Topological Edge & Surface States of Superfluid ^{3}He

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- Jeevak Parpia (Cornell)

- Supported by NSF Grant DMR-1106315

- Topology of Superfluid ^{3}He-A
- Chiral Edge States in 2D ^{3}He-A
- Detecting Chiral Edge Fermions
- ^{3}He-B - 3D Topological SF
- Helical Spin Current
- Signatures of Majorana Modes

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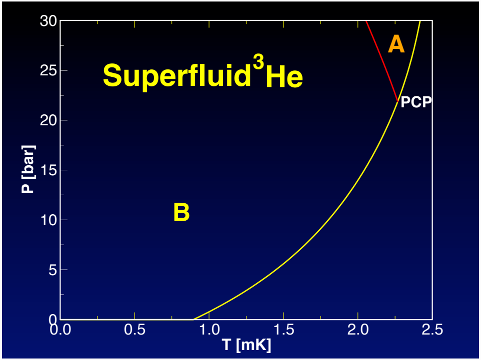
- Hao Wu, J. A. Sauls, Phys. Rev. B 88, 18 184506 (2013)

Symmetry Group of Normal ^{3}*He*: $\mathtt{G} = \mathtt{SO(3)}_S \times \mathtt{SO(3)}_L \times \mathtt{U(1)}_N \times \mathtt{P} \times \mathtt{T}$

Phase Diagram of Bulk ^{3}He



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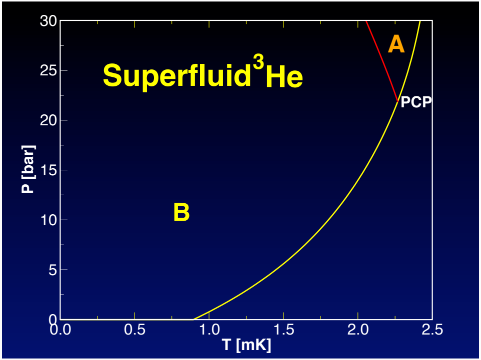

Spin-Triplet, P-wave Order Parameter:

$$\Delta_{\alpha\beta}(\mathbf{p}) = \vec{\mathbf{d}}(\mathbf{p}) \cdot (i\vec{\boldsymbol{\sigma}}\sigma_y)_{\alpha\beta} \rightsquigarrow \mathbf{d}_\mu(\mathbf{p}) = \mathcal{A}_{\mu i}\,\mathbf{p}_i$$

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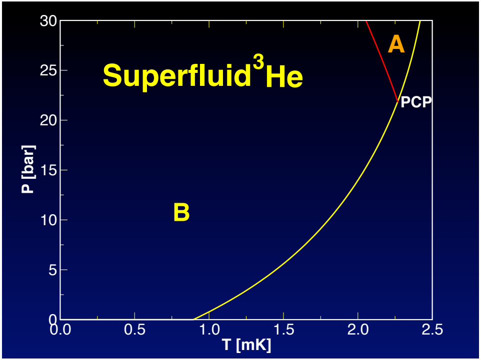
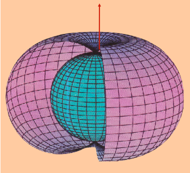


Chiral ABM State $\vec{l} = \hat{\mathbf{m}} \times \hat{\mathbf{n}}$



$$\mathcal{A}_{\mu i} = \Delta\, \hat{\mathbf{d}}_\mu\, (\hat{\mathbf{m}} + i\hat{\mathbf{n}})_i$$

$$L_z = 1\,,\; S_z = 0$$

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Superfluid Phases of ^{3}He

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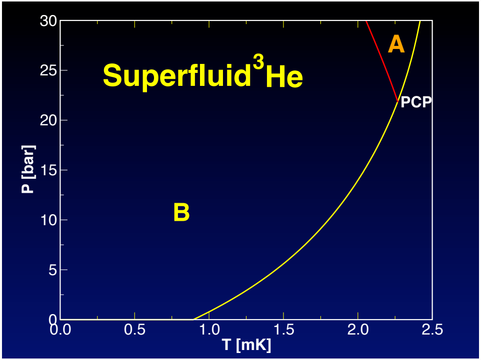

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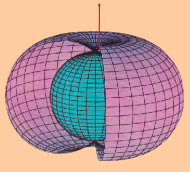

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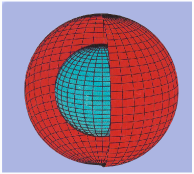
"Isotropic" BW State



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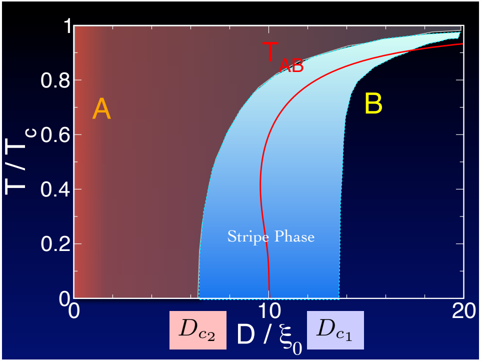
A. Vorontsov & JAS, PRL, 2007



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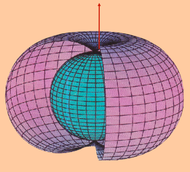

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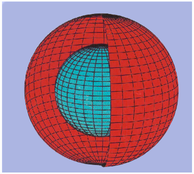

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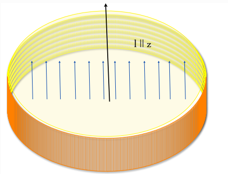
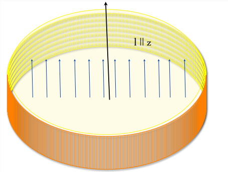
Ground-State for Chiral P-wave BEC Molecules or BCS Pairs Composed of N Fermion atoms:

$$|\,\Phi_N\,\rangle = \left[\iint d\mathbf{r}_1 d\mathbf{r}_2\ \varphi_{s_1 s_2}(\mathbf{r}_1-\mathbf{r}_2)\ \psi^\dagger_{s_1}(\mathbf{r}_1)\psi^\dagger_{s_2}(\mathbf{r}_2)\right]^{N/2} |\,\text{vac}\,\rangle$$



1. $\varphi_{s_1 s_2}(\mathbf{r}) = f(|\mathbf{r}|/\xi)\ (x+iy)\ \chi_{s_1 s_2}(S=1, M_S=0)$
2. Radial size $\equiv \xi$: BEC ($\xi < a$) vs. BCS pairs ($\xi > a$)

Ground-State Angular Momentum of Chiral P-wave Condensates



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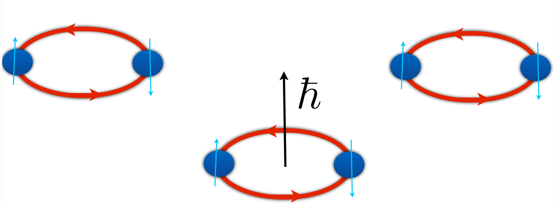

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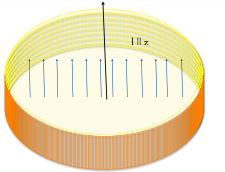

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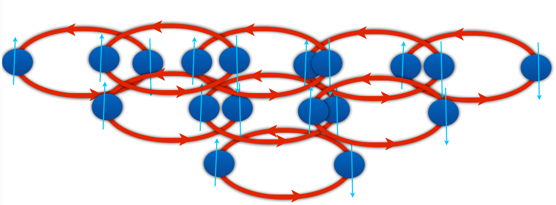

$L_z = (N/2)\hbar\,(a/\xi)^2 \ll (N/2)\hbar$? (P. Morel & P.W. Anderson, 1962, A. Leggett, 1975)

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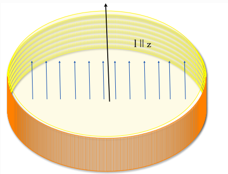

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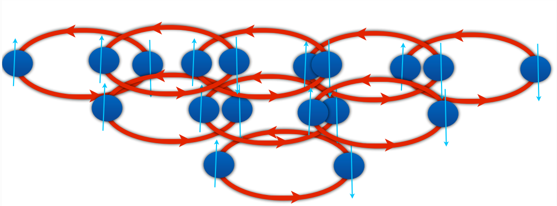

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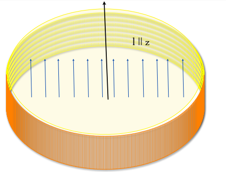

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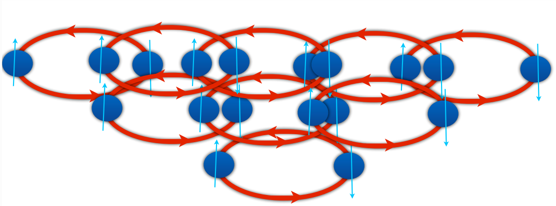

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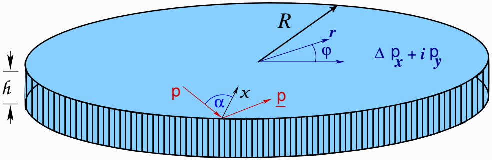
Currents are confined on the Edge

^{3}He-A confined in a cylindrical cavity with $h \ll \xi_0$ and $R \gg \xi_0$.



- 2D Chiral ABM State:
  $\vec{\mathbf{d}}(\mathbf{p}) = \Delta\,\hat{\mathbf{z}}\,(p_x \pm i p_y)/p_f \sim e^{\pm i\varphi_{\mathbf{p}}}$
- Equal-Spin Pairs for all $\mathbf{p}$:
  $\hat{\mathbf{z}} \rightsquigarrow |\rightrightarrows\rangle + |\leftleftarrows\rangle$
- Fully Gapped: $|\vec{\mathbf{d}}(\mathbf{p})|^2 = \Delta^2$

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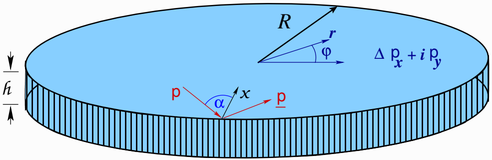

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Bogoliubov Equations for Fermionic Excitations:

$$\left(-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 - \mu\right) u + \sigma_x \frac{\hbar}{i}\left(\Delta_1 \frac{\partial}{\partial x} + i\Delta_2 \frac{\partial}{\partial y}\right) v = \varepsilon\, u$$

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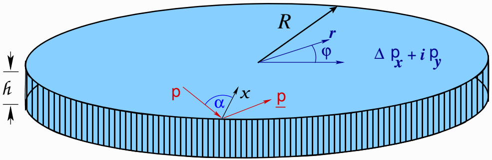

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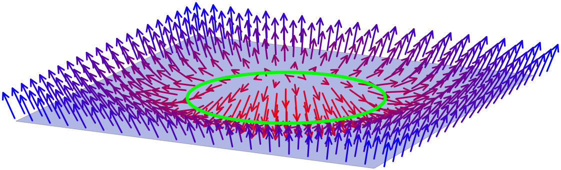
Nambu-Momentum Representation with particle-hole (iso-spin) matrices $\widehat{\vec{\tau}} = (\widehat{\tau}_1, \widehat{\tau}_2, \widehat{\tau}_3)$

$$\widehat{H} = \left(|\mathbf{p}|^2/2m - \mu\right)\widehat{\tau}_3 + \sigma_x \left[\Delta_1\, p_x\, \widehat{\tau}_1 \mp \Delta_2\, p_y\, \widehat{\tau}_2\right]/p_f \;= \vec{\mathbf{m}}(\mathbf{p}) \cdot \widehat{\vec{\tau}}$$

Nambu-Bogoliubov Hamiltonian for 2D ^{3}He-A : $\widehat{H} = \vec{\mathbf{m}}(\mathbf{p}) \cdot \widehat{\vec{\boldsymbol{\tau}}}$

$\rightsquigarrow$ $\vec{\mathbf{m}} = (\, cp_x \,,\, \mp cp_y \,, \xi(\mathbf{p}))$ with $|\vec{\mathbf{m}}(\mathbf{p})|^2 = \left(|\mathbf{p}|^2/2m - \mu\right)^2 + c^2|\mathbf{p}|^2 > 0 \;, \mu \neq 0$



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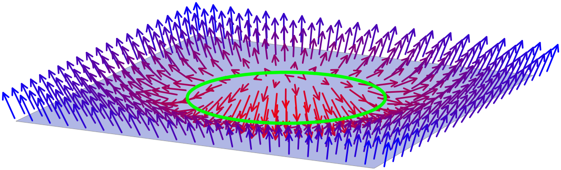

Topological Invariant for 2D ^{3}He-A $\leftrightarrow$ QED in d = 2+1 [G.E. Volovik, JETP 1988]:

$$N_{\text{2D}} = \pi \int \frac{d^2p}{(2\pi)^2}\, \hat{\mathbf{m}}(\mathbf{p}) \cdot \left(\frac{\partial \hat{\mathbf{m}}}{\partial p_x} \times \frac{\partial \hat{\mathbf{m}}}{\partial p_y} \right) = \begin{cases} \pm 1 \;; & \mu > 0 \text{ and } \Delta \neq 0 \\ 0\,; & \mu < 0 \text{ or } \Delta = 0 \end{cases}$$

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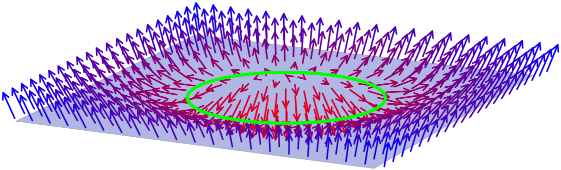

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"Vacuum" $(\Delta = 0)$ with $N_{2D} = 0$ | ^{3}He-A$(\Delta \neq 0)$ with $N_{2D} = 1$

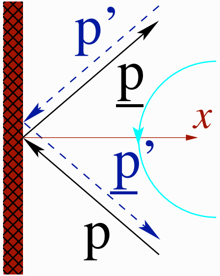
Zero Energy Fermions $\uparrow$ Confined on the Edge

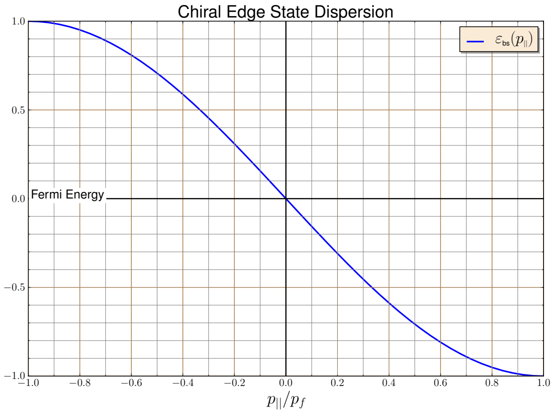
Chiral Edge Fermions in the 2D ^{3}He-A

Propagator for Edge Fermions: $g^{R}_{\text{edge}}(\mathbf{p},\varepsilon;x) = \dfrac{\pi\Delta|\mathbf{p}_x|}{\varepsilon + i\gamma - \varepsilon_{\text{bs}}(\mathbf{p}_{||})}\; e^{-x/\xi_\Delta}$

Confinement on $\xi_\Delta = \hbar v_f/2\Delta \approx 10^3\,\mathring{A} \gg \hbar/p_f$

- $\varepsilon_{\text{bs}} = -c\,p_{||}$ with $c = \Delta/p_f \ll v_f$
- Broken P & T $\rightsquigarrow$ Edge Current





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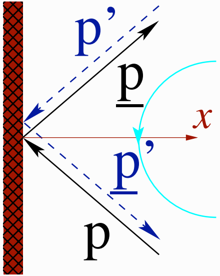

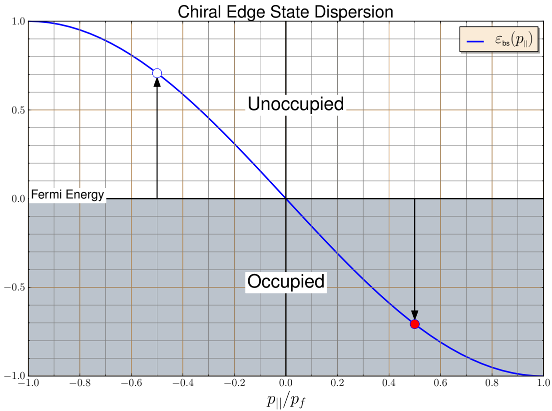

Local Density of States: $N(\mathbf{p}, x; \varepsilon) = -\frac{1}{\pi}\,\mathsf{Im}\, g^{\mathsf{R}}(\mathbf{p}, x; \varepsilon)$

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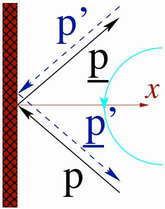
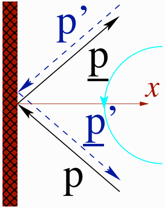


Pair Time-Reversed Trajectories

Spectral Current Density :

$$\vec{J}(\mathbf{p}, x; \varepsilon) = 2N_f\, \vec{v}(\mathbf{p}) \left[N(\mathbf{p}, x; \varepsilon) - N(\mathbf{p}', x; \varepsilon)\right]$$

Local Density of States: $N(\mathbf{p}, x; \varepsilon) = -\frac{1}{\pi}\,\mathrm{Im}\, g^{\mathrm{R}}(\mathbf{p}, x; \varepsilon)$



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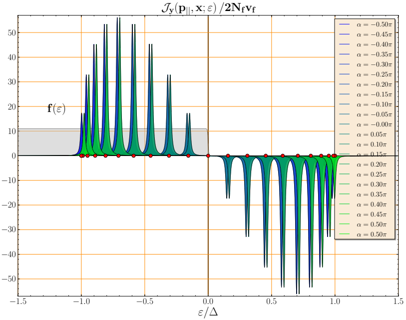

Bound-State Edge Current at $x = 0$

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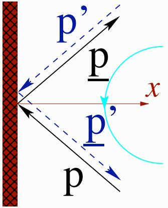

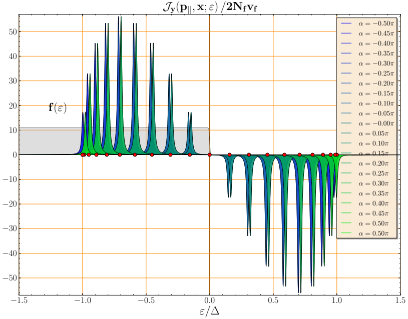

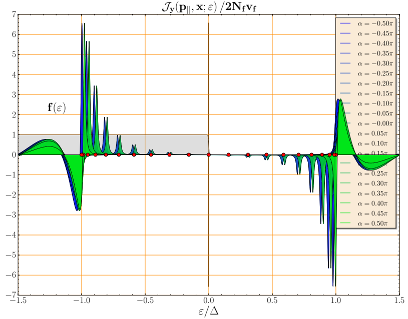

Bound-State Edge Current at $x = 0$

Continuum Edge Current at $x = 10\xi_0$

Ground-State Current Density: $\vec{J}(x) = \int_{-1}^{+1} \frac{dp_{||}}{p_f} \int_{-\infty}^{0} \vec{J}(\mathbf{p}, x; \varepsilon)$

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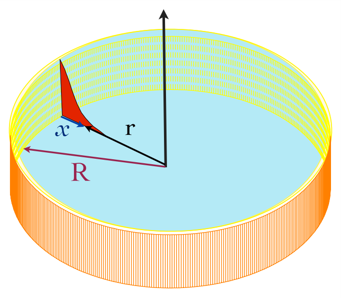

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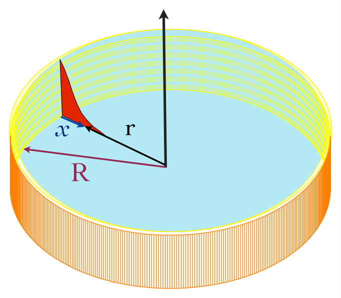

Bound-State Contribution ($R \gg \xi_\Delta$):

$$J_\varphi(\mathbf{p}, x; \varepsilon) = 2N_f\, v_f\, \Delta\, |p_x|\, p_\varphi\, e^{-x/\xi_\Delta}$$
$$\times \left[\delta(\varepsilon - \varepsilon_{\text{bs}}(\mathbf{p}_{||})) - \delta(\varepsilon - \varepsilon_{\text{bs}}(\mathbf{p}'_{||}))\right]$$

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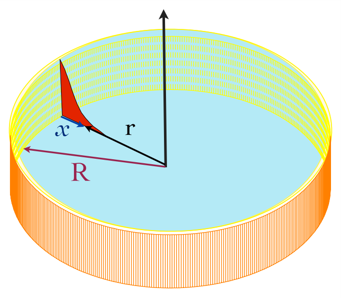

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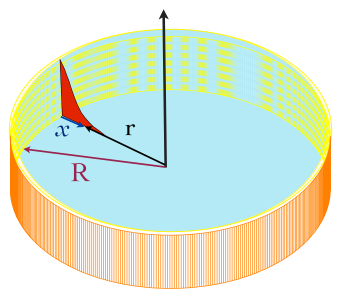

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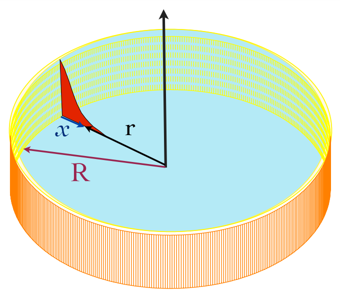

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- $L_z^{\text{bs}} = \int_V d^2r\, [r\, g_\varphi(\mathbf{r})] = N\, \hbar$

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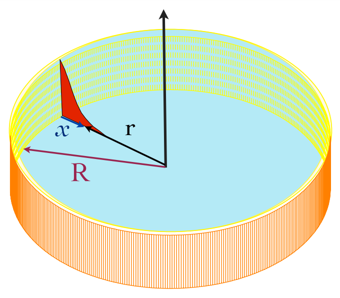

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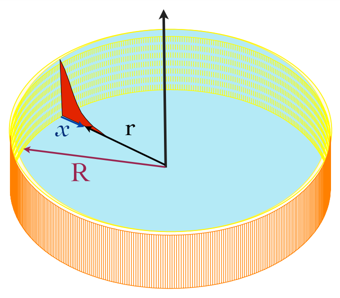

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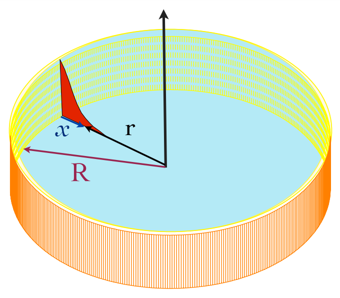

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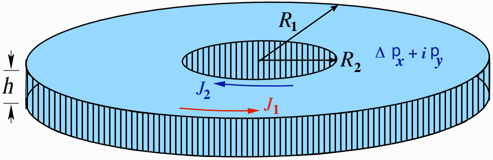
^{3}He-A confined in a toroidal cavity



- $R_1, R_2, R_1 - R_2 \gg \xi_0$
- Volume: $V = h\pi(R_1^2 - R_2^2)$

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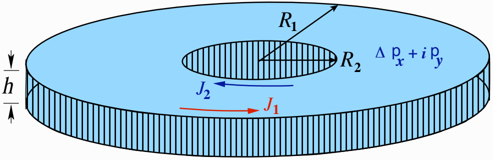

- $R_1, R_2, R_1 - R_2 \gg \xi_0$
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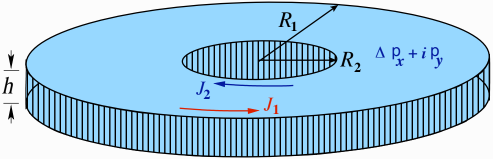

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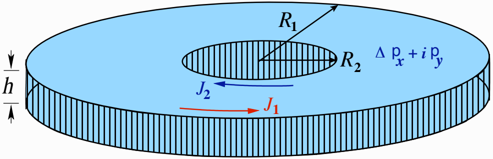

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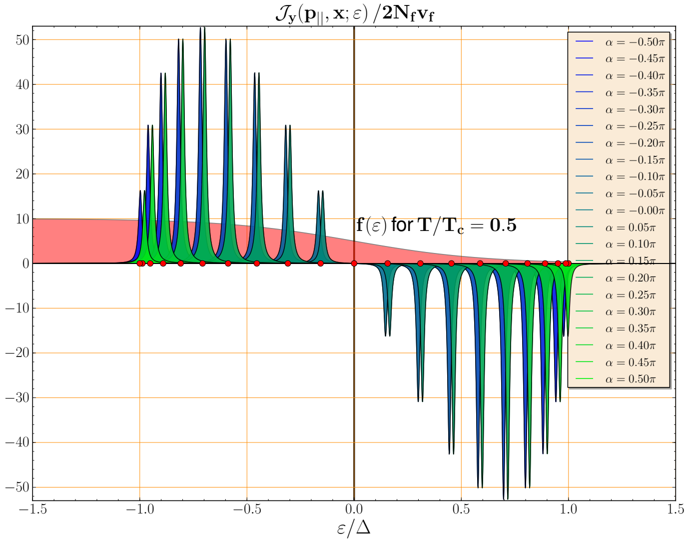
- Angular Momentum:

$$L_z = 2\pi\, h\, (R_1^2 - R_2^2) \times \frac{1}{4}\, n\, \hbar = (N/2)\, \hbar$$

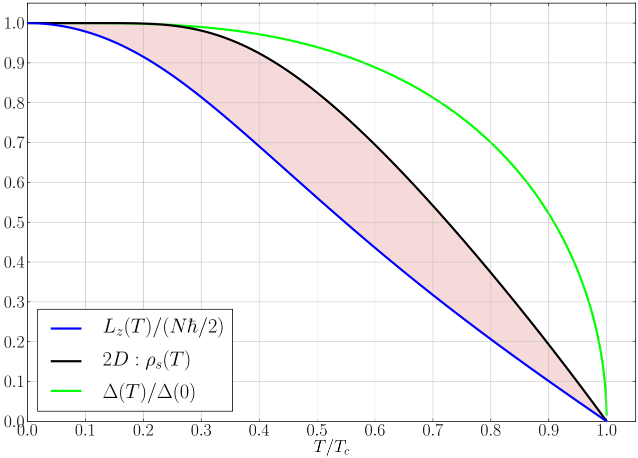
McClure-Takagi Result

Thermally Excited Edge Fermions Carry the Opposite Current



Angular Momentum of ^{3}He-A vs. Temperature

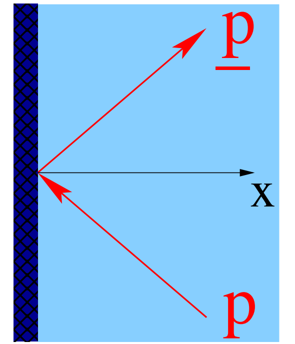
$$L_z = (N/2)\hbar \times \mathcal{Y}_{L_z}(T) \qquad \mathcal{Y}_{L_z}(T) \approx 1 - c\,(T/\Delta)^2\,, \quad T \ll \Delta$$



- JAS, Phys. Rev. B 84, 214509 (2011)
- Y. Tsutsumi et al., PRB 85, 100506(R) (2012)

Edge Currents are Protected by Symmetry, not Topology

Specular Reflection



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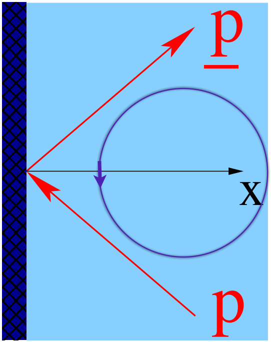

Propagating Chiral Fermions:

$$g^{\mathsf{R}}(\mathbf{p}, \varepsilon; x) = \frac{\pi\Delta|\mathbf{p}_x|}{\varepsilon + i\gamma - \varepsilon_{\mathsf{bs}}(\mathbf{p}_{||})}\, e^{-x/\xi_\Delta}$$

Edge Current: $J = \frac{1}{4}\, n\, \hbar$

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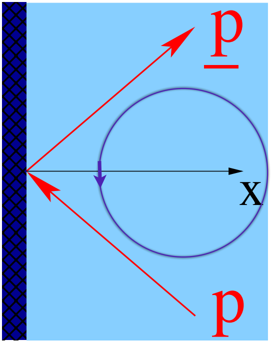
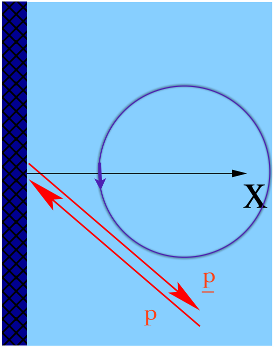


Retro Reflection



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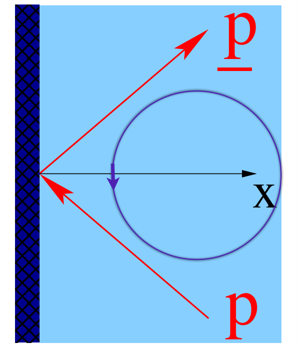

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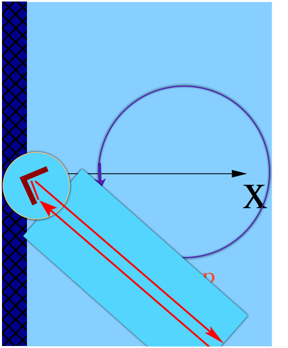

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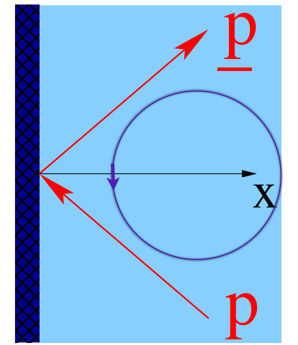

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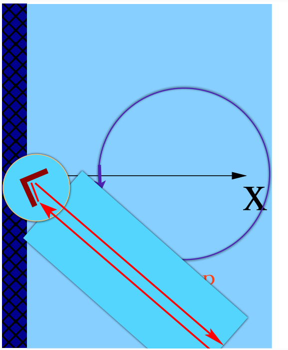
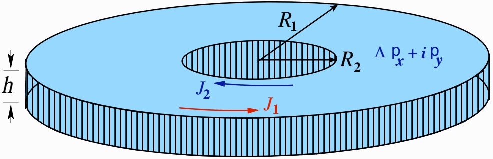


Zero-Energy Fermions for all $\mathbf{p}$:

$$g^{\mathsf{R}}(\mathbf{p}, \varepsilon; x) = \frac{\pi\Delta}{\varepsilon + i\gamma}\, e^{-2\Delta x/v_x}$$

Non-Chiral $\rightsquigarrow$ Edge Current: $J = 0$

Engineered Edges of a Toroidal Cavity



- Sheet Current: $J = f \times \frac{1}{4} n \hbar$
- Non-Specular Surfaces
  $0 \leq f \leq 1$

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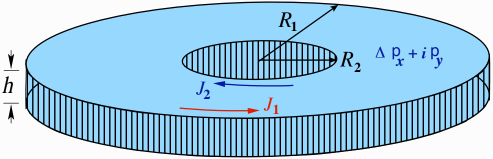

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Incomplete Screening of Counter-Propagating Currents

$$L_z = (N/2)\,\hbar \times \left(\frac{f_1 - r f_2}{1 - r}\right)$$

Non-Extensive Scaling of L_z: $r = (R_2/R_1)^2 \qquad 0 < r < 1$

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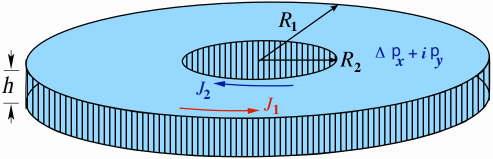

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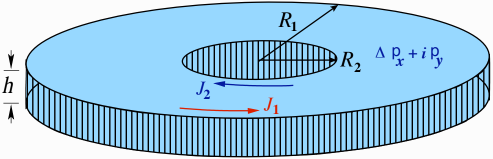
Engineered Edges of a Toroidal Cavity



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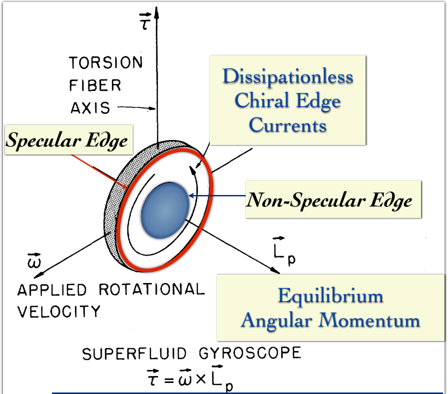
  $$L_z = (N/2)\,\hbar \times \left(\frac{1}{1-r}\right) \gg (N/2)\,\hbar$$
- $f_1 = 0,\ f_2 = 1$
  $$L_z = (N/2)\,\hbar \times \left(\frac{-r}{1-r}\right) \ll -(N/2)\,\hbar$$

- Strong violations of the McClure-Takagi Result

Gyroscopic Experiment to Measure of $L_z(T)$



J. Clow and J. Reppy, *Phys. Rev. A* 5, 424–438

Signatures of Chiral Edge States

▸ Power Law for $T \lesssim 0.5T_c$

$L_z \approx (N/2)\hbar\,(1 - c\,(T/\Delta)^2)$

Toroidal Geometry with Engineered Surfaces

▸ L_z is Non-Extensive :

$L_z > (N/2)\hbar$ or

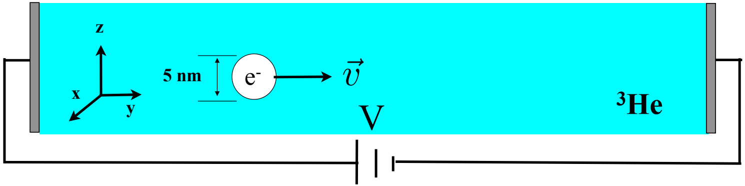
$L_z < -(N/2)\hbar$

Direct Signature of Edge Currents

Detection of Broken *Time-Reversal* Symmetry of Cooper pairs in Superfluid ^{3}He-A

Hiroki Ikegami, Yasumasa Tsutsumi, Kimitoshi Kono, Science 341, 59-62 (2013)

RIKEN, Japan



Electron Mobility:

$$\vec{v} = \widehat{\mu} \cdot \vec{E}$$

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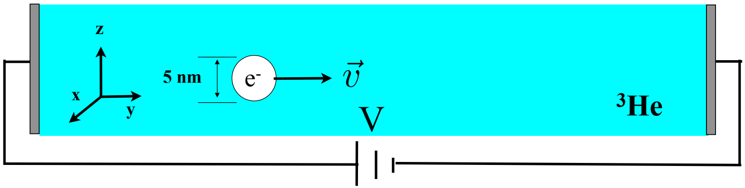


Electron Mobility:

$$\vec{v} = \widehat{\mu} \cdot \vec{E}$$

B-phase Mobility

$$\widehat{\mu} = \mu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

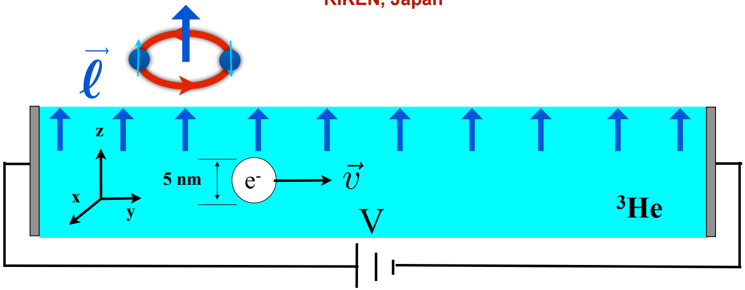
Isotropic

Fully Gapped

Detection of Broken *Time-Reversal* Symmetry of Cooper pairs in Superfluid ^{3}He-A

Hiroki Ikegami, Yasumasa Tsutsumi, Kimitoshi Kono, Science 341, 59-62 (2013)

RIKEN, Japan



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$$\widehat{\mu} = \mu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Isotropic

Fully Gapped

A-phase Mobility

$$\widehat{\mu} = \begin{pmatrix} \mu_{\perp} & \mu_{xy} & 0 \\ -\mu_{xy} & \mu_{\perp} & 0 \\ 0 & 0 & \mu_{\parallel} \end{pmatrix}$$

Anisotropic

Transverse Force

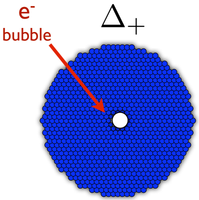
$\vec{\ell} = +\hat{\mathbf{z}}$

Structure of an Ion embedded in ^{3}He-A

$\hbar/p_f \ll R \lesssim \xi_0$

JAS & M. Eschrig, New J. Phys. 11, 075008 (2009)

$(p_x + ip_y)$



Δ_+

$\vec{\ell} = +\hat{\mathbf{z}}$

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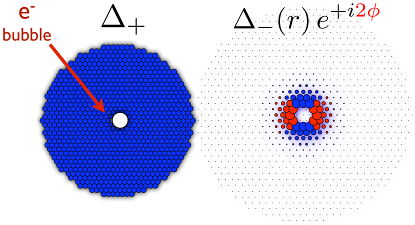
JAS & M. Eschrig, New J. Phys. 11, 075008 (2009)

$(p_x + ip_y)$ $(p_x - ip_y)$



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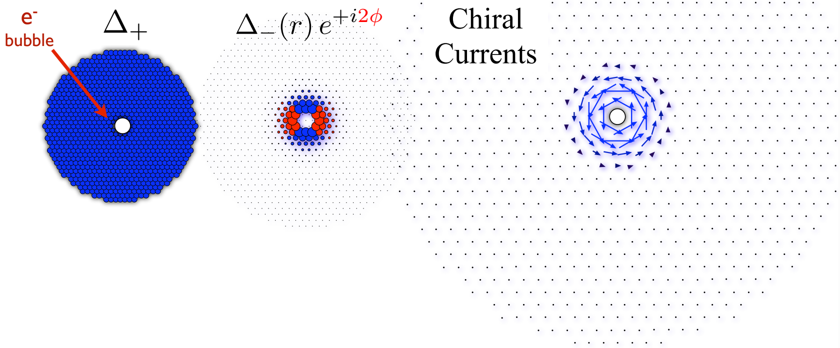

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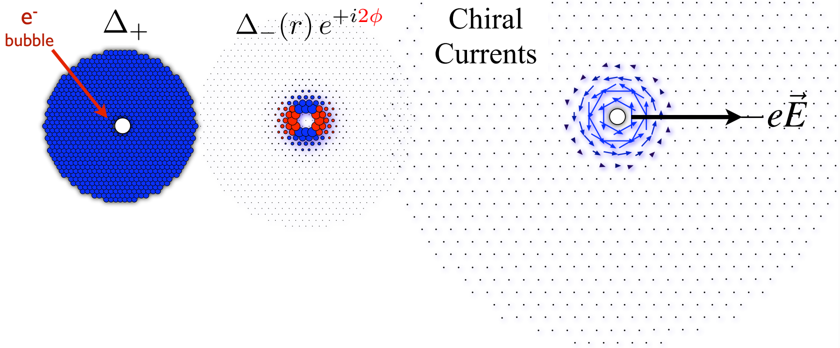

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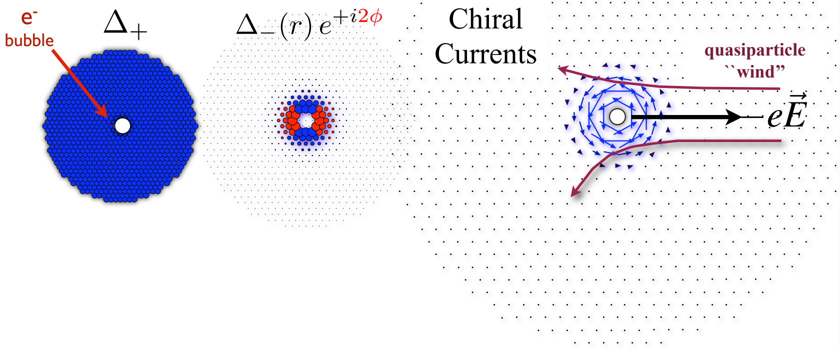

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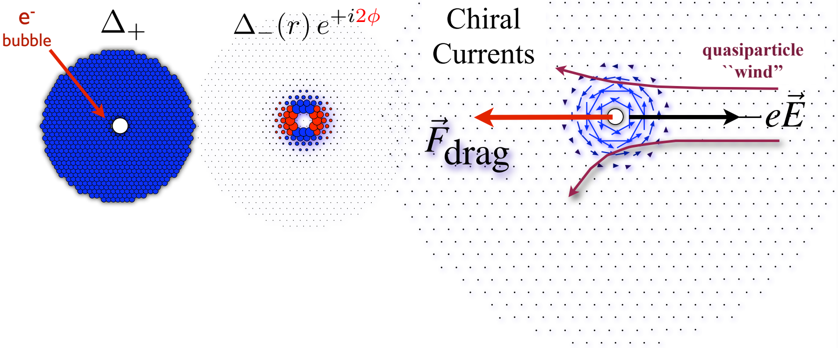

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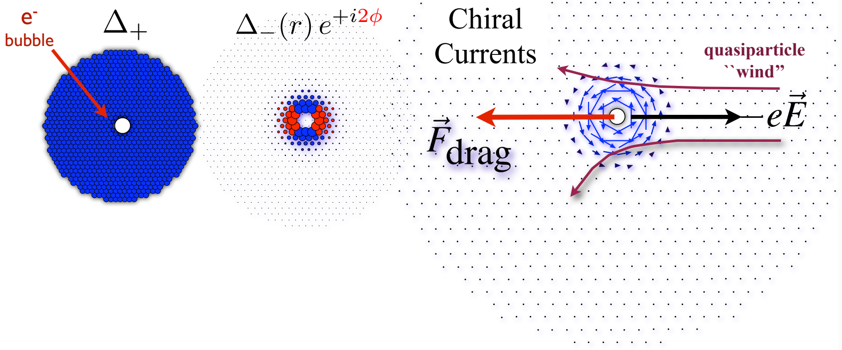

Detection of Broken Time-Reversal Symmetry in ^{3}He-A

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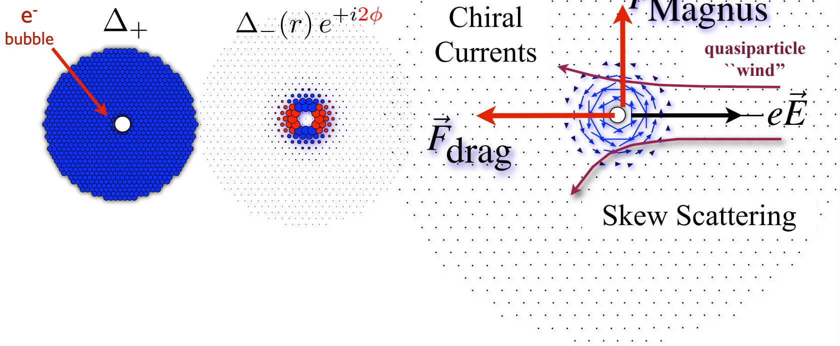

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JAS & M. Eschrig, New J. Phys. 11, 075008 (2009)

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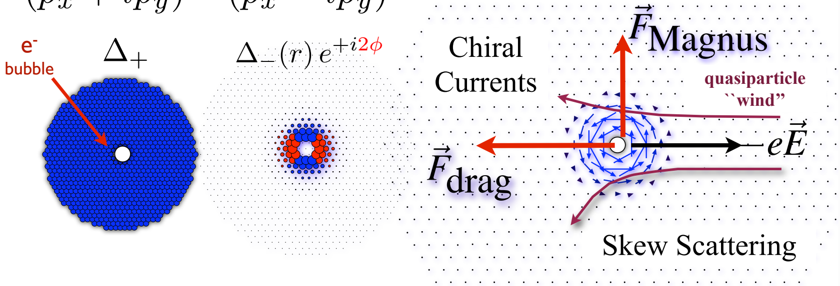
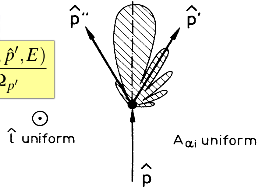


$R \approx$ Å ➡ M.Vuorio and D. Rainer, J. Phys. C Sol. State 10 3093(1977)

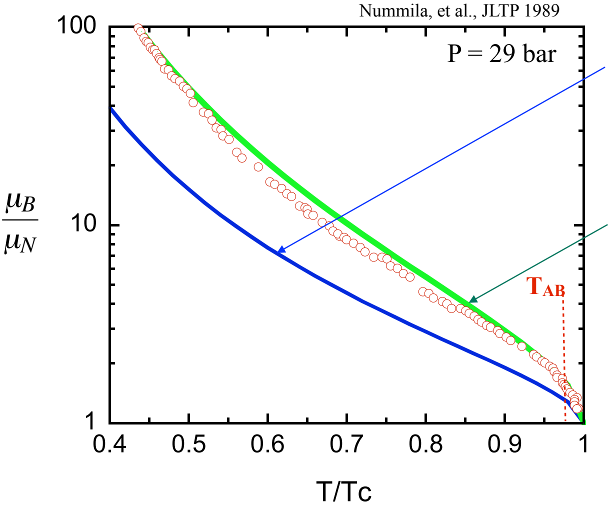
$$e(\mu^{-1})_{ij} = n\,p_f \int \frac{d\Omega_p}{4\pi} \int \frac{d\Omega_{p'}}{4\pi} \int_{-\infty}^{+\infty} dE \left(-\frac{\partial f}{\partial E}\right) (\Delta\hat{p})_i (\Delta\hat{p})_j \frac{\partial\sigma(\hat{p},\hat{p}',E)}{\partial\Omega_{p'}}$$

$$\vec{v} = \left[\mu_{\parallel} \left(\hat{\ell}\cdot\vec{E}\right)\hat{\ell} + \mu_{\perp}\,\hat{\ell}\times\left(\hat{\ell}\times\vec{E}\right) + \mu_{xy}\,\hat{\ell}\times\vec{E}\right]$$



R. Salmelin, M. Salomaa, V. Mineev, Phys. Rev. Lett. 63, 868 (1989)

Mobility of electron bubbles in superfluid ^{3}He-B



constant
$\sigma_{tr} = \pi R^2$

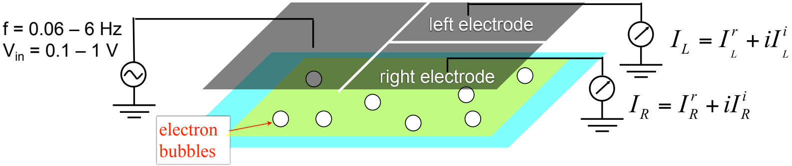
$$\frac{\mu_B}{\mu_N} = 1 + e^{+\Delta(T)/T}$$

R. Bowley, J. Phys. C 1977

*resonant forward scattering
$\sigma_{tr} << = \pi R^2$

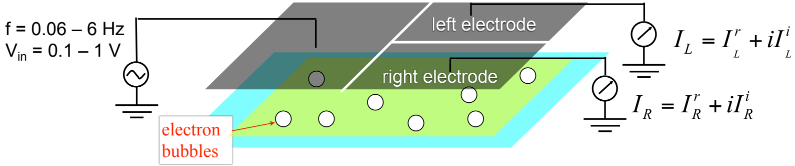
*Theory: G. Baym, C. Pethick, M. Salomaa, PRL 38, 845 1977

Measurement of the Transverse e$^-$ mobility in Superfluid ^{3}He Films



H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

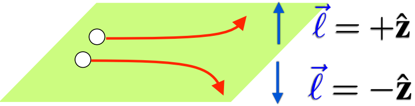
Measurement of the Transverse e⁻ mobility in Superfluid ^{3}He Films



Transverse Force from ***Skew* Scattering**

$$\rightsquigarrow \quad \Delta I = I_R - I_L \neq 0$$

$$\vec{v} = \left[\mu_\perp \vec{E} + \mu_{xy}\, \hat{\ell} \times \vec{E} \right]$$



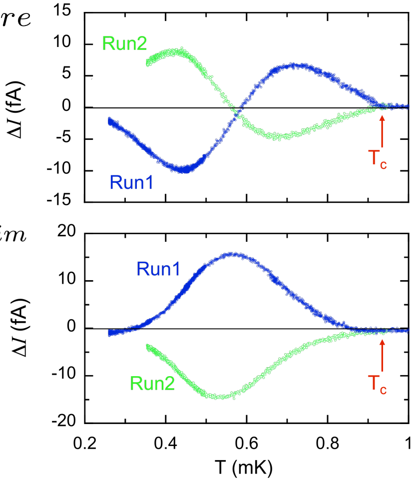
H. Ikegami, Y. Tsutsumi, K. Kono, **Science 341, 59-62 (2013)**

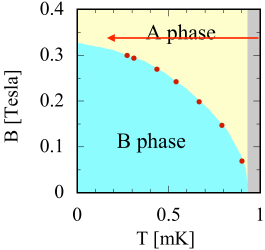
Transverse Current in ^{3}He-A

$$\Delta I = I_R - I_L$$

ΔI^{re}

ΔI^{im}





Single Domains:

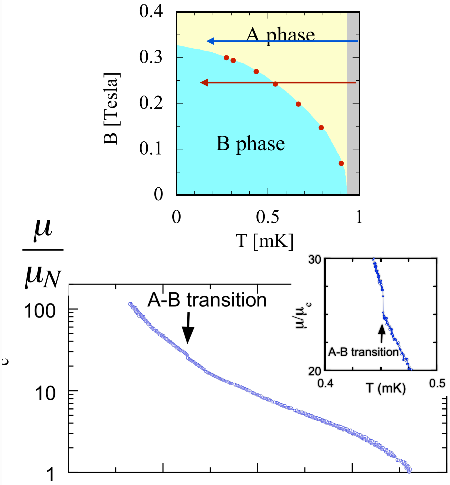
Run 1 $\vec{\ell} = +\hat{\mathbf{z}}$

Run 2 $\vec{\ell} = -\hat{\mathbf{z}}$

$$\frac{|I_R - I_L|}{I_R + I_L} \approx 6\,\%$$

H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

Zero Transverse Current in ^{3}He-B (*T - symmetric phase*)



H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

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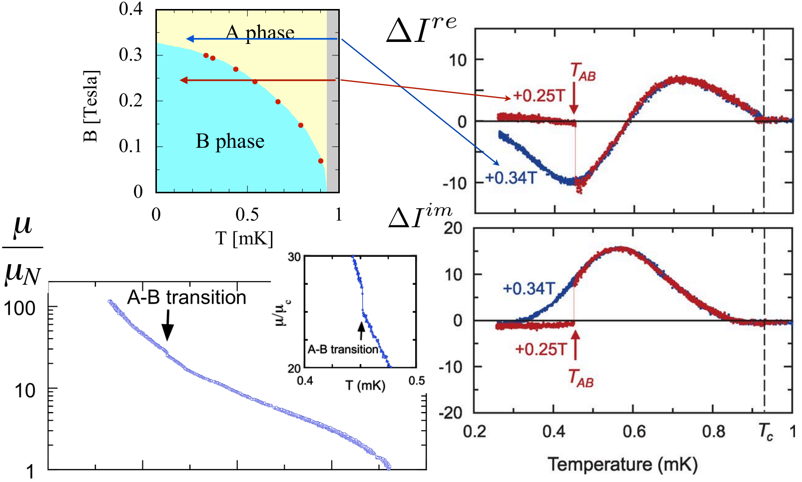


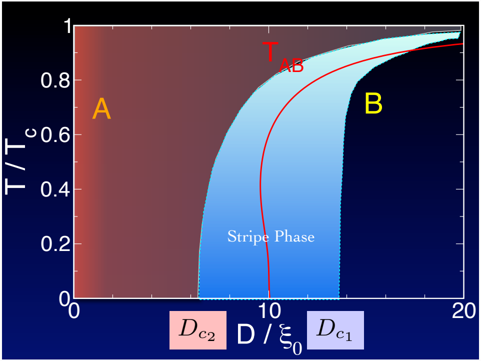
H. Ikegami, Y. Tsutsumi, K. Kono, **Science 341, 59-62 (2013)**

Summary

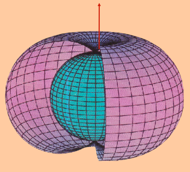
- Phase Diagram of ^{3}He Films
- Topology of Superfluid ^{3}He-A
- Chiral Edge States in 2D ^{3}He-A
- Edge Currents and $L_z(T) - L_z(0) \sim -T^2$: $R \gg \xi_0$
- Edge Currents $\leftrightarrow$ Non-Extensive L_z
- Detection of Chiral Currents via e^- Mobility
- ^{3}He-B - 3D Topological SF
- Helical Spin Current
- Symmetry Protected Majorana Modes
- Signatures of the Majorana Spectrum

Symmetry Group of Normal ^{3}He: $\mathtt{G} = \mathtt{SO(3)_S} \times \mathtt{SO(3)_L} \times \mathtt{U(1)_N} \times \mathtt{P} \times \mathtt{T}$

A. Vorontsov & JAS, PRL, 2007



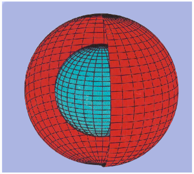
Chiral ABM State $\vec{l} = \hat{\mathbf{m}} \times \hat{\mathbf{n}}$



$$\mathcal{A}_{\mu i} = \Delta\, \hat{\mathbf{d}}_\mu\, (\hat{\mathbf{m}} + i\hat{\mathbf{n}})_i$$

$L_z = 1$, $S_z = 0$

"Isotropic" BW State

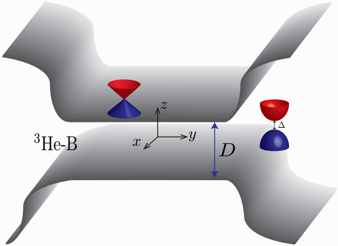


$$\mathcal{A}_{\mu i} = \Delta\, \delta_{\mu i}$$

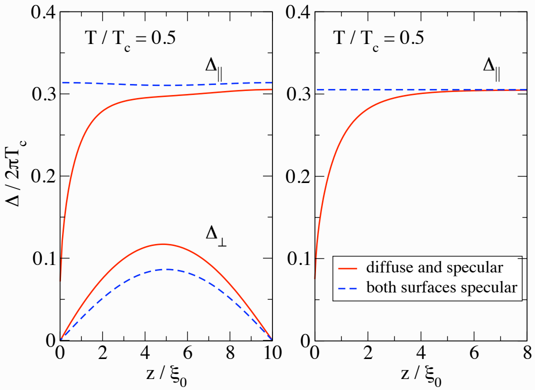
$J = 0$, $J_z = 0$

Spin-Triplet, P-wave Order Parameter:

$$\Delta_{\alpha\beta}(\mathbf{p}) = \vec{\mathbf{d}}(\mathbf{p}) \cdot (i\vec{\boldsymbol{\sigma}}\sigma_y)_{\alpha\beta} \rightsquigarrow \mathbf{d}_\mu(\mathbf{p}) = \mathcal{A}_{\mu i}\, \mathbf{p}_i$$



A. Vorontsov and JAS, PRB 68, 064508 (2003)



$$\hat{\Delta} = \vec{\mathbf{d}}(\mathbf{p}) \cdot (i\vec{\boldsymbol{\sigma}}\sigma_y)$$

$$d_x = \Delta_{\|}(z)\, p_x ,$$
$$d_y = \Delta_{\|}(z)\, p_y ,$$
$$d_z = \Delta_{\perp}(z)\, p_z ,$$

- Residual Symmetry: $\mathtt{G_B} = \mathtt{SO(2)_{L_z+S_z}} \times \mathtt{Z_2^{L+S}} \times \mathtt{T}$

Nambu-Bogoliubov Hamiltonian for Bulk ^{3}He-B:

$$\widehat{H}_{\text{B}} = \xi(\mathbf{p})\hat{\tau}_3 + c\,\mathbf{p}\cdot\vec{\boldsymbol{\sigma}}\,\hat{\tau}_1$$

1. "Relativistic" Fermions: $E(\mathbf{p}) = \sqrt{\xi(\mathbf{p})^2 + c^2|\mathbf{p}|^2}$
2. "light" speed: $c = \Delta/p_f \ll v_f$
3. Emergent *spin-orbit* coupling $\rightsquigarrow$ Helicity eigenstates

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► Topological Invariant for 3D ^{3}He-B protected by $\Gamma = \mathtt{CT}$ symmetry: $\Gamma\widehat{H}_{\text{B}}\Gamma^{\dagger} = -\widehat{H}_{\text{B}}$

Schnyder et al., PRB 78, 195125 (2008); Volovik, JETP Lett. 90, 587 (2009)

$$N_{\text{3D}} = \frac{\pi}{4}\int\frac{d^3p}{(2\pi)^3}\,\epsilon_{ijk}\text{Tr}\left\{\Gamma(\widehat{H}_{\text{B}}^{-1}\partial_{p_i}\widehat{H}_{\text{B}})(\widehat{H}_{\text{B}}^{-1}\partial_{p_j}\widehat{H}_{\text{B}})(\widehat{H}_{\text{B}}^{-1}\partial_{p_k}\widehat{H}_{\text{B}})\right\} = \begin{cases} 0, & \Gamma = 1 \\ 2, & \Gamma = \mathtt{CT} \end{cases}$$

Zero Energy Fermions Confined on a 2D Surface $\uparrow$

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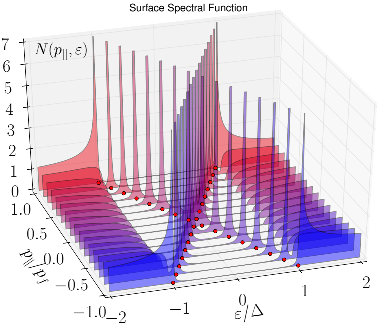
Zero Energy Fermions Confined on a 2D Surface $\uparrow$

- ^{3}He-B is a topological phase for restricted external $\mathtt{T}$-symmetry breaking! :

$$\mathtt{T} \rightarrow \mathtt{T}\times\mathtt{U}(\pi)_{\mathsf{L}_z+\mathsf{S}_z}$$

T. Mizushima, PRB 86 094518, (2012); Hao Wu, JAS, PRB 88, 18 184506 (2013)



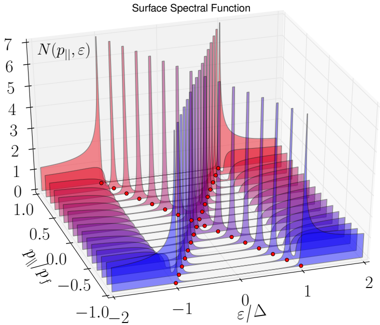
Surface Majorana Modes:

- Ground-state and Excitations:

$$\varepsilon_b^{\pm} = \pm c|\mathbf{p}_{||}|, \quad c = \Delta_{||}/p_f \ll v_f$$

Bound-state spectral weight:

$$N_b(\mathbf{p}, z; \varepsilon) = \frac{\pi}{2}\,\Delta_{\perp}\hat{p}_z\, e^{-2\Delta_{\perp} z/v_f} \times [\delta(\varepsilon - c|\mathbf{p}_{||}|) + \delta(\varepsilon + c|\mathbf{p}_{||}|)]$$



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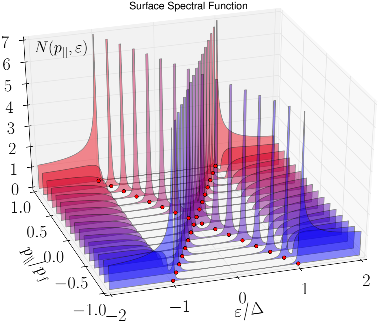

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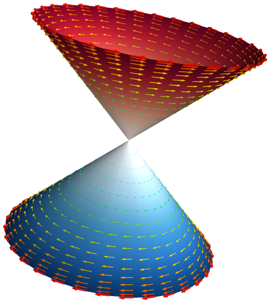
- Helical Spin-Orbit Locking



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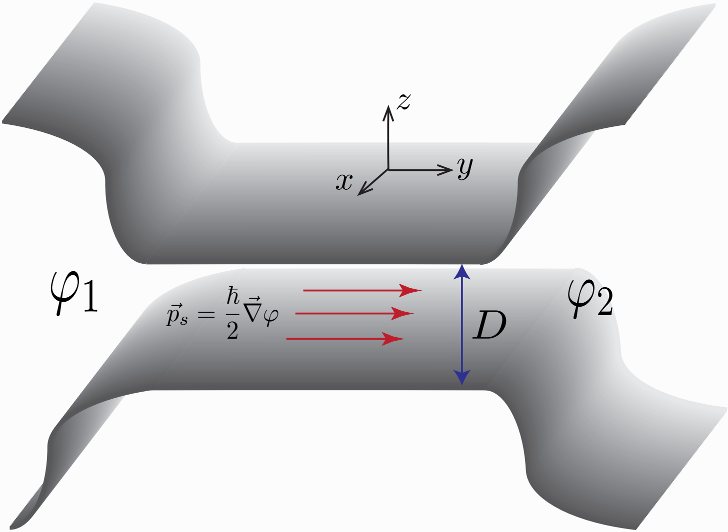
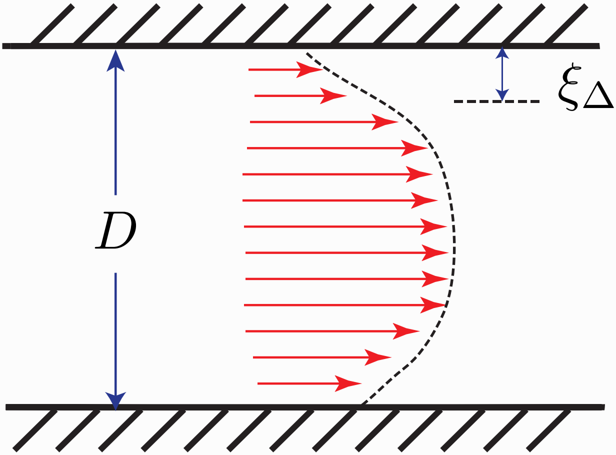
Condensate Flow and Doppler Shift of the Majorana Spectrum



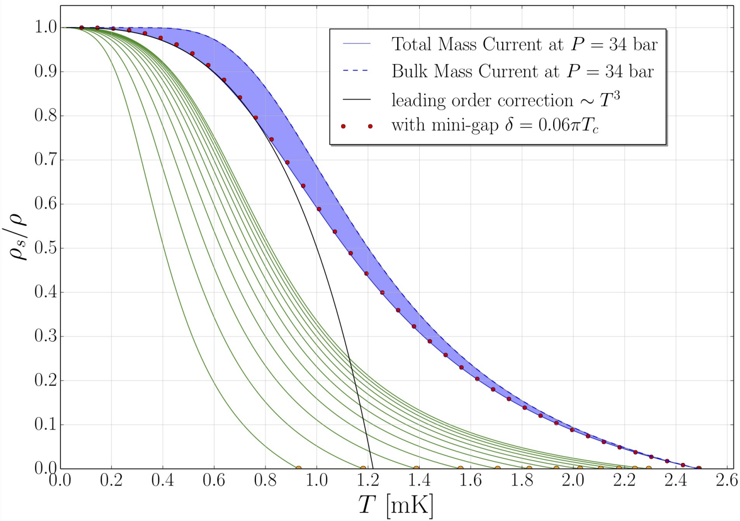
- Doppler Shift from the moving Condensate: $\varepsilon_{\mathbf{p}_{||}} = c|\mathbf{p}_{||}| + \mathbf{p}_{||} \cdot \vec{v}_s$



- Spatial dependence of finite temperature mass current

Superfluid Fraction of a superfluid film of ^{3}He-B of width $D = 13.2\,\xi_\Delta$

$$\rho_\mathrm{s}/\rho \approx 1 - \frac{27\pi\zeta(3)}{2}\,\frac{\xi_\Delta}{D}\,\frac{\Delta_\perp}{\Delta_\parallel}\,\frac{m^*}{m_3}\left(\frac{T}{\Delta_\parallel}\right)^3$$



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► Ongoing: NMR, Acoustic and Flow experiments to investigate: (i) topological protection, (ii) the role of broken symmetry and (iii) non-locality of Majorana pairs