Collective modes and linear response of spin-triplet pairing models of Sr_2RuO_4, UPt_3 and ^{3}He-A

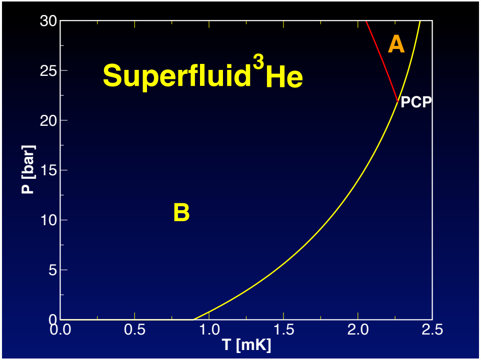
Hao Wu[1], Suk-Bum Chung[2] and J. A. Sauls[1]

[1]Northwestern University and [2] Seoul National University

July 9, 2014

- Supported by NSF Grant DMR-1106315

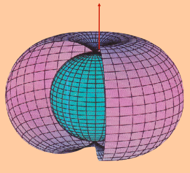
Phase Diagram of Bulk ^{3}He



Spin-Triplet, P-wave Order Parameter:

$$\Delta_{\alpha\beta}(\mathbf{p}) = \vec{\mathbf{d}}(\mathbf{p}) \cdot (i\vec{\sigma}\sigma_y)_{\alpha\beta} \rightsquigarrow \mathbf{d}_\mu(\mathbf{p}) = A_{\mu i}\,\mathbf{p}_i$$

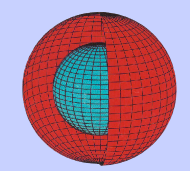
Chiral ABM State $\vec{l} = \hat{\mathbf{m}} \times \hat{\mathbf{n}}$



$$A_{\mu i} = \Delta\,\hat{\mathbf{d}}_\mu\,(\hat{\mathbf{m}} + i\hat{\mathbf{n}})_i$$

$$L_z = 1\,,\ S_z = 0$$

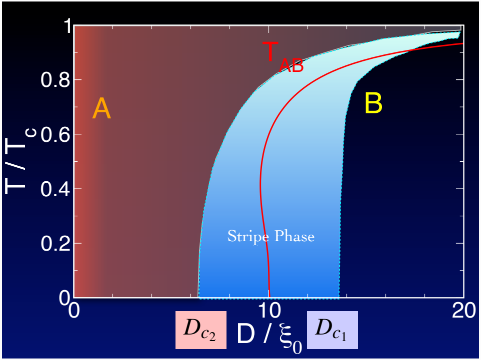
"Isotropic" BW State



$$A_{\mu i} = \Delta\,\delta_{\mu i}$$

$$J = 0\,,\ J_z = 0$$

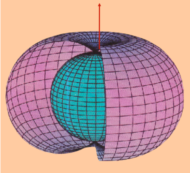
A. Vorontsov & JAS, PRL, 2007



Spin-Triplet, P-wave Order Parameter:

$$\Delta_{\alpha\beta}(\mathbf{p}) = \vec{\mathbf{d}}(\mathbf{p}) \cdot (i\vec{\sigma}\sigma_y)_{\alpha\beta} \rightsquigarrow \mathbf{d}_\mu(\mathbf{p}) = A_{\mu i}\,\mathbf{p}_i$$

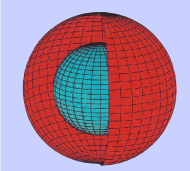
Chiral ABM State $\vec{l} = \hat{\mathbf{m}} \times \hat{\mathbf{n}}$



$$A_{\mu i} = \Delta\,\hat{\mathbf{d}}_\mu\,(\hat{\mathbf{m}} + i\hat{\mathbf{n}})_i$$

$L_z = 1$, $S_z = 0$

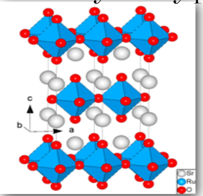
“Isotropic” BW State



$$A_{\mu i} = \Delta\,\delta_{\mu i}$$

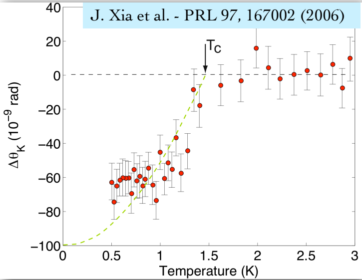
$J = 0$, $J_z = 0$

Are there electronic superconductors with broken symmetry phases analogous to ^{3}He?

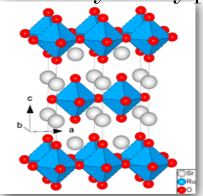


Sr_2RuO_4 T_c=1.15K

- S=1 (NMR)
- "p-wave", E_{1u}
- Broken T-symmetry ✔ Kerr rotation



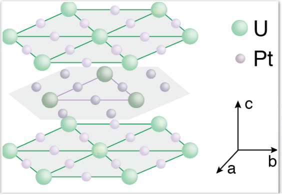
Are there electronic superconductors with broken symmetry phases analogous to ^{3}He?

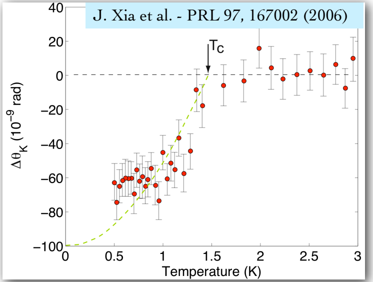


Sr_2RuO_4 T_c=1.15K

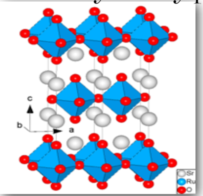
- S=1 (NMR)
- "p-wave", E_{1u}
- Broken T-symmetry ✔ Kerr rotation

UPt_3 T_c=0.56 K





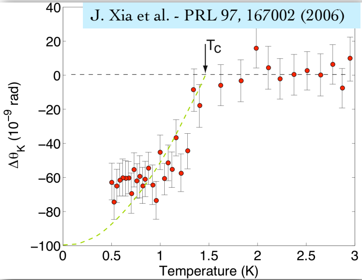
Are there electronic superconductors with broken symmetry phases analogous to ^3He?



Sr_2RuO_4 T_c=1.15K

- S=1 (NMR)
- "p-wave", E_{1u}
- Broken T-symmetry ✔ Kerr rotation

J. Xia et al. - PRL 97, 167002 (2006)



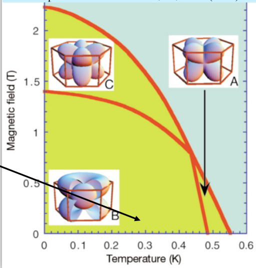
UPt_3 T_c=0.56 K

- S=1 (NMR & H_{c2})
- "f-wave", E_{2u}
- Broken T-symmetry

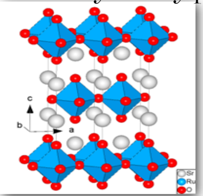
multiple SC phases

JAS, Adv. Phys. 43, 113(1994)

A. Huxley et al. - Nature, 406, 160 (2000)
T. Champel & V. Mineev - PRL, 86, 4903 (2001)



Are there electronic superconductors with broken symmetry phases analogous to ^{3}He?



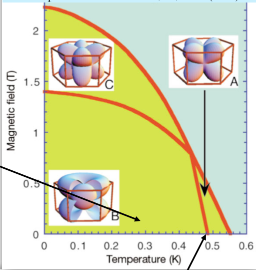
Sr_2RuO_4 T_c=1.15K
- S=1 (NMR)
- "p-wave", E_{1u}
- Broken T-symmetry ✔ Kerr rotation

UPt_3 T_c=0.56 K
- S=1 (NMR & H_{c2})
- "f-wave", E_{2u}
- Broken T-symmetry ✔

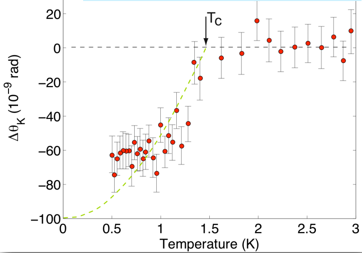
multiple SC phases

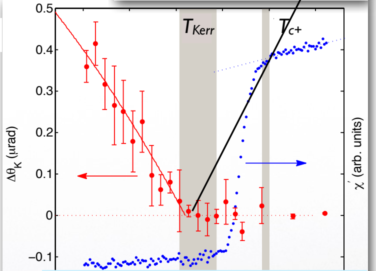
JAS, Adv. Phys. 43, 113(1994)

A. Huxley et al. - Nature, 406, 160 (2000)
T. Champel & V. Mineev - PRL, 86, 4903 (2001)



J. Xia et al. - PRL 97, 167002 (2006)

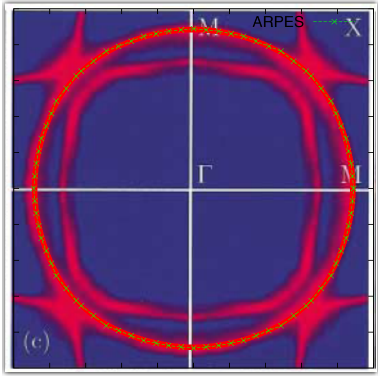




E. Schemm et al. - Science (to be published) 2014

Pairing Models

ARPES Fermi Surface



A. Damascelli, et al. Phys. Rev. Lett. 85, (2000).

$S = 1$, *p-wave* Cooper Pairs (E_{1u})

$\approx 2D$ Fermi surface (γ-band)

- T. M. Rice and M. Sigrist, J. Phys. Cond. Mat. 7, L643 (1995).

$$\vec{d}(\mathbf{p}) = \hat{d}\,\left(A_x\ \hat{p}_x + A_y\ \hat{p}_y\right)$$

Anisotropic E_{1u} Cooper Pairs:

- S Raghu et al, J. Phys. (2013); T. Scaffidi et al. arXiv:1401.0016
- Q. H. Wang et al. Eur.Phys. Lett. 104, 17013(2013)

$$\hat{p}_x \to Y_x(\mathbf{p}) \sim \hat{p}_x I(\mathbf{p})$$
$$\hat{p}_y \to Y_y(\mathbf{p}) \sim \hat{p}_y I(\mathbf{p})$$

$I(\hat{p}_x, \hat{p}_y)$ is invariant under D_{4h}

$$I(\mathbf{p}) = \left(1 - \varepsilon\,|\hat{p}_x^2 - \hat{p}_y^2|\right)/\left(1 + \varepsilon^2/2\right)$$

$$0 \le \varepsilon \le 1$$

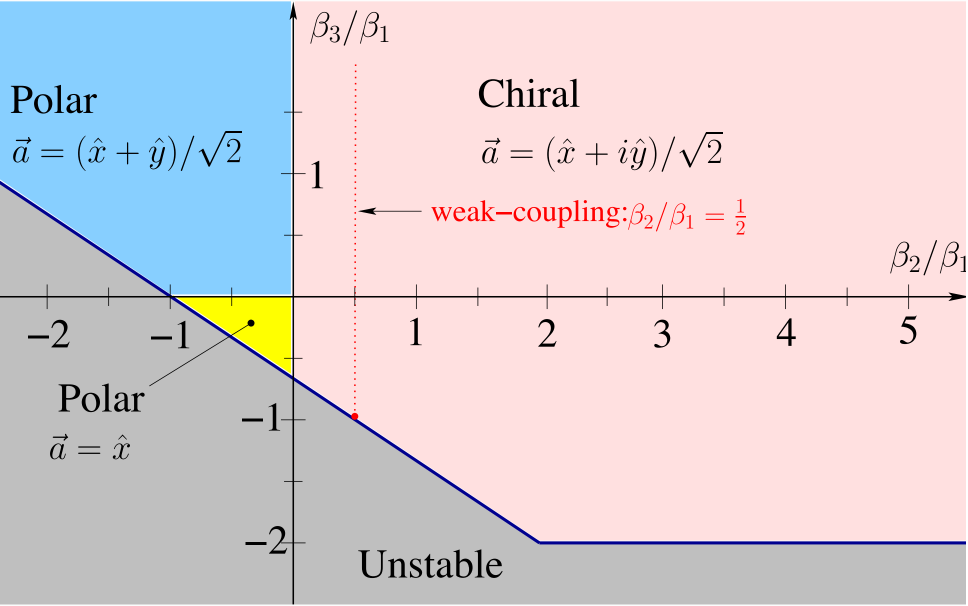
- Multi-component order + Anisotropy + Disorder $\rightsquigarrow$ Spectroscopy of Pairing Symmetry

- Maximal Group Sr_2RuO_4: $\mathtt{G} = \mathtt{D_{4h}} \times \mathtt{SO(3)_S} \times \mathtt{U(1)_N} \times \mathtt{T}$
- Equal-Spin-Pairing: $$\vec{d}(\mathbf{p}) = \hat{d}\,\Big(A_x\;Y_x(\mathbf{p}) + A_y\;Y_y(\mathbf{p})\Big)$$
- $\vec{A}$ transforms as a complex vector under $\mathtt{D_{4h}}$
- Ginzburg-Landau Functional
  - D. W. Hess et al, J. Phys. Cond. Mat. 1, 43 (1989).

$$\mathscr{F}[\vec{A}] = \alpha(T)\,|\vec{A}|^2 + \beta_1\,|\vec{A}|^4 + \beta_2\,|\vec{A}\cdot\vec{A}|^2 + \beta_3\,\left[|A_x|^4 + |A_x|^4\right]$$

- $\beta_2 > 0 \rightsquigarrow$ Broken Time-Reversal
- $\beta_3 \neq 0 \rightsquigarrow$ Tetragonal Anisotropy

Phase Diagram for and ESP E_{1u} Superconductor $\sim Sr_2RuO_4$



- Chiral basis vectors: $\hat{\mathbf{x}}_\pm = (\hat{\mathbf{x}} \pm i\hat{\mathbf{y}})/\sqrt{2}$ with $\hat{\mathbf{x}}_\pm \cdot \hat{\mathbf{x}}_\pm^* = 1\,,\ \hat{\mathbf{x}}_+ \cdot \hat{\mathbf{x}}_- = 1\,,\ \hat{\mathbf{x}}_\pm \cdot \hat{\mathbf{x}}_\pm = 0$

- $\vec{A}(\mathbf{r},t) = \Delta\hat{\mathbf{x}}_+$ (Ground State) + $\vec{\mathscr{A}}(\mathbf{r},t)$ (Cooper Pair Fluctuations)

$$\vec{\mathscr{A}}(\mathbf{r},t) = D(\mathbf{r},t)\,\hat{\mathbf{x}}_+ + E(\mathbf{r},t)\,\hat{\mathbf{x}}_-$$

- Normal Modes: $D^\pm \equiv D \pm D^*$ $\quad E^\pm \equiv E \pm E^*$

- Potential Energy of Pair Fluctuations: $\mathscr{U}[\vec{\mathscr{A}}] = \int dV\ \delta\mathscr{F}[\vec{\mathscr{A}}]$

$$\delta\mathscr{F}[\vec{\mathscr{A}}] = 4\Delta^2\left[\beta_1\,(D^+)^2 + \beta_2\,((E^+)^2 + E^-)^2) + \frac{1}{2}\beta_3\,((D^+)^2 + E^+)^2)\right] + \mathscr{O}(\mathscr{A}^3)$$

- D^- is *absent* $\rightsquigarrow$ Goldstone Mode (Phase Fluctuations)
- Lagrangian for Cooper Pair Fluctuations - Bosonic Excitations

$$\mathscr{L}[\vec{\mathscr{A}},\dot{\vec{\mathscr{A}}}] = \int dV\left\{\frac{1}{2}\,\mu\,\partial_t\vec{\mathscr{A}}\cdot\partial_t\vec{\mathscr{A}}^* - \delta\mathscr{F}[\vec{\mathscr{A}}]\right\}$$

$\mu = \beta_1 + \frac{1}{2}\beta_3$ (BCS Theory)

► G. E. Volovik and M. V. Kazan, JETP 60, 276 (1984) ► S. Theodorakis, PRB 37, 3318 (1988)

- Amplitude Mode ($\hat{x}_+$) - "Higgs Mode"

$$\partial_t^2 D^+ + 4\Delta^2 D^+ = 0 \quad \rightsquigarrow \quad \omega_{\text{D+}} = 2\Delta$$

  - c.f. $NbSe_2$ $\rightsquigarrow$ theory SC+CDW: P. Littlewood and C. Varma, PRL 47, 811 (1981)

- Anderson-Bogoliubov Phase Mode ($\hat{x}_+$) - "Goldstone Mode"

$$\partial_t^2 D^- = 0 \quad \rightsquigarrow \quad \omega_{\text{D-}} = 0$$

- Anderson-Higgs Mechanism - Charge coupling to a gauge field, $\text{A}_\mu = (\text{A}_0, \vec{\text{A}})$

$$\omega_{\text{D-}} = 0 \quad \xrightarrow{\partial_\mu \to \partial_\mu - ie\text{A}_\mu} \quad \omega_{\text{pl}} = \sqrt{4\pi ne^2/m^* c^2} \text{ (plasmon)}$$

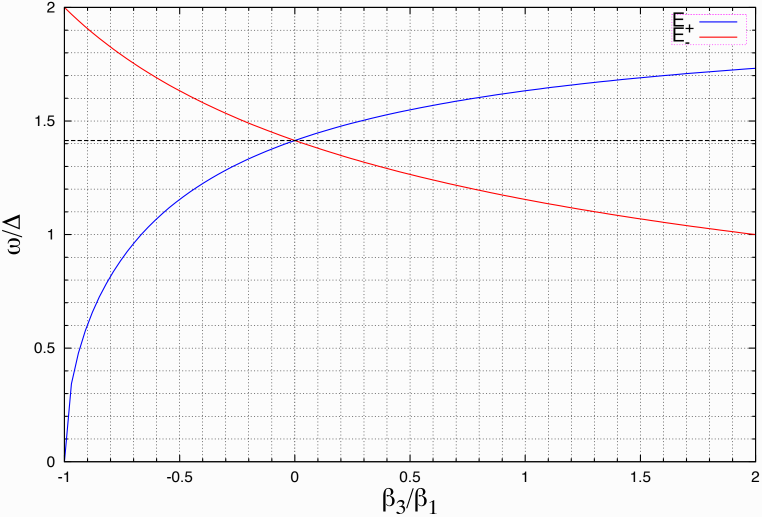
- Anderson-Higgs Modes ($\hat{x}_-$) - "Opposite Chirality"

$$\partial_t^2 E^+ + 4\Delta^2 \frac{2\beta_2 + \beta_3}{2\beta_1 + \beta_3} E^+ = 0 \quad \rightsquigarrow \quad \omega_{\text{E+}} = 2\Delta\sqrt{\frac{2\beta_2 + \beta_3}{2\beta_1 + \beta_3}} \quad \xrightarrow{\beta_3 = 0} \quad \sqrt{2}\Delta$$

$$\partial_t^2 E^- + 4\Delta^2 \frac{2\beta_2}{2\beta_1 + \beta_3} E^- = 0 \quad \rightsquigarrow \quad \omega_{\text{E-}} = 2\Delta\sqrt{\frac{2\beta_2}{2\beta_1 + \beta_3}} \quad \xrightarrow{\beta_3 = 0} \quad \sqrt{2}\Delta$$

  - Degeneracy of $E^\pm$ Modes is lifted by Anisotropy!

Collective Mode Frequencies vs. Anisotropy



$$\omega_{\text{E-}} = 2\Delta\sqrt{\frac{2\beta_2}{2\beta_1+\beta_3}} \xrightarrow{\beta_3\to-2\beta_2} 2\Delta \quad \text{and} \quad \omega_{\text{E+}} = 2\Delta\sqrt{\frac{2\beta_2+\beta_3}{2\beta_1+\beta_3}} \xrightarrow{\beta_3\to-2\beta_2} 0$$

Microscopic Theory

- Keldysh green's function $\hat{g}^K = \begin{pmatrix} g & \vec{f}\cdot(i\vec{\sigma}\sigma_y) \\ \vec{\bar{f}}\cdot(i\sigma_y\vec{\sigma}) & \bar{g} \end{pmatrix}$
- Quasiclassical Transport Equation

$$(\varepsilon\hat{\tau}_3 - \hat{\Sigma}_{\text{ext}} - \hat{\Sigma}^R)\circ\hat{g}^K - \hat{g}^K\circ(\varepsilon\hat{\tau}_3 - \hat{\Sigma}_{\text{ext}} - \hat{\Sigma}^A) - \hat{\Sigma^K}\circ\hat{g}^R$$
$$+\hat{g}^A\circ\hat{\Sigma}^K + i\vec{v}_f\cdot\nabla\hat{g}^K = 0\,,$$

- Mean field approximation

$$\hat{\Sigma}^A = \hat{\Sigma}^R = \widehat{\Delta}, \qquad \hat{\Sigma}^K = 0.$$

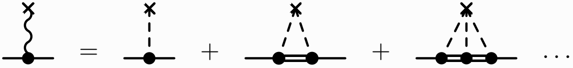
- Linearized dynamical equation

$$(\varepsilon + \frac{\omega}{2})\tau_3\hat{g}^K - \hat{g}^K(\varepsilon - \frac{\omega}{2})\tau_3 + i\vec{v}_f\cdot\nabla\hat{g}^K - (\hat{\Delta} + \hat{\Sigma}_{\text{ext}})\circ\hat{g}^K + \hat{g}^K\circ(\hat{\Delta} + \hat{\Sigma}_{\text{ext}}) = 0$$

- Time-dependent BCS "gap equation"

$$\vec{\Delta}(\vec{p}_f, \vec{R}; t) = \int \frac{d\varepsilon}{4\pi i}\int d\vec{p}\,'_f V^t(\vec{p}_f, \vec{p}\,'_f)\vec{f}(\vec{p}\,'_f, \vec{R}; \varepsilon, t)\,,$$

- Disorder (random field - impurity T-matrix) $\rightsquigarrow \ell = v_f\tau$, $\ell^{-1} = n_{\text{imp}}\,\sigma$

$\hat{\Sigma}_{\text{imp}}(\vec{p}_f, \vec{R}; \epsilon, t) = n_{\text{imp}}$ 

- Chiral basis: $\hat{x}_{\pm} = (\hat{p}_x \pm i\hat{p}_y)/\sqrt{2}$, equilibrium state $\sim \Delta\hat{x}_+$
- Electromagnetic coupling

$$\widehat{\Sigma}_{\text{ext}} = \frac{e}{c}\vec{v}_{\mathbf{p}} \cdot \vec{A}(\vec{q},\omega)\,\widehat{\tau}_3$$

- Paring self-energy (Nambu)

$$\widehat{\Delta}(\mathbf{p}) = \begin{pmatrix} 0 & \hat{d}\cdot(i\vec{\sigma}\sigma_y)d(\mathbf{p}) \\ \hat{d}\cdot(i\sigma_y\vec{\sigma})d'(\mathbf{p}) & 0 \end{pmatrix}$$

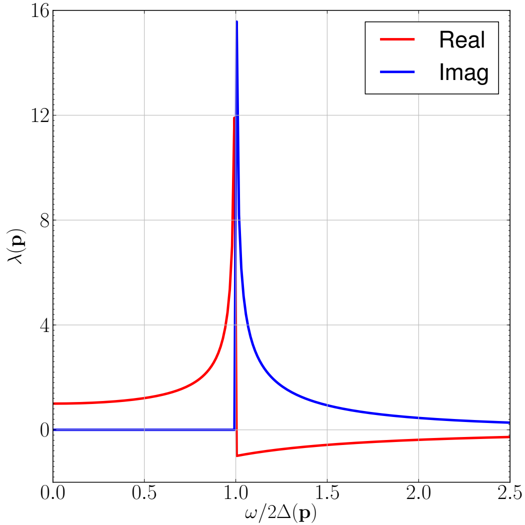
- Dynamic gap: $d(\mathbf{p}) \to d(\mathbf{p},\vec{q},\omega)$
- Decompose into Chiral basis:

$$d(\mathbf{p},\vec{q},\omega) = \Delta\hat{x}_+ + D(\vec{q},\omega)\,\hat{x}_+ + E(\vec{q},\omega)\,\hat{x}_-$$

$$d'(\mathbf{p},\vec{q},\omega) = \Delta'\hat{x}_+ + D^*(\vec{q},\omega)\,\hat{x}_+ + E^*(\vec{q},\omega)\,\hat{x}_-$$

Dynamical Equation – time dependent gap equation $\eta(\mathbf{p},\vec{q}) = \vec{v}_{\mathbf{p}} \cdot \vec{q}$

- $d(p) = D\hat{x}_+ + E\hat{x}_-$
$$d(p) = \frac{1}{2}\int \frac{d\phi'}{2\pi} V(p,p') \left\{ -\frac{1}{2}\eta'\bar{\lambda}\Delta(p')\left[\frac{2e}{c}\vec{v}_{\mathbf{p}}\cdot\vec{A}\right] + \left[\gamma(p') + \frac{1}{2}(\omega^2 - \eta'^2)\bar{\lambda}\right] d(p') - \bar{\lambda}\left[|\Delta(p')|^2 d(p') + \Delta(p')^2 d'(p')\right]\right\}$$
- $d'(p) = D^*\hat{x}_+ + E^*\hat{x}_-$
$$d'(p) = \frac{1}{2}\int \frac{d\phi'}{2\pi} V(p,p') \left\{ +\frac{1}{2}\eta'\bar{\lambda}\Delta^*(p')\left[\frac{2e}{c}\vec{v}_{\mathbf{p}}\cdot\vec{A}\right] + \left[\gamma(p') + \frac{1}{2}(\omega^2 - \eta'^2)\bar{\lambda}\right] d'(p') - \bar{\lambda}\left[|\Delta(p')|^2 d'(p') + \Delta^*(p')^2 d(p')\right]\right\}$$
- Normal Modes: $D^{\pm} = D \pm D^*$ $E^{\pm} = E \pm E^*$
    - S.K. Yip and JAS, J. Low Temp. Phys. 86, 257 (1992).



- $x = \omega/2|\Delta(\mathbf{p})|$
- Define moments

$$\lambda_{mn} = \int \frac{d\phi}{2\pi} \lambda(\omega, \mathbf{p})$$
$$\times I(\phi)^{2m} \cos^{2n}(\phi)$$

- Anderson-Bogoliubov (Goldstone) Mode

$$\omega^2 D_- = 0$$

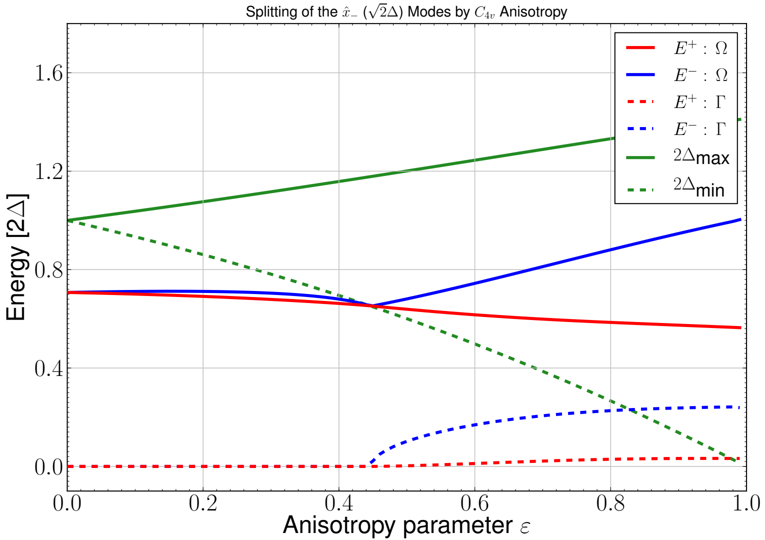
- Anderson-Higgs Mode ($\hat{x}_+$)

$$\left(\omega^2 - \frac{4\lambda_{10}\Delta(T)^2}{\lambda_{00}}\right) D_+ = 0$$

- Anderson-Higgs Mode ($\hat{x}_-$)

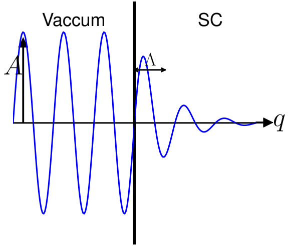
$$\left(\omega^2 - \frac{4\Delta(T)^2\lambda_{11}}{\lambda_{00}}\right) E_+ = 0$$

$$\left(\omega^2 - \frac{4\Delta(T)^2(\lambda_{10} - \lambda_{11})}{\lambda_{00}}\right) E_- = 0$$

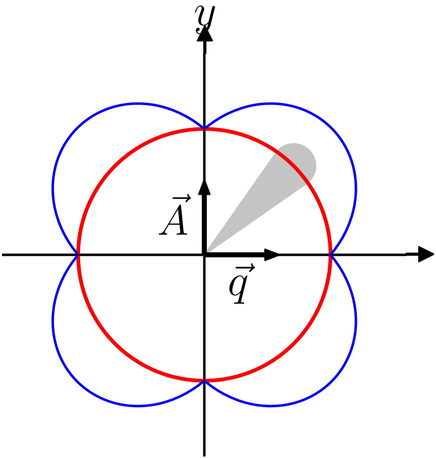


Splitting of the $\hat{x}_-$ ($\sqrt{2}\Delta$) Modes by C_{4v} Anisotropy
Energy [2Δ]
Anisotropy parameter ε
1.6
1.2
0.8
0.4
0.0
0.0
0.2
0.4
0.6
0.8
1.0
$E^+ : \Omega$
$E^- : \Omega$
$E^+ : \Gamma$
$E^- : \Gamma$
2Δmax
2Δmin

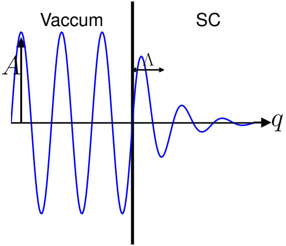
Mode coupling to a transverse EM field - polarization $[100]$



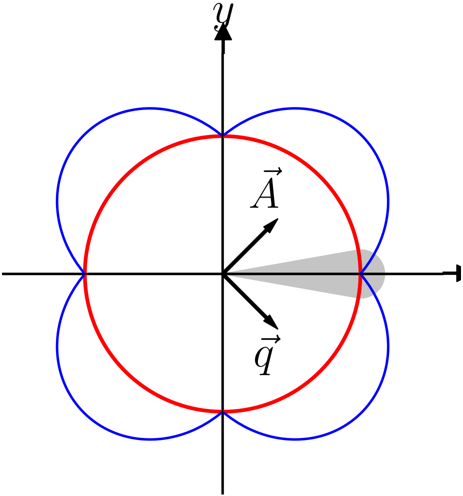
- EM field along [100]



$$\left[(\omega+i\Gamma)^2-\omega_{\mathrm{E+}}^2\right] E_{+} = i\frac{e}{c}\, \Lambda_{100}(\omega) q v_f^2 \Delta \mathrm{A}(q,\omega)$$



- EM field along [110]



$$\left[(\omega+i\Gamma)^2-\omega_{\text{E-}}^2\right] E_{-} = \frac{e}{c}\,\Lambda_{110}(\omega) q v_f^2 \Delta \mathrm{A}(q,\omega)$$

- Current Response: quasiparticle excitation

$$J^{\text{ex}} = \frac{eN_f}{2}\int\frac{d\phi}{2\pi}\vec{v}_{\mathbf{p}}\left[1+\frac{\eta^2}{\omega^2-\eta^2}(1-\lambda)\right]\left(\frac{2e}{c}\vec{v}_{\mathbf{p}}\cdot\vec{A}\right)$$

- Current Response: collective mode

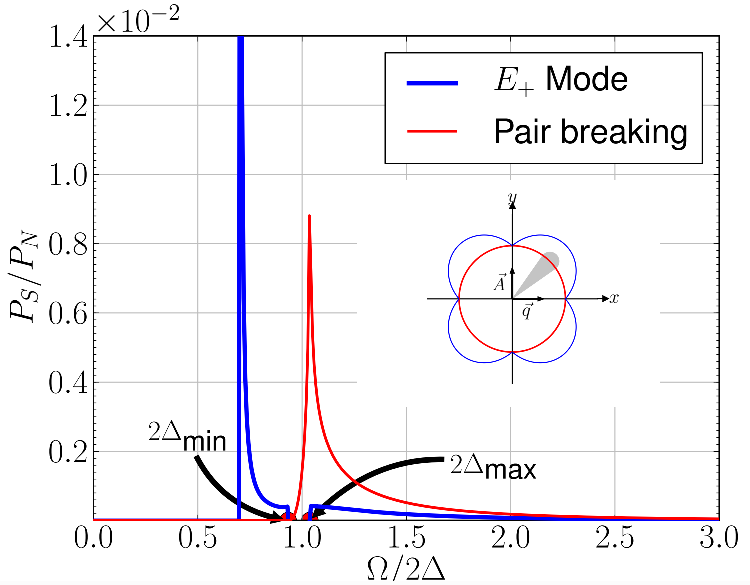
$$J^{\text{mode}} = \frac{eN_f}{2}\int\frac{d\phi}{2\pi}\vec{v}_{\mathbf{p}}\,\eta\,\bar{\lambda}\left(\vec{\Delta}_R\cdot\vec{d}^{-}-i\vec{\Delta}_I\cdot\vec{d}^{+}\right)$$

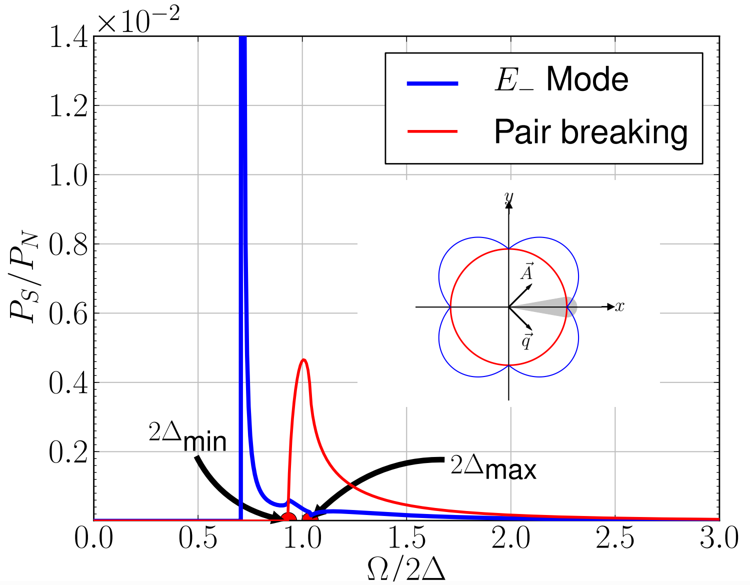
- Power absorption $\vec{J}=\widehat{K}\vec{A}$

$$P_S(\omega)=P_N(\omega)\frac{\xi_0}{\Lambda}\int\frac{dq}{2\pi}\frac{Im\,K(q,\omega)}{\left|q^2+\frac{4\pi}{c}K(q,\omega)\right|^2}$$

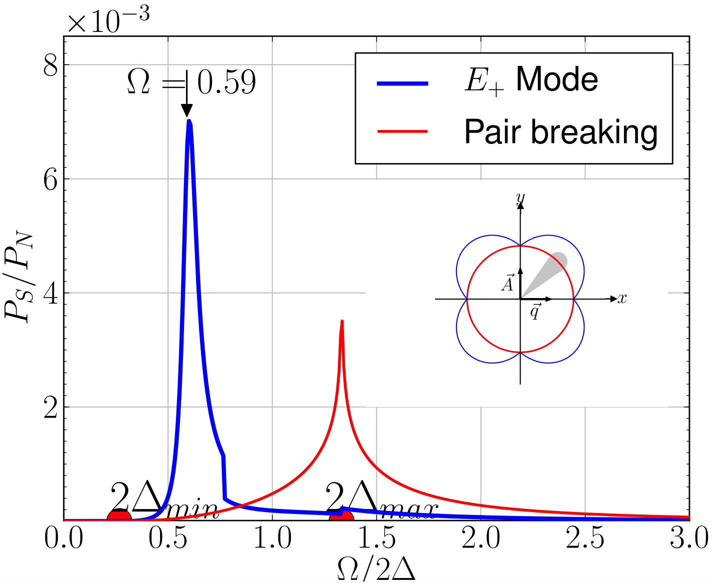
- P. J. Hirschfeld et al, Phys. Rev. B 40, 6695 (1989).

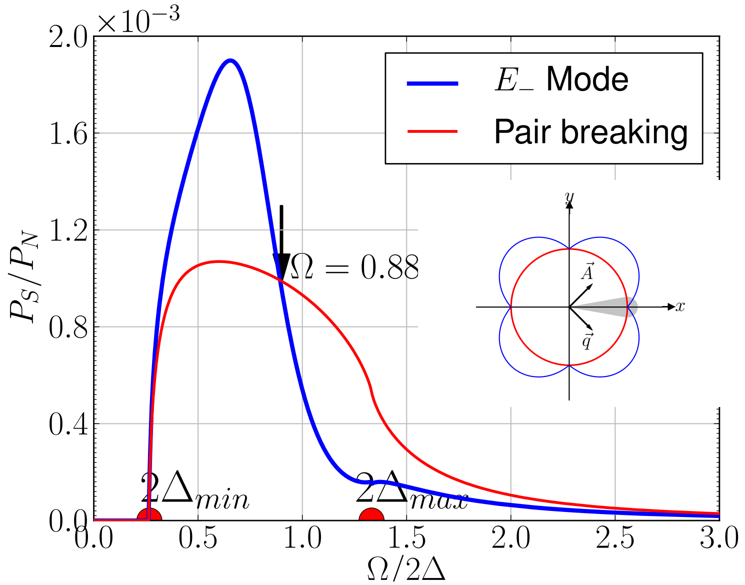
Power absorption: anisotropy $\varepsilon = 0.1$ - polarization $[100]$




$\times 10^{-2}$
1.4
1.2
1.0
0.8
0.6
0.4
0.2
P_S/P_N
E_- Mode
Pair breaking
y
x
$\vec{A}$
$\vec{q}$
2Δmin
2Δmax
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\Omega/2\Delta$

Power absorption: anisotropy $\varepsilon = 0.8$ - polarization $[100]$

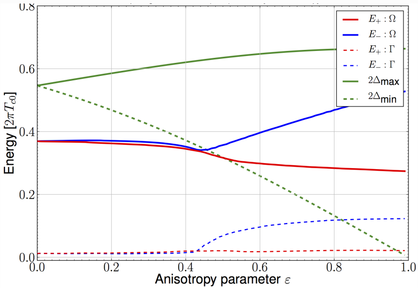


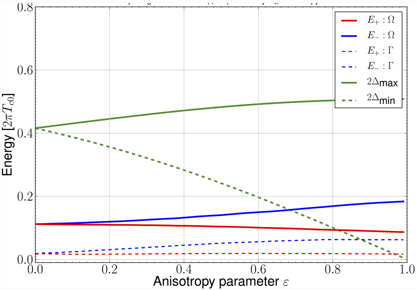

$\times 10^{-3}$
2.0
1.6
1.2
0.8
0.4
0.0
P_S/P_N
E_- Mode
Pair breaking
$\Omega = 0.88$
$2\Delta_{min}$
$2\Delta_{max}$
y
x
$\vec{A}$
$\vec{q}$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\Omega/2\Delta$

Effect of disorder on mode spectrum

- Low disorder
  $\hbar/2\tau\Delta = 0.01$

- High disorder
  $\hbar/2\tau\Delta = 0.1$

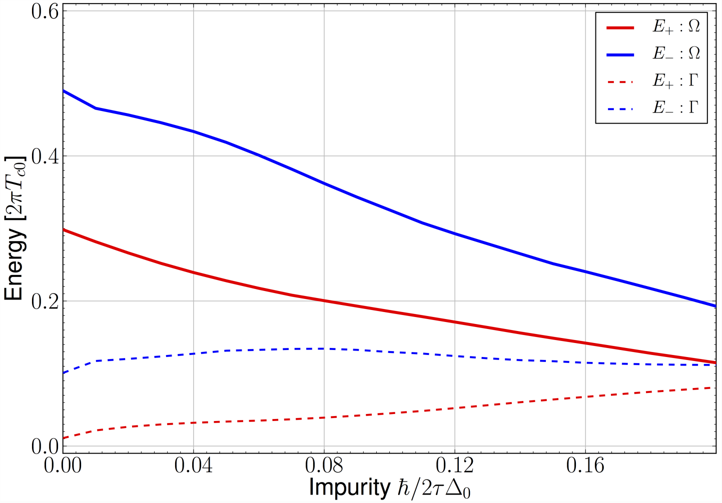




Effect of disorder on mode spectrum

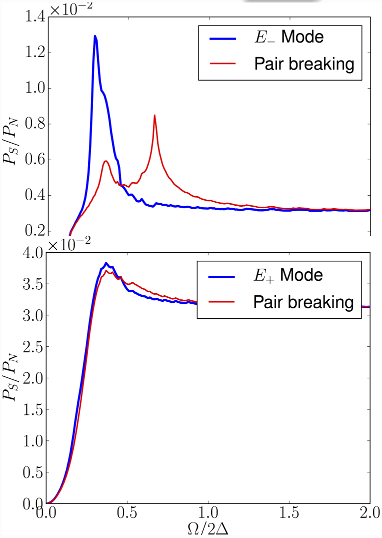
- Mode spectrum dependence on impurity

  Unitary limit $\sigma = 1.0$, and high anisotropy $\varepsilon = 0.8$

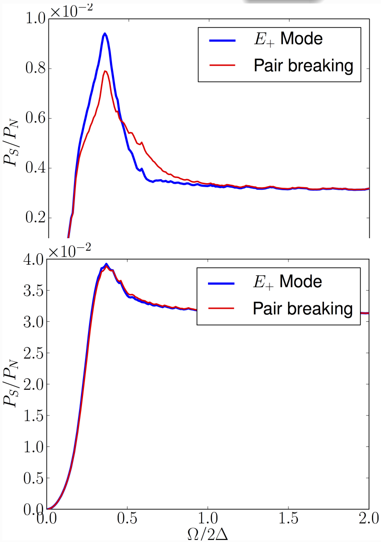


Effect of disorder on power absorption, Unitary limit $\sigma = 1.0$, high anisotropy $\varepsilon = 0.8$

- E_+, polarization [100]



- E_-, polarization [110]



Summary and Conclusions

- Multi-Component Superconductors
  Multiple Phases *and* a Spectrum of Cooper Pair Excitations
- TDGL Theory - Spectrum of Anderson-Higgs Modes with $\omega < 2\Delta$
- Anisotropy $\rightsquigarrow$
  - Splitting of degenerate Bosonic Modes for E_{1u} Pairing
  - Coupling of Fermions to Bosons $\rightsquigarrow$ Lifetime of AH Modes
- Coupling to Transverse EM Fields $\rightsquigarrow$
  - Sub-gap ($\omega \approx 10$GHz) EM absorption
    $\rightsquigarrow$ Signatures of Broken T-symmetry for E_{1u} Pairing
  - Selection Rules for coupling to AH Modes $\rightsquigarrow$
    $\rightsquigarrow$ Signatures of Pairing Symmetry
- Signatures of AH Modes in $P(\omega)$ survive weak disorder and strong anisotropy