Microwave Response of Superconductors with Paramagnetic Impurities

Mehdi Zarea and James A. Sauls

Hearne Institute of Theoretical Physics, Louisiana State University ,Department of Physics & Astronomy, Baton Rouge, LA 70803, USA

arXiv-2512.20565v2

DE-AC02-07CH11359

HITP
Hearne Institute of Theoretical Physics
Louisiana State University

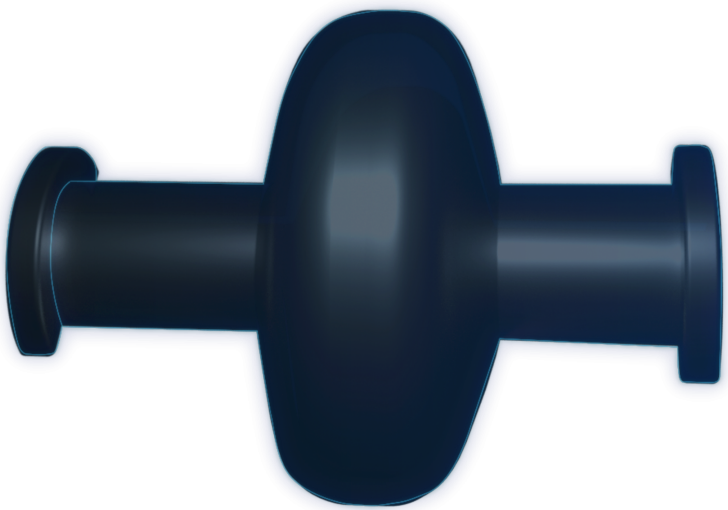
SRF cavities

- High quality factor (Q) of the N-doped Nb SRF cavities: $Q \approx 2 \times 10^{11}$ with $E \approx 50\,\mathrm{MV/m}$.

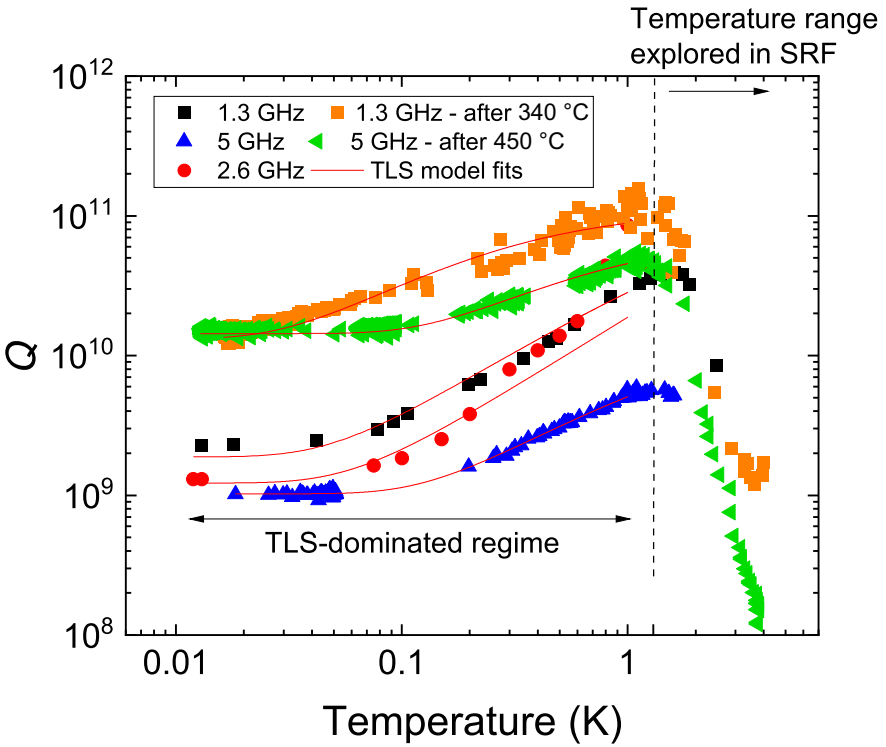
  A Grassellino et al 2017 Supercond. Sci. Technol. 30 094004

  Nb $T_c \approx 9.3K$ $\hbar\omega/\Delta \ll 1$

- *high energy particle accelerators* , *quantum computing*, *quantum sensing*



Sources of dissipation at low temperatures



A.Romanenko *et al*, Phys. Rev .Applied, 13:034032,2020

The sources of dissipation at low temperatures?

- Two-level systems (TLS) in the oxide layer:

  TLS dissipation is present even after removing Nb_2O_5.

- TLS in the superconductor: Theory under development

  A. Abogoda *et al*, Phys. Rev. B 111, L020103 W.T Lin , Poster session Thursday, P68

- Non-equilibrium quasiparticle excitations: K. Serniak *et al*, PRL. 121, 157701

The Message

- ***Low concentration*** (even less than $1ppm$) of magnetic impurities in ***strong coupling*** regime causes significant dissipation in SRF cavities
- **Sub-gap states** near Fermi energy contribute to:
  - ***residual resistance*** at low temperatures
  - ***anomalies in quality factor behavior*** at low temperatures
  - ***anomalies in frequency shift*** near $T = 0$ and T_c
  - ***coherence peak in conductivity*** near T_c

$$\widehat{g}(\hat{p}, \mathrm{r}; \varepsilon_n, t) \equiv \begin{pmatrix} g(\hat{p}, \mathrm{r}; \varepsilon_n)\hat{1} & f(\hat{p}, \mathrm{r}; \varepsilon_n) i\sigma_y \\ \bar{f}(\hat{p}, \mathrm{r}; \varepsilon_n) i\sigma_y & \bar{g}(\hat{p}, \mathrm{r}; \varepsilon_n)\hat{1} \end{pmatrix}$$

where $\hat{p}$ is the direction momentum on the Fermi surface

- Eilenberger equation: $\widehat{g}^2 = -\pi^2 \widehat{1}$

$$\left[i\varepsilon_n \widehat{\tau}_3 - \widehat{\Delta} - \widehat{\Sigma}_{imp}, \widehat{g} \right] + i\hbar \mathrm{v}_f \cdot \boldsymbol{\nabla} \widehat{g} = 0 \qquad \widehat{\Delta} = \Delta \widehat{\tau}_1 i\sigma_y,$$

$$\Delta = 2\pi\lambda \sum_n \int d\hat{p} f(\hat{p}; \varepsilon_n), \qquad \lambda : \text{e-phonon coupling constant}$$

- Find $\widehat{\Sigma}_{imp}(\widehat{g})$ self-consistently using T-matrix

Magnetic impurities in weak coupling regime

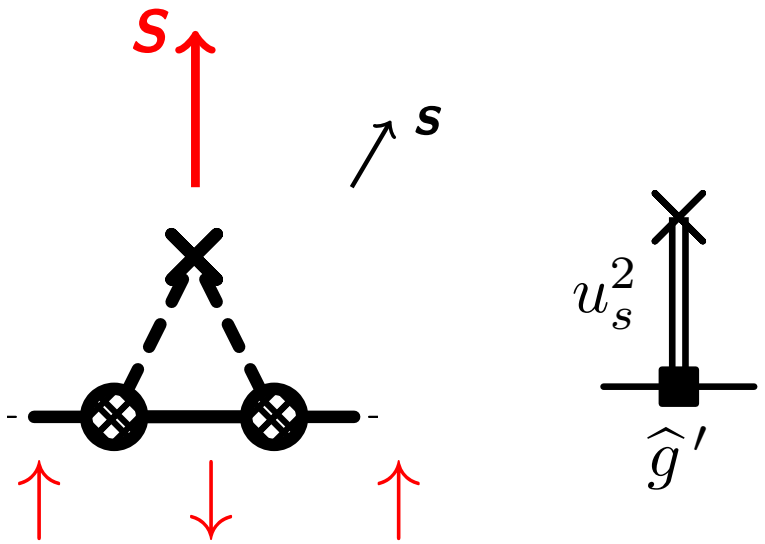
A.A. Abrikosov and L.P. Gorkov, JETP 12:1243, 1961

uncorrelated, classical spin

$H' = J\vec{S}\cdot\vec{s}$,

$\vec{S}$ impurity spin

$\vec{s} = (\hbar/2)\vec{\sigma}$ electron spin

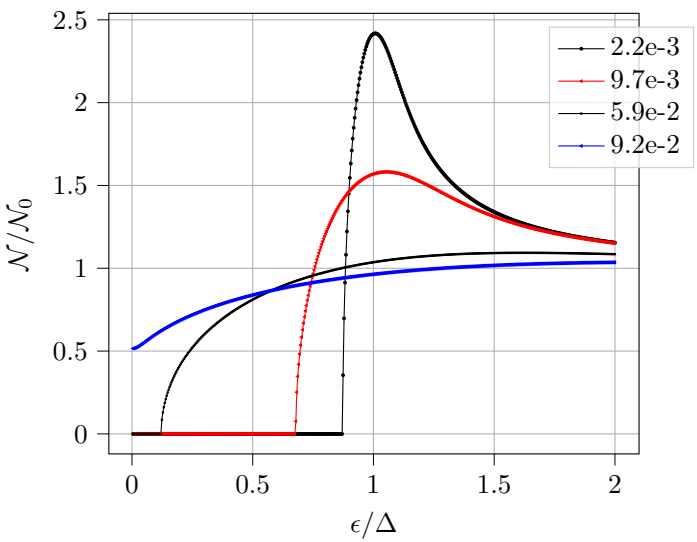


- Δ and T_c are reduced
- impurity states expanded

  from the gap edges

  Gapless superconductor

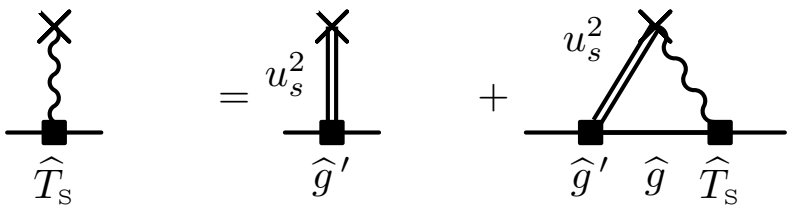
  Pair breaking rate: $1/\tau_s$

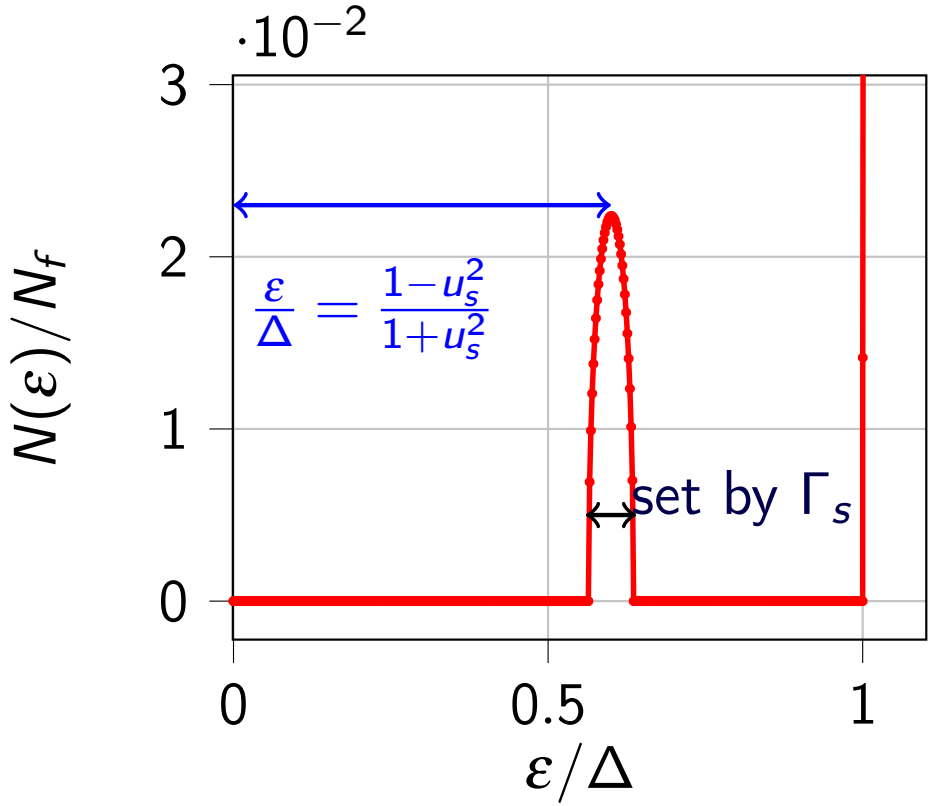


$\alpha \equiv 1/2\pi T_{c_0}\tau_s$ pair-breaking rate/pair-formation rate

Spin impurities, strong coupling limit

L.Yu, Acta Physica Sinica, 21(1):75, 1965
H.Shiba, PTP, 40(3):435–451, 1968
A.I.Rusinov, JETP Lett., 9:146, 1969





- impurity **concentration**

$\Gamma_s = \frac{c_s}{2\pi N_f}$ $\quad \frac{\Gamma_s}{T_{c0}} \approx \frac{N_s}{N_{at}} \frac{T_f}{T_{c0}}$

- **coupling strength**

$\bar{u}_s^2 = (\pi N_f J)^2 S(S+1)$

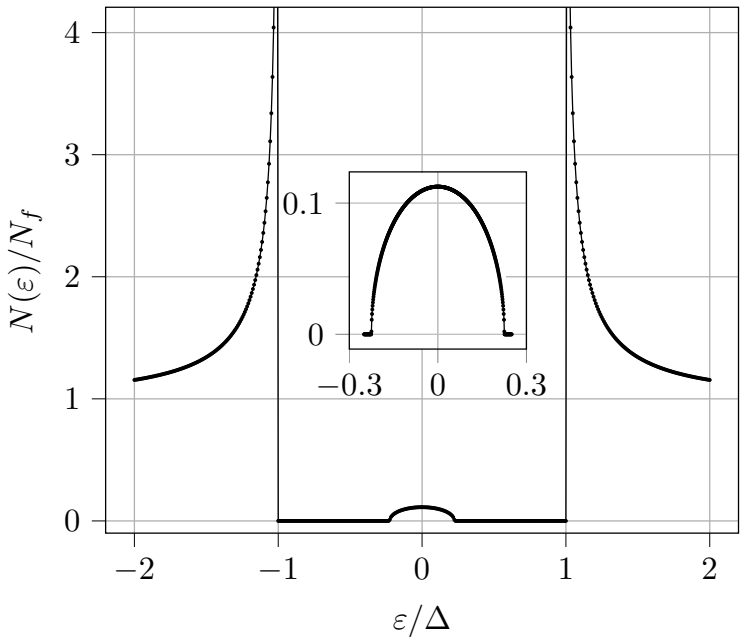
$\frac{1}{2\tau_s} = \Gamma_s \frac{\bar{u}_s^2}{(1+\bar{u}_s^2)^2 - 4f(\varepsilon)^2}$

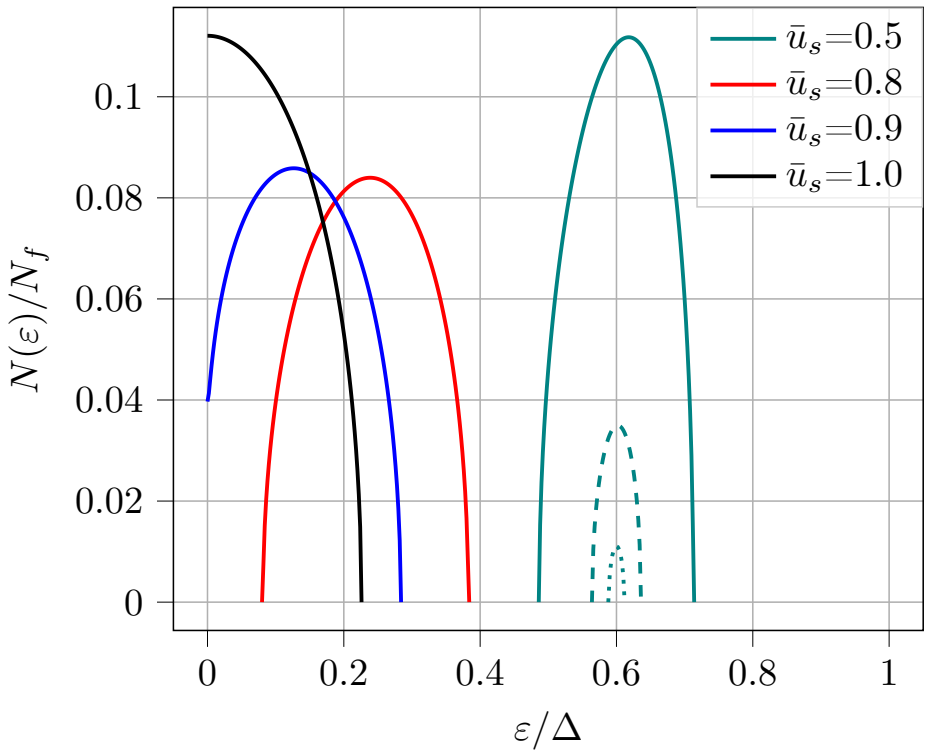
Contribution to Drude σ_n:

$\delta(\frac{1}{\tau_n}) = \Gamma_s \frac{2\bar{u}_s^4}{(1+\bar{u}_s^2)^2 - 4f^2}$

At $\bar{u}_S = 1$ the subgap is at Fermi level

Impurity states inside the gap





$T/T_c = 0.02 \qquad \bar{u}_S = 1$

Solid lines: $\Gamma_S/2\pi T_{c_0} = 3.5 \times 10^{-3} \rightarrow N_{imp}/N_{atom} \approx 10^{-5}$

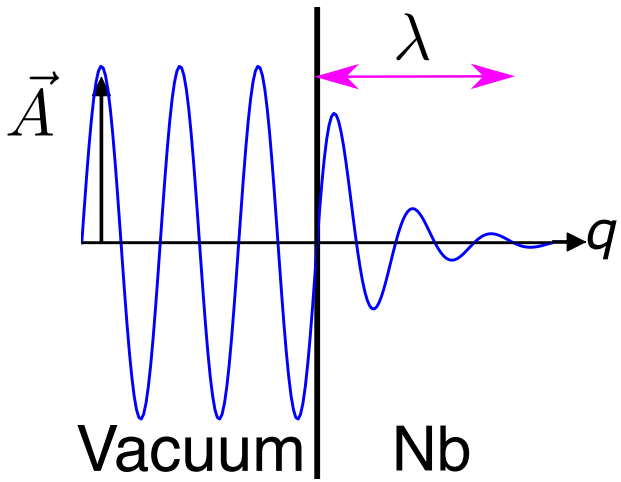
Dashed line: $\Gamma_S/2\pi T_{c_0} = 3.5 \times 10^{-4}$ Dotted line: $\Gamma_S/2\pi T_{c_0} = 3.5 \times 10^{-5}$

The Current Linear Response Function

$$\widehat{\Sigma}_{\mathrm{EM}}(\mathrm{r},t) = -\frac{e}{c}\mathrm{v}_f \cdot \mathrm{A}(\mathrm{r},t)\widehat{\tau}_3$$

$$\mathrm{J}^{\mathrm{R}}(\mathrm{q},\omega) = -\frac{c}{4\pi} K(\omega)\,\mathrm{A}(\mathrm{q},\omega)\,.$$

$$\begin{aligned} K(\omega,T) &= \frac{1}{\lambda^2(\omega,T)} - i\frac{4\pi\omega}{c^2}\sigma_1(\omega,T) \\ &= \frac{4\pi e^2}{mc^2} n_s - i\frac{4\pi\omega}{c^2}\sigma_1(\omega,T) \end{aligned}$$



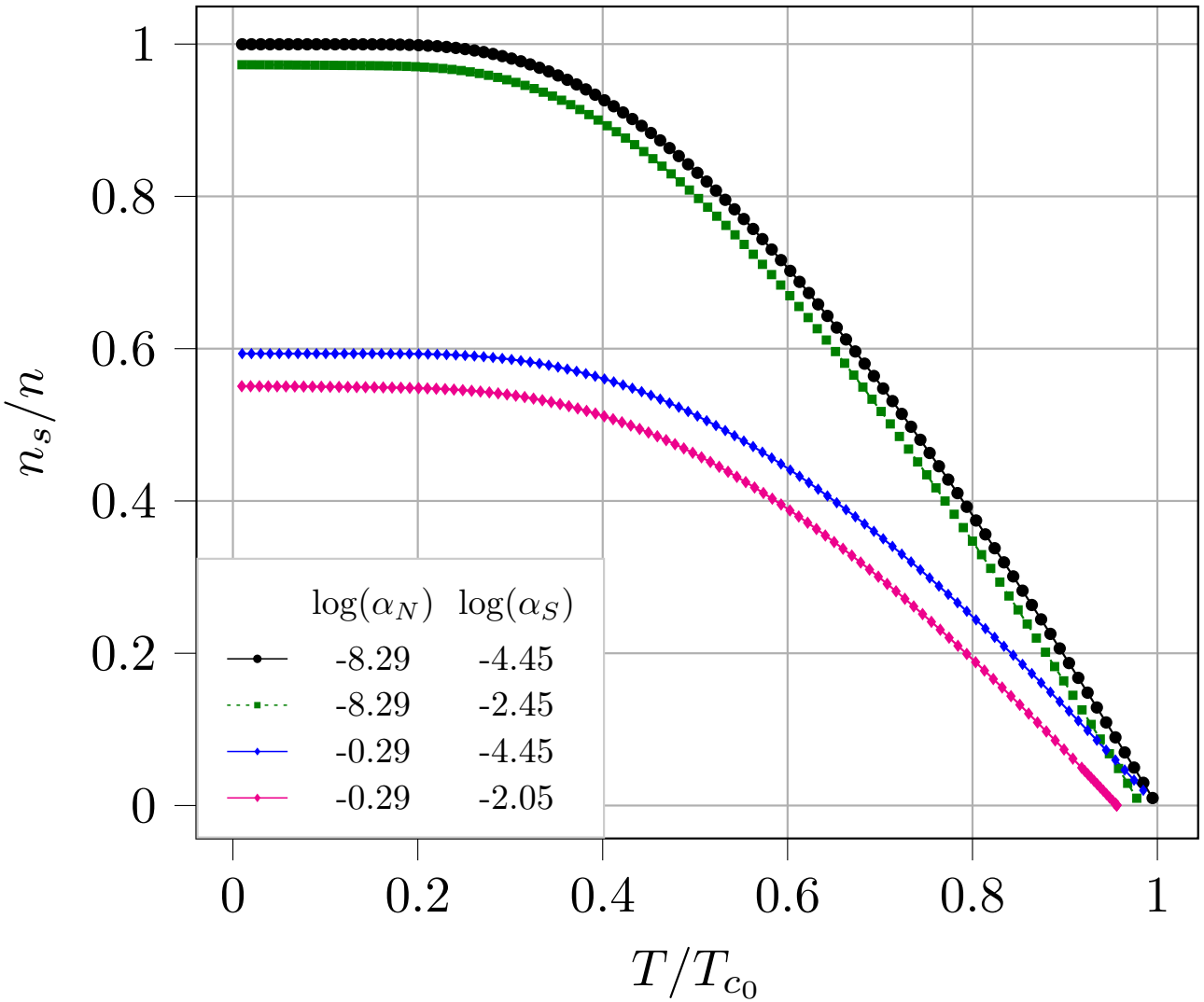
$\lambda(\omega,T)$: penetration depth

n_s superfluid fraction

σ_1: the dissipation of the EM field

$\hbar\omega/\Delta \ll 1$

Superfluid fraction

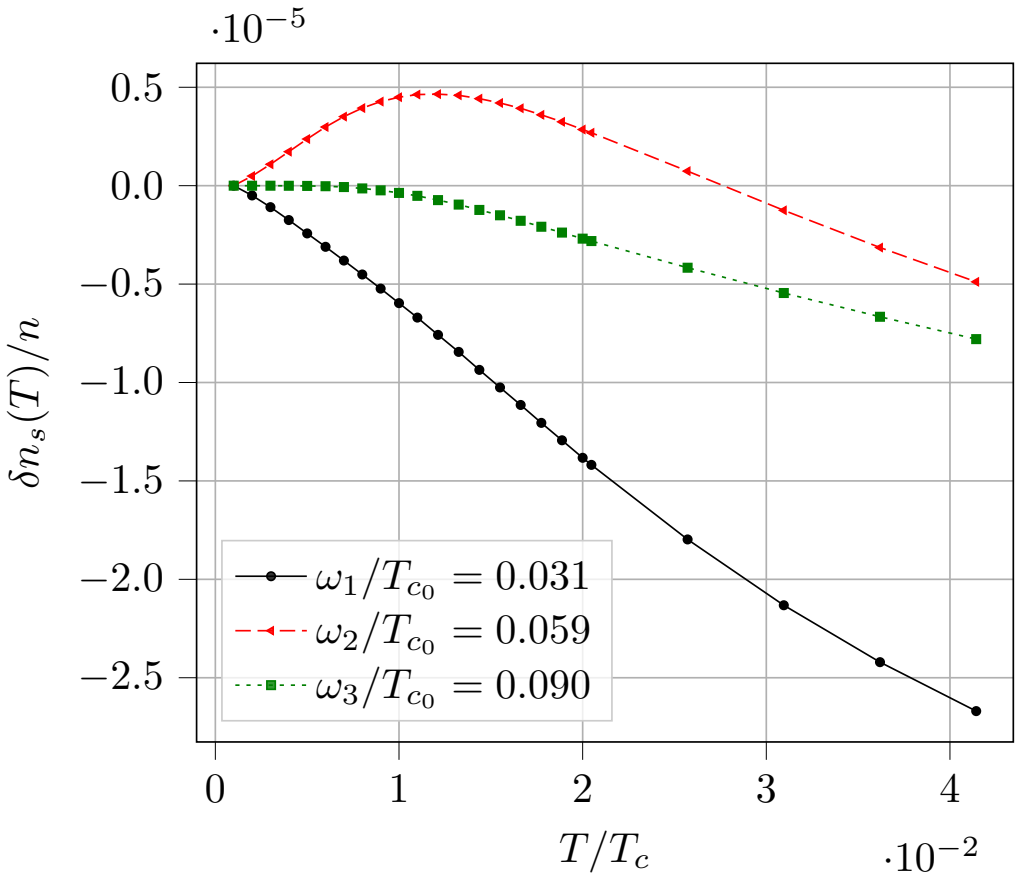


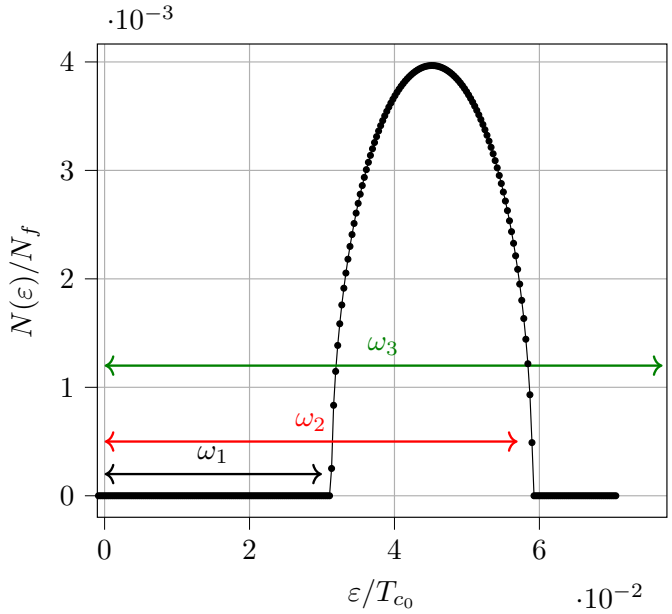
$\omega = 0.005\, T_{c_0}$

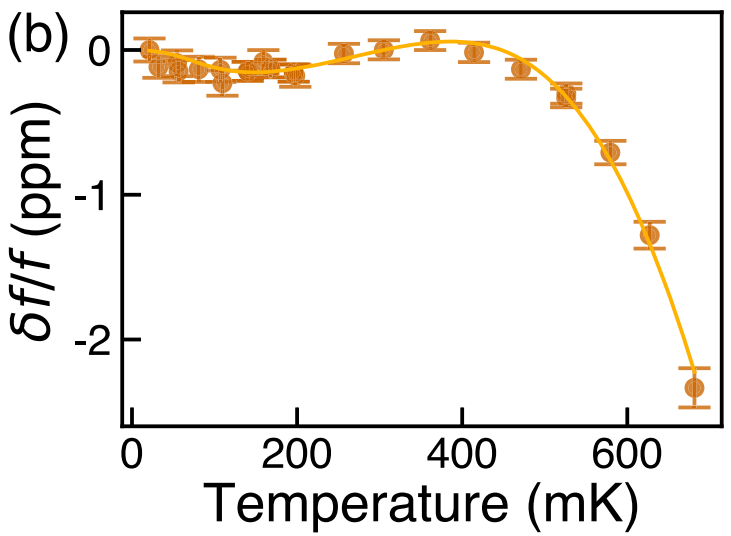
$(f \approx 1 GHz)$

$\alpha_N = \Gamma_N/\pi T_{c_0}$ $\alpha_S = \Gamma_S/2\pi T_{c_0}$ $\bar{u}_S = 1.$

Superfluid fraction near zero temperature





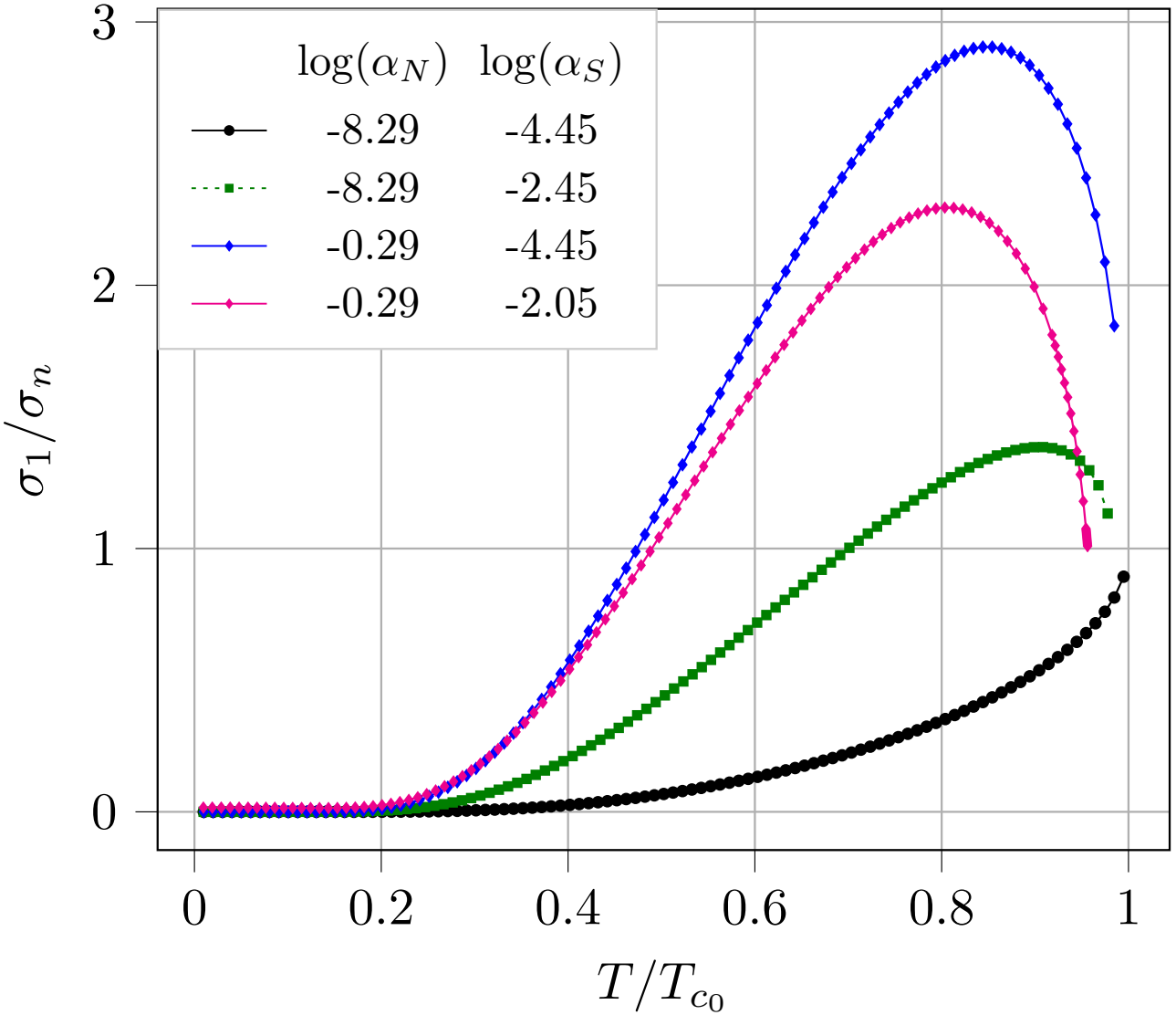


$\bar{u}_s = 0.975$ and $\Gamma_s/2\pi T_{c_0} = 8.9 \times 10^{-6}$

$$\frac{f - f_0}{f_0} = -\frac{1}{R}\mathrm{Re}\left(\frac{1}{\sqrt{K}}\right) \approx \frac{\lambda}{R}$$

[K.D Crowley *et al*,Phys.Rev.X, 13:041005,

Dissipative conductivity - near T_c



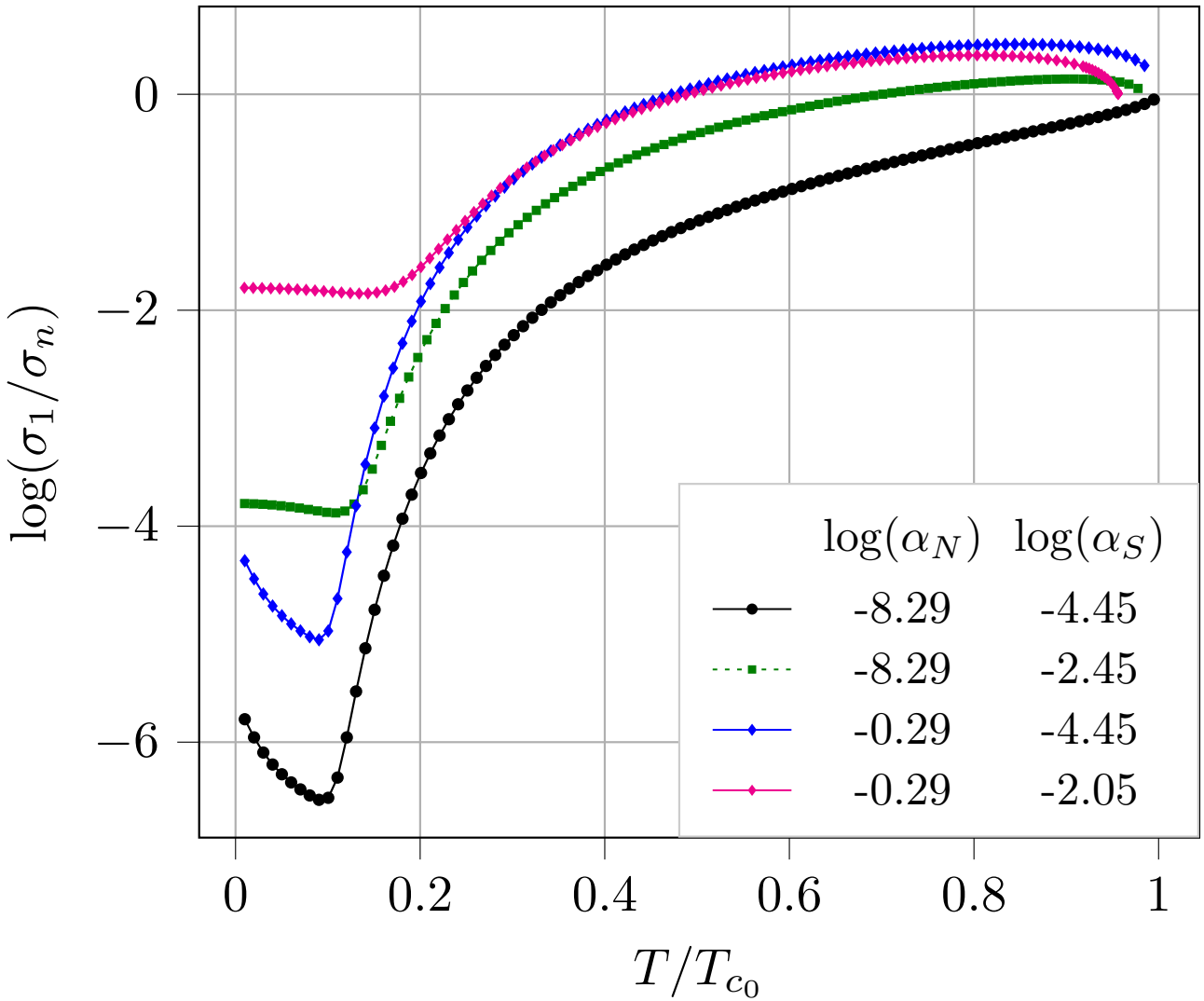
$\bar{u}_s = 1$

$\hbar\omega/k_B T_{c_0} = 0.005$

$(f \approx 1GHz)$.

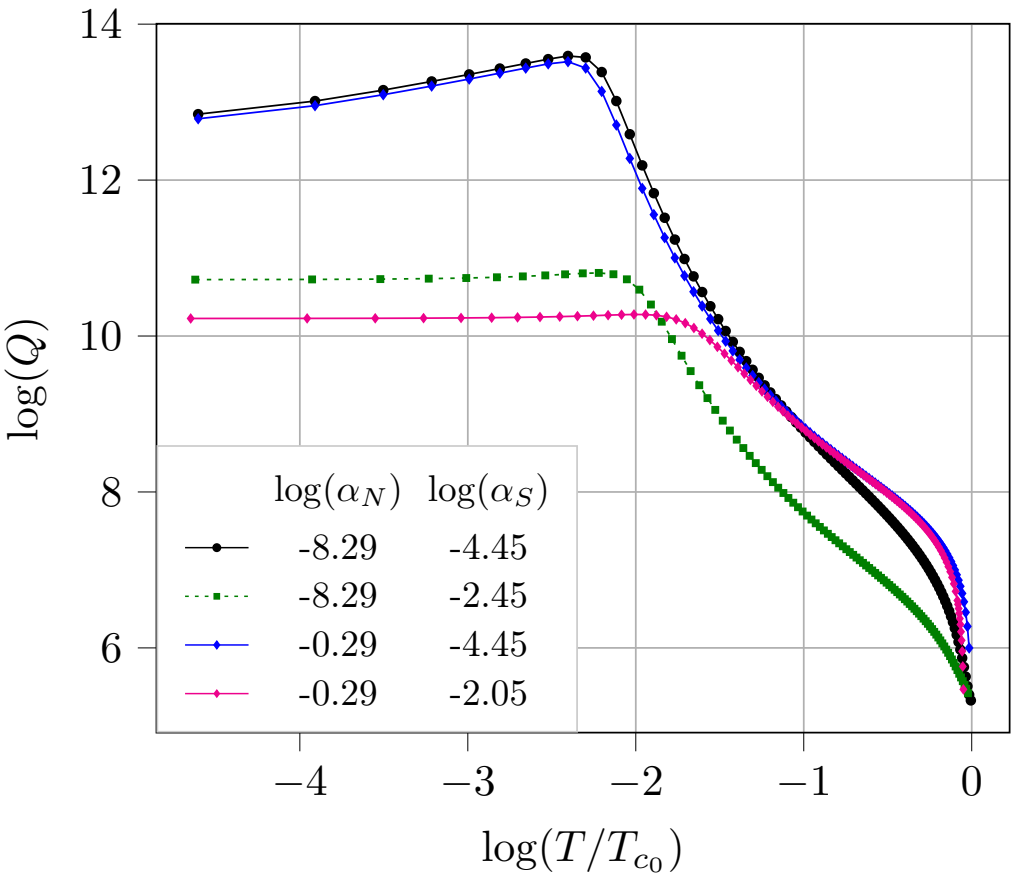
$\sigma_n \equiv \sigma_1(T = T_c)$

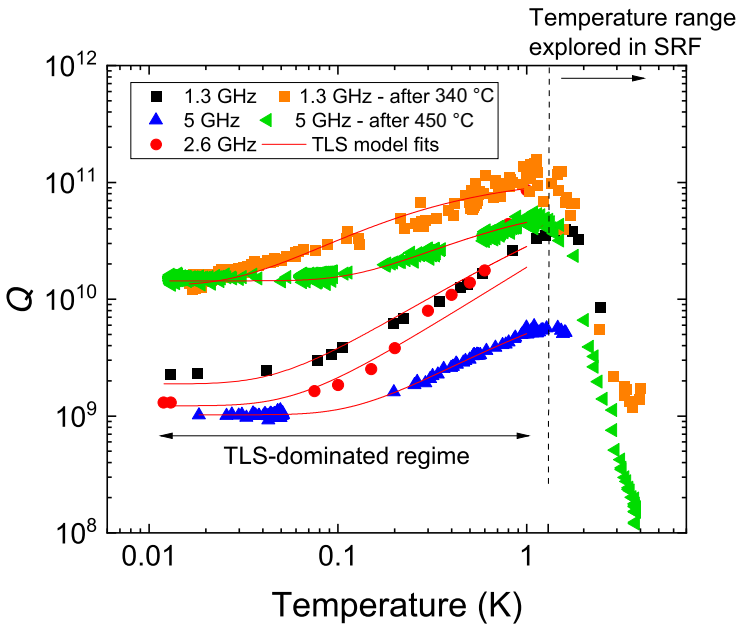
Conductivity at low temperatures



The step represents dissipation from subgap states near the Fermi level.

Quality factor



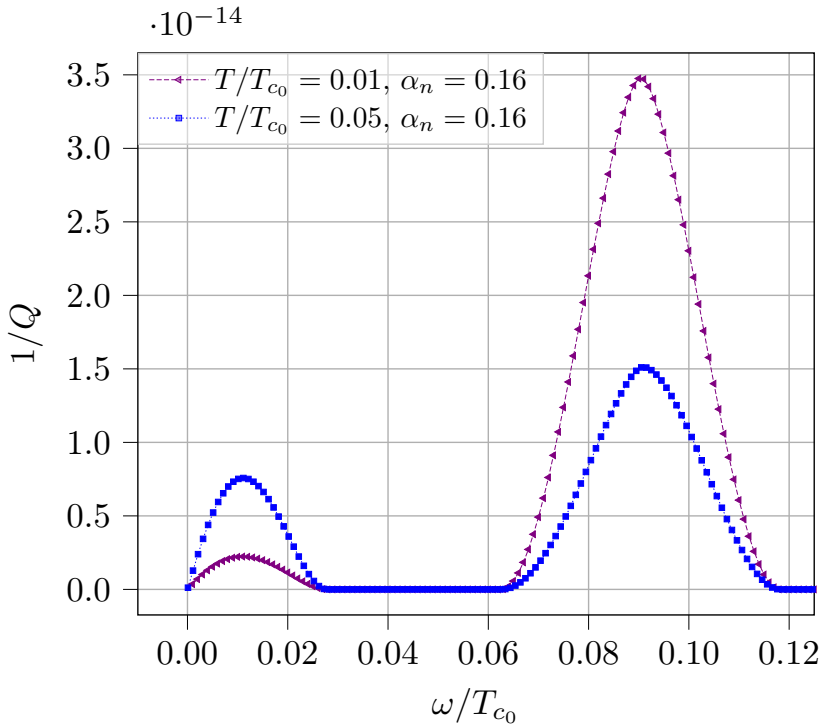


[A. Romanenko *et al*, Phys.Rev.Applied, 13:034032]

[J. He *et al*, Prog.Theor.Exp.Phys, 2025:063I01]

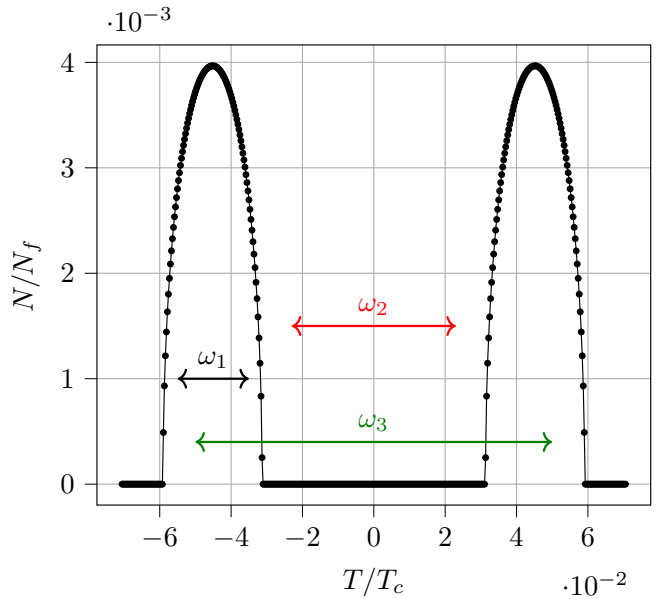
- dissipation from impurity sub-gap quasi-particles not TLS,
- provides physical mechanism for the residual resistivity,

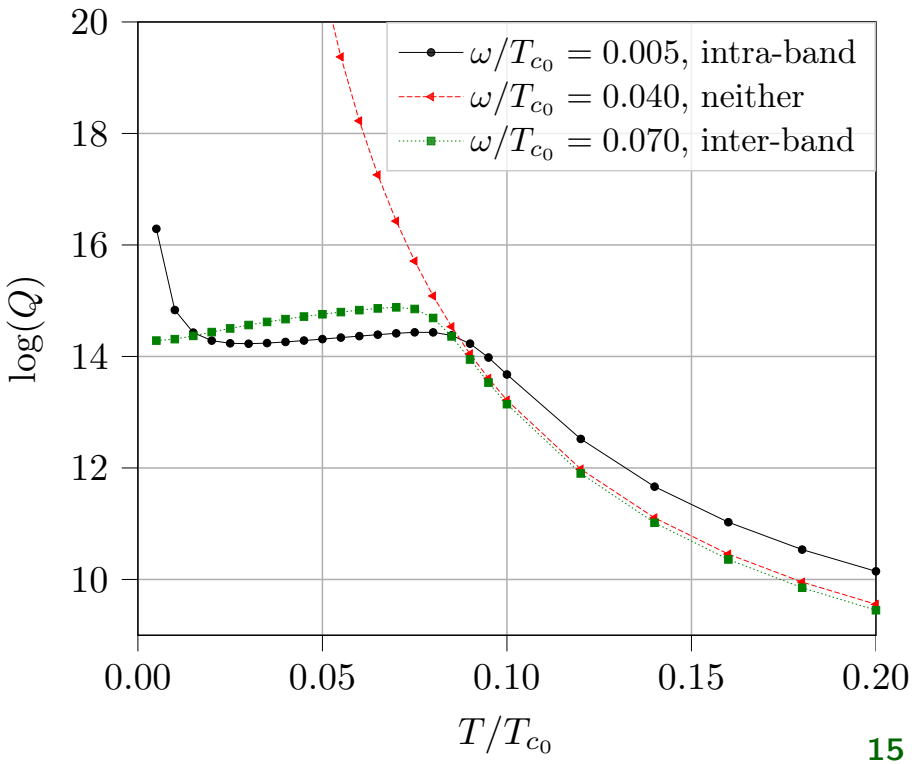
The residual resistivity



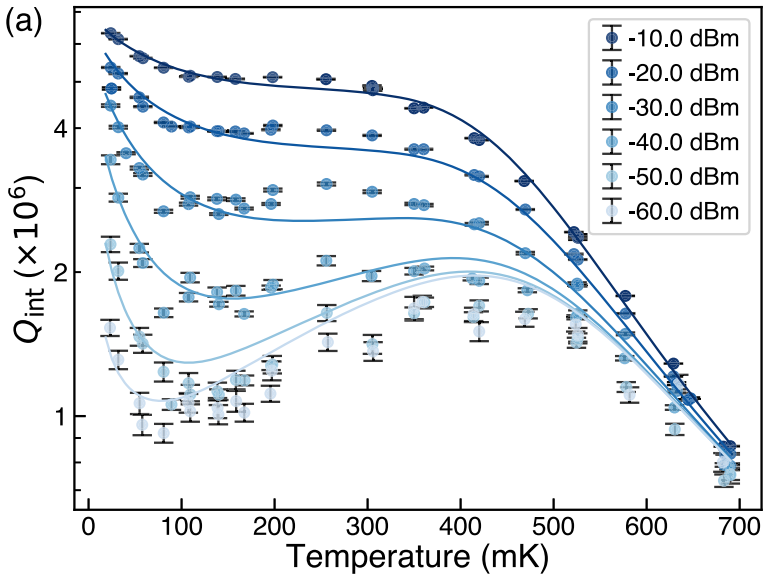
$$\Gamma_S/2\pi T_{c_0} = 8.9 \times 10^{-6}$$

$$\bar{u}_S = 0.975$$

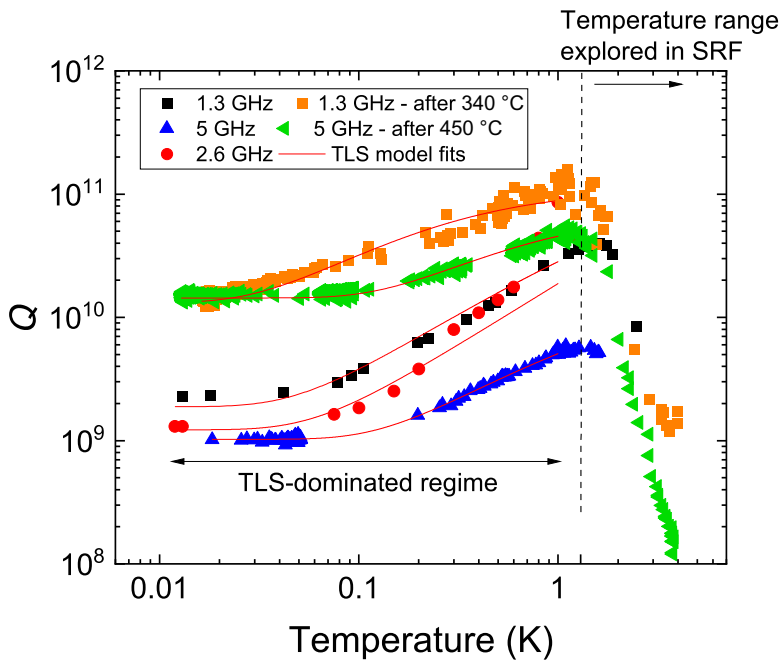


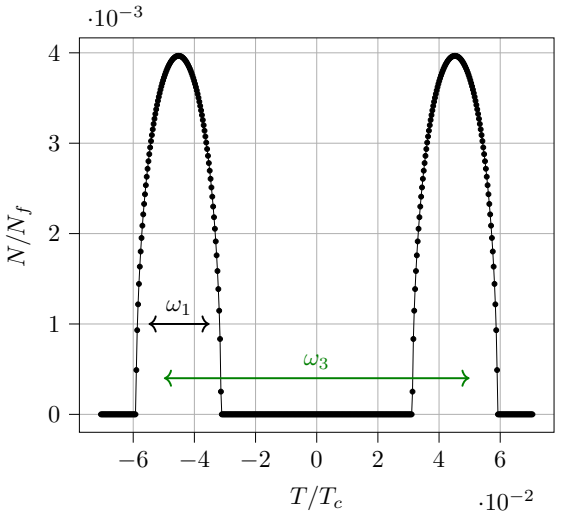


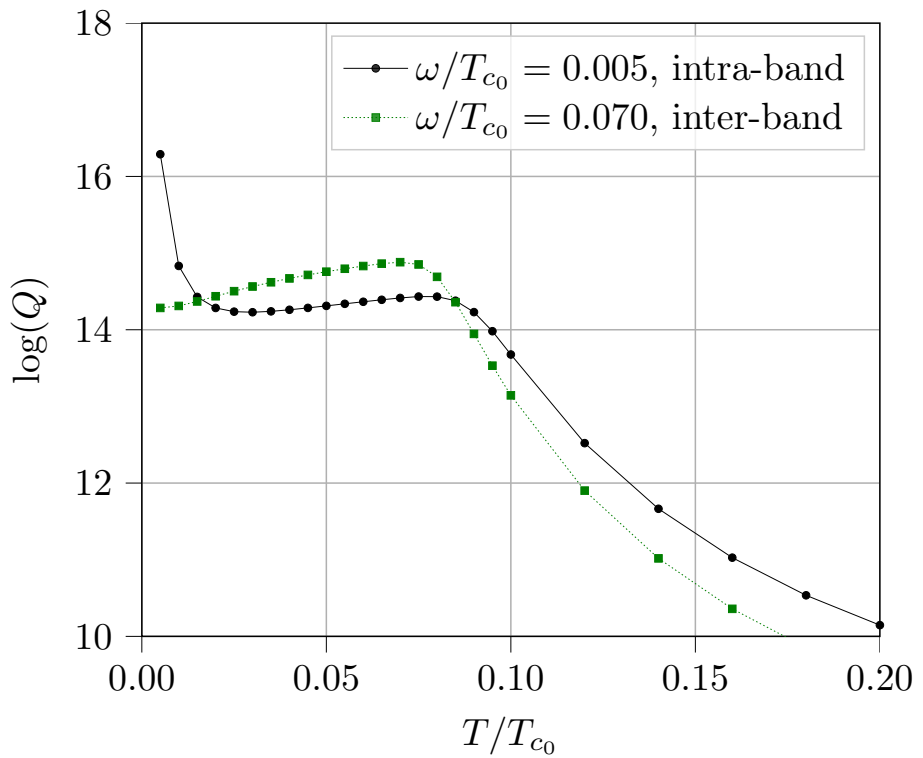
Quality factor and the sub-gap position



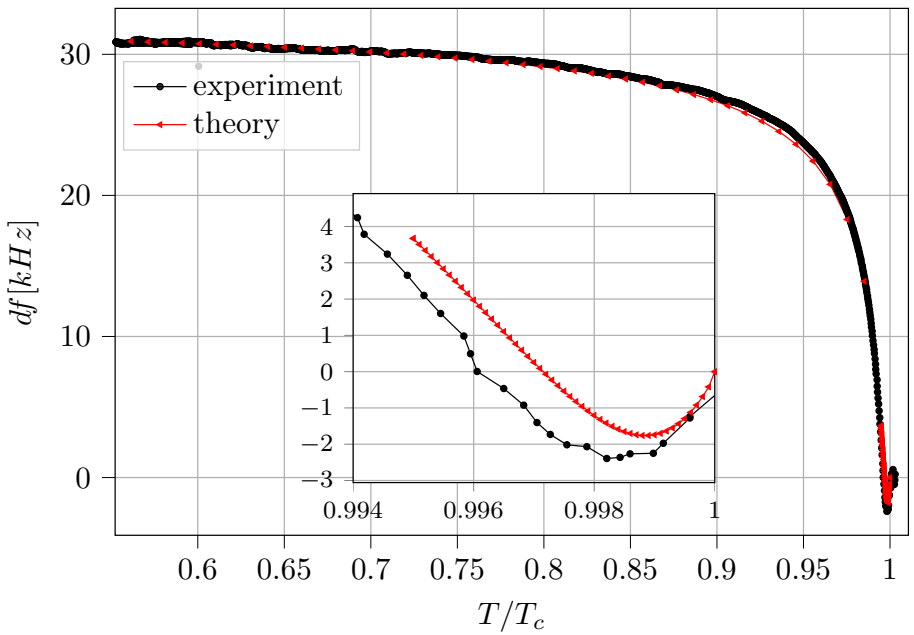
[K.D Crowley *et al*,Phys.Rev.X, 13:041005, 2023]

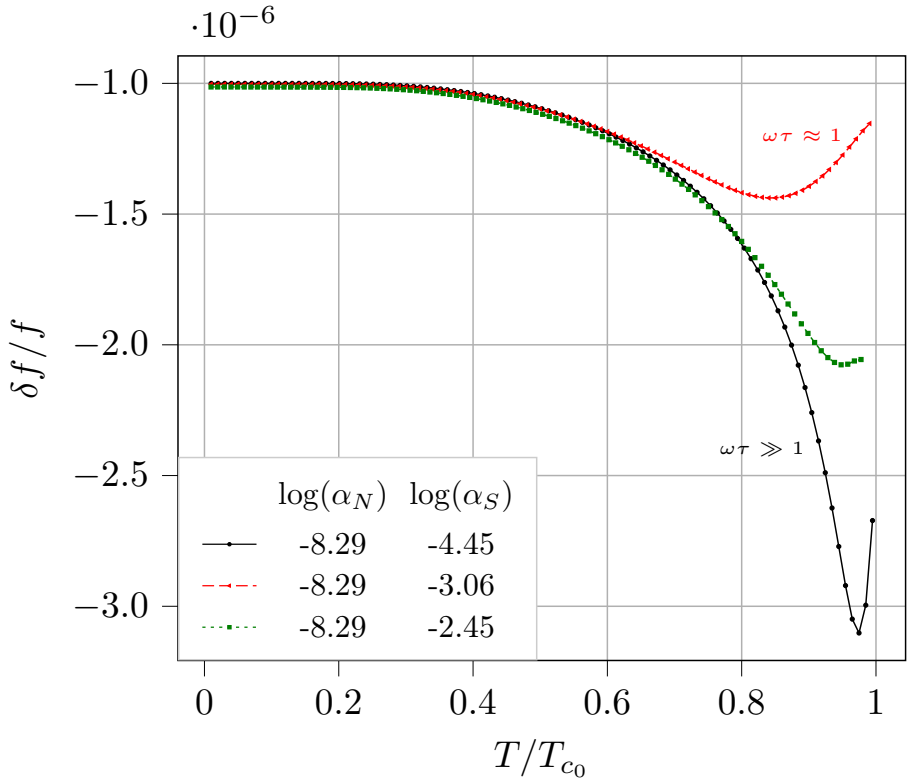






The anomaly in the frequency shift





[Bafia *et al*, Phys. Rev. Appl., 23:054052]

[Zarea *et al* Front. Electron. Mater. 3:1259401]

[H. Ueki *et al* Prog. Theor. Exp. Phys., 2025:053102]

The width and the depth depend on:

- $\omega\tau$ $(1/\tau$ diagonal scattering rate)
- fraction of τ_S/τ_N

Conclusion

The sub-gap spectrum of quasiparticles, generated by dilute concentrations of paramagnetic impurities,

- limits $Q(T)$ of SRF cavities operating at microwave frequencies at low temperatures.
- provides a theory for the zero-temperature residual resistance
- creates anomalies in the frequency shift which are signature of paramagnetic impurities

THANK YOU!

arXiv-2512.20565v2