R - I_L$$





Single Domains:

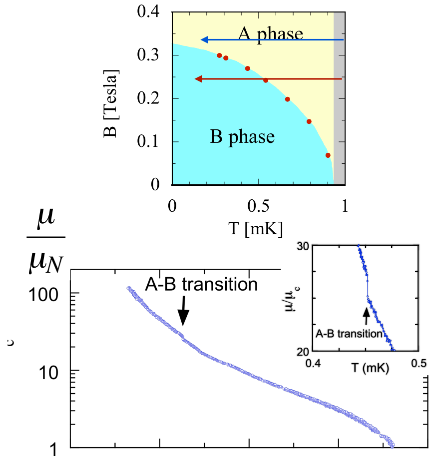
Run 1 $\vec{\ell} = +\hat{\mathbf{z}}$

Run 2 $\vec{\ell} = -\hat{\mathbf{z}}$

$$\frac{|I_R - I_L|}{I_R + I_L} \approx 6\,\%$$

H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

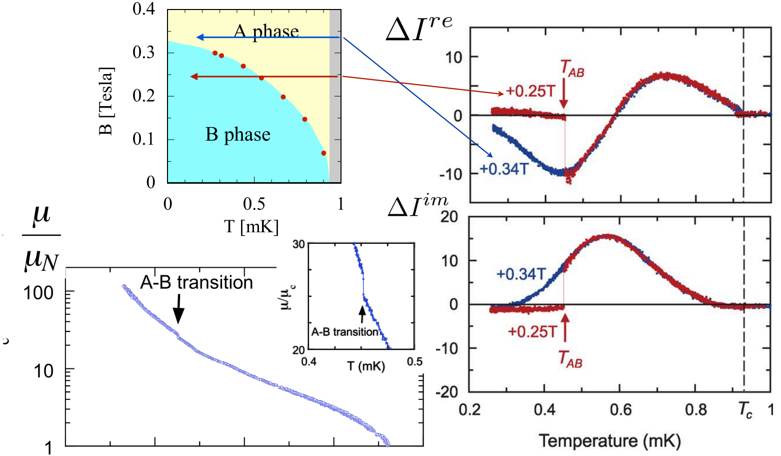
Zero Transverse e$^-$ current in ^{3}He-B (*T - symmetric phase*)



H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

Detection of Broken Time-Reversal & Mirror Symmetry in ^{3}He-A

Zero **Transverse e$^-$ current in ^{3}He-B (*T - symmetric phase*)**



H. Ikegami, Y. Tsutsumi, K. Kono, **Science 341, 59-62 (2013)**

- Structure of Electrons in Superfluid ^{3}He-A
- Forces of Moving Electrons in Superfluid ^{3}He-A

$$\Downarrow$$

- Scattering Theory of ^{3}He Quasiparticles by Electron Bubbles

Forces on the Electron bubble in ^{3}He-A:

- $M\frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} + \mathbf{F}_{\mathrm{QP}}$, $\mathbf{F}_{QP}$ – force from quasiparticle collisions

Forces on the Electron bubble in ^{3}He-A:

- $M\frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} + \mathbf{F}_{\mathrm{QP}}$, $\mathbf{F}_{QP}$ – force from quasiparticle collisions
- $\mathbf{F}_{QP} = -\overleftrightarrow{\eta} \cdot \mathbf{v}$, $\overleftrightarrow{\eta}$ – generalized Stokes tensor
- $\overleftrightarrow{\eta} = \begin{pmatrix} \eta_\perp & \eta_{\mathsf{AH}} & 0 \\ -\eta_{\mathsf{AH}} & \eta_\perp & 0 \\ 0 & 0 & \eta_\parallel \end{pmatrix}$ for broken P & T symmetries with $\hat{\mathbf{l}} \parallel \mathbf{e}_z$

Forces on the Electron bubble in ^{3}He-A:

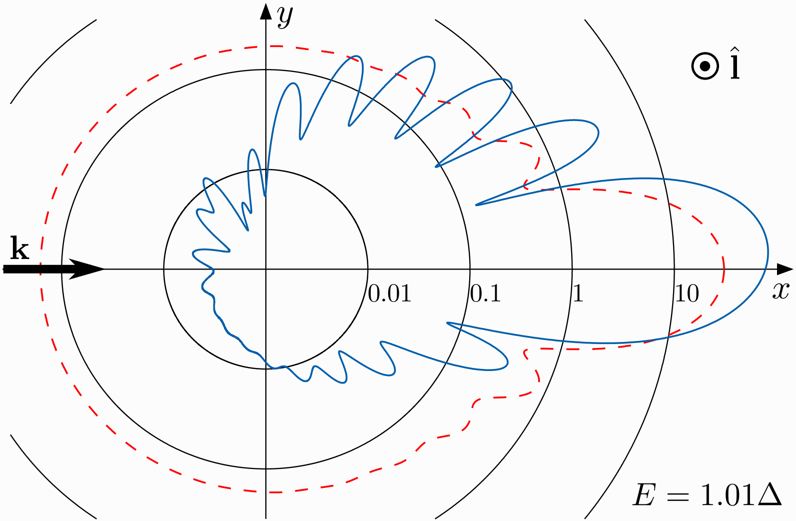
- $M\frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} + \mathbf{F}_{\text{QP}}$, $\mathbf{F}_{QP}$ – force from quasiparticle collisions
- $\mathbf{F}_{QP} = -\overleftrightarrow{\eta}\cdot\mathbf{v}$, $\overleftrightarrow{\eta}$ – generalized Stokes tensor
- $\overleftrightarrow{\eta} = \begin{pmatrix} \eta_\perp & \eta_{\text{AH}} & 0 \\ -\eta_{\text{AH}} & \eta_\perp & 0 \\ 0 & 0 & \eta_\parallel \end{pmatrix}$ for broken P & T symmetries with $\hat{\mathbf{l}} \parallel \mathbf{e}_z$
- $M\frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} - \eta_\perp\mathbf{v} + \frac{e}{c}\mathbf{v}\times\mathbf{B}_{\text{eff}}$, for $\boldsymbol{\mathcal{E}} \perp \hat{\mathbf{l}}$
- $\mathbf{B}_{\text{eff}} = -\frac{c}{e}\eta_{\text{AH}}\hat{\mathbf{l}}$ $\quad B_{\text{eff}} \simeq 10^3 - 10^4$ T !!!

Forces on the Electron bubble in ^{3}He-A:

- $M\frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} + \mathbf{F}_{\rm QP}$, $\mathbf{F}_{QP}$ – force from quasiparticle collisions
- $\mathbf{F}_{QP} = -\overleftrightarrow{\eta}\cdot\mathbf{v}$, $\overleftrightarrow{\eta}$ – generalized Stokes tensor
- $\overleftrightarrow{\eta} = \begin{pmatrix} \eta_\perp & \eta_{\rm AH} & 0 \\ -\eta_{\rm AH} & \eta_\perp & 0 \\ 0 & 0 & \eta_\parallel \end{pmatrix}$ for broken P & T symmetries with $\hat{\mathbf{l}} \parallel \mathbf{e}_z$
- $M\frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} - \eta_\perp\mathbf{v} + \frac{e}{c}\mathbf{v}\times\mathbf{B}_{\rm eff}$, for $\boldsymbol{\mathcal{E}} \perp \hat{\mathbf{l}}$
- $\mathbf{B}_{\rm eff} = -\frac{c}{e}\eta_{\rm AH}\hat{\mathbf{l}}$ $B_{\rm eff} \simeq 10^3 - 10^4$ T !!!
- Mobility: $\mathbf{v} = \overleftrightarrow{\mu}\boldsymbol{\mathcal{E}}$, where $\overleftrightarrow{\mu} = e\overleftrightarrow{\eta}^{-1}$

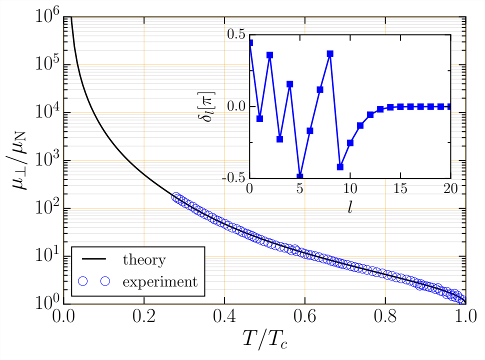
▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

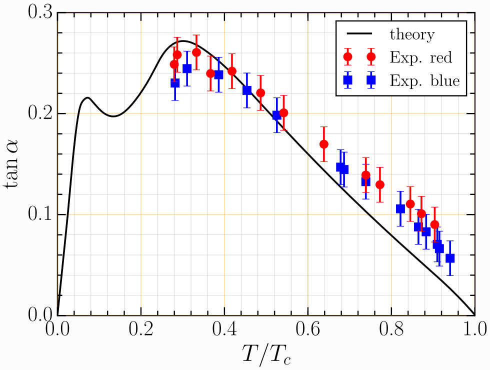
Differential cross section for Bogoliubov QP-Ion Scattering $k_f R = 11.17$



▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Comparison between Theory and Experiment for the Drag and Transverse Forces





- $\mu_\perp = e\,\dfrac{\eta_\perp}{\eta_\perp^2 + \eta_{AH}^2}$
- $\mu_{AH} = -e\,\dfrac{\eta_{AH}}{\eta_\perp^2 + \eta_{AH}^2}$

- $\tan\alpha = \left|\dfrac{\mu_{AH}}{\mu_\perp}\right| = \dfrac{\eta_{AH}}{\eta_\perp}$
- Electron Bubble Radius: $k_f R = 11.17$

▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

▶O. Shevtsov and JAS, JLTP 187, 340–353 (2017)

Summary

- Electrons in ^{3}He-A are "dressed" by a spectrum of Chiral Fermions
- Electrons are "Left handed" in a Right-handed Chiral Vacuum $\rightsquigarrow L_z \approx -100\,\hbar$
- Experiment: RIKEN mobility experiments $\rightsquigarrow$ Observation an AHE in ^{3}He-A
- Origin: Broken Mirror & Time-Reversal Symmetry
- Theory: Scattering of Bogoliubov QPs by the dressed Ion $\rightsquigarrow$

  • Drag Force $(-\eta_\perp \mathbf{v})$ • Transverse Force $(\frac{e}{c}\mathbf{v} \times \mathbf{B}_{\text{eff}})$
- *Anomalous Hall Field:* • $\mathbf{B}_{\text{eff}} \approx \frac{\Phi_0}{3\pi^2} k_f^2 \, (k_f R)^2 \left(\frac{\eta_{\text{AH}}}{\eta_{\text{N}}}\right) \mathbf{l} \simeq 10^3 - 10^4 \, \text{T} \, \mathbf{l}$

Fundamental Connections between Physics at Different Scales

Fundamental Connections between Physics at Different Scales

Dynamical Consequences of Spontaneous Symmetry Breaking

New Bosonic Excitations

- "It is only slightly overstating the case to say that physics is the study of symmetry" – P. W. Anderson

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN)

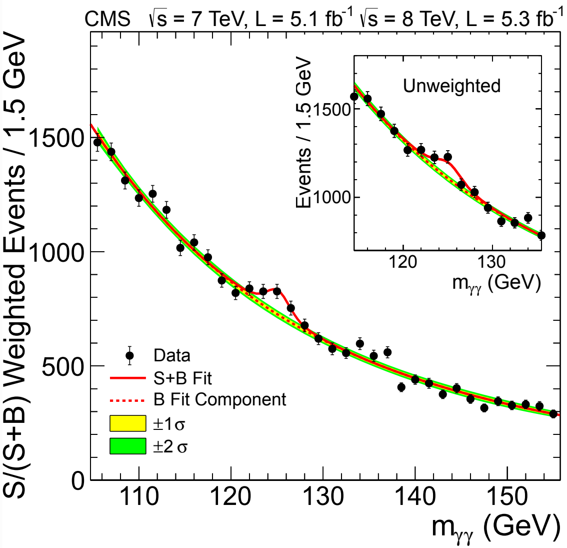
CERN-PH-EP/2012-220
2013/01/29

CMS-HIG-12-028

Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC

The CMS Collaboration

Higgs Boson with mass $M = 125$ GeV

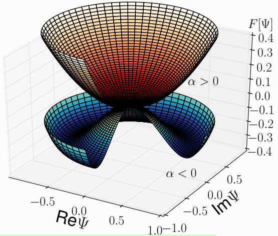


Dynamical Consequences of Spontaneous Symmetry Breaking

Scalar Higgs Boson (spin $J = 0$) [P. Higgs, PRL 13, 508 1964]

Energy Functional for the Higgs Field

$$\mathcal{U}[\Delta] = \int dV \left\{ \alpha \, |\Delta|^2 + \beta \, |\Delta|^4 + \frac{1}{2}c^2 \, |\boldsymbol{\nabla}\Delta|^2 \right\}$$



▶ Broken Symmetry State: $\Delta = \sqrt{|\alpha|/2\beta}$

Space-Time Fluctuations about the Broken Symmetry Vacuum State

$\Delta(\mathbf{r}, t) = \Delta + D(\mathbf{r}, t)$ ▶ Eigenmodes: $D^{(\pm)} = D \pm D^*$ (Conjugation Parity)

- $$\mathcal{L} = \int d^3r \left\{ \frac{1}{2}[(\dot{D}^{(+)})^2 + (\dot{D}^{(-)})^2] - 2\Delta^2 \, (D^{(+)})^2 - \frac{1}{2}[c^2(\boldsymbol{\nabla} D^{(+)})^2 + c^2(\boldsymbol{\nabla} D^{(-)})^2] \right\}$$

▶ $\partial_t^2 D^{(-)} - c^2 \boldsymbol{\nabla}^2 \, D^{(-)} = 0$

Massless Nambu-Goldstone Mode

▶ $\partial_t^2 D^{(+)} - c^2 \boldsymbol{\nabla}^2 \, D^{(+)} + 4\Delta^2 \, D^{(+)} = 0$

Massive Higgs Mode: $M = 2\Delta$

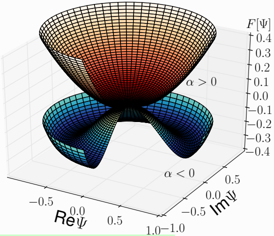
Dynamical Consequences of Spontaneous Symmetry Breaking

BCS Condensation of Spin-Singlet ($S = 0$), S-wave ($L = 0$) "Scalar" Cooper Pairs

Ginzburg-Landau Functional

$$F[\Delta] = \int dV \left\{ \alpha \, |\Delta|^2 + \beta \, |\Delta|^4 + \kappa \, |\nabla \Delta|^2 \right\}$$

▶ Order Parameter: $\Delta = \sqrt{|\alpha|/2\beta}$



Space-Time Fluctuations of the Condensate Order Parameter

$\Delta(\mathbf{r}, t) = \Delta + D(\mathbf{r}, t)$ ▶ Eigenmodes: $D^{(\pm)} = D \pm D^*$ (Fermion "Charge" Parity)

- $$\mathcal{L} = \int d^3r \left\{ \frac{1}{2}[(\dot{D}^{(+)})^2 + (\dot{D}^{(-)})^2] - 2\Delta^2 \, (D^{(+)})^2 - \frac{1}{2}[v^2(\nabla D^{(+)})^2 + v^2(\nabla D^{(-)})^2] \right\}$$

▶ $\partial_t^2 D^{(-)} - v^2 \nabla^2 D^{(-)} = 0$

Anderson-Bogoliubov Mode

▶ $\partial_t^2 D^{(+)} - v^2 \nabla^2 D^{(+)} + 4\Delta^2 \, D^{(+)} = 0$

Amplitude Higgs Mode: $M = 2\Delta$

Dynamical Consequences of Spontaneous Symmetry Breaking

- Observation of Higgs Modes in Superfluid ^{3}He-B

Observation of a New Sound-Attenuation Peak in Superfluid ^{3}He-*B*

R. W. Giannetta,[a] A. Ahonen,[b] E. Polturak, J. Saunders,
E. K. Zeise, R. C. Richardson, and D. M. Lee
Laboratory of Atomic and Solid State Physics and Materials Science Center, Cornell University, Ithaca, New York 14853
(Received 25 March 1980)

Results of zero-sound attenuation measurements in ^{3}He-*B*, at frequencies up to 60 MHz and pressures between 0 and 20 bars, are reported. At frequencies of 30 MHz and above, a new attenuation feature is observed which bears the signature of a collective mode of the superfluid.

VOLUME 45, NUMBER 4 PHYSICAL REVIEW LETTERS 28 JULY 1980

Measurements of High-Frequency Sound Propagation in ^{3}He-*B*

D. B. Mast, Bimal K. Sarma, J. R. Owers-Bradley, I. D. Calder,
J. B. Ketterson, and W. P. Halperin
Department of Physics and Astronomy and Materials Research Center, Northwestern University, Evanston, Illinois 60201
(Received 10 April 1980)

Measurements of the attenuation and velocity of pulsed high-frequency sound have been performed up to 133 MHz in superfluid ^{3}He-*B*. A new collective mode of the order parameter was discovered at a frequency extrapolated to T_c of $\omega = (1.165 \pm 0.05)\Delta_{BCS}(T_c)$, where $\Delta_{BCS}(T)$ is the energy gap in the weak-coupling BCS theory. The group velocity has been observed to decrease by as much as $\frac{2}{3}$ of the zero-sound velocity.

- Retrospective (1961 - 2022) on the impact of this discovery:
J. A. Sauls, J.Low Temp. Phys. 208, 1/2, 87-118 (2022).

Field Theory of the Bosonic Excitations of Superfluid ^{3}He-B

^{3}He-B: $B_{\alpha i} = \frac{1}{\sqrt{3}}\Delta\,\delta_{\alpha i}$ $L=1$, $S=1$ $\rightsquigarrow$ $J=0$ $C=+1$

- Symmetry of ^{3}He-B: $\mathtt{H} = \mathtt{SO(3)}_{\mathtt{J}} \times \mathtt{T}$
- Fluctuations: $\mathcal{D}_{\alpha i}(\mathbf{r},t) = A_{\alpha i}(\mathbf{r},t) - B_{\alpha i} = \sum_{J,m} D_{J,m}(\mathbf{r},t)\, t^{(J,m)}_{\alpha i}$
- Lagrangian:

$$\mathcal{L} = \int d^3r \left\{ \tau\,\mathrm{Tr}\left\{\dot{\mathcal{D}}\dot{\mathcal{D}}^\dagger\right\} - \alpha\,\mathrm{Tr}\left\{\mathcal{D}\mathcal{D}^\dagger\right\} - \sum_{p=1}^{5}\beta_p\,u_p(\mathcal{D}) - \sum_{l=1}^{3} K_l\,v_l(\partial\mathcal{D}) \right\}$$

$$\partial_t^2 D^{(\mathtt{C})}_{J,m} + E^{(\mathtt{C})}_{J,m}(\mathbf{q})^2\, D^{(\mathtt{C})}_{J,m} = \frac{1}{\tau}\eta^{(\mathtt{C})}_{J,m}$$

with $J=\{0,1,2\}\,,\; m=-J\ldots+J\,,\; \mathtt{C}=\pm 1$

- *Nambu's Boson-Fermion Mass Relations for ^{3}He-B*: JAS & T. Mizushima, Phys. Rev. B 95, 094515 (2017)

Spectrum of Bosonic Modes of Superfluid ^{3}He-B : Condensate is $J^{\mathtt{C}} = 0^+$

▶4 Nambu-Goldstone Modes & 14 Higgs modes

$$E^{(\mathtt{C})}_{J,m}(\mathbf{q}) = \sqrt{M^2_{J,\mathtt{C}} + \left(c^{(\mathtt{C})}_{J,|m|}|\mathbf{q}|\right)^2}$$

Mode	Symmetry	Mass	Name
$D^{(+)}_{0,m}$	$J = 0,\ \mathtt{C} = +1$	2Δ	Amplitude Higgs
$D^{(-)}_{0,m}$	$J = 0,\ \mathtt{C} = -1$	0	NG Phase Mode
$D^{(+)}_{1,m}$	$J = 1,\ \mathtt{C} = +1$	0	NG Spin-Orbit Modes
$D^{(-)}_{1,m}$	$J = 1,\ \mathtt{C} = -1$	2Δ	Spin-Orbit Higgs Modes
$D^{(+)}_{2,m}$	$J = 2,\ \mathtt{C} = +1$	$\sqrt{\frac{8}{5}}\Delta$	2^+ Higgs Modes
$D^{(-)}_{2,m}$	$J = 2,\ \mathtt{C} = -1$	$\sqrt{\frac{12}{5}}\Delta$	2^- Higgs Modes

▶Vdovin, Maki, Wölfle, Serene, Nagai, Schopohl, JAS, Volovik, R. Fishman, R. McKenzie, G. Moores, ...

▶*Broken Symmetry & Nonequilibrium Superfluid ^{3}He*, Les Houches Lectures, arXiv:cond-mat/9910260 (1999), J.A. Sauls

Bosonic Mode Spectrum for ^{3}He-B

Bosonic Excitations of ^{3}He-B

Goldstone Mode w/ J=0⁻ → $D^{(-)}_{00} = i|\Delta|\, \underbrace{\varphi(q,\omega)}$

phase mode

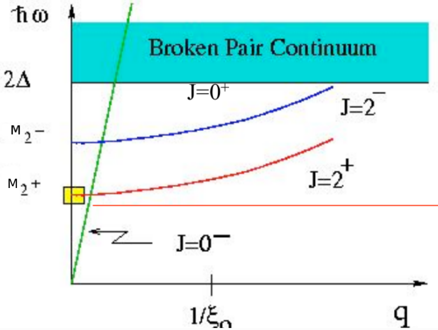
$$\left(\partial_t^2 - c_{00}^2 \nabla^2\right) D^{(-)}_{00} = \dots$$

Pair Excitons w/ J=2⁺/⁻

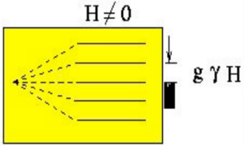
$$\left(\partial_t^2 + \Omega_{JC}^2\right) D^{(C)}_{JM} = \dots$$

Anderson-Higgs Modes

coupling to internal & external fields



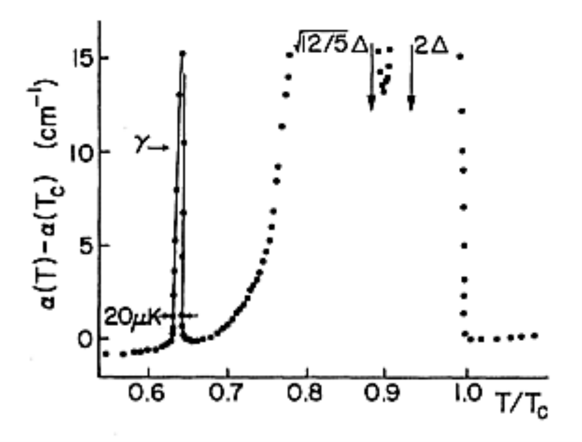
Nuclear Zeeman levels

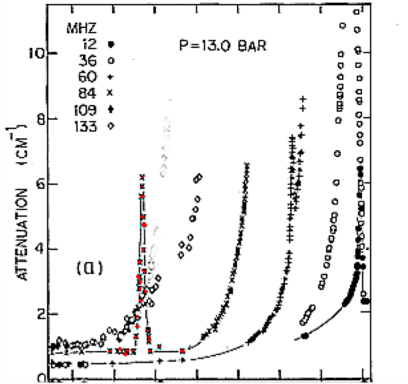


JAS & J. Serene, PRL 1982

Dynamical Consequences of Spontaneous Symmetry Breaking

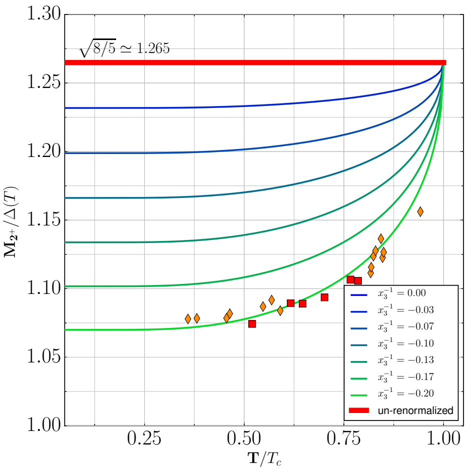
Higgs Mode with mass: $M = 500$ neV and spin $J = 2$ at Cornell & Northwestern





▶R. Giannetta et al., PRL 45, 262 (1980) ▶D. Mast et al. Phys. Rev. Lett. 45, 266 (1980)

Vacuum Polarization ⇝ Mass shift of the $J^{\mathrm{C}} = 2^{+}$ Higgs Mode in ^{3}He-B



- Measurements: D. Mast et al. PRL 45, 266 (1980)
- Vacuum polarization in both ph and pp Channels
- Exchange p-h channel: $F_2^a = -0.88$
  R. Fishman and JAS PRB B 33, 6068, 1986.
- *Attractive* f-wave pairing interaction

$\Downarrow$

- Higgs Modes with $J = 4^{\pm}$ with $M \lesssim 2\Delta$!

$\Downarrow$

Discovery of an Excited Pair State in ^{3}He-B
J. Davis et al., Nature Physics 4, 571 (2008).

▶JAS and J. W. Serene, Coupling of Order-Parameter Modes with L>1 to Zero Sound in ^{3}He-B, Phys. Rev. B 23, 4798 (1982)
▶JAS and T. Mizushima, On Nambu's Boson-Fermion Mass Relations, Phys. Rev. B 95, 094515 (2017)

Published in *Journal of Low Temperature Physics, Vol. 91, 13-37 (1993).*

Transverse Waves in Superfluid ^{3}He-B

G. F. Moores[a] and J. A. Sauls[a,b]

[a] *Department of Physics & Astronomy, Northwestern University, Evanston, Illinois 60208, USA*
[b] *Nordita, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*

(Received October 7, 1992)

- Liquid Helium flows without resistance
- … but also behaves like a solid !
- Sound waves propagate like polarized light

- *Emergence* of quanta that transport momentum & angular momentum
- *Spin=2 Higgs Boson of the Superfluid Vacuum*

▶ "Electromagnetic Absorption in Anisotropic Superconductors", P. Hirschfeld et al., PRB 40, 6695 (1989)

Transverse Sound Waves Propagate in Superfluid ^{3}He

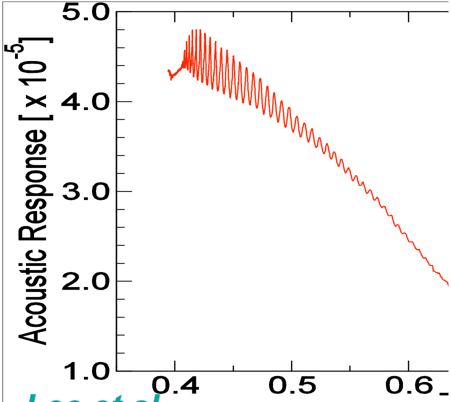
Low Temperature Physic Group

Northwestern University

Yoonseok Lee
Florida

Tom Haard
Berkeley

Bill Halperin
Northwestern



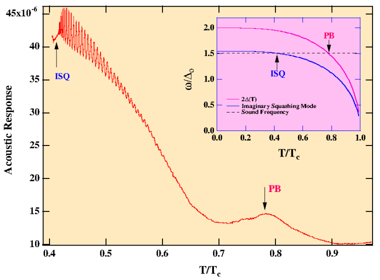
Lee et al.
Nature 400, *431 (1999).*

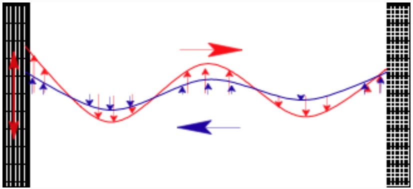
$J = 2^-$, $m = \pm 1$ Higgs Modes Transport Mass and Spin

- "Transverse Waves in Superfluid ^{3}He-B", G. Moores and JAS, JLTP 91, 13 (1993)
- "Electromagnetic Absorption in Anisotropic Superconductors", P. Hirschfeld et al., PRB 40, 6695 (1989)

$$C_t(\omega) = \sqrt{\frac{F_1^s}{15}} v_f \left[\rho_n(\omega) + \frac{2}{5}\rho_s(\omega)\underbrace{\left\{\frac{\omega^2}{(\omega + i\Gamma)^2 - \frac{12}{5}\Delta^2 - \frac{2}{5}(q^2 v_f^2)}\right\}}_{D^{(-)}_{2,\pm 1}}\right]^{\frac{1}{2}}$$

Transverse Zero Sound Propagation in Superfluid ^{3}He-B: *Cavity Oscillations of TZS*





- Y. Lee et al. Nature 400 (1999)

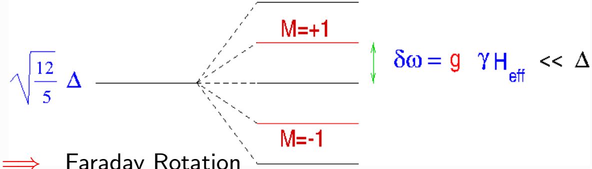
$\mathbf{B}$ $\longrightarrow$

Faraday Rotation: Magneto-Acoustic Birefringence of Transverse Currents

- "Magneto-Acoustic Rotation of Transverse Waves in ^{3}He-B", J. A. Sauls et al., Physica B, 284,267 (2000)

$$C_{\substack{\text{RCP}\\ \text{LCP}}}(\omega) = v_f \left[\frac{F_1^s}{15}\rho_n(\omega) + \frac{2F_1^s}{75}\rho_s(\omega) \underbrace{\left\{ \frac{\omega^2}{(\omega+i\Gamma)^2 - \Omega_{2,\pm}^{(-)}(\mathbf{q})} \right\}}_{D_{2,\pm 1}^{(-)}} \right]^{\frac{1}{2}}$$

$$\Omega_{2,\pm}^{(-)}(\mathbf{q}) = \sqrt{\tfrac{12}{5}}\Delta \pm g_{2-}\,\gamma H_{\text{eff}}$$



- Circular Birefringence $\Longrightarrow$ $C_{\text{RCP}} \neq C_{\text{LCP}}$ $\Longrightarrow$ Faraday Rotation

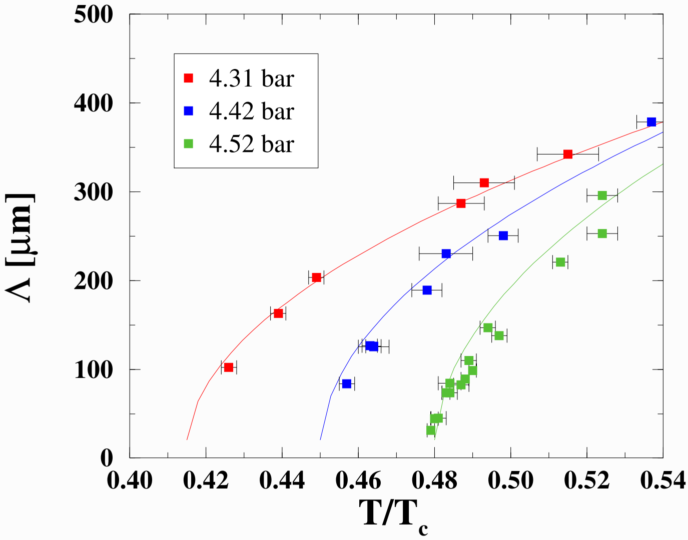
$$\left(\frac{C_{\text{RCP}} - C_{\text{LCP}}}{C_t}\right) \simeq g_{2-} \left(\frac{\gamma H_{\text{eff}}}{\omega}\right)$$

- Faraday Rotation Period ($\gamma H_{\text{eff}} \ll (\omega - \Omega_2^{(-)})$):

$$\Lambda \simeq \frac{4\pi C_t}{g_{2-}\gamma H} \simeq 500\,\mu m\,, \quad H = 200\,G$$

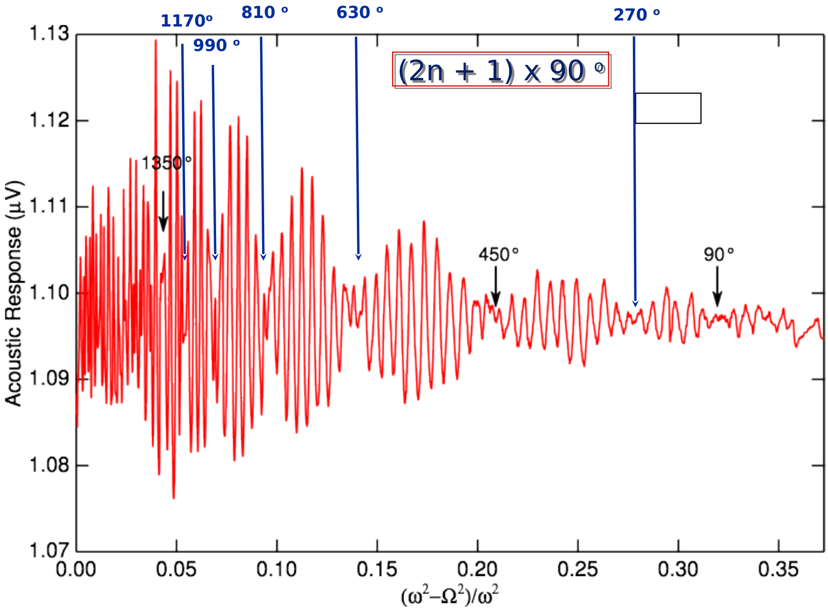
- *Discovery of the acoustic Faraday effect in superfluid ^{3}He-B*, Y. Lee, et al. Nature 400, 431 (1999)

Faraday Rotation: Magneto-Acoustic Birefringence of Transverse Currents



▶ "Broken Symmetry & Non-Equilibrium Superfluid ^{3}He", J. A. Sauls, *Lecture Notes - Les Houches 1999*, Eds. H. Godfrin & Y. Bunkov, Elsievier (2000)

Large Faraday Rotations vs. ``Blue Tuning'' B = 1097 G



Why I came and built a career in Physics at Northwestern

The Reason Northwestern is a Great Place to Pursue Research in Physics

Northwestern Ultra-Low Temperature Lab

The Coldest Place in North America is in the Basement of Tech!

Bill Halperin's Laboratory

T = 0.0003 K !

Low Temperatures enable

Technologies

From

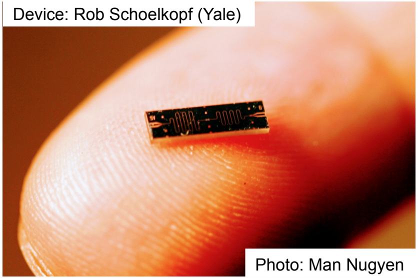
High Energy Accelerators

To

Quantum Sensors and Quantum Computers

Superconducting Qubits & Quantum Circuits

Engineered ``Atoms" you can hold in your hand!



Superconducting Qubit
Coupled to Microwave Resonator

~ 4 * 10^{-5} eV

↓

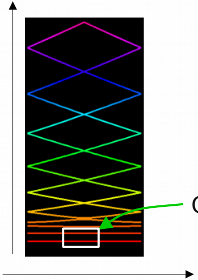
f ~ 10 GHz

Microwave Photons

↓

T < 0.5 Kelvin

Quantized Energy Levels



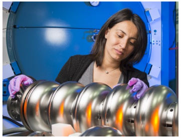
The Qubit

Tuning parameter:
External magnetic flux
Gate voltage

Jens Koch

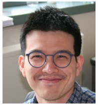
Quantum Computing with SRF Technology

Anna Grassellino



superconducting Niobium RF cavities

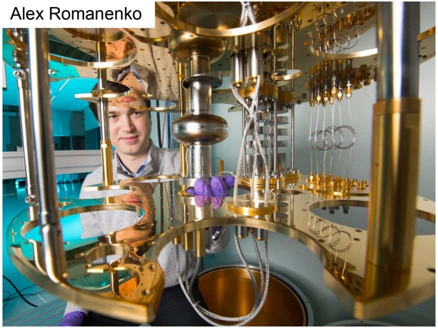
$Q = 4 \times 10^{11}$



Nik Zhelev

Wave Ngampruetikorn
CAPST

Alex Romanenko



3D SRF architecture for long coherence Qubits

SRF cavities coupled to Josephson junctions

Understand SRF cavities Operating at the single photon level

SRF cavities at ULT for Dark Matter detection

State of the art Blue Fors Cryogenic platforms:
"push button" T = 6 mK

NU-Fermi Center for Applied Physics & Superconducting Technologies

Recent Members of the Theory Group

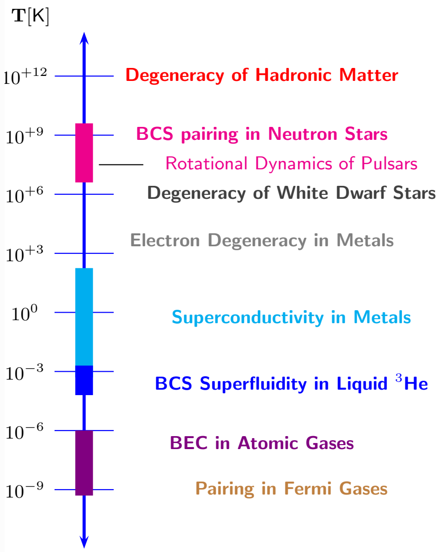
Kathy Burgess

Thank You!

The End

Extra Slides

Bardeen-Cooper-Schrieffer (BCS) Theory from 10^{-9} K to 10^{+9} K



Key Discoveries

1908 Helium is liquified

1911 Superconductivity is discovered in Hg
1933 Diamagnetism - Meissner Effect
1935 London's Theory
1950 Ginzburg-Landau Theory

1956 Copper Instability
1957 BCS Pairing Theory
1957 Landau Fermi Liquid Theory
1957 Abrikosov's Theory of Type II SC
1958 Pairing in Nuclei and Nuclear Matter
1959 Gauge-Invariant Pairing Theory
1959 Field Theory formulation of BCS Theory
1961 Theory of Spin-Triplet Pairing
1962 Josephson Effect

1967 Pulsars discovered - Hewish & Bell
1969 Pulsar Glitches observed in Vela
1980 Superfluid hydrodynamics of NS

1972 Discovery of Triplet, P-wave Superfluid ^{3}He
1979 Discovery of Heavy Electron Superconductors
1982 Exotic Pairing in U-based Heavy Fermions
1986 High T_c CuO Superconductivity
1994 Exotic Pairing discovered in Sr_2RuO_4
1995 D-wave Pairing identified in $YBa_2Cu_3O_{7-x}$
2001 Co-existent Ferromagnetism & Superconductivty
2008 Fe-based Superconductors discovered

1995 Discovery of BEC in cold atomic ^{87}Rb
1998 Degeneracy of Cold Fermionic Gases: ^{6}Li
2007 BEC-BCS Condensation in ^{6}Li, ^{40}K

2008 *Topological* Superfluids & Superconductors

Observation of the Higgs Mode in a BCS Superdonductor

VOLUME 45, NUMBER 8 PHYSICAL REVIEW LETTERS 25 AUGUST 1980

Raman Scattering by Superconducting-Gap Excitations and Their Coupling to Charge-Density Waves

R. Sooryakumar and M. V. Klein

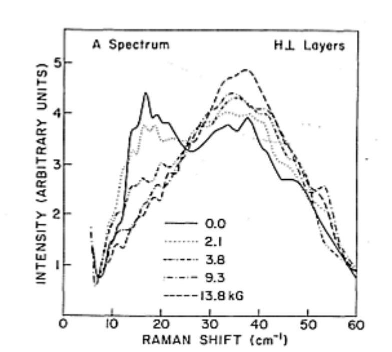
Department of Physics and Materials Research Laboratory, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801

(Received 24 March 1980)

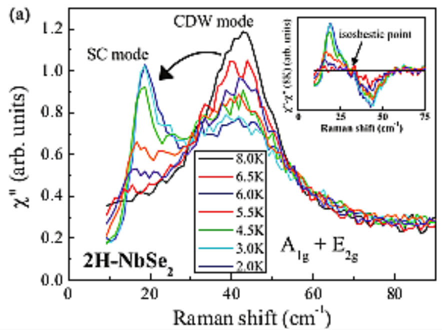
$2H$-$NbSe_2$ undergoes a charge-density-wave (CDW) distortion at 33 K which induces A and E Raman-active phonon modes. These are joined in the superconducting state at 2 K by new A and E Raman modes close in energy to the BCS gap 2Δ. Magnetic fields suppress the intensity of the new modes and enhance that of the CDW-induced modes, thus providing evidence of coupling between the superconducting-gap excitations and the CDW.

Higgs Mode with mass: $M = 3$ meV and spin $J = 0$ in $NbSe_2$

Raman Absorption in $NbSe_2$



R. Sooyakumar & M. Klein, PRL 45, 660 (1980)



M. Meásson et al. PRB B 89, 060503(R) (2014)

- $\hbar\omega_{\gamma_1} = \hbar\omega_{\gamma_2} + 2\Delta$
- Amplitude Higgs - CDW Phonon Coupling

▶ Theory: P. Littlewood & C. Varma, PRL 47, 811 (1981)

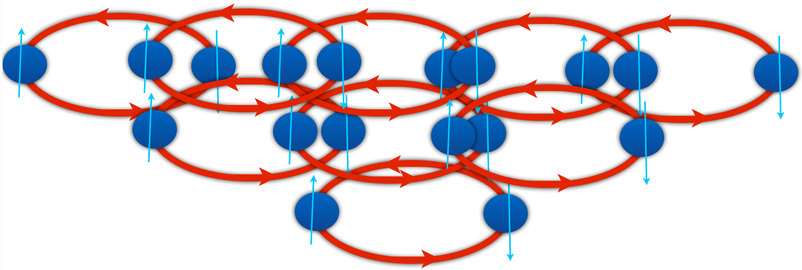
Parity Violation in a Superfluid Vacuum of Liquid ^{3}He

Chiral P-wave BCS Condensate

$$|\,\Psi_N\,\rangle = \left[\iint d\mathbf{r}_1 d\mathbf{r}_2\; \Psi_{s_1 s_2}(\mathbf{r}_1 - \mathbf{r}_2)\; \psi^\dagger_{s_1}(\mathbf{r}_1)\psi^\dagger_{s_2}(\mathbf{r}_2)\right]^{N/2} |\,\text{vac}\,\rangle$$

$$\Psi_{s_1 s_2}(\mathbf{r}) = f(|\mathbf{r}|/\xi)\;(x+iy)\;\chi^{(1,0)}_{s_1 s_2}$$

▶P.W. Anderson & P. Morel, Phys. Rev. 123, 1911 (1961)



$$\mathtt{SO(3)_S} \times \mathtt{SO(3)_L} \times \mathtt{U(1)_N} \times \mathtt{T} \times \mathtt{P} \longrightarrow \mathtt{SO(2)_S} \times \mathtt{U(1)_{N\text{-}L_z}} \times \mathtt{Z_2}$$

Realized as the Ground State of Superfluid ^{3}He

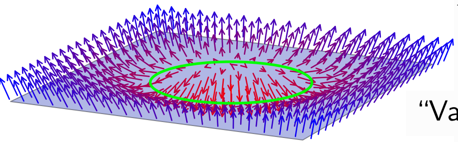
Fermionic Hamiltonian for 2D Chiral Superfluid (^{3}He-A Thin Film & Sr_2RuO_4?):

$$\widehat{H} = \begin{pmatrix} (|\mathbf{p}|^2/2m^* - \mu) & c(p_x + ip_y) \\ c(p_x - ip_y) & -(|\mathbf{p}|^2/2m^* - \mu) \end{pmatrix} = \vec{\mathbf{m}}(\mathbf{p}) \cdot \widehat{\vec{\boldsymbol{\tau}}}$$

$$\vec{\mathbf{m}} = (\, cp_x \, , \, \mp cp_y \, , \xi(\mathbf{p})) \text{ with } |\vec{\mathbf{m}}(\mathbf{p})|^2 = \left(|\mathbf{p}|^2/2m - \mu\right)^2 + c^2|\mathbf{p}|^2 > 0 \; , \mu \neq 0$$

▶Topological Invariant for 2D chiral SC $\leftrightarrow$ QED in d = 2+1 [G.E. Volovik, JETP 1988]:

$$N_{\text{C}} = \int \frac{d^2p}{4\pi}\, \hat{\mathbf{m}}(\mathbf{p}) \cdot \left(\frac{\partial \hat{\mathbf{m}}}{\partial p_x} \times \frac{\partial \hat{\mathbf{m}}}{\partial p_y} \right) = \begin{cases} \pm 1 \, ; & \mu > 0 \text{ and } \Delta \neq 0 \\ 0 \, ; & \mu < 0 \text{ or } \Delta = 0 \end{cases}$$



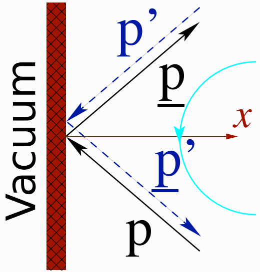
"Vacuum" $(\Delta = 0)$ & $N_{\text{C}} = 0$ | ^{3}He-A $(\Delta \neq 0)$ with $N_{\text{C}} = 1$

Zero Energy Fermions ↑ Confined on the Edge

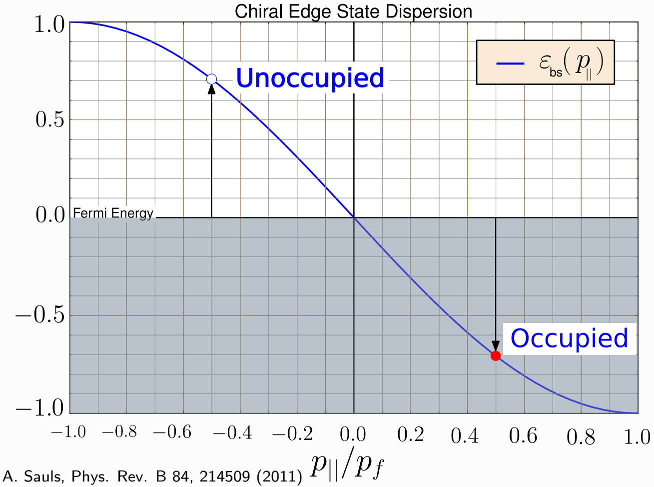
Massless Chiral Fermions in the 2D ^{3}He-A Films

Edge Fermions: $G^{R}_{\text{edge}}(\mathbf{p},\varepsilon;x) = \dfrac{\pi\Delta|\mathbf{p}_x|}{\varepsilon + i\gamma - \varepsilon_{\text{bs}}(\mathbf{p}_{||})}$ e^{-x/ξ_Δ} $\xi_\Delta = \hbar v_f/2\Delta \approx 10^2\,\text{Å} \gg \hbar/p_f$

- $\varepsilon_{\text{bs}} = -c\,p_{||}$ with $c = \Delta/p_f \ll v_f$

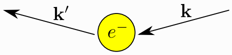


- Broken P & T $\rightsquigarrow$ Edge Current



- M. Stone, R. Roy, PRB 69, 184511 (2004)
- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

T-matrix description of Quasiparticle-Ion scattering



- Lippmann-Schwinger equation for the T-matrix ($\varepsilon = E + i\eta\,;\ \eta \to 0^+$):

$$\hat{T}^R_S(\mathbf{k}',\mathbf{k},E) = \hat{T}^R_N(\mathbf{k}',\mathbf{k}) + \int \frac{d^3k''}{(2\pi)^3} \hat{T}^R_N(\mathbf{k}',\mathbf{k}'')\left[\hat{G}^R_S(\mathbf{k}'',E) - \hat{G}^R_N(\mathbf{k}'',E)\right]\hat{T}^R_S(\mathbf{k}'',\mathbf{k},E)$$

$$\hat{G}^R_S(\mathbf{k},E) = \frac{1}{\varepsilon^2 - E^2_{\mathbf{k}}}\begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad E_{\mathbf{k}} = \sqrt{\xi_k^2 + |\Delta(\hat{\mathbf{k}})|^2},\ \ \xi_k = \frac{\hbar^2 k^2}{2m^*} - \mu$$

- Normal-state T-matrix:

$$\hat{T}^R_N(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = \begin{pmatrix} t^R_N(\hat{\mathbf{k}}',\hat{\mathbf{k}}) & 0 \\ 0 & -[t^R_N(-\hat{\mathbf{k}}',-\hat{\mathbf{k}})]^\dagger \end{pmatrix}$$ in p-h (Nambu) space, where

$$t^R_N(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = -\frac{1}{\pi N_f}\sum_{l=0}^{\infty}(2l+1)e^{i\delta_l}\sin\delta_l P_l(\hat{\mathbf{k}}'\cdot\hat{\mathbf{k}}),$$ $P_l(x)$ – Legendre function

- Hard-sphere potential $\rightsquigarrow \tan\delta_l = j_l(k_f R)/n_l(k_f R)$ – spherical Bessel functions

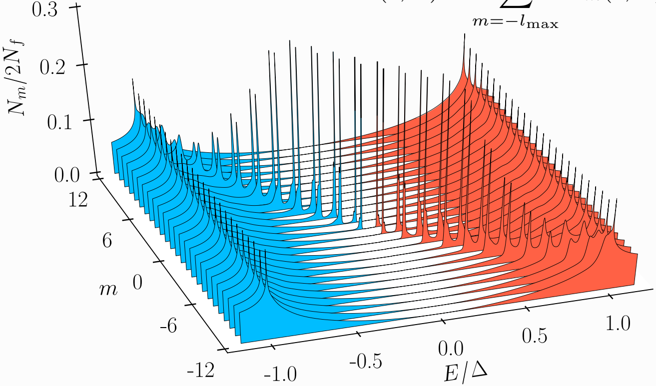
  - $k_f R$ – determined by the Normal-State Mobility $\rightsquigarrow$ $k_f R = 11.17$ ($R = 1.42\,\text{nm}$)

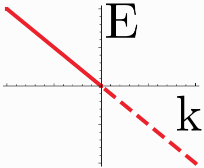
Weyl Fermion Spectrum bound to the Electron Bubble

$$\mu_{\mathsf{N}} = \frac{e}{n_3 p_f \sigma_{\mathsf{N}}^{\mathsf{tr}}} \quad \Leftarrow \quad \mu_{\mathsf{N}}^{\mathsf{exp}} = 1.7 \times 10^{-6} \, \frac{m^2}{V\, s}$$

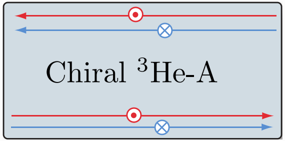
$$\tan \delta_l = j_l(k_f R)/n_l(k_f R) \quad \Rightarrow \quad \sigma_{\mathsf{N}}^{\mathsf{tr}} = \frac{4\pi}{k_f^2} \sum_{l=0}^{\infty} (l+1) \sin^2(\delta_{l+1} - \delta_l) \quad \rightsquigarrow \quad k_f R = 11.17$$

$$N(\mathbf{r}, E) = \sum_{m=-l_{\text{max}}}^{l_{\text{max}}} N_m(\mathbf{r}, E), \quad l_{\text{max}} \simeq k_f R$$

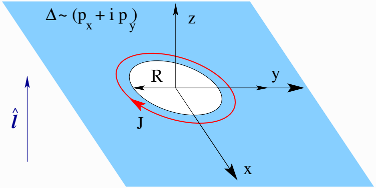


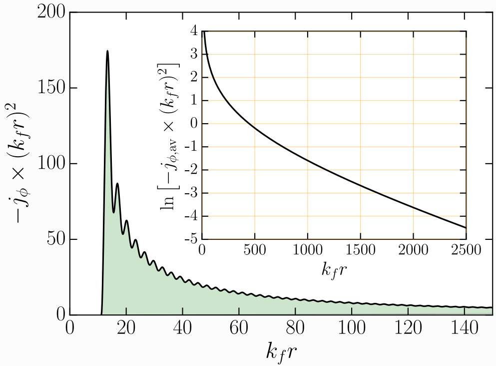


Current bound to an electron bubble ($k_f R = 11.17$)

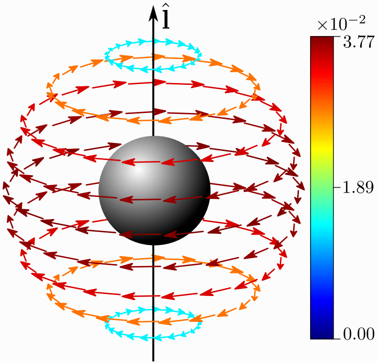


$\Longrightarrow$





$\Longrightarrow$



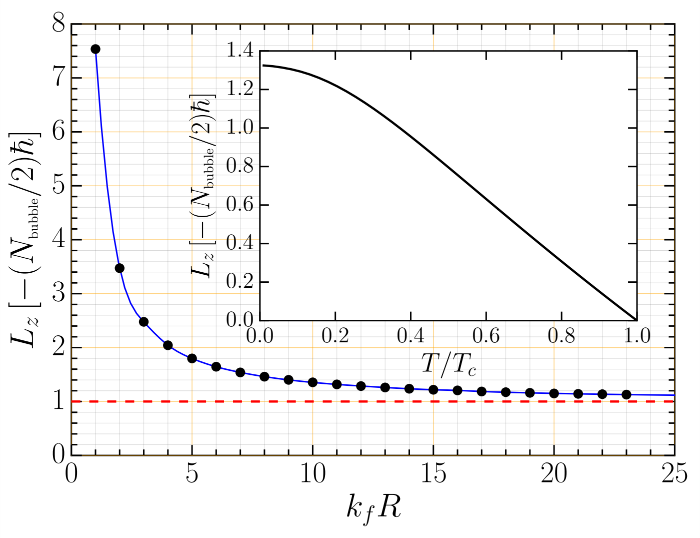
$$\mathbf{j}(\mathbf{r})/v_f N_f k_B T_c = j_\phi(\mathbf{r})\hat{\mathbf{e}}_\phi$$

$$\mathbf{L}(T \to 0) \approx -\hbar N_{\mathrm{bubble}}/2\,\hat{\mathbf{l}} \approx -100\,\hbar\,\hat{\mathbf{l}}$$

▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Angular momentum of an electron bubble in ^{3}He-A ($k_f R = 11.17$)

$$\mathbf{L}(T \to 0) \approx -\hbar N_{\text{bubble}} \hat{\mathbf{l}}/2\,; \qquad N_{\text{bubble}} = n_3\,\frac{4\pi}{3} R^3 \approx 200\ {}^3\text{He atoms}$$



Determination of the Stokes Tensor from the QP-Ion T-matrix

(i) Fermi's golden rule and the QP scattering rate:

$$\Gamma(\mathbf{k}',\mathbf{k}) = \frac{2\pi}{\hbar} W(\hat{\mathbf{k}}',\hat{\mathbf{k}})\delta(E_{\mathbf{k}'} - E_{\mathbf{k}}), \;\; W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = \frac{1}{2}\sum_{\tau'\sigma';\tau\sigma} |\overbrace{\langle\,\mathbf{k}',\sigma',\tau'\,|}^{\text{outgoing}}\hat{T}_S\overbrace{|\,\mathbf{k},\sigma,\tau\,\rangle}^{\text{incoming}}|^2$$

(ii) Drag force from QP-ion collisions (linear in $\mathbf{v}$): ▶Baym et al. PRL **22**, 20 (1969)

$$\mathbf{F}_{\text{QP}} = -\sum_{\mathbf{k},\mathbf{k}'} \hbar(\mathbf{k}'-\mathbf{k})\left[\hbar\mathbf{k}'\mathbf{v}f_{\mathbf{k}}\left(-\frac{\partial f_{\mathbf{k}'}}{\partial E}\right) - \hbar\mathbf{k}\mathbf{v}(1-f_{\mathbf{k}'})\left(-\frac{\partial f_{\mathbf{k}}}{\partial E}\right)\right]\Gamma(\mathbf{k}',\mathbf{k})$$

(iii) Microscopic reversibility condition: $W(\hat{\mathbf{k}}',\hat{\mathbf{k}}:+\mathbf{l}) = W(\hat{\mathbf{k}},\hat{\mathbf{k}}':-\mathbf{l})$

Broken T and mirror symmetries in ^{3}He-A $\Rightarrow$ fixed $\hat{\mathbf{l}}$ $\rightsquigarrow$ $W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) \neq W(\hat{\mathbf{k}},\hat{\mathbf{k}}')$

(iv) Generalized Stokes tensor:

$$\mathbf{F}_{\text{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v} \;\;\rightsquigarrow\;\; \eta_{ij} = n_3 p_f \int_0^\infty dE\left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E)\;, \;\; \overleftrightarrow{\eta} = \begin{pmatrix} \eta_\perp & \eta_{\text{AH}} & 0 \\ -\eta_{\text{AH}} & \eta_\perp & 0 \\ 0 & 0 & \eta_\parallel \end{pmatrix}$$

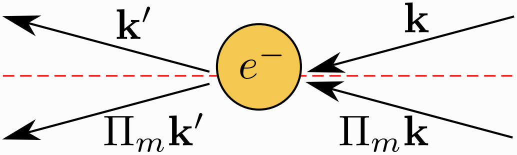
$n_3 = \dfrac{k_f^3}{3\pi^2}$ – ^{3}He particle density, $\sigma_{ij}(E)$ – transport scattering cross section,

$f(E) = [\exp(E/k_BT)+1]^{-1}$ – Fermi Distribution

Mirror-symmetric scattering $\Rightarrow$ longitudinal drag force

$$\mathbf{F}_{\text{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^\infty dE\left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E)$$

Subdivide by mirror symmetry:



$$W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}),$$

$$\sigma_{ij}(E) = \sigma_{ij}^{(+)}(E) + \sigma_{ij}^{(-)}(E),$$

$$\sigma_{ij}^{(+)}(E) = \frac{3}{4}\int_{E\geq|\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'}\int_{E\geq|\Delta(\hat{\mathbf{k}})|}\frac{d\Omega_{\mathbf{k}}}{4\pi}\left[(\hat{\mathbf{k}}'_i-\hat{\mathbf{k}}_i)(\hat{\mathbf{k}}'_j-\hat{\mathbf{k}}_j)\right]\frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E)$$

Mirror-symmetric cross section: $W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W(\hat{\mathbf{k}},\hat{\mathbf{k}}')]/2$

$$\frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2\frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}}')|^2}}W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}})\frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}})|^2}}$$

$\rightsquigarrow$ Stokes Drag $\eta_{xx}^{(+)} = \eta_{yy}^{(+)} \equiv \eta_\perp,\ \eta_{zz}^{(+)} \equiv \eta_\parallel$, No transverse force $\left[\eta_{ij}^{(+)}\right]_{i\neq j} = 0$

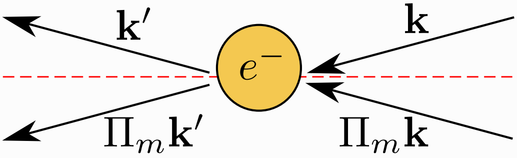
Mirror-antisymmetric scattering $\Rightarrow$ transverse force

$$\mathbf{F}_{\mathsf{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^\infty dE\left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E)$$

Subdivide by mirror symmetry:

$$W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}),$$

$$\sigma_{ij}(E) = \sigma_{ij}^{(+)}(E) + \sigma_{ij}^{(-)}(E),$$



$$\sigma_{ij}^{(-)}(E) = \frac{3}{4}\int_{E\geq|\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'}\int_{E\geq|\Delta(\hat{\mathbf{k}})|}\frac{d\Omega_{\mathbf{k}}}{4\pi}\left[\epsilon_{ijk}(\hat{\mathbf{k}}'\times\hat{\mathbf{k}})_k\right]\frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E)\left[f(E)-\frac{1}{2}\right]$$

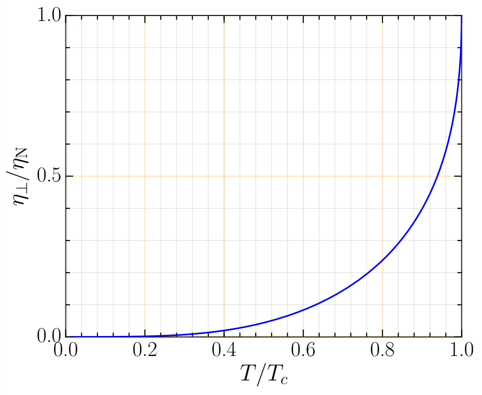
Mirror-antisymmetric cross section: $W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) - W(\hat{\mathbf{k}},\hat{\mathbf{k}}')]/2$

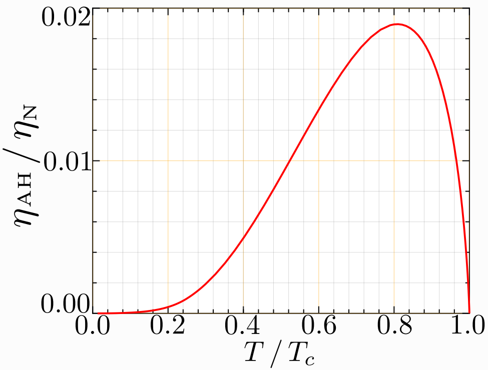
$$\frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2\frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}}')|^2}}W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}})\frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}})|^2}}$$

Transverse force $\quad \eta_{xy}^{(-)} = -\eta_{yx}^{(-)} \equiv \eta_{\mathsf{AH}} \quad \Rightarrow \quad$ anomalous Hall effect

▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Theoretical Results for the Drag and Transverse Forces





- $\Delta p_x \approx p_f \qquad \sigma^{\text{tr}}_{xx} \approx \sigma^{\text{tr}}_{\text{N}} \approx \pi R^2$
- $F_x \approx n\, v_x\, \Delta p_x\, \sigma^{\text{tr}}_{xx}$
  $\approx n\, v_x\, p_f\, \sigma^{\text{tr}}_{\text{N}}$

$$|F_y/F_x| \approx \frac{\hbar}{p_f R}\,(\Delta(T)/k_B T_c)^2$$

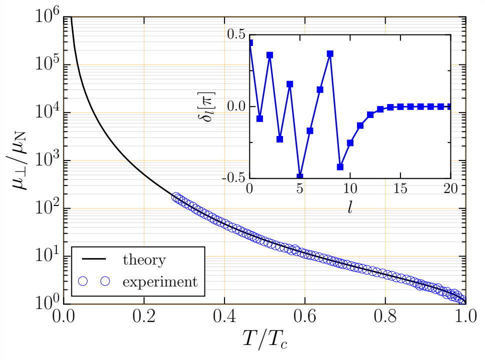
- $\Delta p_y \approx \hbar/R \quad \sigma^{\text{tr}}_{xy} \approx (\Delta(T)/k_{\text{B}}T_c)^2 \sigma^{\text{tr}}_{\text{N}}$
- $F_y \approx n\, v_x\, \Delta p_y\, \sigma^{\text{tr}}_{xy}$
  $\approx n\, v_x\, (\hbar/R)\, \sigma^{\text{tr}}_{\text{N}} (\Delta(T)/k_{\text{B}}T_c)^2$

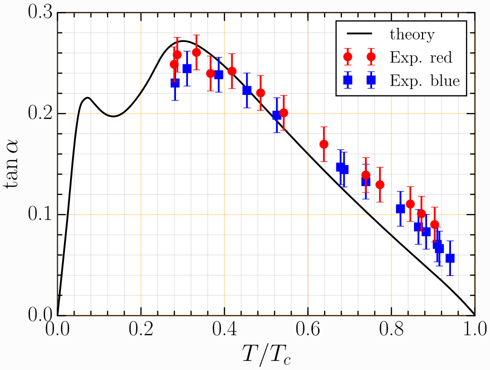
$k_f R = 11.17$

Branch Conversion Scattering in a Chiral Condensate

▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Comparison between Theory and Experiment for the Drag and Transverse Forces





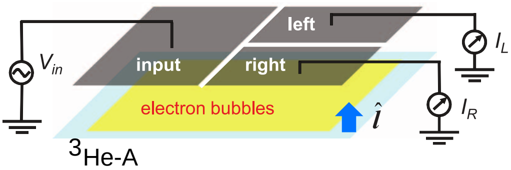
- $\mu_\perp = e\,\frac{\eta_\perp}{\eta_\perp^2 + \eta_{\mathsf{AH}}^2}$
- $\mu_{\mathsf{AH}} = -e\,\frac{\eta_{\mathsf{AH}}}{\eta_\perp^2 + \eta_{\mathsf{AH}}^2}$

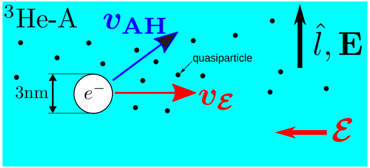
- $\tan\alpha = \left|\frac{\mu_{\mathsf{AH}}}{\mu_\perp}\right| = \frac{\eta_{\mathsf{AH}}}{\eta_\perp}$
- Electron Bubble Radius: $k_f R = 11.17$

▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

▶O. Shevtsov and JAS, JLTP 187, 340–353 (2017)

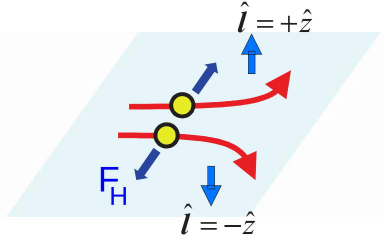
Anomalous Hall Effect for Electron Transport in ^{3}He-A





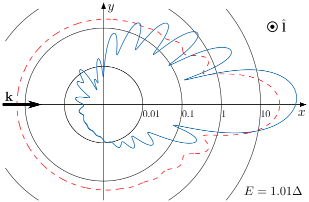
$$\vec{v} = \mu_{||}\,\vec{E} + \mu_{AH}\,\vec{E}\times\hat{l}$$

R. Salmalin, M. Salomaa, V. Mineev, Phys. Rev. Lett.63, 868, (1989)



Ikegami, Tsutsumi & Kono, Science 341, 59 (2013)

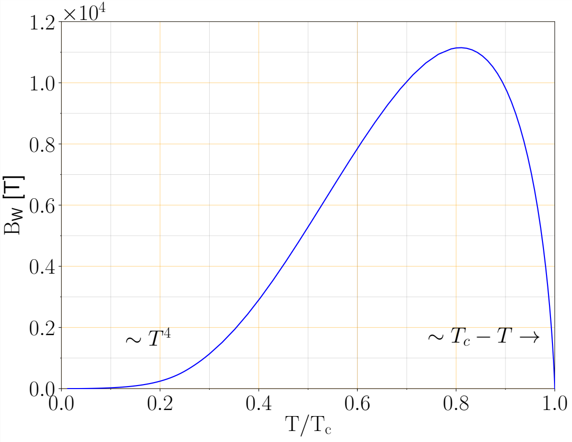
Resonant QP Skew Scattering by Chiral Edge States



O. Shevtsov & J. A. Sauls, Phys. Rev. B, 94, 064511, (2016)

Vanishing of the Effective Magnetic Field for $T \to 0$

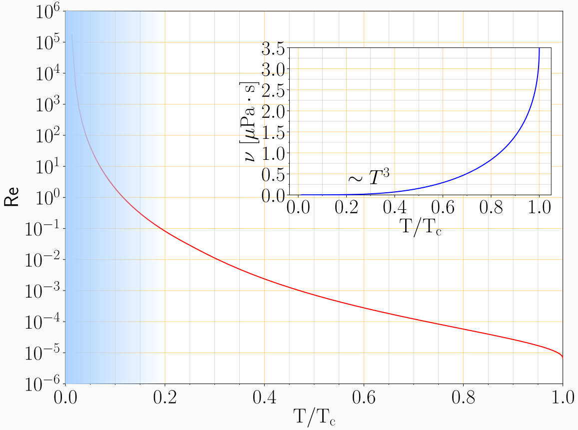
$$B_{\mathsf{W}} = 5.9 \times 10^5\ \mathsf{T} \left(\frac{\eta_{xy}}{\eta_{\mathsf{N}}}\right)$$



$$\eta_{xy}/\eta_{\mathsf{N}}|_{T=0.8\,T_c} \approx \frac{\hbar}{p_f R}$$

Breakdown of Laminar Flow

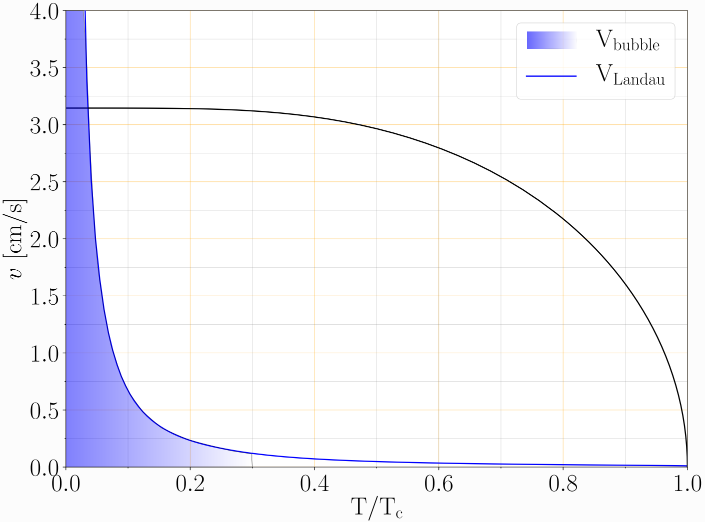
$$\mathsf{R}e = \mathsf{R}e_{\mathsf{N}} \left(\frac{\eta_{\mathsf{N}}}{\eta}\right)^{3/2} \xrightarrow[T\to 0]{} \sim \left(\frac{T_c}{T}\right)^{9/2}$$



$$\mathsf{R}e_{\mathsf{N}} = 6.7 \times 10^{-6}$$

Generation of a Turbulent Tangle of Quantized Vortices from the Chiral Vacuum

Breakdown of Scattering Theory for $T \to 0$



Electron Bubble Velocity

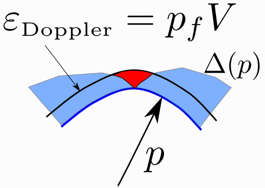
- $V_{\mathsf{N}} = \mu_{\mathsf{N}} E_{\mathsf{N}} = 1.01 \times 10^{-4}\,\mathsf{m/s}$
- $V = \mu_{\mathsf{N}} E_{\mathsf{N}} \sqrt{\frac{\eta_{\mathsf{N}}}{\eta}}$

Maximum Landau critical velocity

- $V_c^{\mathsf{max}} \approx 155 \times 10^{-4}\,\mathsf{m/s}\, \frac{\Delta_{\mathsf{A}}(T)}{k_b T_c}$

Nodal Superfluids:

- $V_c = \Delta(p)/p_f \to 0$ for $p \to p_{\mathsf{node}}$

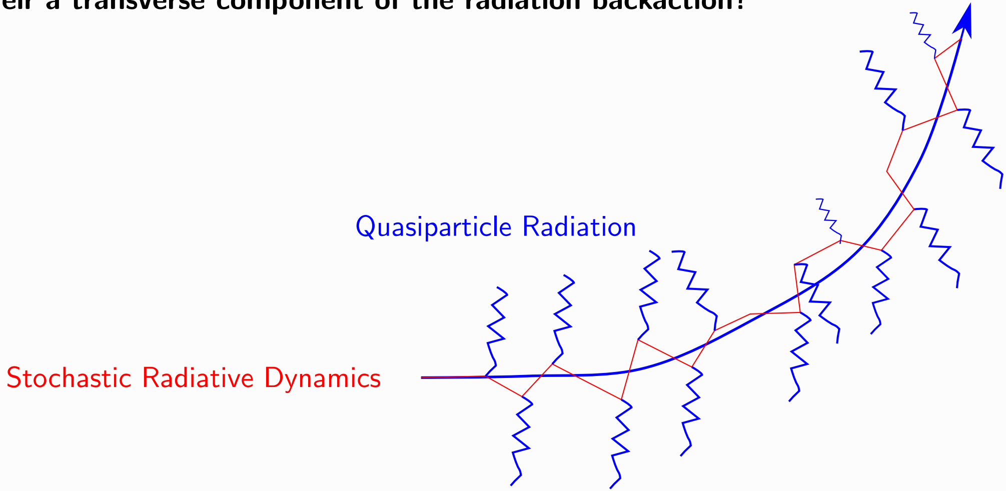


- Radiation Dominated Damping: $T \lesssim 0.1 T_c$

Radiation of Weyl Fermions from the Chiral Vacuum

Radiation Damping - Pair-Breaking at $T \to 0$

Is their a transverse component of the radiation backaction?



$\rightsquigarrow$ **Asymmetry in the Radiation of Chiral Fermions from a Chiral Vacuum**