Searching for Axions and Nonlinear QED with Superconducting RF Cavities

Hikaru Ueki*, Mehdi Zarea†, and J. A. Sauls‡

Hearne Institute of Theoretical Physics, Department of Physics and Astronomy,
Louisiana State University, Baton Rouge, LA 70803, USA

Single SRF Cavity Detection

- Niobium superconducting radio-frequency (SRF) cavities have quality factors $Q \sim 10^{10} - 10^{12}$ for resonant frequencies, $f \sim 50$ MHz $-$ 4 GHz, and can support field energies corresponding to $N \sim 10^{25}$ photons [1,2].
- Nb SRF cavities originally designed for high-energy particle accelerators [3], are being developed for quantum computing [4] and sensing to capture physics beyond the Standard Model [5-7].
- Bogorad *et al.* proposed SRF cavities as a platform for detecting axions and photon interactions in QED - the Euler-Heisenberg (EH) interaction [5].
- The axion field is generated via $\mathbf{E}\cdot\mathbf{B}$ by pumping two resonant modes (ω_1 and ω_2) of the cavity. The resulting axion source current is cubic in the EM field, as is the EH source current,

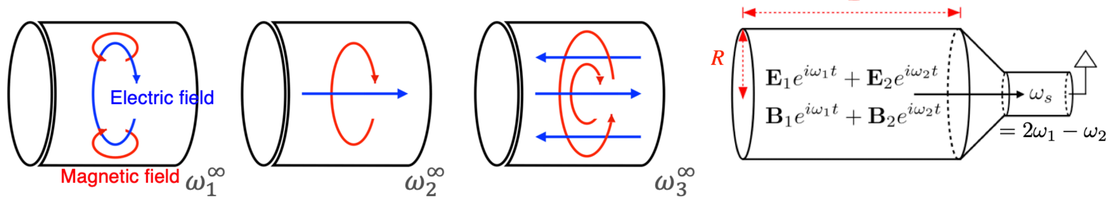
$$\mathbf{j}_a = \frac{c}{4\pi} g_{a\gamma\gamma}\left(\mathbf{E}\times\boldsymbol{\nabla}a - \mathbf{B}\frac{1}{c}\frac{\partial a}{\partial t}\right), \quad \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla}^2 + \frac{m_a^2c^2}{\hbar^2}\right)a = g_{a\gamma\gamma}\mathbf{E}\cdot\mathbf{B},$$

$$\mathbf{j}_{\rm EH} = -\frac{c}{4\pi}\xi\boldsymbol{\nabla}\times\left[2(E^2-B^2)\mathbf{B} - 7(\mathbf{E}\cdot\mathbf{B})\mathbf{E}\right] + \frac{c}{4\pi}\xi\frac{1}{c}\frac{\partial}{\partial t}\left[2(E^2-B^2)\mathbf{E} + 7(\mathbf{E}\cdot\mathbf{B})\mathbf{B}\right],$$

  $g_{a\gamma\gamma}$ is the axion-photon coupling, and $\xi = \hbar e^4/45\pi m_e^4 c^7$ is the EH coupling.
- The axion field $a \propto \mathbf{E}\cdot\mathbf{B}$ can source photons at the intermodulation at frequency $\omega_s = 2\omega_1 - \omega_2$ in a cavity simultaneously pumped with photons in modes with resonant frequencies ω_1 and ω_2. Observation of these photons at ω_s is the signature of the axion field or the EH photon-photon interaction.
- For a cylindrical cavity the relevant modes and frequencies for a "perfect conductor" are: $\omega_1^\infty = \omega_{011}^{\rm TE}$, $\omega_2^\infty = \omega_{010}^{\rm TM}$, and $\omega_3^\infty = 2\omega_1^\infty - \omega_2^\infty = \omega_{020}^{\rm TM}$ [5].

Pump Signal

- $\rm TE_{011}$ mode
- $\rm TM_{010}$ mode
- $\rm TM_{020}$ mode



Signal contamination: Nonlinear Meissner currents & leakage noise

- Superconductors are not *perfect* conductors: The field penetrates into the superconductor, generates currents that confine the cavity field. The Meissner current is a nonlinear function of the field at the vacuum-superconducting interface (NLM effect), sources photons at the signal frequency, and thus generates false positive signals for axion or EH detection [8].

$$\mathbf{j}_s = -\frac{c}{4\pi\lambda_{\rm L}}\left(1 - \frac{\theta}{A_c^2}|\mathbf{A}|^2\right)\mathbf{A}\,,\ A_c \equiv H_c\lambda_{\rm L}\,,$$

H_c : Critical field , $\lambda_{\rm L}$: London depth , θ : pair-breaking strength .

- Expand the EM field in the basis of modes of an ideal cavity [9]:

$$\mathbf{E}_i(\mathbf{r},t) = \mathbf{e}_i(\mathbf{r})\int \mathbf{E}(\mathbf{r}',t)\cdot\mathbf{e}_i(\mathbf{r}')dv',\ \text{basis functions}: \{\mathbf{e}_i(\mathbf{r}),\mathbf{b}_i(\mathbf{r})\}$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\omega_i^{\infty 2}}{c^2}\right)\int \mathbf{E}(\mathbf{r},t)\cdot\mathbf{e}_i(\mathbf{r})dv$$

$$= -\frac{4\pi}{c^2}\frac{d}{dt}\int \mathbf{j}(\mathbf{r},t)\cdot\mathbf{e}_i(\mathbf{r})dv - \frac{\omega_3^\infty}{c}\int_S [\mathbf{E}(\mathbf{r},t)\times\mathbf{b}_i(\mathbf{r})]\cdot d\mathbf{a}. \qquad (1)$$

  The first term in Eq. (1) is the vacuum term with contributions from the axion/EH currents [5]. The second term is the surface term with contributions from the NLM currents and leakage current.
- For pumping period t_p the NLM field $\mathbf{E}_i(\mathbf{r},t) = {\rm Re}\mathbf{E}_i(\mathbf{r})e^{-i(\omega_3^\infty+\delta\omega_i)t - \frac{\omega_3^\infty}{2Q_s}t}$ at the axion/EH signal frequency ω_s has the form:

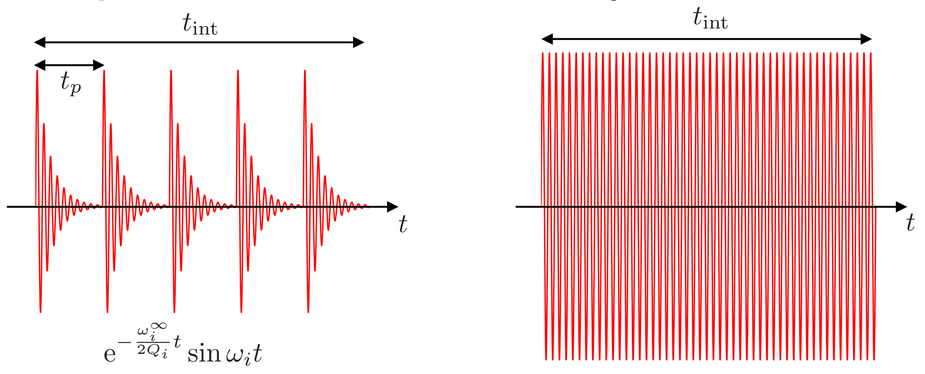
$$\mathbf{E}_{\rm NLM}(\mathbf{r}) = iC_{\rm NLM}\frac{cQ_s}{\omega_3^\infty}\mathbf{e}_3(\mathbf{r})\int_S[\mathbf{E}_3(\mathbf{r}')\times\mathbf{b}_3(\mathbf{r}')]\cdot d\mathbf{a}',$$

$$C_{\rm NLM} = \frac{-i\omega_3^\infty/2Q_s}{\omega_{\rm NLM} - \omega_s - i\omega_3^\infty/2Q_s},\quad \omega_{\rm NLM} = \omega_3^\infty + \delta\omega_{\rm NLM},$$

$$\delta\omega_{\rm NLM} = -\frac{\omega_3^\infty\lambda_{\rm L}}{2}\int_S|\mathbf{b}_3(\mathbf{r})|^2 da\left[1 + \frac{\theta}{16H_c^2}\frac{1-e^{-\frac{3\omega_3^\infty}{2Q_3}t_p}}{1-e^{-\frac{\omega_3^\infty}{2Q_3}t_p}}|\mathbf{B}_{\rm NLM}(R)|^2\right].$$

- Schematic depiction of the pumping periodic time t_p and experimental integration (total measurement) time $t_{\rm int}$:

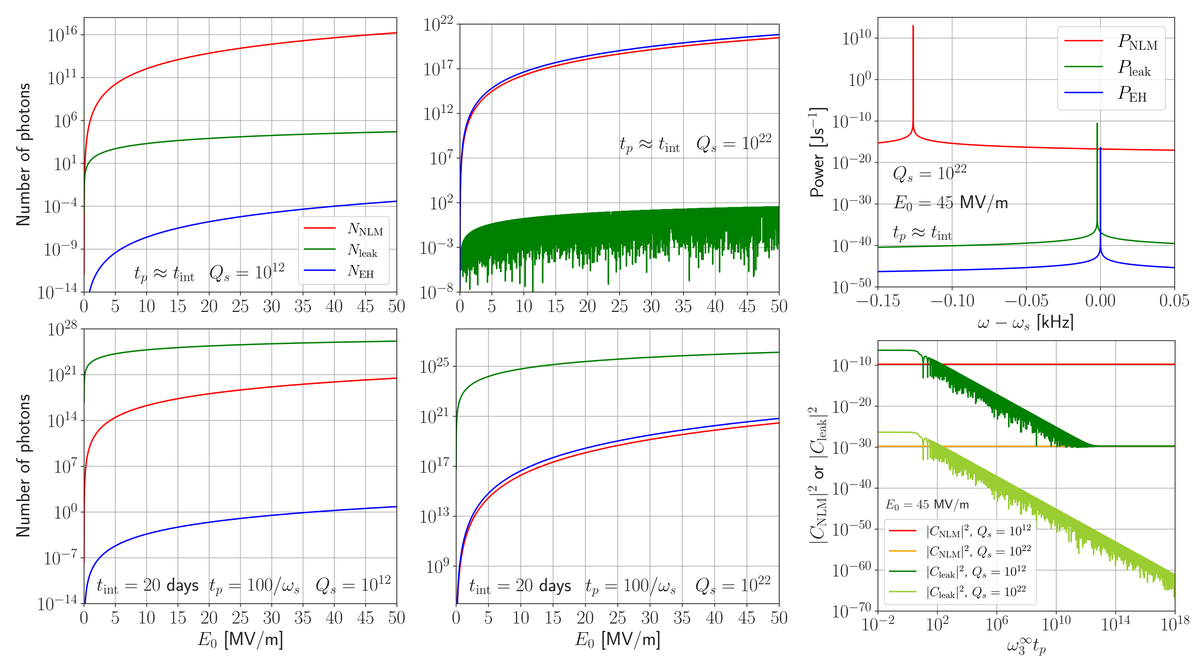
(a) $\omega_i^\infty t_p \gg Q_i$ (b) $\omega_i^\infty t_p \ll Q_i$ Damping rate $\sim 1/t_{\rm int}$



- We also calculate the leakage of photons in the pump mode resonance into the signal mode (equations not shown) [7] using the same formalism.

Results: Number of Signal Photons versus False Positives

- $N_i = \frac{1}{\hbar\omega_s}\int\frac{dv}{8\pi}|\mathbf{E}_i(\mathbf{r})|^2\frac{1}{t_p}\int_0^{t_p}dt e^{-\frac{\omega_i^\infty}{Q_i}t}$, Radius: $R = 0.5$ m, Disorder: $\theta = 0.001$

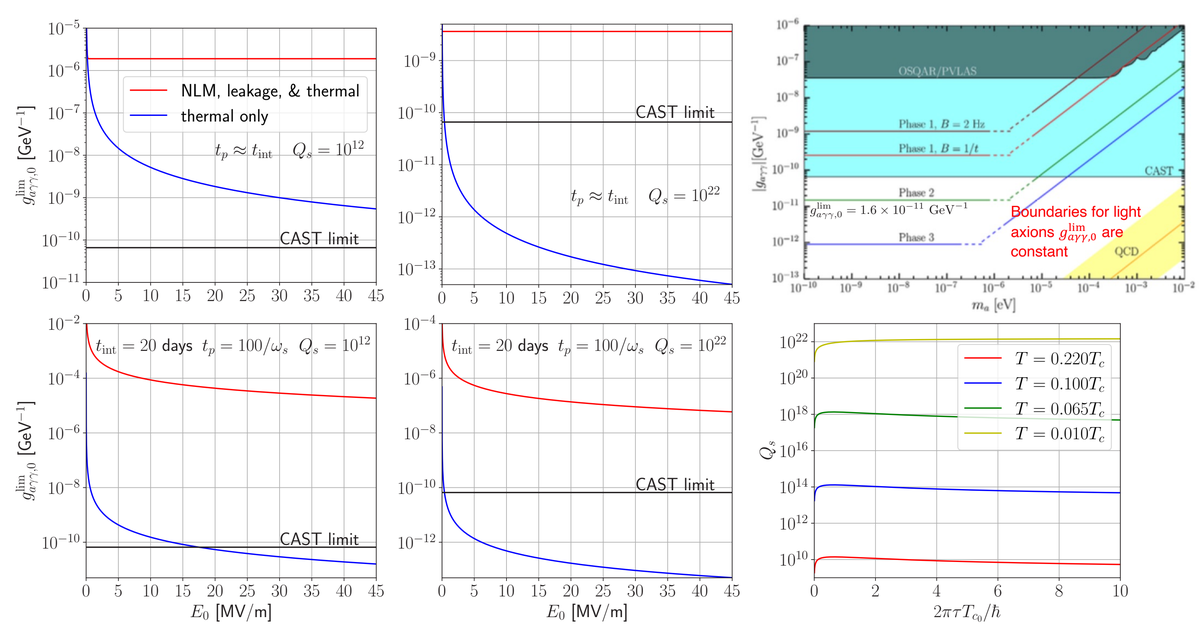


- Lower limit of sensitivity to light axions at SNR $= 1$

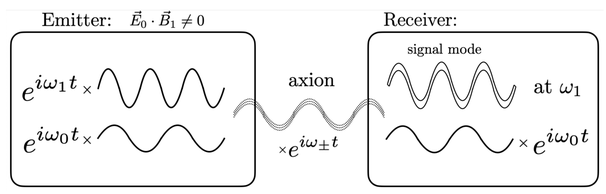
$$g_{a\gamma\gamma,0}^{\rm lim} = 1.6\times10^{-11}\ {\rm GeV}^{-1}\left(\frac{10^{12}}{Q_s}\right)^{1/4}\left(\frac{45\ {\rm MV/m}}{E_0}\right)^{3/2}(t_{\rm int}\times B)^{1/8}$$

$$\times\left(\frac{N_{\rm th}+N_{\rm NLM}+N_{\rm leak}}{N_{\rm th}}\right)^{1/4}\left(\frac{\omega_3^\infty t_p/Q_s}{1-e^{-\frac{\omega_3^\infty}{Q_s}t_p}}\right)^{1/4}\left(\frac{1-e^{-\frac{\omega_3^\infty}{2Q_s}t_p}}{\omega_3^\infty t_p/2Q_s}\frac{\omega_1^\infty t_p/Q_1 + \omega_2^\infty t_p/2Q_2}{1-e^{-\frac{\omega_1^\infty}{Q_1}t_p - \frac{\omega_2^\infty}{2Q_2}t_p}}\right)^{1/2},$$

Cavity bandwidth: $B = \begin{cases}\omega_3^\infty/Q_s, & \omega_3^\infty t_p \gg Q_s\,,\\ 1/t_{\rm int}, & \omega_3^\infty t_p \ll Q_s\,.\end{cases}$



Two SRF Cavity Detection



- Emitter: Pump two modes with $\mathbf{E}\cdot\mathbf{B} \neq 0$ at resonant frequencies ω_0 and ω_1. Axions are created at $\omega_\pm = |\omega_0 \pm \omega_1|$ and propagate to the receiver.
- Receiver: Pump the mode at ω_0. Signal: Axions covert to photons at ω_1.
- NLM photons only at $3\omega_0$ are non-resonant with signal photons.
- Leakage is suppressed: $\mathbf{E}_{\rm leak}(\mathbf{r}) \propto \mathbf{e}_1(\mathbf{r})\int_S[\mathbf{E}_0(\mathbf{r}')\times\mathbf{b}_1(\mathbf{r}')]\cdot d\mathbf{a}'$ vanishes for these modes: $\omega_0 = \omega_{011}^{\rm TE}$ and $\omega_1 = \omega_{010}^{\rm TM}$ or $\omega_0 = \omega_{010}^{\rm TM}$ and $\omega_1 = \omega_{011}^{\rm TE}$.

Conclusions

- Cavities with a ultra-high $Q \sim 10^{22}$, with weak but finite disorder, are required for detection of the EH effect. Such a high Q Niobium cavity may be achievable with removal of all surface Nb-oxides, $\theta \simeq 10^{-3}$ and $T \simeq 0.1$ K.
- The two-cavity emitter/receiver setup can avoid the NLM background and leakage noise, but requires two identical SRF cavities for mode matching.

Acknowledgements & References

We thank Y. Kahn and R. Harnik for discussing the detection of axions and EH interaction using SRF cavities. This research was supported by the U.S. Department of Energy, Office of Science, National Quantum Information Science Research Centers, Superconducting Quantum Materials and Systems Center (SQMS) under contract number DE-AC02-07CH11359, and the Center for Computation and Technology at Louisiana State University.

[1] A. Grassellino *et al.*, Sci. Technol. **26**, 102001 (2013). [2] A. Romanenko *et al.*, Appl. Phys. Lett. **105** , 234103 (2014). [3] H. Padamsee *et al.*, *RF Superconductivity for Accelerators*, 2nd ed. (Wiley-VCH Verlag GmbH and Co., KGaA, Weinheim, 2008). [4] A. Romanenko *et al.*, Phys. Rev. Applied **13**, 034032 (2020). [5] Z. Bogorad *et al.*, Phys. Rev. Lett. **123**, 021801 (2019). [6] A. Berlin *et al.*, J. High Energ. Phys. **2020**, 88 (2020). [7] C. Gao and R. Harnik, J. High Energ. Phys. **2021**, 53 (2021). [8] J. A. Sauls, Prog. Theor. Exp. Phys. **2022**, 033I03 (2022). [9] J. C. Slater, Rev. Mod. Phys. **18**, 441 (1946).

*uekih@lsu.edu, †zarea.mehdi@gmail.com, ‡sauls@lsu.edu.