Pairing Fluctuation Effects on Quasiparticle Transport

Quasiparticle-Pair Fluctuation Scattering Contribution to Zero Sound Attenuation

Wei-Ting Lin[1,2] and J. A. Sauls[1,3]

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[3] Hearne Institute of Theoretical Physics, Louisiana State University

- https://arxiv.org/abs/2110.14073
- https://arxiv.org/abs/2110.12445
- https://arxiv.org/abs/2110.10339

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Long-lived Bosonic Modes in Fermi Liquids - I

❖Proximity to a Magnetic Instability ⇝ ***Paramagnons*** ⇝ Corrections to Fermi liquid theory

$$C_v = \gamma_S T + \beta T^3 \ln T$$

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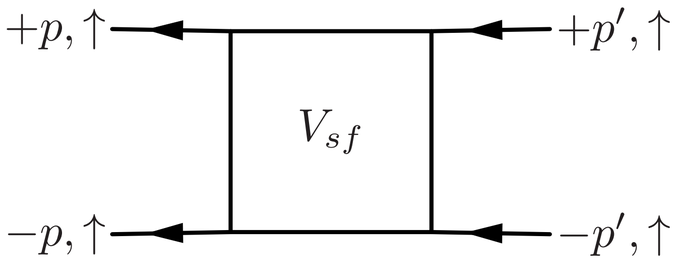

$$= \frac{-I}{1 - I\chi(|\vec{p} - \vec{p}'|)}$$

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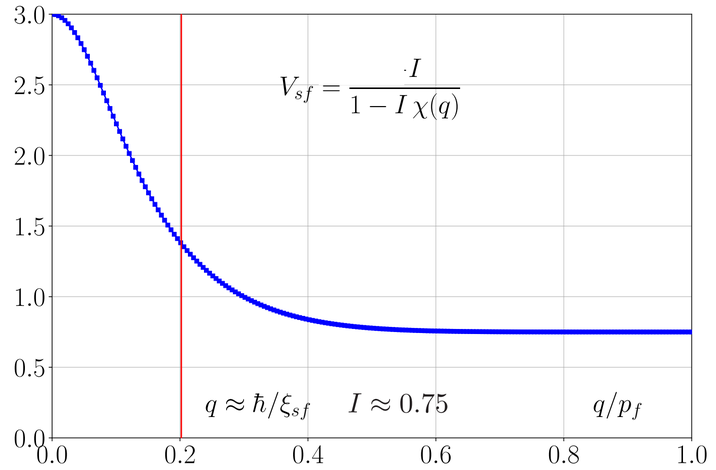

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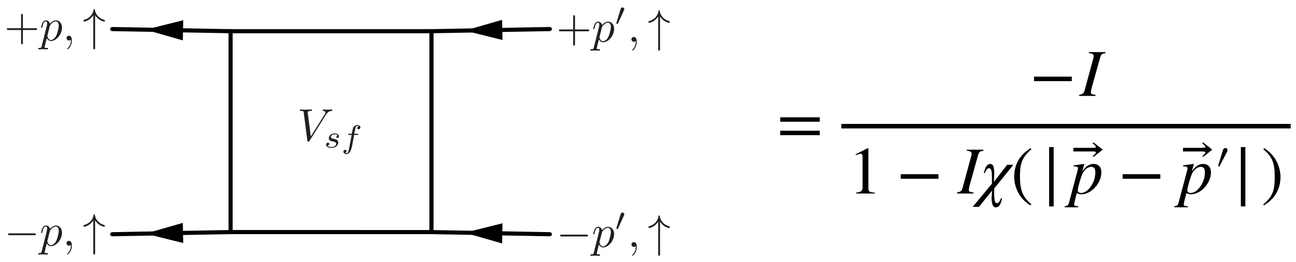

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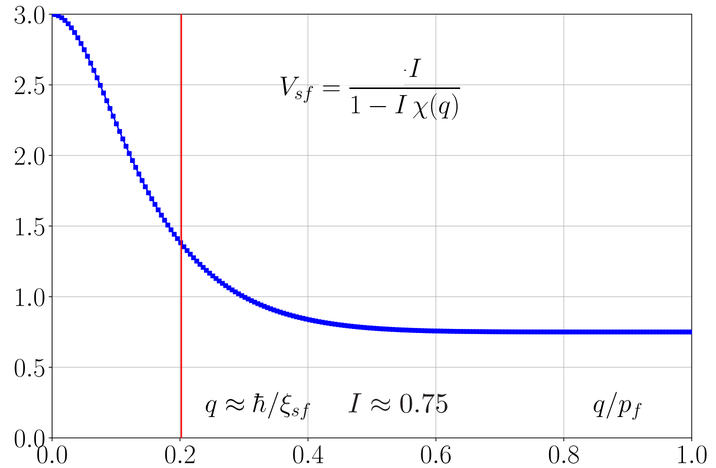


Dominant pairing channel:

$l = 1$ (p-wave) $\qquad S = 1$ (triplet)

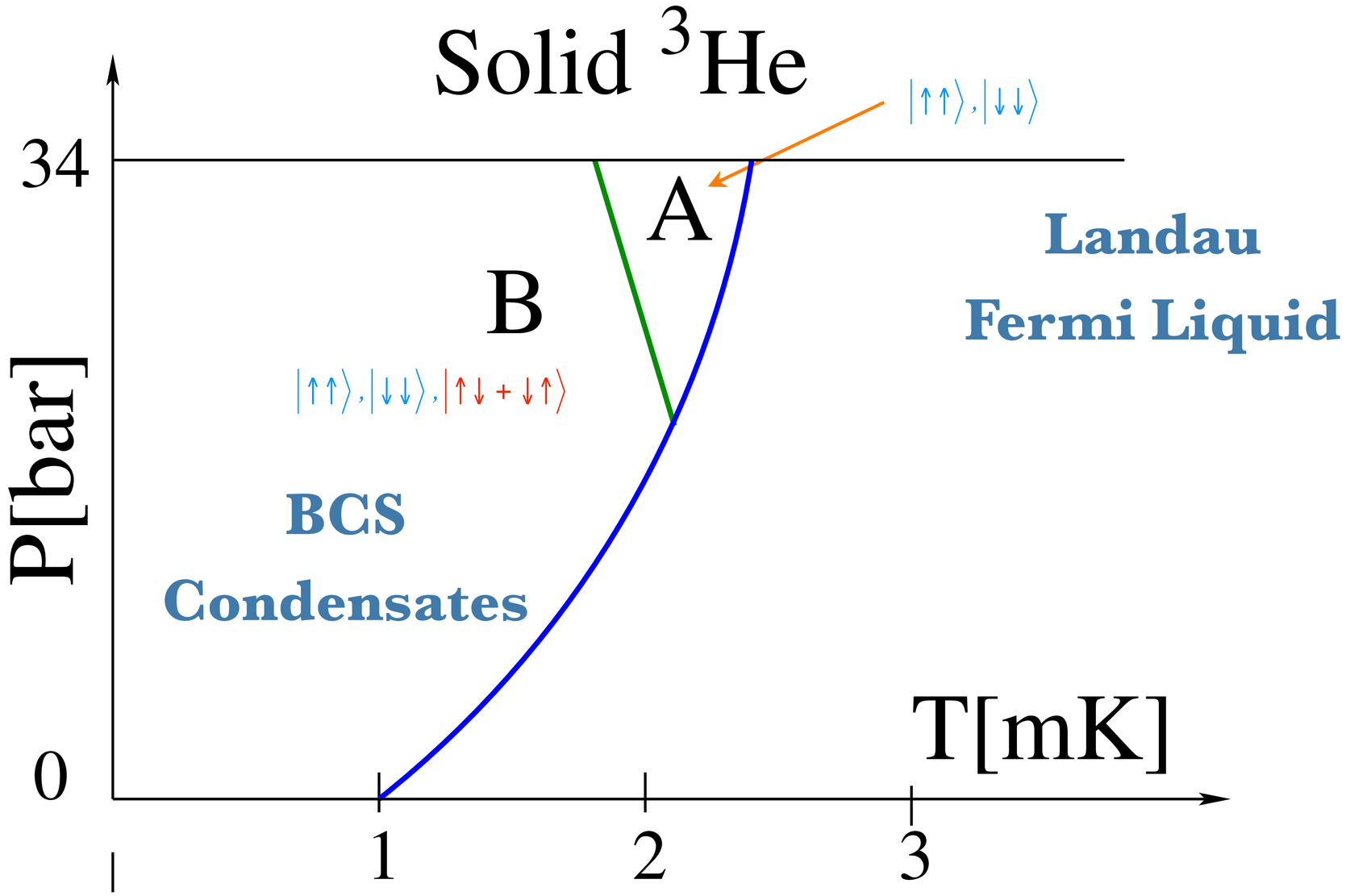
$$\hat{p}_x + i\hat{p}_y \sim \sin\theta_{\hat{p}}\, e^{+i\phi_{\hat{p}}} \qquad |\uparrow\uparrow\rangle$$

$$\hat{p}_z \sim \cos\theta_{\hat{p}} \qquad |\uparrow\downarrow + \downarrow\uparrow\rangle$$

$$\hat{p}_x - i\hat{p}_y \sim \sin\theta_{\hat{p}}\, e^{-i\phi_{\hat{p}}} \qquad |\downarrow\downarrow\rangle$$

❖ **9 degenerate pairing fluctuation channels for $T \gtrsim T_c$**

Low Temperature Phases of ^{3}He



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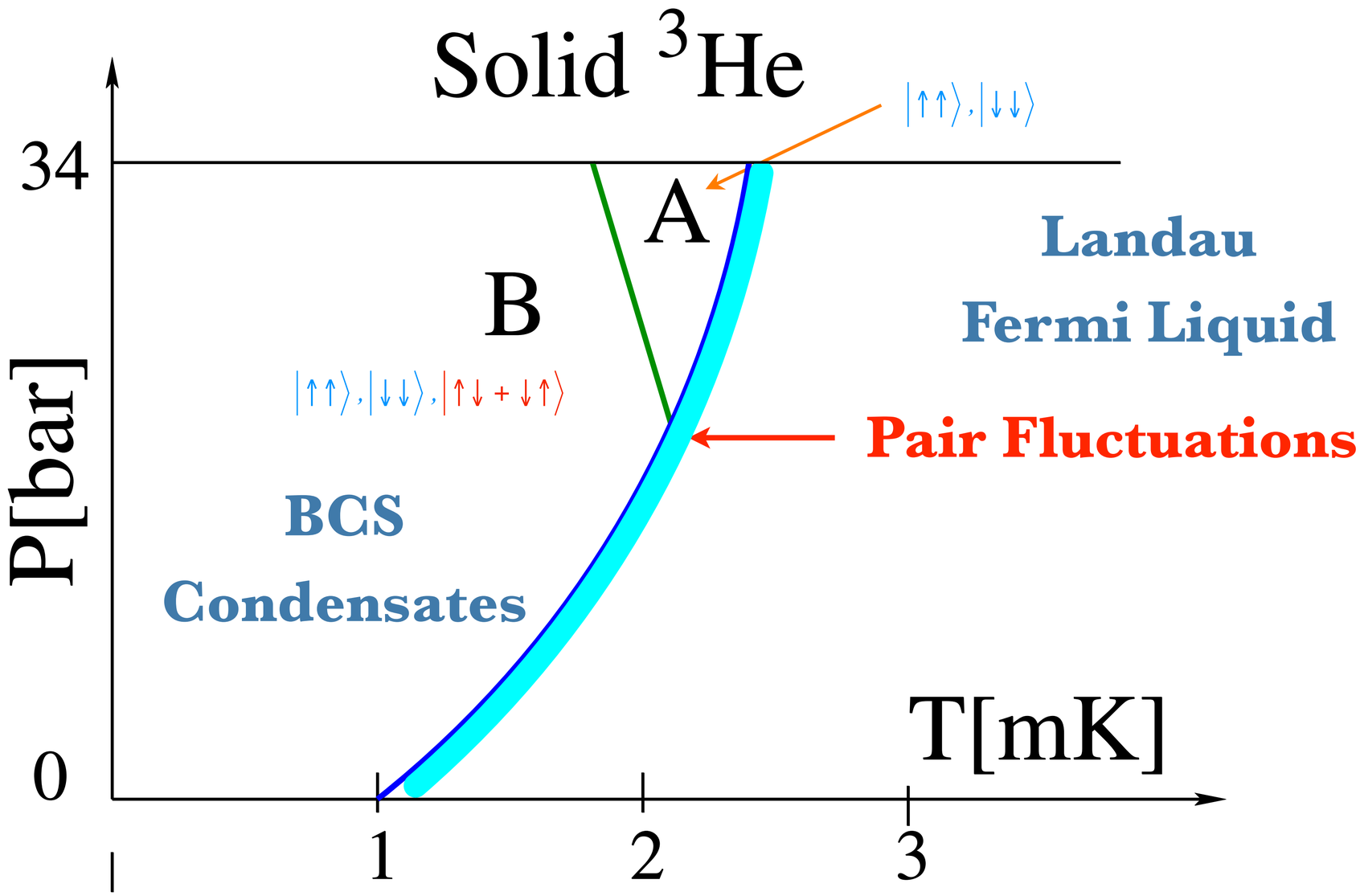
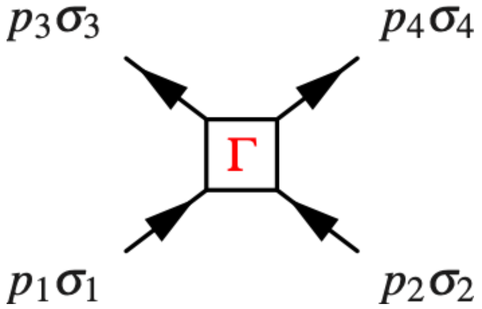


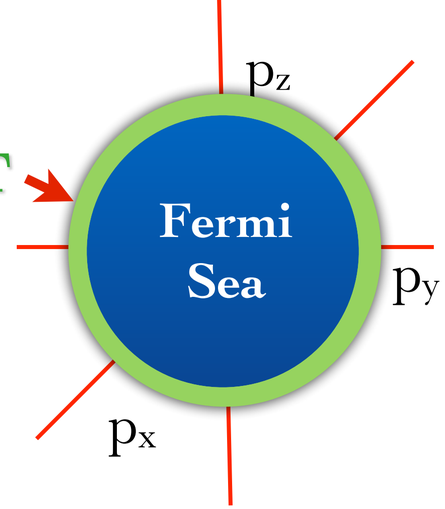
Breakdown of Hydrodynamics in Liquid ^{3}He



Binary collision frequency

$$\frac{\hbar}{\tau} = E_f \left\langle |\Gamma|^2 \right\rangle_{FS} \left(\frac{k_B T}{E_f} \right)^2$$

Landau (1958)



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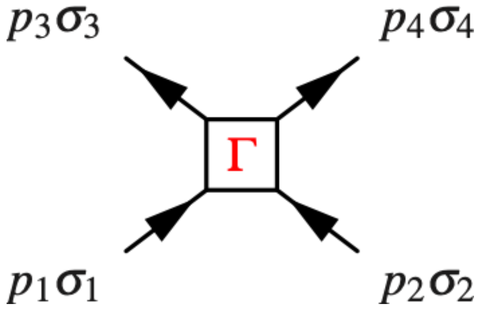

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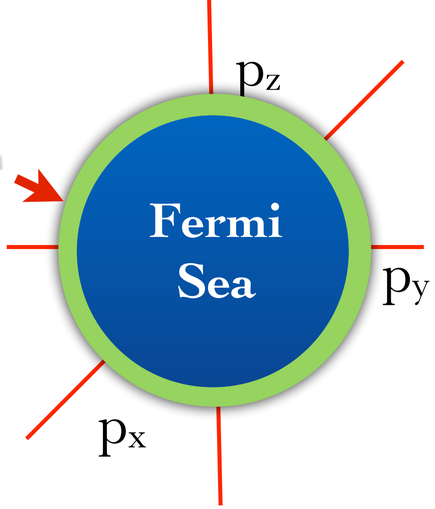

Shear viscosity

$$\eta = \frac{1}{5} n p_f (v_f \tau) \propto \left(\frac{1}{T^2} \right)$$

Damping of hydrodynamic sound

$$\alpha_1 = \frac{2}{3} \left(\frac{\eta \omega^2}{\rho c_1^2} \right) \propto \left(\frac{1}{T^2} \right) \uparrow$$

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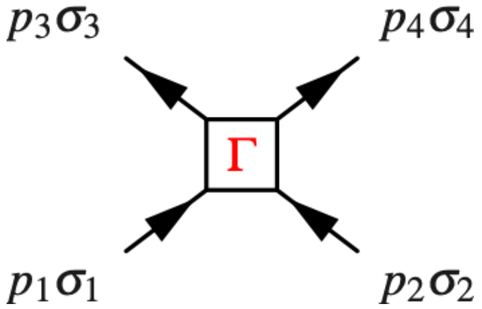

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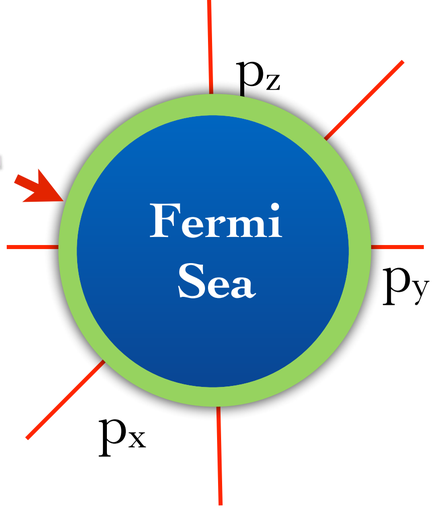

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Crossover to collisionless dynamics

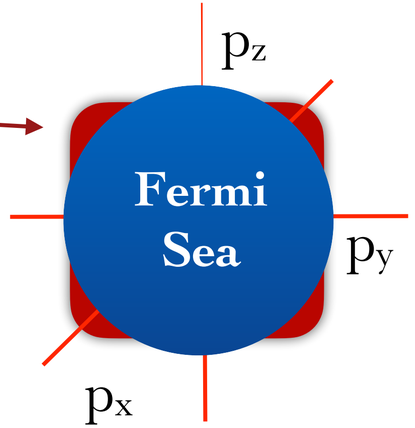
$$\omega\tau \left(T^* \right) \approx 1$$

$$T \leq T^* \approx 10\,\mathrm{mK} \quad \text{at} \; \frac{\omega}{2\pi} = 20\,\mathrm{MHz}\,(P = 15\,\mathrm{bar})$$

$T = 0$: Zero Sound is an eigenmode of Fermi surface deformations

Landau (1958)

``Elastic Deformation'' of the Fermi Surface



$$\nu_{\hat{\mathbf{p}}} = \sum_{lm} \nu_{lm}\, Y_{lm}(\hat{\mathbf{p}})$$

⇝ **Change in a quasiparticle's energy**

$$\delta\varepsilon_{\hat{\mathbf{p}}} = \sum_{lm} F_l^s\, \nu_{lm}\, Y_{lm}(\hat{\mathbf{p}})$$

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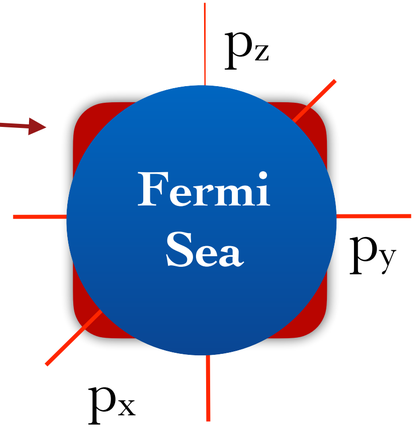

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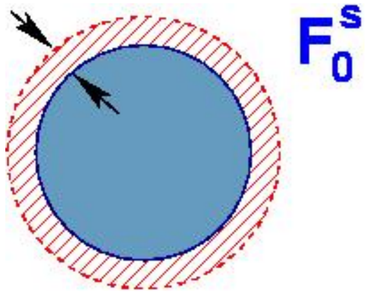
Longitudinal (zero) sound

$$\omega = c_0\, q$$

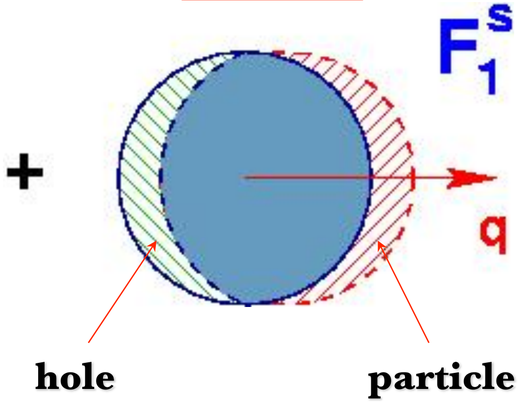
$\nu_{00} \propto \delta n$

dilitation



$\nu_{10} \propto \hat{\mathbf{q}} \cdot \mathbf{J}$

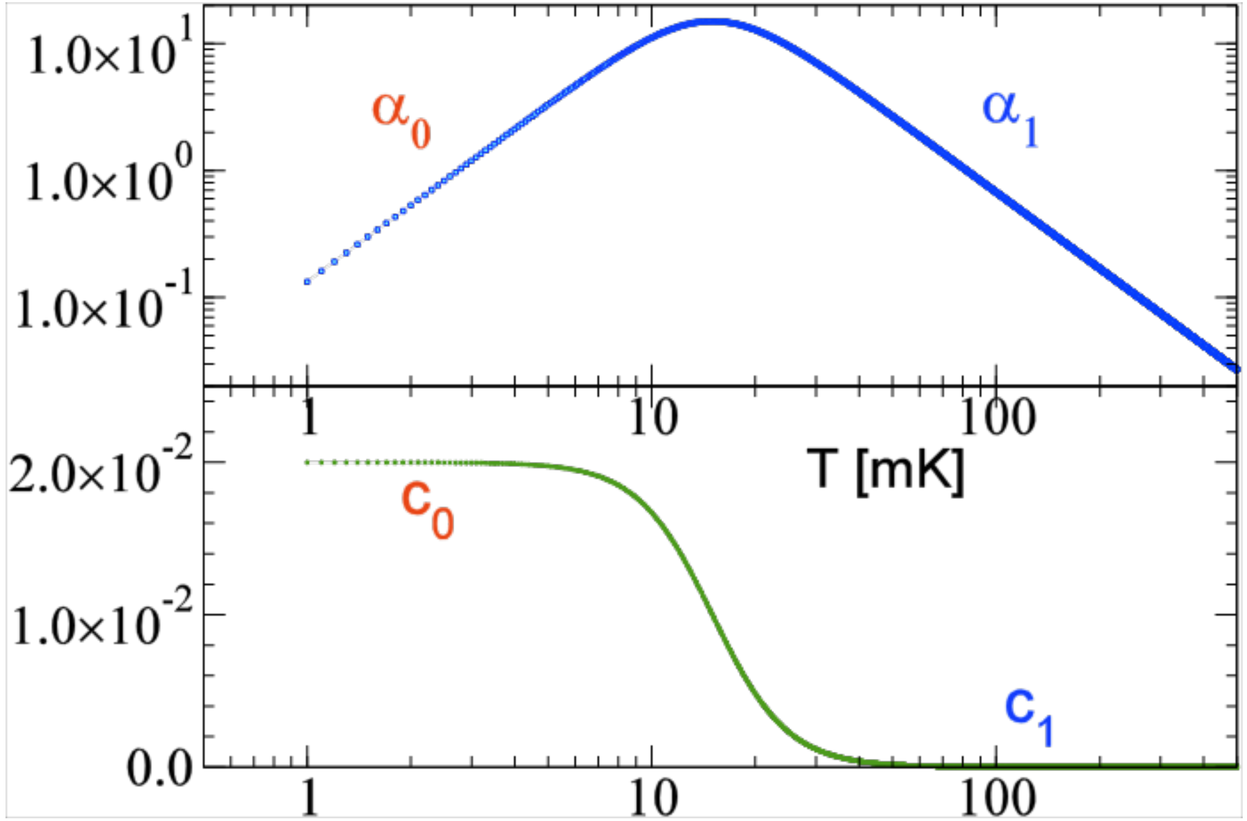
dipole



❖ Zero Sound is a *coherent* particle & hole excitation

Cross-Over from Hydrodynamic to Collisionless (Zero) Sound

Abel, Anderson & Wheatley (1966)



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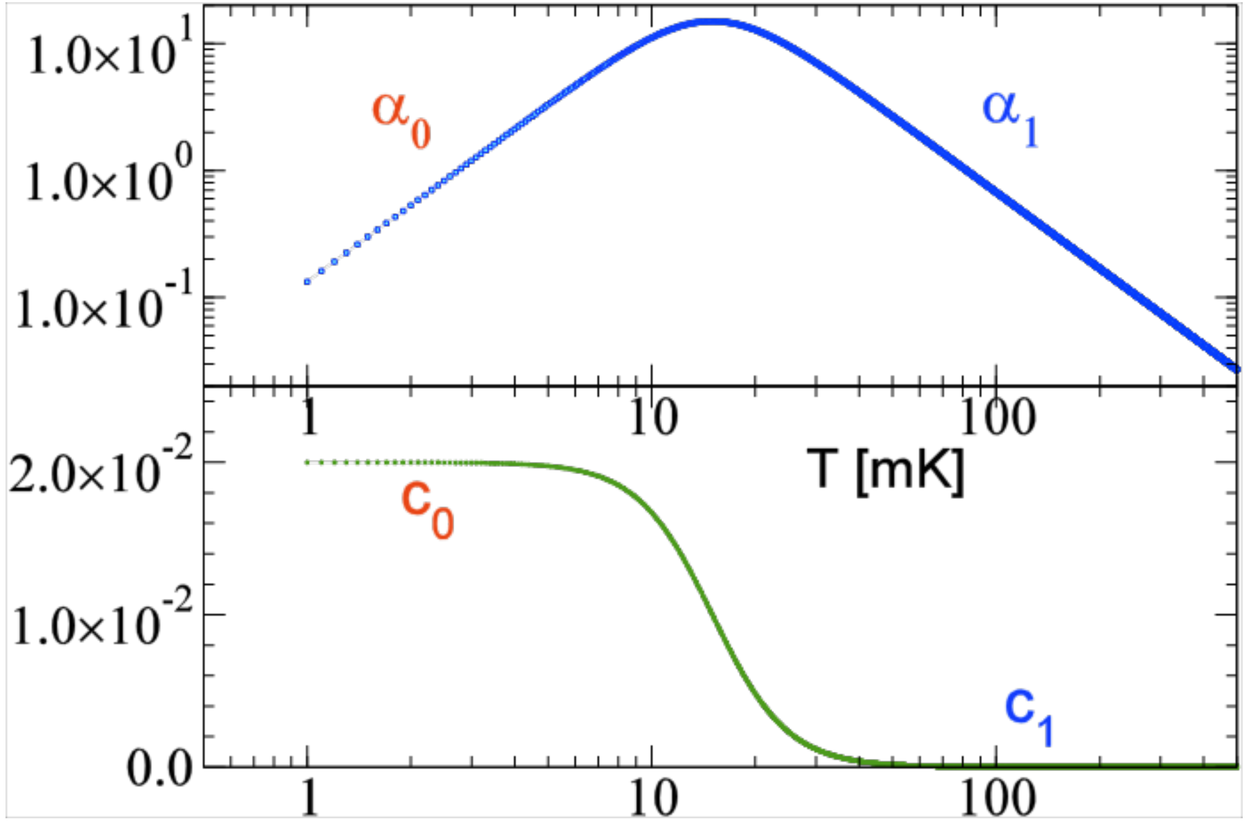
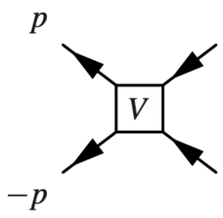


Long-lived Bosonic Modes in Fermi Liquids - II

❖ Proximity to a BCS Pairing Instability ⇝ Pair Fluctuations ⇝ **Enhanced Scattering of QPs**

Attractive pairing interaction



$$\rightsquigarrow T_c = 1.13\omega_c\, e^{-1/V}$$

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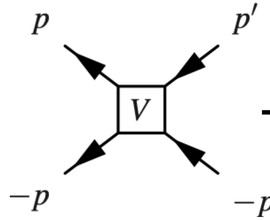

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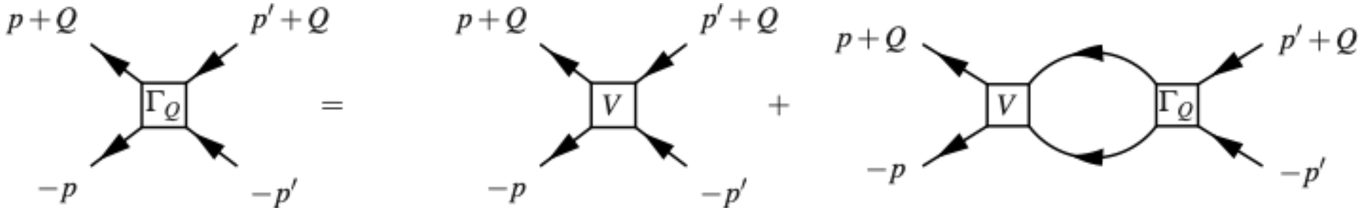
Cooper Pair amplitude above T_c



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$$\Gamma_Q = -\frac{2}{N(0)}\left\{\frac{1}{\ln(T/T_c) - i\omega\tau_0 + |\mathbf{Q}|^2\xi_0^2}\right\}$$

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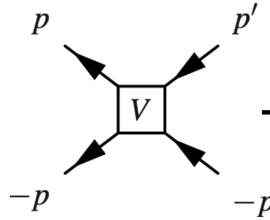

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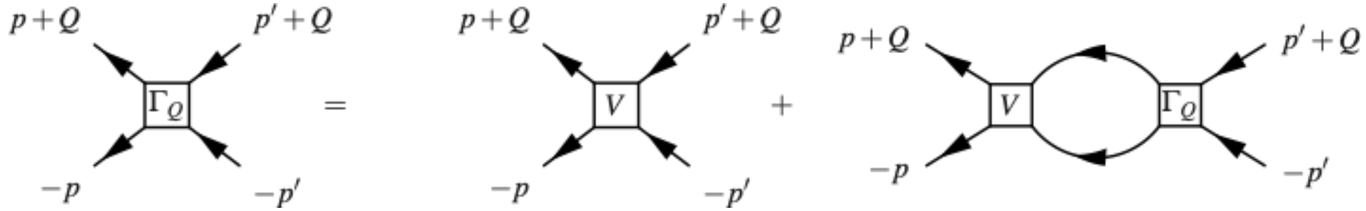

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$$\tau(\mathbf{Q}) = \frac{\tau_0}{(T/T_c - 1) + |\mathbf{Q}|^2\xi_0^2}$$

$$\Gamma(\mathbf{Q},t) = -\frac{2}{N(0)\tau_0}e^{-t/\tau(\mathbf{Q})}$$

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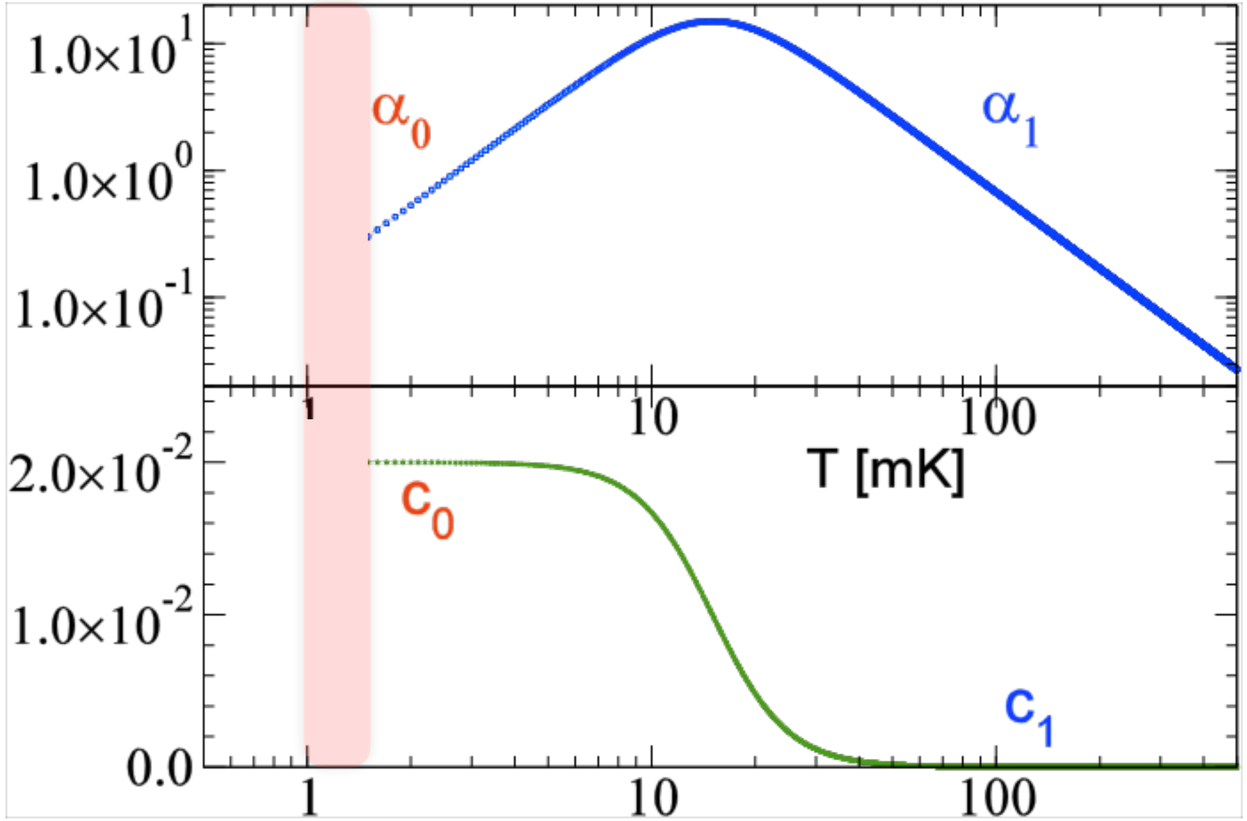

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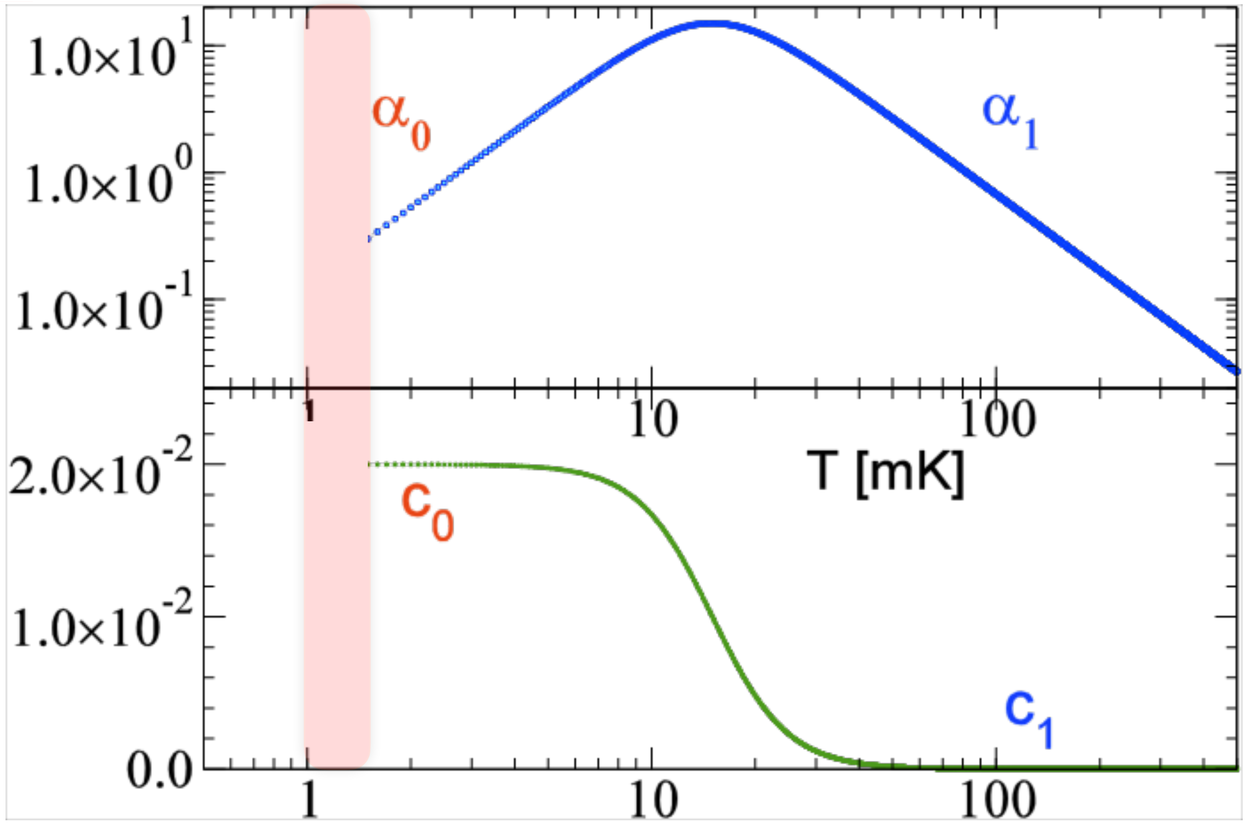

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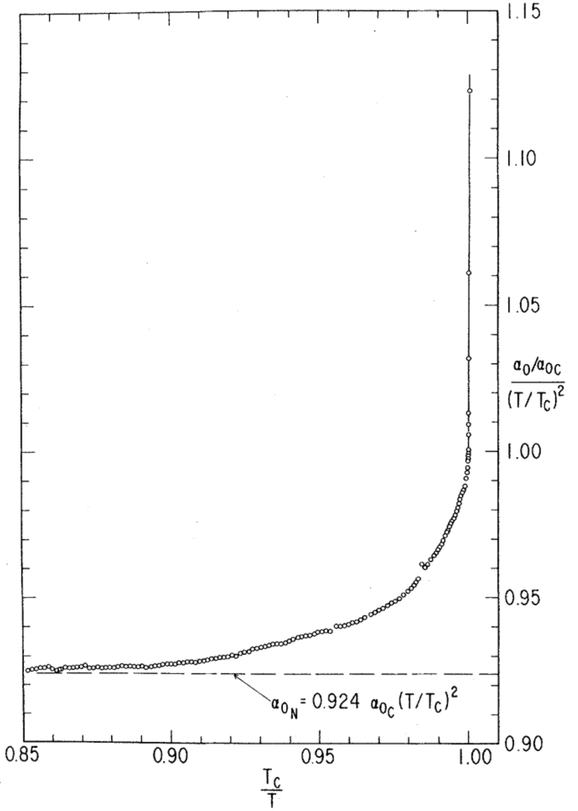

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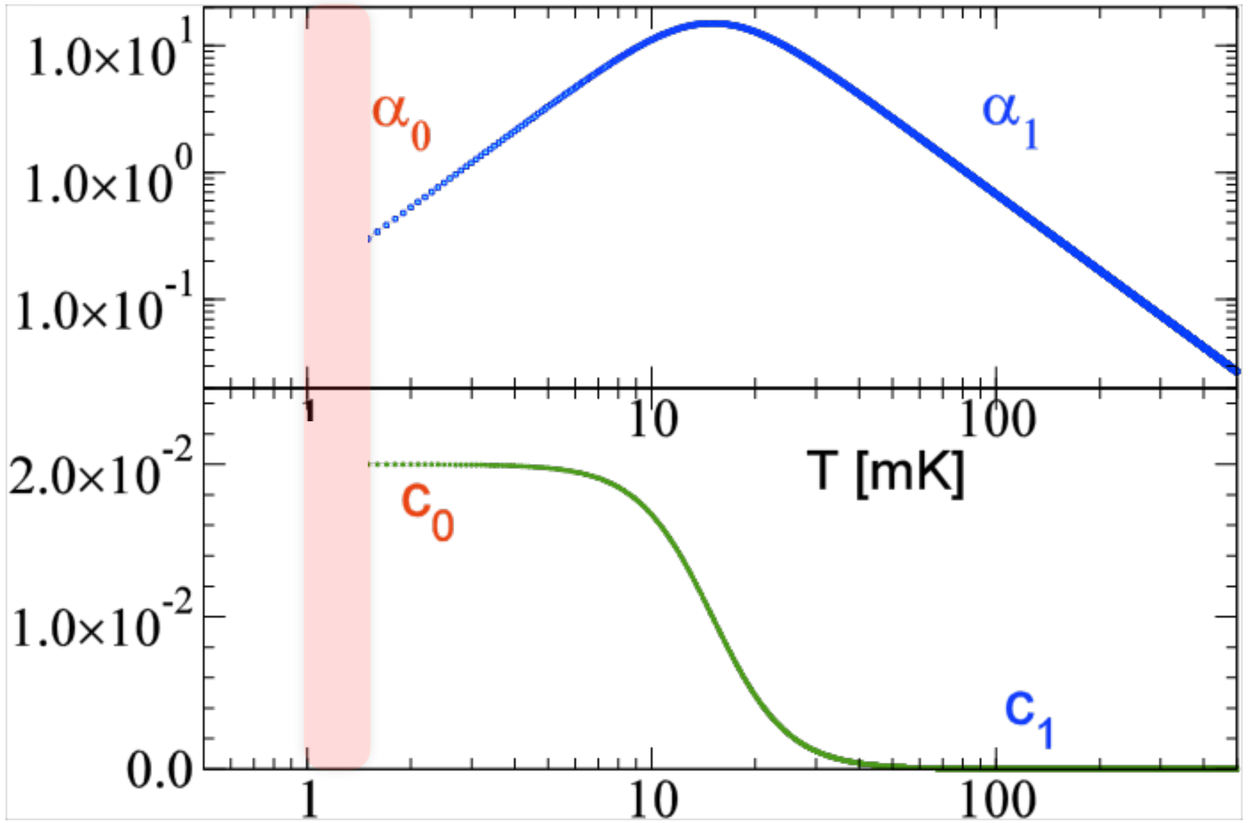

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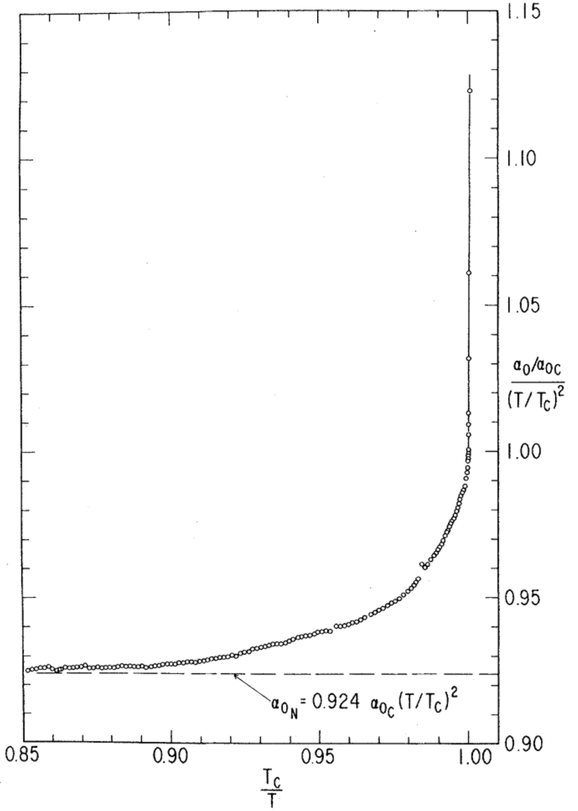

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First-principles theory of Cooper Pair Fluctuation Corrections to Quasiparticle Transport

Effects of Incipient Pairing on Non-equilibrium Quasiparticle Transport in Fermi Liquids.
Prog. Theor. Exp. Phys., 2022(2):1--28, 2022. Wei-Ting Lin and J. A. Sauls

Zero Sound is an eigenmode of the Landau-Boltzmann Equation

Keldysh Formulation of the LBE with Collision Processes Calculated Systematically

$$\frac{\partial}{\partial t} n_{\mathbf{p}} + \mathbf{v}_{\mathbf{p}} \cdot \boldsymbol{\nabla}_{\mathbf{R}} n_{\mathbf{p}} - \boldsymbol{\nabla}_{\mathbf{R}} \Sigma^{\mathrm{MF}}_{\mathbf{p}} \cdot \boldsymbol{\nabla}_{\mathbf{p}} n_{\mathbf{p}} = I_{\mathbf{p}}[n_p]$$

Fermi Velocity

Internal Forces (Landau)

Internal Forces (Collisions)

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Fermi Velocity — Internal Forces (Landau) — Internal Forces (Collisions)

$$I_{\mathbf{p}}[n_p] \equiv -i \left(\Sigma^{21}_{\mathbf{p}} n_{\mathbf{p}} + \Sigma^{12}_{\mathbf{p}} (1 - n_{\mathbf{p}}) \right)$$

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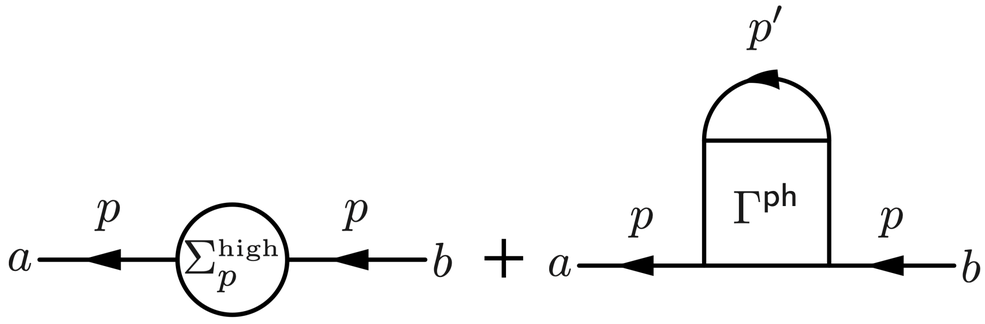

Fermi Velocity - m*/m | Landau Mean Field

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$$\frac{1}{\tau} = A T^2$$

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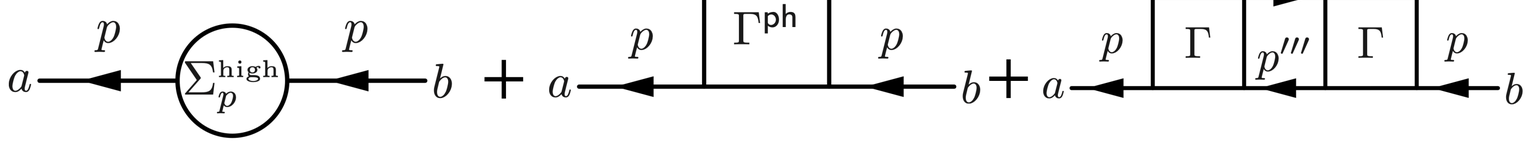


Fermi Velocity - m*/m | Landau Mean Field | QP - QP Scattering

Zero Sound is an eigenmode of the Landau-Boltzmann Equation

Keldysh Formulation of the LBE with Collision Processes Calculated Systematically

$$\frac{\partial}{\partial t} n_{\mathbf{p}} + \mathbf{v}_{\mathbf{p}} \cdot \boldsymbol{\nabla}_{\mathbf{R}} n_{\mathbf{p}} - \boldsymbol{\nabla}_{\mathbf{R}} \Sigma^{\mathrm{MF}}_{\mathbf{p}} \cdot \boldsymbol{\nabla}_{\mathbf{p}} n_{\mathbf{p}} = I_{\mathbf{p}}[n_p]$$

Fermi Velocity — Internal Forces (Landau) — Internal Forces (Collisions)

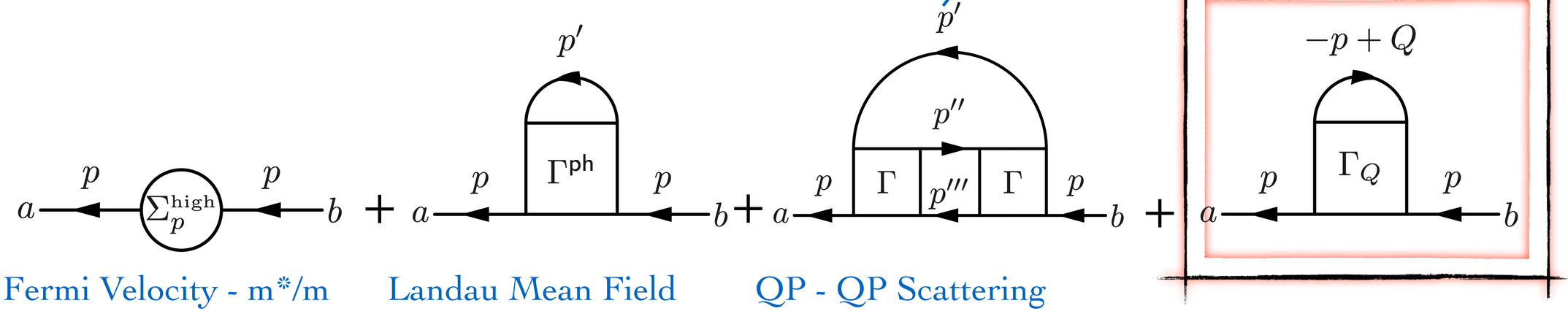
$$I_{\mathbf{p}}[n_p] \equiv -i \left(\Sigma^{21}_{\mathbf{p}} n_{\mathbf{p}} + \Sigma^{12}_{\mathbf{p}} (1 - n_{\mathbf{p}}) \right)$$

$$\frac{1}{\tau} = A T^2$$

$$\Gamma(Q, \omega) \propto \frac{1}{\ln(T/T_c) + \xi_0^2 Q^2 - i\pi\omega/8T}$$

QP - Pair Fluctuation Scattering

$$\Sigma^{ab}(\mathbf{p}, \varepsilon) =$$



Fermi Velocity - m*/m — Landau Mean Field — QP - QP Scattering

QP-Pair Fluctuation Collision Integral

$$\frac{\partial n_{\mathbf{p}}}{\partial t} + \mathbf{v}_{\mathbf{p}}\varepsilon_p \cdot \boldsymbol{\nabla}_{\mathbf{r}} n_{\mathbf{p}} - \boldsymbol{\nabla}_{\mathbf{p}} n_p \cdot \boldsymbol{\nabla}_{\mathbf{r}} \Sigma^{\mathrm{MF}}_{\mathbf{p}} = I^{\mathrm{Landau}}_{\mathbf{p}}[n_p] + I^{\mathrm{C}}_{\mathbf{p}}[n_p]$$

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Binary Collision Scattering $\frac{1}{\tau} = A\,T^2$

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$$I^{\mathrm{C}}_{\mathbf{p}} = \int \frac{d^3p'}{(2\pi\hbar)^3} \int \frac{d^3Q}{(2\pi\hbar)^3} W_{\mathrm{C}}(\mathbf{p},\mathbf{p}';\mathbf{Q},\varepsilon_{\mathbf{p}'}+\varepsilon_{-\mathbf{p}'+\mathbf{Q}}) \times \delta(\varepsilon_{\mathbf{p}'}+\varepsilon_{-\mathbf{p}'+\mathbf{Q}}-\varepsilon_{\mathbf{p}}-\varepsilon_{-\mathbf{p}+\mathbf{Q}})$$
$$\left[n(\varepsilon_{\mathbf{p}'})n(\varepsilon_{-\mathbf{p}'+\mathbf{Q}})\left(1-n(\varepsilon_{\mathbf{p}'}+\varepsilon_{-\mathbf{p}'+\mathbf{Q}}-\varepsilon_{\mathbf{p}})\right)(1-n(\varepsilon_{\mathbf{p}}))\right.$$
$$\left.-\left(1-n(\varepsilon_{\mathbf{p}'})\right)\left(1-n(\varepsilon_{-\mathbf{p}'+\mathbf{Q}})\right)n(\varepsilon_{\mathbf{p}'}+\varepsilon_{-\mathbf{p}'+\mathbf{Q}}-\varepsilon_{\mathbf{p}})n(\varepsilon_{\mathbf{p}})\right]$$

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❖ Collision rate for QPs of momentum **p** scattering off Cooper Pair fluctuations of momentum **Q** and energy ω

$$W_{\mathrm{C}}(\mathbf{p},\mathbf{p}';\mathbf{Q},\omega) = \pi\left|\frac{4\pi}{N(0)}\right|^2 (2S+1) \sum_{m=-l}^{+l} \left| Y_{lm}(\hat{\mathbf{p}}) \frac{1}{\ln(T/T_c) - i\frac{\pi\omega}{8T} + |\mathbf{Q}|^2\xi^2_{lm}} Y^*_{lm}(\hat{\mathbf{p}}') \right|^2$$

❖N.B. $(2S+1)\times(2l+1) = 9$ fluctuation modes - QP-CP Scattering Rates Add

Perturbation theory for zero sound dispersion

$$(\omega - \mathbf{v_p}\cdot\mathbf{k})\psi_{\hat{\mathbf{p}}} - \mathbf{v_p}\cdot\mathbf{k}\int \frac{d\Omega_{\mathbf{p}'}}{4\pi} F^s(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')\,\psi_{\hat{\mathbf{p}}'} = i\int d\varepsilon_{\mathbf{p}}\,\left\{I^L_{\mathbf{p}}[\psi_{\hat{\mathbf{p}}'}] + I^{\mathrm{C}}_{\mathbf{p}}[\psi_{\hat{\mathbf{p}}'}]\right\}$$

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QP-QP Scattering

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High Frequency Limit

$$\frac{1}{\omega\tau} \ll 1$$

$$\psi_{\hat{\mathbf{p}}} = \underbrace{\nu_{\hat{\mathbf{p}}}}_{\mathcal{O}(1)} + \underbrace{\delta\nu_{\hat{\mathbf{p}}}}_{\mathcal{O}(1/\omega\tau)}$$

- J. S. Corruccini, et al., Phys. Rev. 180, 225 (1969).

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Attenuation

$$\alpha_0 = a_0\,T^2 + \mathcal{A}_0\,\Theta(T/T_c - 1)$$

- J. S. Corruccini, et al., Phys. Rev. 180, 225 (1969).

QP-Pair Fluctuation Correction to Zero Sound attenuation

$$\alpha_0^C = \mathcal{A}_0\,\Theta(\vartheta)$$

$$\mathcal{A}_0 = \frac{2.24}{10\pi}\frac{1}{N(0)\xi_0^3 T_c}\left(\frac{v_f}{c_0}\right)^3\frac{m^*}{m}\left(1+\tfrac{1}{5}F_2^s\right)^2\frac{T_c}{v_f}$$

Determines Pressure dependence†

$$F_2^s = 1.39$$

$$\Theta(\vartheta) = \int_{-\infty}^{\infty}\frac{dx}{\pi}\,\mathcal{F}(x;\vartheta)\,\mathrm{sech}^4(2x/\pi)$$

Reduced temperature

$$\vartheta = T/T_c - 1$$

$$\mathcal{F} \equiv \xi_{1m}^3\int_0^{\infty} Q^2 dQ\,|\Gamma_{1m}(Q,\omega)|^2 = \frac{\pi}{2}\frac{\mathrm{Im}\,\sqrt{\vartheta + i|x|}}{|x|}$$

Scaled frequency

$$x \equiv \pi\omega/8T$$

Cooper Pair Fluctuation Probability density

† T. Haard, 3He Calculator, ULT@Northwestern Online (2000), http://spindry.phys.northwestern.edu/he3.htm

QP-Pair Fluctuation Correction to Zero Sound attenuation

$$\alpha_0^{\mathrm{C}} = \mathcal{A}_0\,\Theta(\vartheta) + \alpha_\infty^{\mathrm{C}}$$

Short-wavelength counter term

$$\alpha_\infty^{\mathrm{C}} = -0.215 \times \alpha_0^{\mathrm{total}}(\vartheta = 0)$$

$$\mathcal{A}_0 = \frac{2.24}{10\pi}\frac{1}{N(0)\xi_0^3 T_c}\left(\frac{v_f}{c_0}\right)^3 \frac{m^*}{m}\left(1+\tfrac{1}{5}F_2^s\right)^2 \frac{T_c}{v_f}$$

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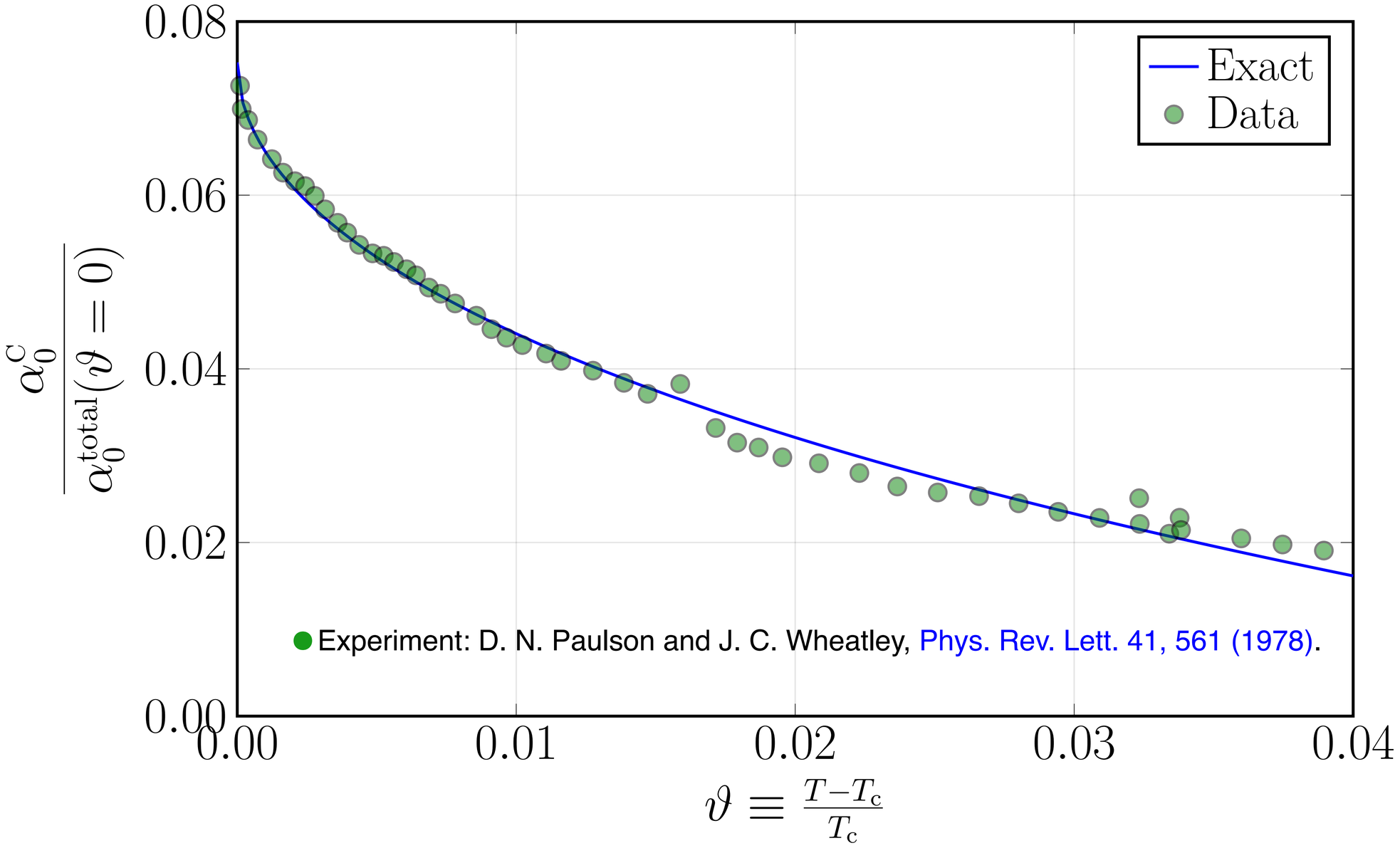

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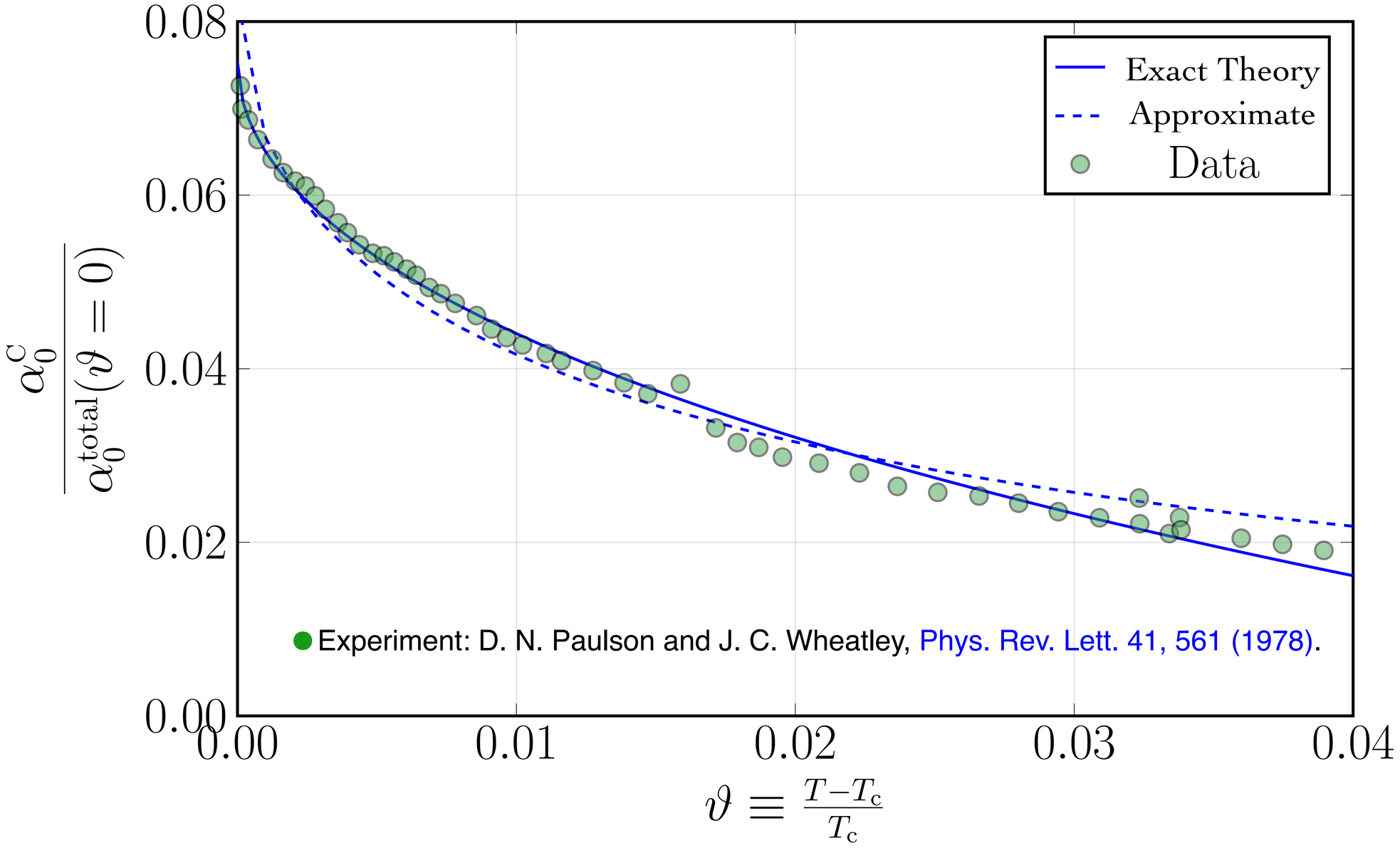

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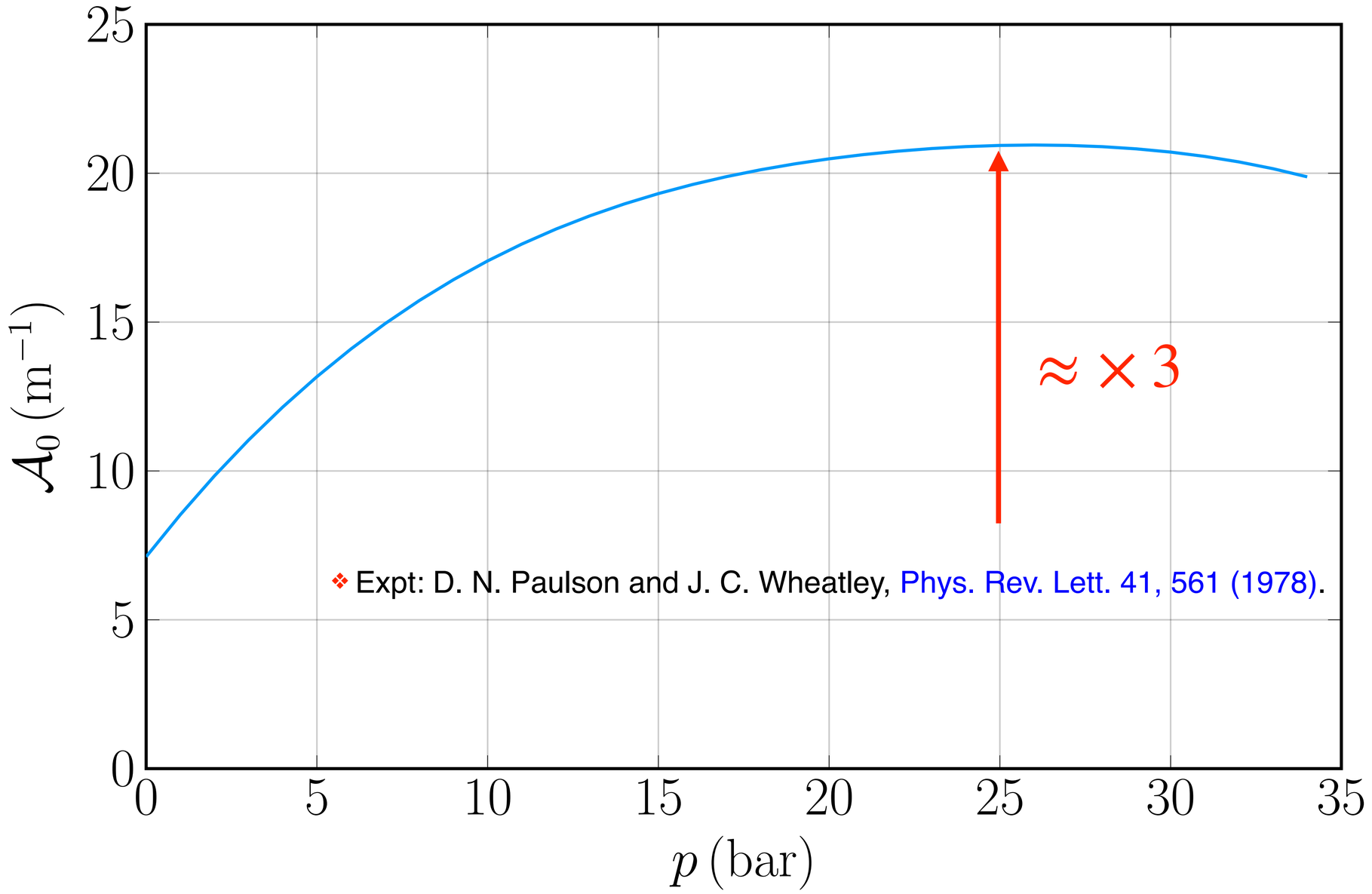
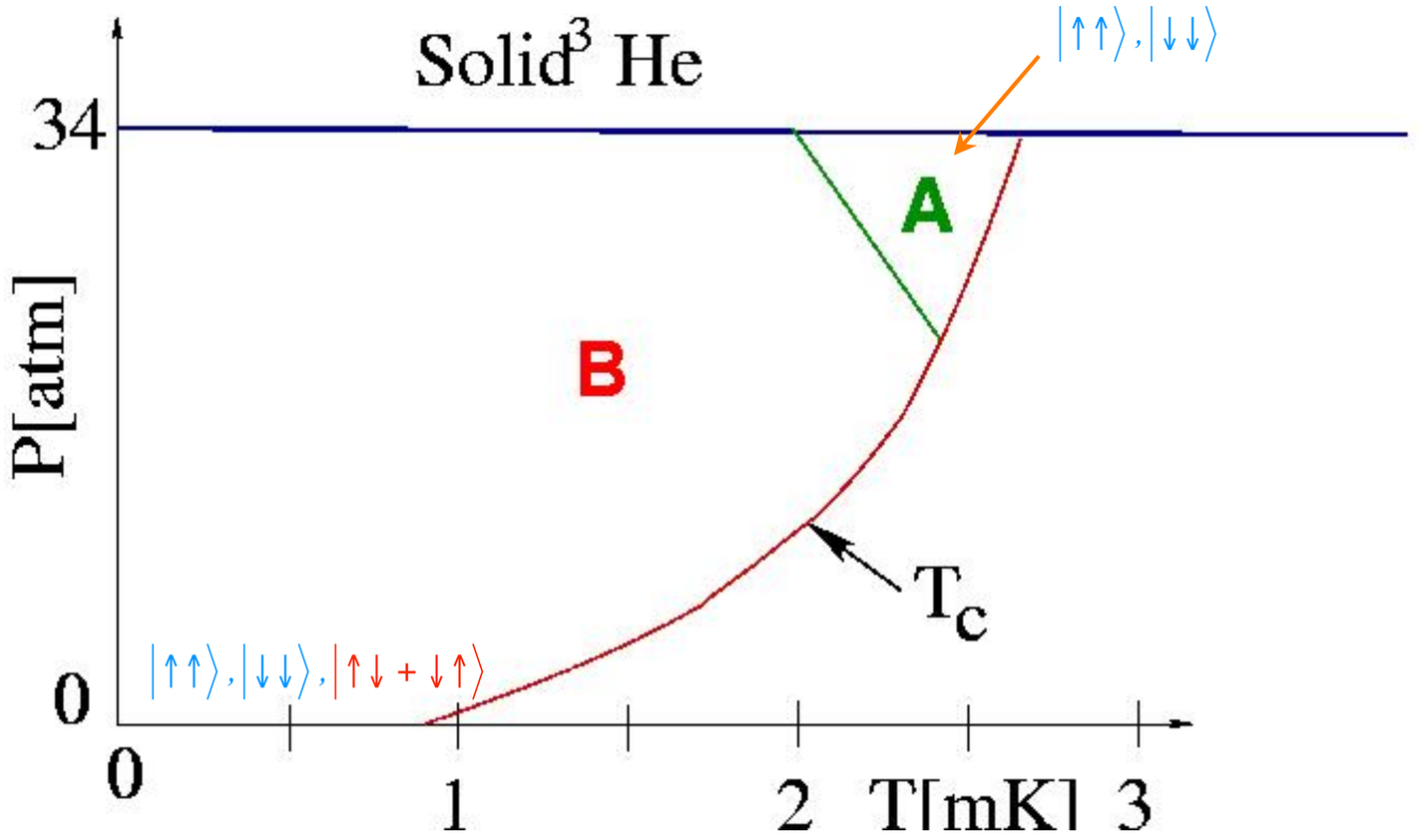


Nuclear Zeeman energy lifts the degeneracy of the Triplet States



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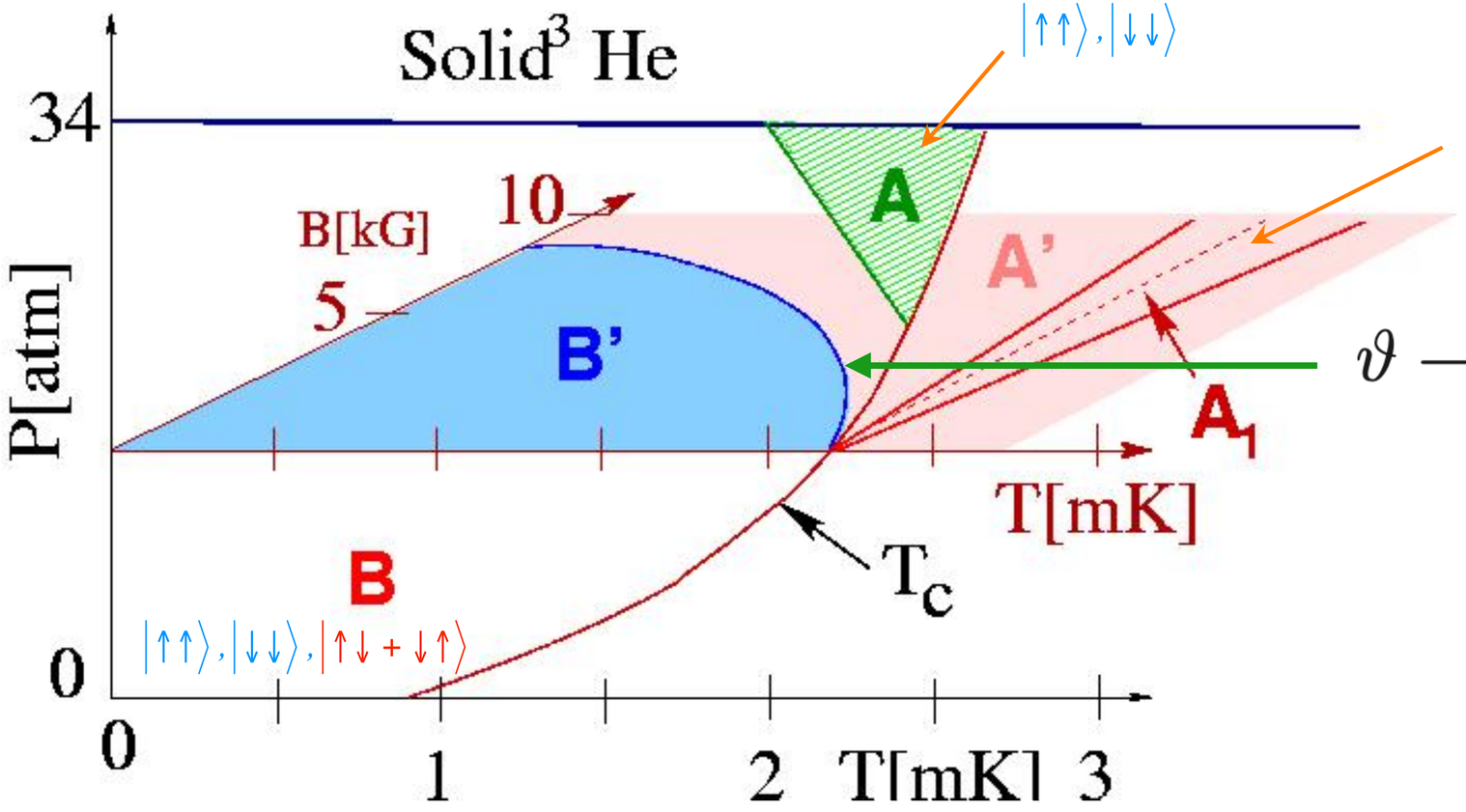

$$\vartheta \to \vartheta_{m_s} = (T - T_{c_{m_s}})/T_{c_{m_s}}$$

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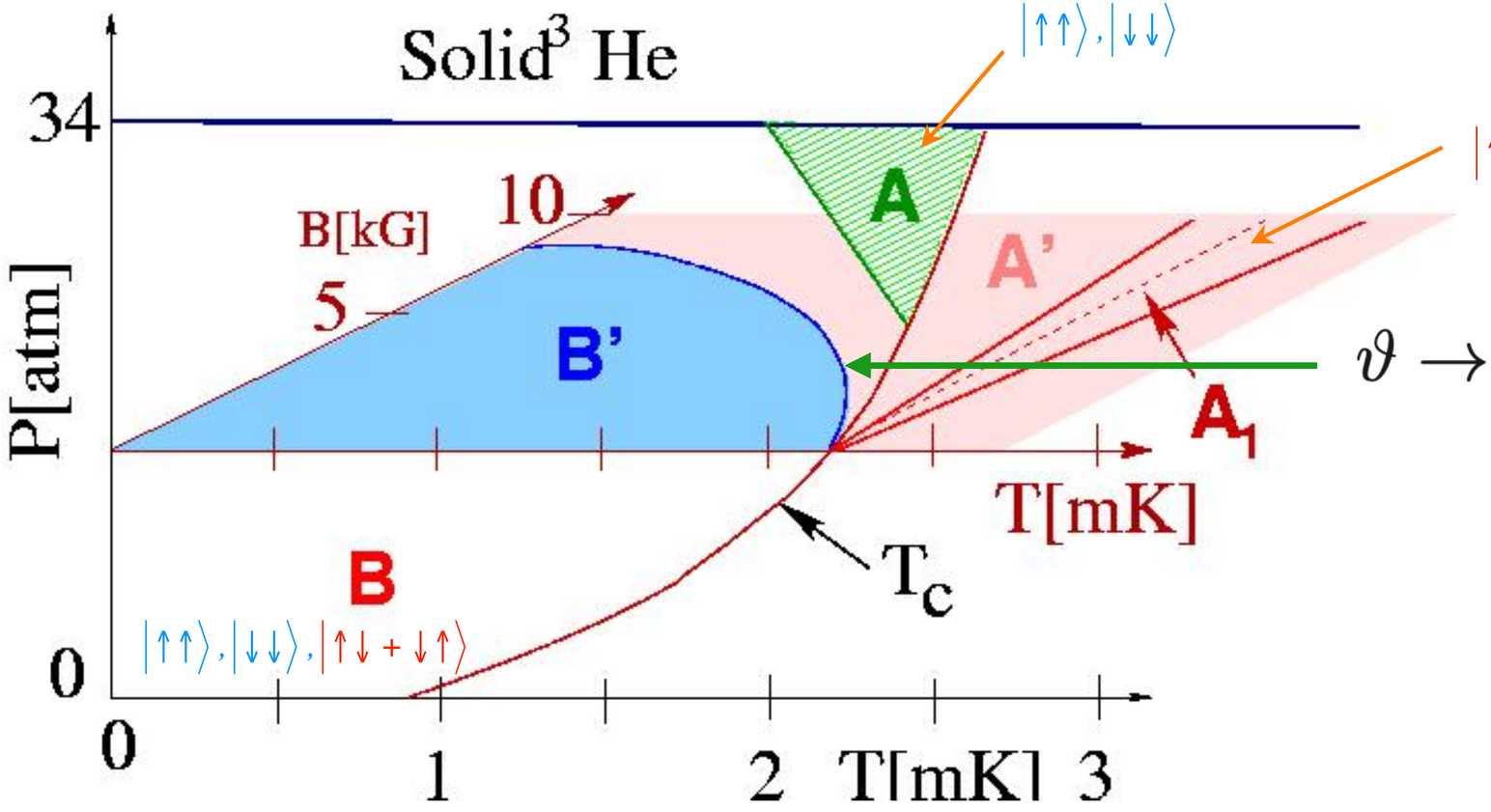

$$T_{c0} = T_c - \eta\, B^2$$

$$\eta = \simeq 2.4\,\mu\mathrm{K/kG}^2$$

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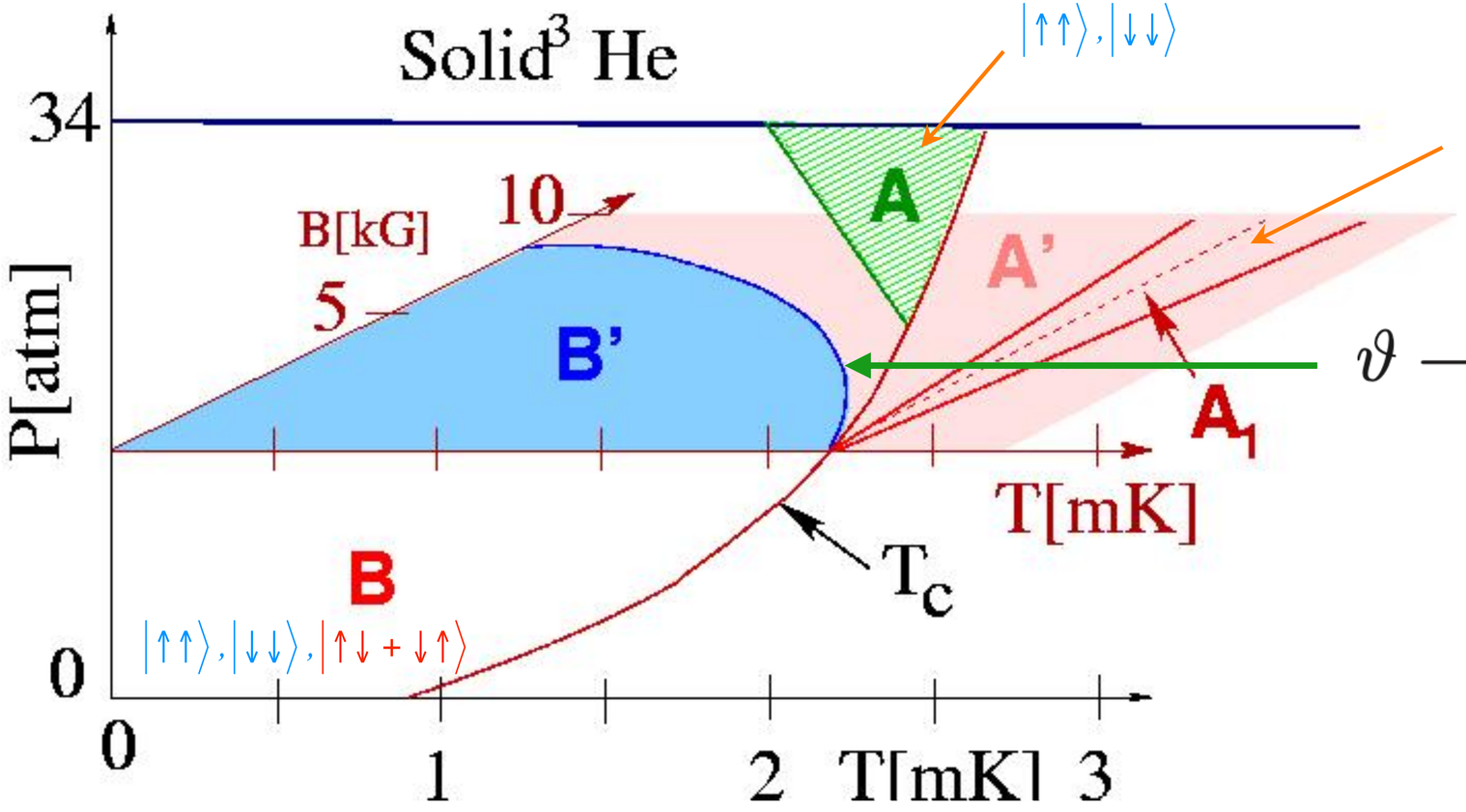

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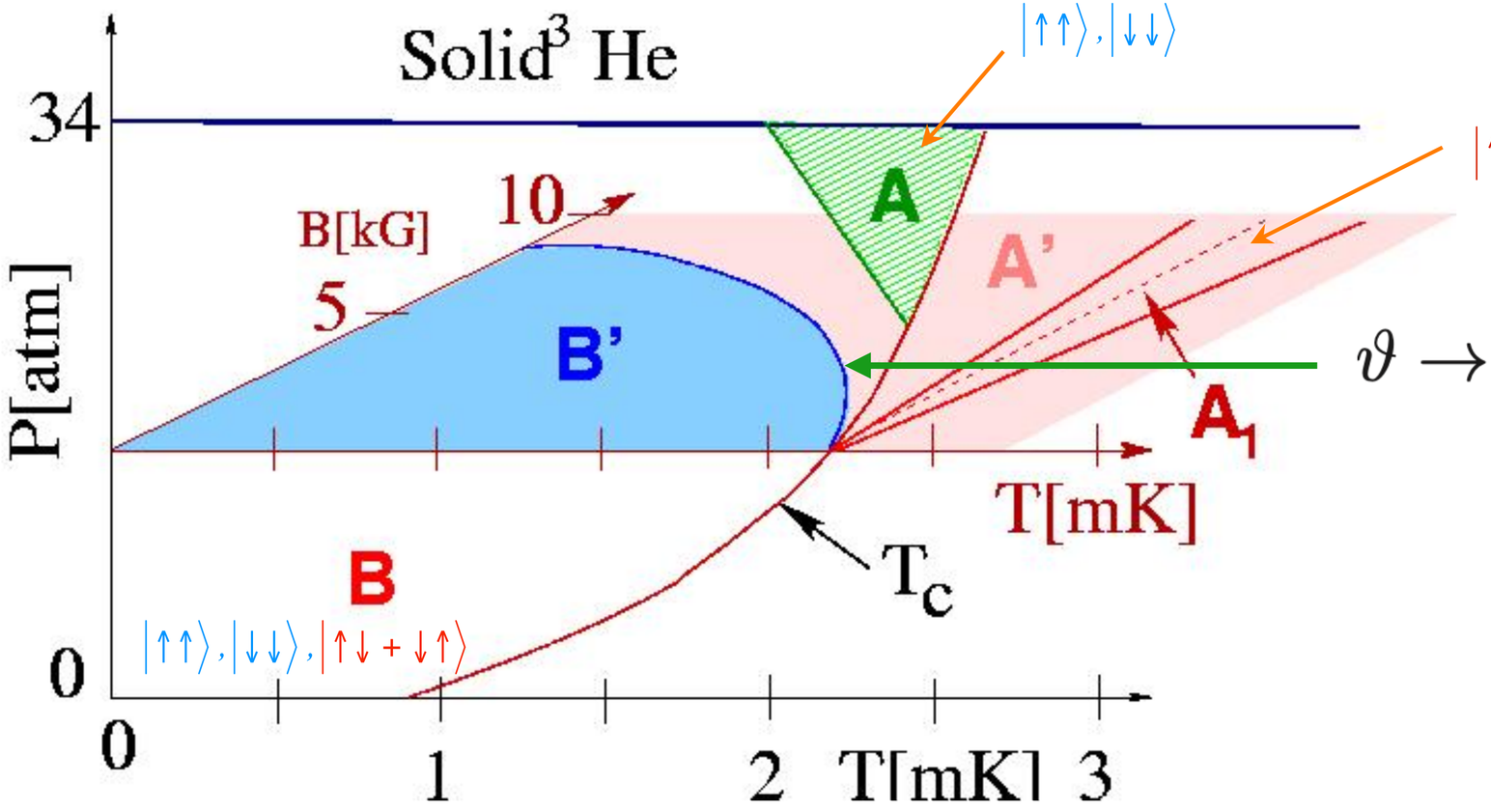

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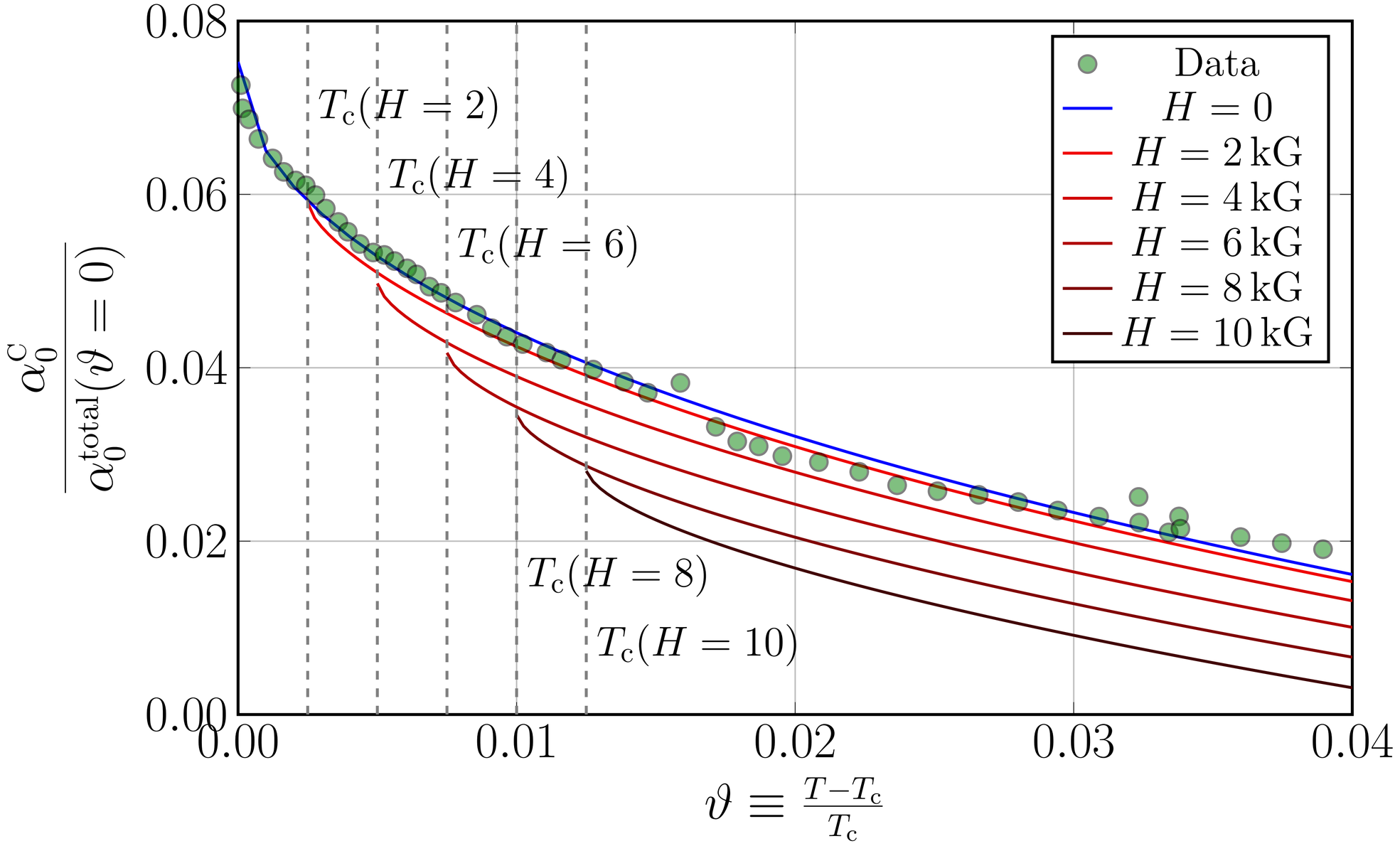
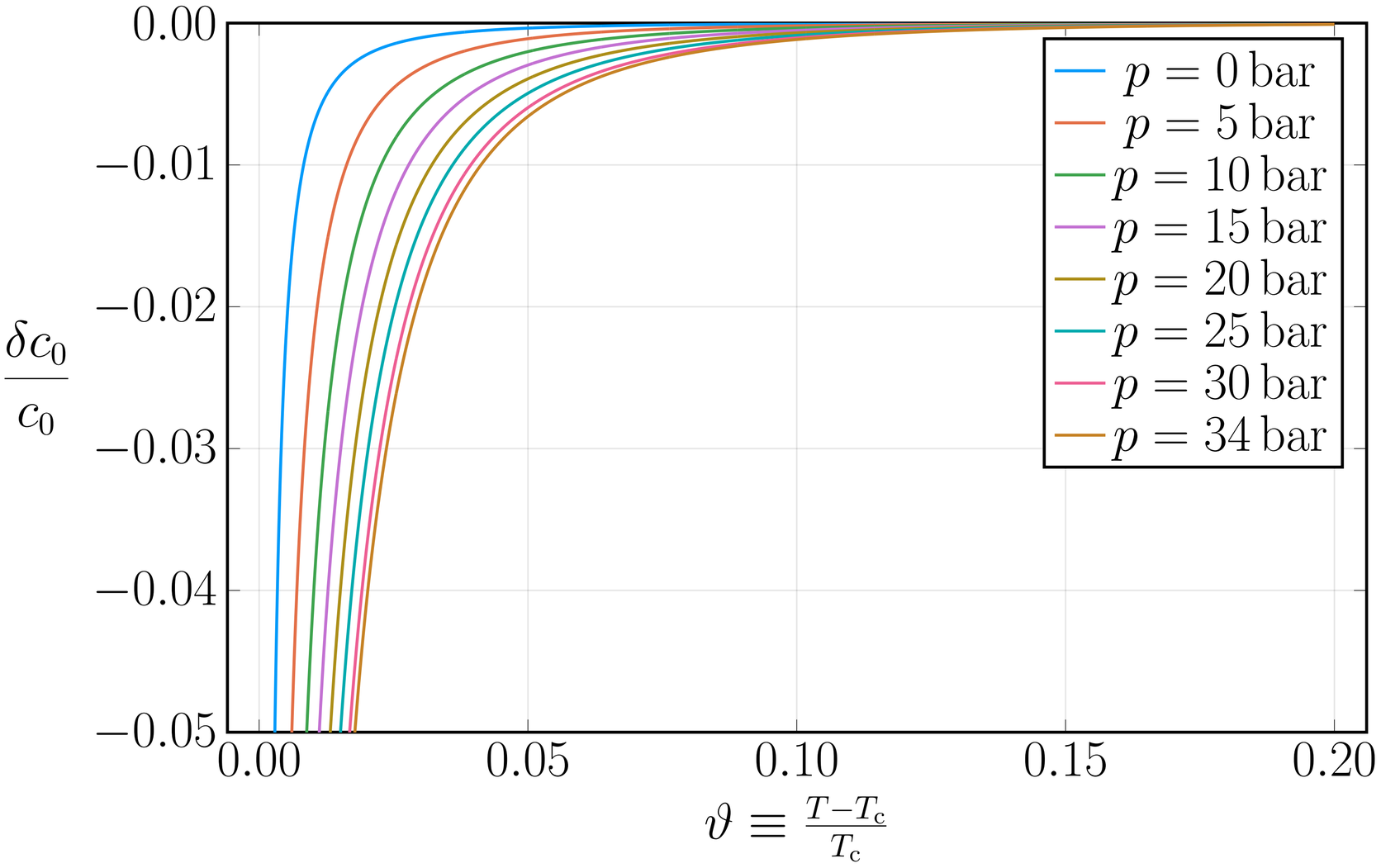
- **Suppresses the lower $T_{c_{m_s}}$ fluctuations**

$$\Gamma_{lm}(Q,\omega) \to \Gamma^{m_s}_{lm}(Q,\omega) = \frac{1}{\vartheta_{m_s} - i\omega\tau_0 + |\mathbf{Q}|^2\xi^2_{lm}}$$

Effect of the Zeeman Splitting of Spin-triplet pairs on the Fluctuation Correction to zero sound attenuation



Pairing Fluctuation Correction to Zero Sound Velocity



- Y. Lee, et al., JLTP **103**(5), 265 (1996)
- Observed $\delta c_0/c_0 \approx -30$ ppm
- Our theory is based on the thermal population of pair fluctuations
- Lee et al.: $\omega/2\pi = 113$ MHz
- Pair formation rate: $2\pi k_B T_c/\hbar$
- $\therefore\ \hbar\omega/2\pi k_B T_c = 0.86$
- Acoustic field likely suppresses pair fluctuations …
- Extend theory of fluctuations to the nonequilibrium regime $\hbar\omega \sim 2\pi k_B T$

❖Pairing Fluctuation Corrections to the Kinetic Theory of Liquid ^{3}He, W.-T.Lin & J. A. Sauls, Journal of Low Temperature Physics 208, 331–340 (2022)

Summary

- We derived corrections to the Landau-Boltzmann Transport equation for an interacting Fermi liquid in the vicinity of the Cooper instability based on Keldysh field theory and the quasiparticle self energies to $\mathcal{O}(\mathbf{s}^3)$ in the expansion parameters of Fermi liquid theory.

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- **Validation:** Enhanced attenuation of zero sound in liquid ^{3}He provides strong support for the feedback role of quasiparticle-pair-fluctuation scattering near T_c, and is in excellent agreement with the experimental report by Paulson and Wheatley.

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V. Emery's Heuristic Theory of Pair Fluctuation Corrections to Quasiparticle Transport

V. J. Emery , Fluctuations above the superfluid transition in liquid ^{3}He, Journal of Low Temperature Physics 22, 467–478 (1976)

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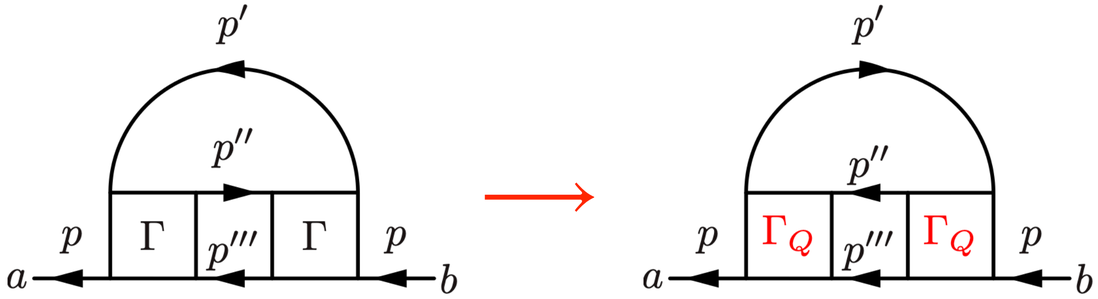
- In terms of the leading order Fermi liquid Self Energies Emery's method corresponds to this adhoc replacement:



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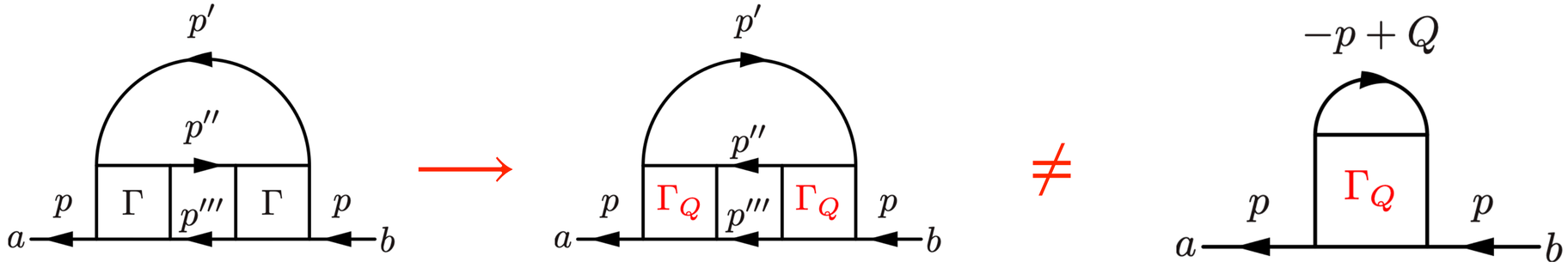

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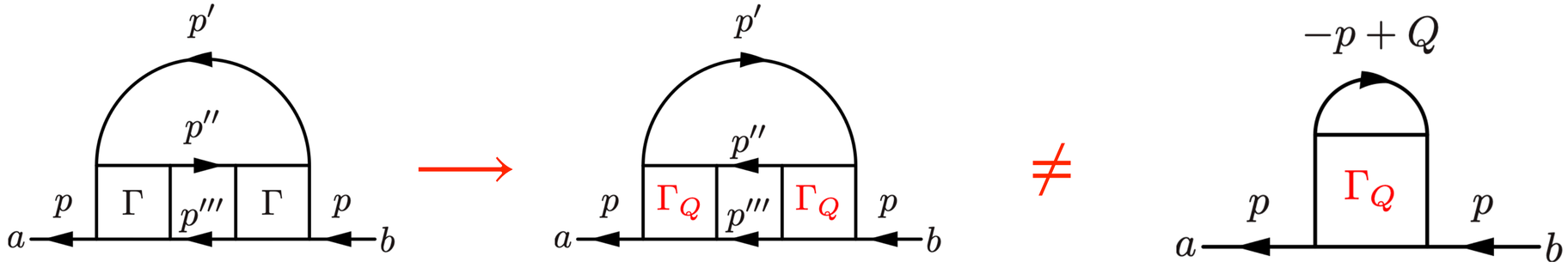

- Emery's result does not follow from Keldysh + Quasiclassical Transport Theory
- The adhoc replacement double counts the role of Cooper Pair Fluctuations
- Emery's result incorrectly *adds pairing fluctuation amplitudes coherently* in different channels (S, l) above T_c

Pairing Fluctuations and Transport in ^{3}He

Early Papers

- V. J. Emery, “Fluctuations above the superfluid transition in liquid ^{3}He”, (J. Low Temp. Phys. 22, 467, 1976)
- J .M. Parpia et al., “Viscosity of liquid ^{3}He-B near the superfluid transition”, (Phys. Rev. Lett. 40, 565, 1978)
- D. N. Paulson and J. C. Wheatley, “Incipient superfluidity in liquid ^{3}He above the superfluid transition temperature”, (Phys. Rev. Lett. 41, 561, 1978)
- V. K. Samalam and J. W. Serene, “Zero-sound attenuation from order-parameter fluctuations in liquid ^{3}He”, (Phys. Rev. Lett. 41, 497, 1978)
- A. Pal and P. Bhattacharyya, “Fluctuation contribution to the velocity and damping of sound in liquid ^{3}He above the superfluid transition temperature”,(J. Low Temp. Phys. 37, 379, 1979)
- Lee et al., “High frequency acoustic measurements in ^{3}He near the transition temperature”, (J. Low Temp. Phys. 103, 265, 1996)