Electron teleportation in p-wave superconducting wires

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Introduction

- Kitaev model of $p-$wave superconducting wires [1] supports zero energy Majorana edge states.
- Semenoff *et al*[2, 3] proposed that Majorana states can be used for performing *electron teleportation* from one to the other edge of the wire. Detailed numerical calculations [4] did not support this idea.
- Modified proposal by [5] suppresses Andree's reflection by including charging energy into the wire.
- We show that including charging energy prevents teleportation.
- Our proposed model of including charging energy only between two edge site has the advantage of being length-independent.
- None of above models supports the possibility of electron teleportation in the wire.

Majorana fermions in Kitaev's wire

The Hamiltonian of a p-wave spinless superconducting wire [1] is

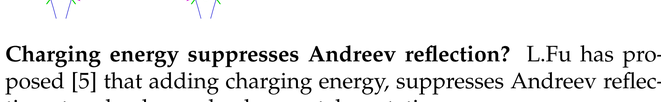
$$\mathcal{H}=\sum_{s=1}^{N-1}(ta_s^\dagger a_{s+1}+\Delta a_s a_{s+1}+h.c.)+\sum_{s=1}^{N}\mu(2a_s^\dagger a_s-1) \quad (1)$$

where N *is the length* of the wire, t *is the hopping term* between neighboring sites , Δ is *Cooper pair term* and μ is the chemical potential.
At Kitaev's point $t=\pm\Delta$ and $\mu=0$, the Hamiltonian reads

$$H=\frac{i}{2}\sum_{s=1}^{N-1}2t\big(\eta_s\xi_{s+1}\big)=t\big(\eta_1\xi_2+\eta_2\xi_3+\ldots\eta_{N-1}\xi_N\big)$$

which is represented in terms of Majorana operators $a_s=\xi_s+i\eta_s$. $\eta_s\xi_{s+1}$ are degenerate bulk states.
ξ_1 and η_N are isolated edge Majorana fermions. They form zero energy Dirac fermion $c_0=(\xi_1+i\eta_N)/2$ which relates the even and odd parity ground states $c_0^\dagger|\mathbb{0}\rangle=|\mathbb{1}\rangle$ where

- $|\mathbb{0}\rangle$ is the ground state with even number of electrons,
  $|\mathbb{1}\rangle$ is the ground state with odd number of electrons



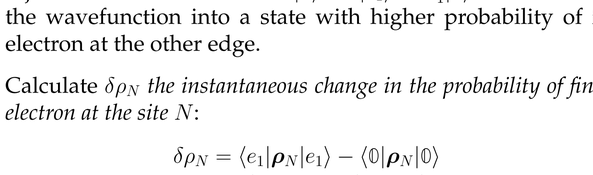
Teleportation proposal

- **Original teleportation proposal:** Semenoff *et al*[2, 3] proposed that sine a single electron is delocalized at two edges and bulk states are separated by a gap $\Delta\approx t$: *If an electron is injected at first site $s=1$ it can be instantaneously detected at the other edge $s=N$, independent of the length of the wire N.* They show this by calculating edge site fermions correlations which are *independent of the wire length*

$$\langle a_N a_1^\dagger\rangle=\langle a_N^\dagger a_1\rangle=\langle a_N a_1\rangle=\langle a_N^\dagger a_1^\dagger\rangle=1/4. \quad (2)$$

- **Andreev's reflection of the injected electron as a hole:** Bloch *et al* [4] show by numerical calculations using nonequilibrium Green functions that *there is no correlation between tunneling edge currents* into trapped Majorana states near two vortices.

  Due to Andreev reflection the injected electron reflects back as a hole from the same site, instead of teleporting to the other side.



- **Charging energy suppresses Andreev reflection?** L.Fu has proposed [5] that adding charging energy, suppresses Andreev reflection at each edge and enhances teleportation.

Quantifying the success of electron teleportation

The correlation functions of edge sites fermions are independent of N but they do not measure the success the success of teleportation; it is also important to discern Andreev reflection from actual electron teleportation.
The teleportation proposal relies on the existence of an entangled state of two electrons (at edge sites) $|\Phi\rangle=|1\ldots0\rangle+|0\ldots1\rangle$ which is isolated from other states of the system by a constant energy gap Δ. If an electron is injected into the first site $|\Phi\rangle\rightarrow|1\ldots1\rangle$ the probability of finding electron at site N instantaneously increases to one independent of the distance.
We first check *whether the ground state of the Kitaev wire form a proper entangled state which can be used for teleportation?*
For measuring and quantifying the success of teleportation we apply the following procedure which is a minimal requirement for performing teleportation:

- Inject an electron at first site $|\mathbb{0}\rangle\rightarrow|e_1\rangle=a_1^\dagger|\mathbb{0}\rangle$ This must collapse the wavefunction into a state with higher probability of finding electron at the other edge.
- Calculate $\delta\rho_N$ *the instantaneous change in the probability of finding an electron at the site N*:

$$\delta\rho_N=\langle e_1|\boldsymbol{\rho}_N|e_1\rangle-\langle\mathbb{0}|\boldsymbol{\rho}_N|\mathbb{0}\rangle$$
$$\boldsymbol{\rho}_N=a_N^\dagger a_N-a_N a_N^\dagger=2a_N^\dagger a_N-1$$

- if teleportation is possible, there must be an instantaneous change in $\delta\rho_N$ independent of the length of the wire N.

Ground states of Kitaev wire

$|\mathbb{0}\rangle,|\mathbb{1}\rangle$ are completely uniform superposition of all states with the same amplitude! Arranging by edge sites occupation:

$$|\mathbb{0}\rangle=\frac{1}{2}(\underbrace{|0\mathbb{1}1\rangle+|1\mathbb{1}0\rangle}_{\text{particle-hole entangled}}+\underbrace{|0\mathbb{0}0\rangle+|1\mathbb{0}1\rangle}_{\text{Cooper-pair entangled}}) \quad (3)$$

$$|\mathbb{1}\rangle=\frac{1}{2}(\underbrace{|0\mathbb{0}1\rangle+|1\mathbb{0}0\rangle}_{\text{particle-hole entangled}}+\underbrace{|0\mathbb{1}0\rangle+|1\mathbb{1}1\rangle}_{\text{Cooper-pair entangled}}) \quad (4)$$

$|\mathbb{o}\rangle$ has the exact same structure of $|\mathbb{0}\rangle$ but with sites $1,N$ eliminated. Likewise is defined $|\mathbb{1}\rangle$.
$\mathbb{0}\rangle$ and $\mathbb{1}$ are *mean field ground states, but the presence of Cooper pair entangled states along the particle-hole entangled state prevents teleportation*:

$$|e_1\rangle\equiv a_1^\dagger|\mathbb{0}\rangle=\underbrace{|1\mathbb{1}1\rangle}_{from\ particle-hole}+\underbrace{|1\mathbb{0}0\rangle}_{from\ Cooper-pair}$$
$$\delta\rho_N\equiv\langle e_1|\boldsymbol{\rho}_N|e_1\rangle-\langle\mathbb{0}|\boldsymbol{\rho}_N|\mathbb{0}\rangle=0$$

Effects of charging energy

To check L.Fu[5] model we add the charging energy to the Kitaev Hamiltonian

$$H_C=\frac{e^2}{2C}(\Sigma n_s-N/2)^2=u(\Sigma\rho_s)^2 \qquad \Sigma n_s \text{ is total number of electrons} \quad (5)$$

Though after injecting electron at first site the probability of finding electron at the other edge increased $\delta\rho_N\neq0$ but *it is length-dependent*; the charging energy couples all bulk states to each other and to the ground state.

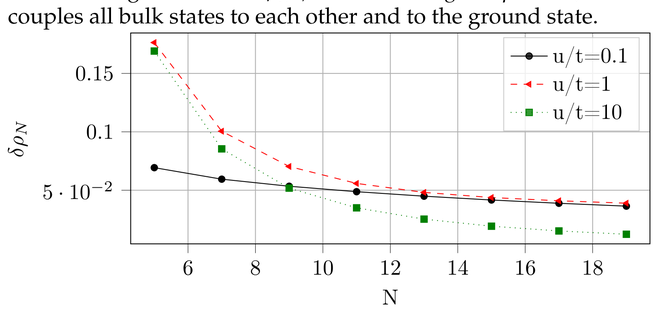
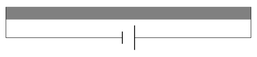


Figure 1: The change in the probablity density of electron at the other edge $\delta\rho_N$, including the charging energy in the Hamiltonian. $\delta\rho_N$ does depend on N and it vanishes as $N\rightarrow\infty$.

Coulomb interaction between edges

We propose a simpler and more efficient model in which the Coulomb charging energy is set only between two edge sites of the wire.



$$H_{u_r}=2u_r\rho_1\rho_N=-2u_r\xi_1\eta_1\xi_N\eta_N \quad (6)$$

The ground state would change to

$$|\mathbb{0}\rangle=\big[c(|1\mathbb{1}0\rangle+|0\mathbb{1}1\rangle)+s(|1\mathbb{0}1+|0\mathbb{0}0\rangle)\big]/\sqrt{2}$$
$$|\mathbb{1}\rangle=\big[c(|1\mathbb{0}0\rangle+|0\mathbb{0}1\rangle)+s(|1\mathbb{1}1+|0\mathbb{1}0\rangle)\big]/\sqrt{2} \quad (7)$$

where c^2-s^2 monotonically inreasing function of u_r/Δ
The amplitude of the Cooper entagled state is suppressed and the resulting *change in probality of finding electron, is independent of N.*

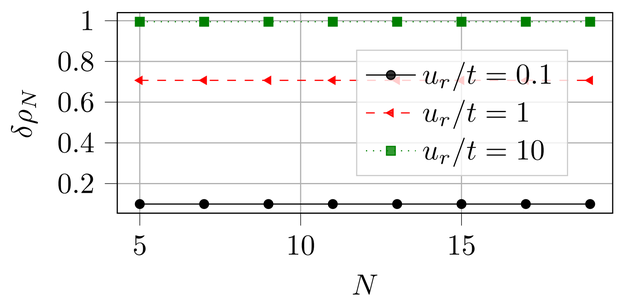


Figure 2: The instantaneous change in the probability of detecting electron at the other edge $\delta\rho_N$ as function of N, for different values of edge site charging energy model Eq.(6). The indecued change $\delta\rho_N$ *is independent of* N.

Limitations of charging models

For both charging models Eq.(5) and Eq.(6) $\delta\rho_N\neq0$. However this is the result of introducing Coulomb repulsion between two edges; without this optical connection $\delta\rho_N$ vanishes. This has to be contrasted to the original spin teleportation schemes [6] where the receiver of the quantum information is physically disconnected from the sender: the spin teleportation is unambiguously the result of nonlocal quantum correlations between two sites. Another evidence in supporting this idea is that this effect is also observed for normal metal wire.

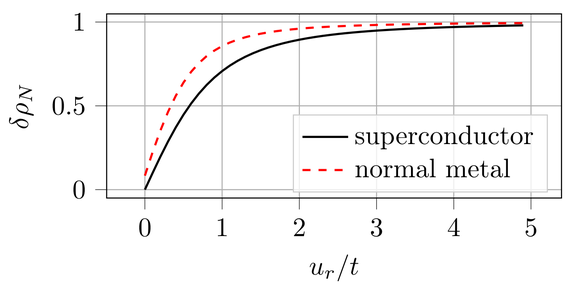


Figure 3: $\delta\rho_N$ as a function of $u_r/t=$ for fixed value of N. $\delta\rho_N$ is higher for normal metal $\Delta=0$ compared to the superconductor $\Delta=t$.

Time evolution of induced density

It is important to note that, in all teleportation models studied here, $\delta\rho_N$ *is time-dependent*. *The time evolution of ρ_N is essentially independent of ρ_1*, each being a fast oscillating function with the frequency of the order of the hoping term $t=\Delta$.
Coulomb interaction introduces a long time-scale correlation between two edges. The time-evolution of our edge sites charging model Eq.(6) is particularly simple:

- The long time scale $\sim1/u_r$ is set by the Coulomb repulsion between two edges densities.
- The short time scale $\sim1/\Delta$ is the result of hoping between neighboring sites.
- The instantaneous induced change in the density $\delta\rho_N$ is a transient effect: the information received by site N lost after the time at which an electron can hop from site $N-1$ to N. Therefore the transient nature of $\delta\rho_N$ is the consequence of casuality.

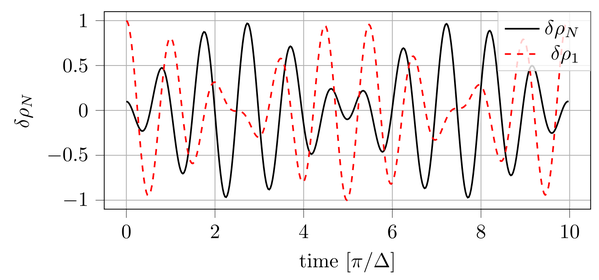


Figure 4: Time-dependent induced charge at first and last sites for $u_r/t=0.1$ of Kitaev wire. Short time scale is set by Cooper pair energy $1/\Delta$ and the long one by the Coulomb energy $1/u_r$.

Conclusion

The apparent feasibility of electron teleportation using Majorana fermions, originates from invoking the mean-field solution for the ground state, which violates the conservation of particles number. The same mean-field nature of the ground state prevents any attempt to suppress Andreev reflection from the same edge in favor of actual teleportation of the injected electron.

- Kitaev wire, without Coulomb interaction, does not have the proper entangled ground state for performing teleportation.
- Including the global Coulomb charging energy [5] results in a *length-dependent* induced charge $\delta\rho_N$. The term playing the crucial rule in improving the result is the Coulomb interactions between two edges.
- In the improved model, Coulomb repulsion is merely between edge sites. The advantage of this model is that the $\delta\rho_N$ is independent of the length.
- Nevertheless nonzero value of $\delta\rho_N$ in both charging models:
  - is the result of introducing optical connection between edges.
  - is a time-dependent (transient) effect.
  - is present even in a normal metal.

These limitations argue for impossibility of teleportation via Majorana fermions even in the presence of Coulomb interactions.

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