# Two-Level Systems and Quasiparticle Dissipation in Superconductors

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## Introduction

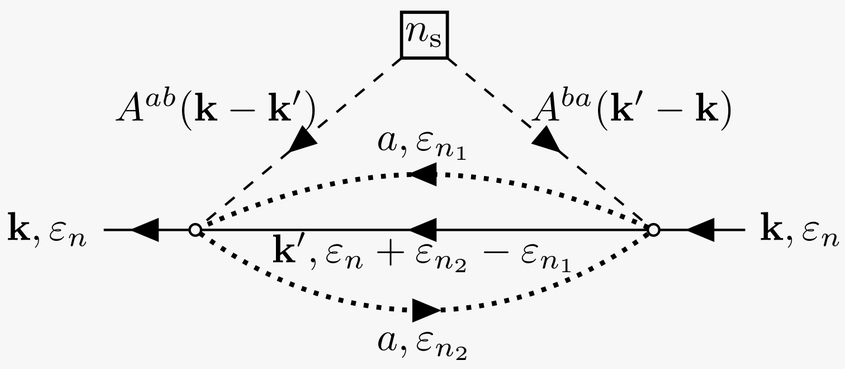
- Two-level system (TLS) defects embedded in superconducting devices limit their performance under nonequilibrium conditions.
- We developed a theory for electron-TLS interaction to study superconducting device performance at low temperatures and GHz frequencies [1].

## Theory of electron-TLS interaction

- The total Hamiltonian is $H = H_{\rm BCS} + H_{\rm TLS} + H_{\rm e-TLS}$, where $H_{\rm BCS}$ is the s-wave BCS mean-field Hamiltonian, $H_{\rm TLS} = \sum_j \frac{E_j}{2}\sigma_z(j)$ is the TLS Hamiltonian with $E_j$ the energy splitting for each TLS, and

$$H_{\rm e-TLS} = \sum_\alpha \int {\rm d}^3 r\, \psi^\dagger_\alpha(\mathbf{r}) \left[ \sum_j v_j(\mathbf{r})\,\mathbb{1} + m_j(\mathbf{r})\,\sigma_z(j) + n_j(\mathbf{r})\,\sigma_x(j) \right] \psi_\alpha(\mathbf{r})\,.$$

- The relevant 2nd-order self-energy diagram is




where the dotted lines are local-fermions, and we get

$$\widehat{\Sigma}(\epsilon_n) = \frac{1}{2\pi\tau_{\rm in}} \int dJ\, p(J) \times N_{ge}(J) \times T \sum_{m\neq 0} \underbrace{\frac{2J}{\omega_m^2 + J^2}}_{\text{bosonic propagator}} \widehat{\mathfrak{G}}(\epsilon_n - \omega_m)\,,$$

where $\tau_{\rm in}^{-1}$ is the inelastic scattering rate, $p(J)$ is the TLS energy distribution, and $N_{ge}(J)$ is the relative occupation of TLSs, which is similar to fermion statistics (see below).

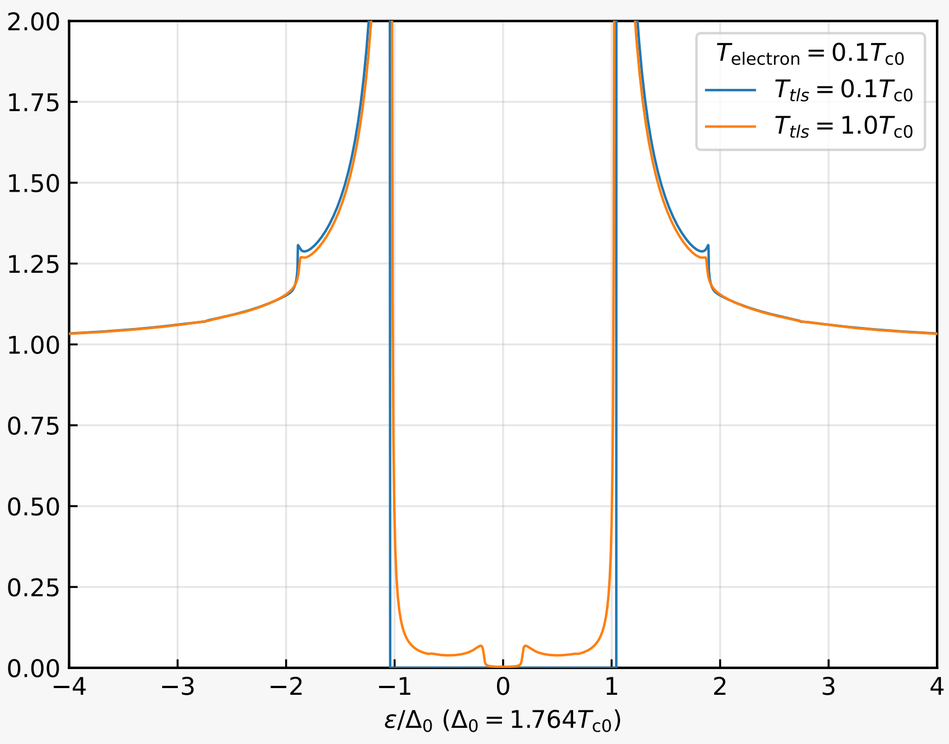
## Density of States (DOS)

- By analytic continuation, we can fully separate the TLS occupation from the bath,

$$\widehat{\Sigma}^{\rm R}(\epsilon) = \frac{1}{2\pi\tau_{\rm in}} \int p(J) dJ \left\{ \widehat{\mathfrak{G}}^{\rm R}(\epsilon - J)\, N_g + \widehat{\mathfrak{G}}^{\rm R}(\epsilon + J)\, N_e - N_{ge} \int_{-\infty}^{\infty} \frac{d\epsilon'}{2\pi i} \frac{2J}{J^2 - (\epsilon + i0 - \epsilon')^2} 2i\,\mathfrak{Im}\widehat{\mathfrak{G}}^{\rm R}(\epsilon') n_{\rm F}(\epsilon') \right\},$$

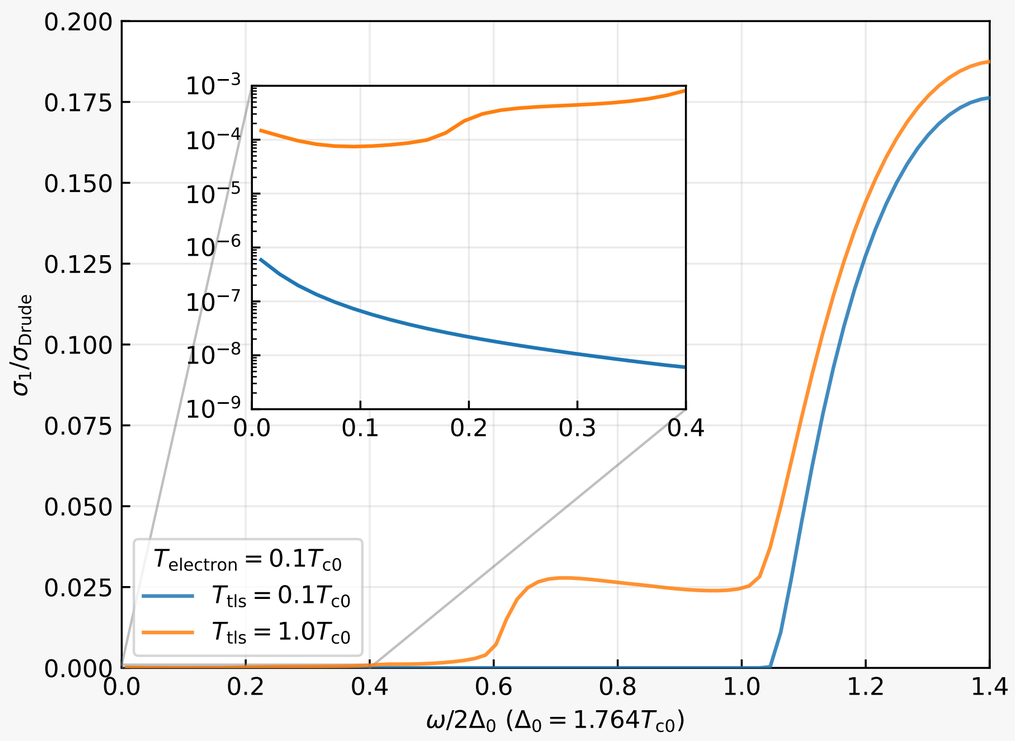
where $N_{ge} \equiv N_g - N_e$ is the difference between the TLS's ground and excited state occupations.

- Note that although a bosonic propagator appears, the TLS occupation factors $N_{g,e}(J)$ are fermionic.
- We can then change the TLS occupation out of equilibrium, as the relaxation of TLSs can be significantly affected by external conditions [2].
- When the TLS temperature $T_{\rm tls} > T$ where $T$ is the electron temperature, the DOS near the Fermi level is substantially enhanced.

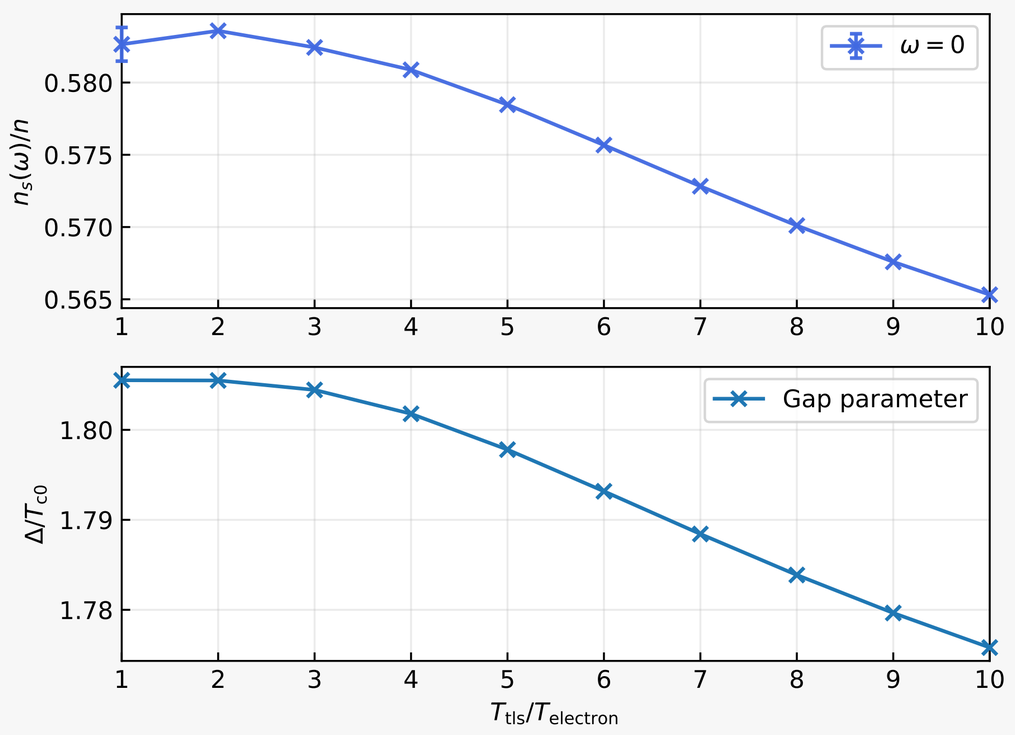



## AC Conductivity and Superfluid Density

- The existence of sub-gap states can increase the dissipation, which is related to the enhanced conductivity [3].
- We calculate the electromagnetic response of a superconductor [4] at $T = 0.1\,T_{\rm c0}$, with TLS energy splitting fixed at $J = 1.5\,T_{\rm c0}$ and the effective temperature of TLSs increased from the electron temperature to 10 times bigger.
- The nonequilibrium TLSs increase the conductivity by orders of magnitude.




- Superfluid density, in contrast, only changes by a few percent. The gap parameter follows the similar trend.




## Summary and Discussion

- We develop a microscopic theory for superconductors with the electron-TLS coupling.
- The e-TLS coupling gives rise to a bosonic mode similar to phonons in conventional superconductivity theory, which can enhances the gap parameter.
- When the TLS defects are excited by a higher effective temperature, the sub-gap spectral weight in the DOS are largely enhanced, leading to the increased conductivity.

## References and Acknowledgments

[1] J. He, W.-T. Lin, and J. A. Sauls, *Prog. Theor. Exp. Phys.* **2025**, 063I01 (2025).

[2] M. Lucas et al., *Nat. Commun.* **14**, 3522 (2023).

[3] M. Zarea and J. A. Sauls, arXiv:2512.20565 (2025).

[4] D. Rainer and J. A. Sauls, in *Superconductivity: From Basic Physics to New Developments* (World Scientific, Singapore, 1995), pp. 45–78. (arXiv:1809.05264)

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