Anomalous Thermal Hall Effects in Chiral Superconductors & ^{3}He

NSF DMR-1508730

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- Priya Sharma (Royal Holloway)

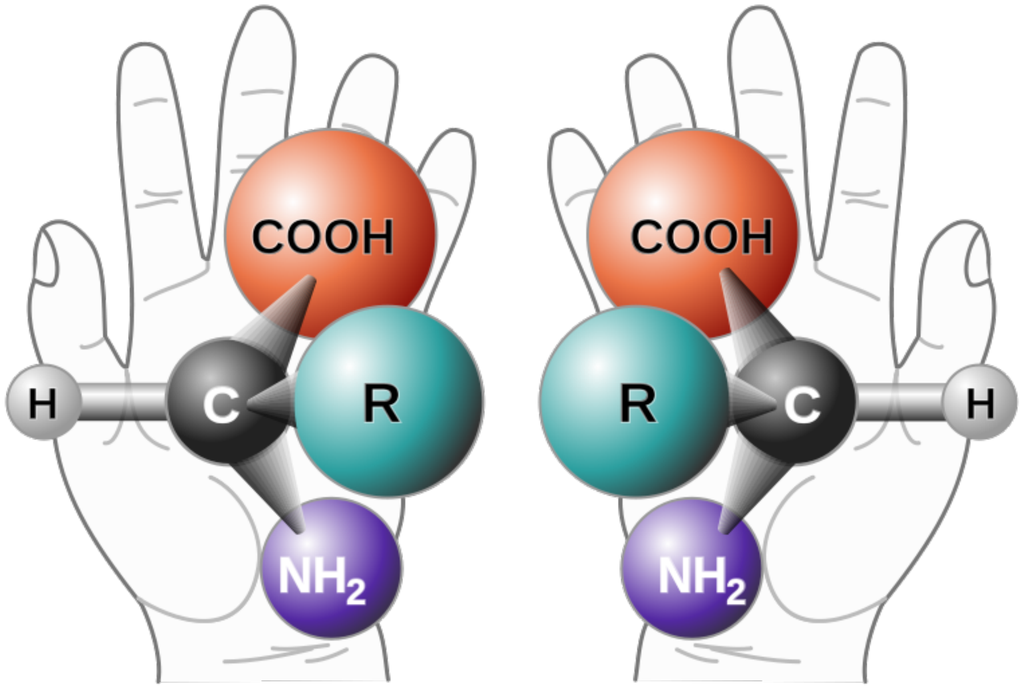
- K. Kono & H. Ikegami (RIKEN)
- W. Halperin (Northwestern)
- J. Parpia (Cornell)
- J. Saunders (Royal Holloway)

✓ *Electrons & Weyl Fermions in Superfluid ^{3}He-A*, O. Shevtsov & J. A. Sauls, Phys. Rev. B, 94, 064511, (2016)

✓ *Anomalous Thermal Hall Effect in Chiral Superconductors*, Wave Ngampruetikorn & J. A. Sauls, Phys. Rev. Lett., **124**, 157002 (2020)

✓ *Anomalous Thermal Hall Effect in Chiral Phases of ^{3}He-Aerogel*, Priya Sharma & J. A. Sauls, unpublished (2021)

Chiral Enantiomers



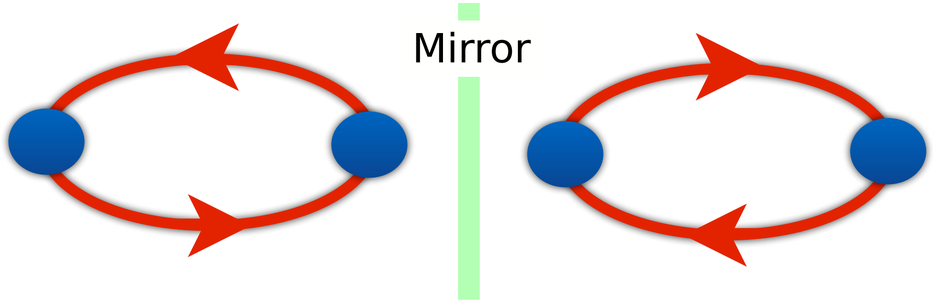
★ **Poly-atomic molecules**

Handedness: broken mirror symmetry

✳ **P. W. Anderson & P. Morel, PR 123 1911 (1961)**

Chiral Superfluids

$$\Psi = f(r)\ (x+iy)^{\nu}$$



- **Broken mirror symmetry**

$$\Pi_{xz}\Psi = f(r)\ (x-iy)^{\nu}$$

- **Broken time-reversal symmetry**

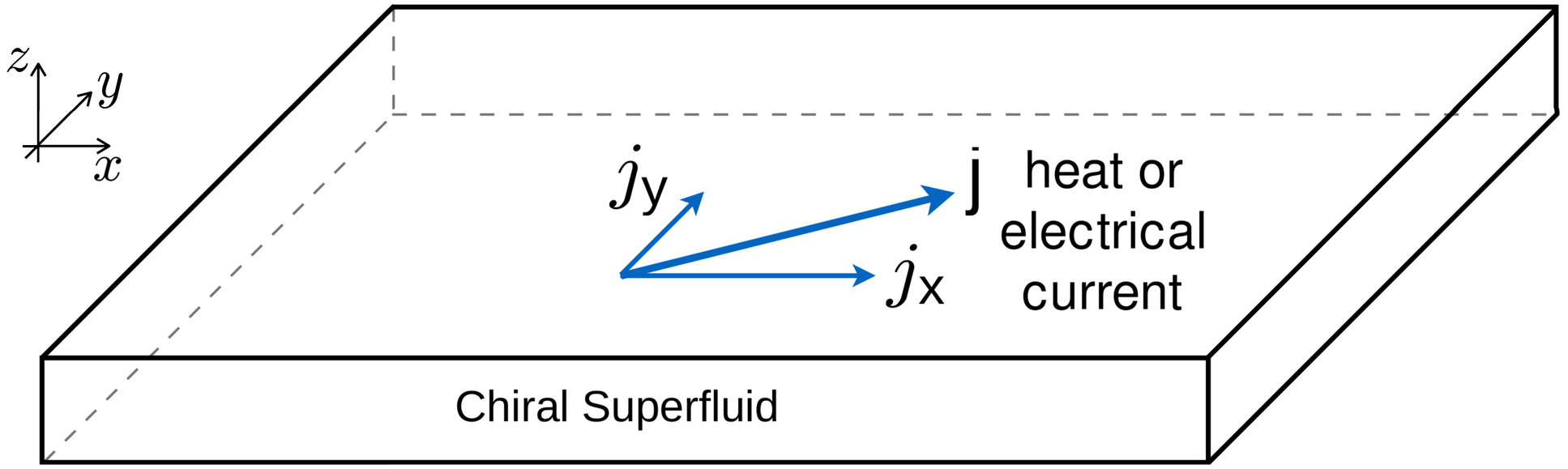
$$\mathrm{T}\Psi = f(r)\ (x-iy)^{\nu}$$

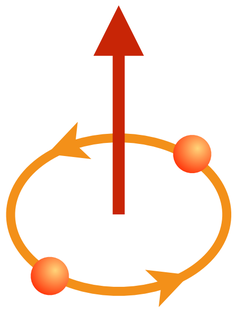
- **Chiral symmetry**

$$\mathrm{T}\Pi_{xz}\Psi = f(r)\ (x+iy)^{\nu}$$

✳ **Realized in Superfluid ^{3}He-A**

Anomalous Hall & Thermal Hall Effects in Chiral Superfluids





Cooper pairs

$\vec{E}$ or $-\nabla T$

with broken T and Π_{xz} symmetries

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{yy} \end{pmatrix} = \begin{pmatrix} E \\ 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} \kappa_{xx} & \kappa_{xy} \\ -\kappa_{xy} & \kappa_{yy} \end{pmatrix} = \begin{pmatrix} -dT/dx \\ 0 \end{pmatrix}$$

Anomalous Hall Effects are *Definitive Signatures of Chirality*

★ Hall effects require

➡ **Broken time-reversal symmetry**

➡ **Broken mirror symmetry**

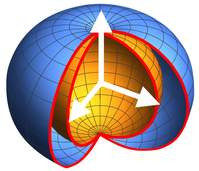
Both are broken in Chiral Superconductors

★ Combined with theory, Hall transport measurements can be used to

➡ **Identify Order parameter**

➡ **Chiral pairing & Chern number** ν

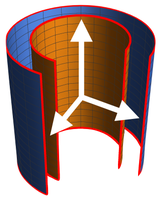
- — *Hall transport depends on both* ***winding number and gap structure***



^{3}He-A

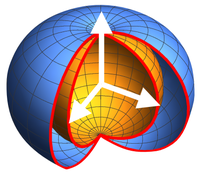
$|\nu| = 1$

2D
(**Sr_2RuO_4**)

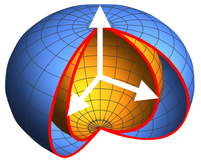


Eg_1 or Eg_2

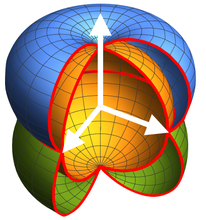
3D
(**UPt_3**)



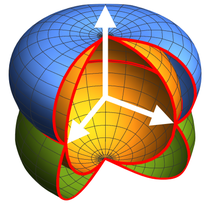
Eg_1



Eg_2

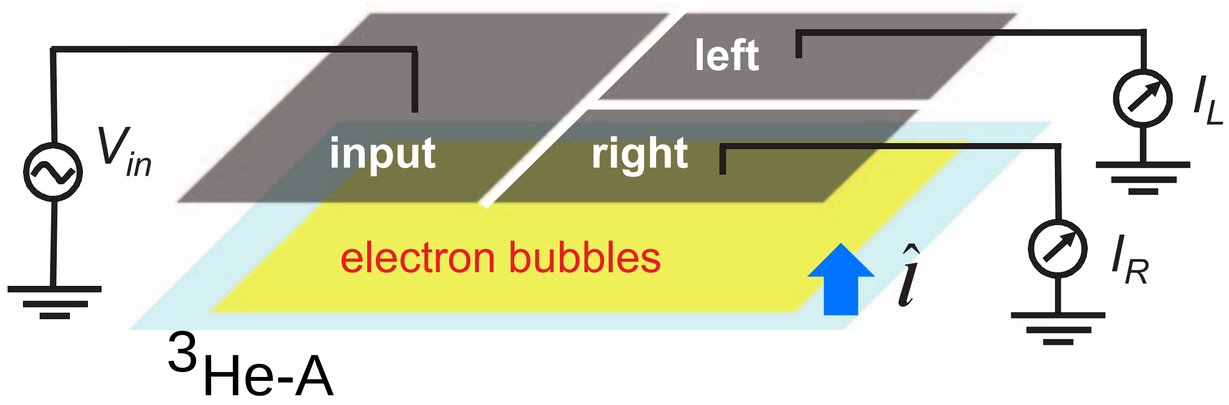


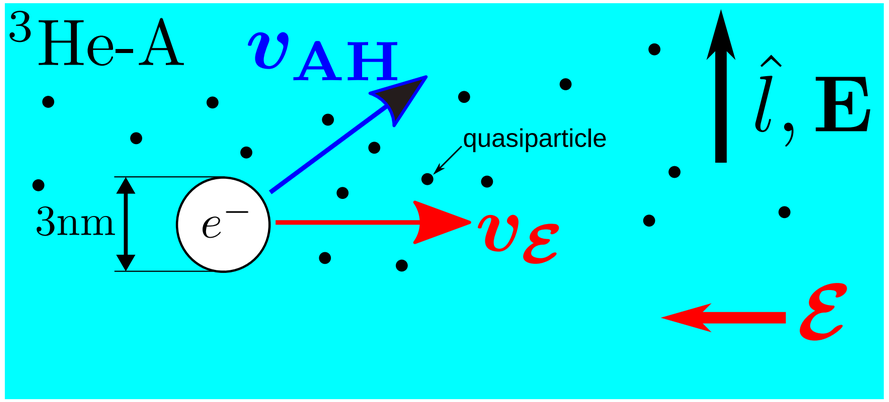
Eu_1



Eu_2

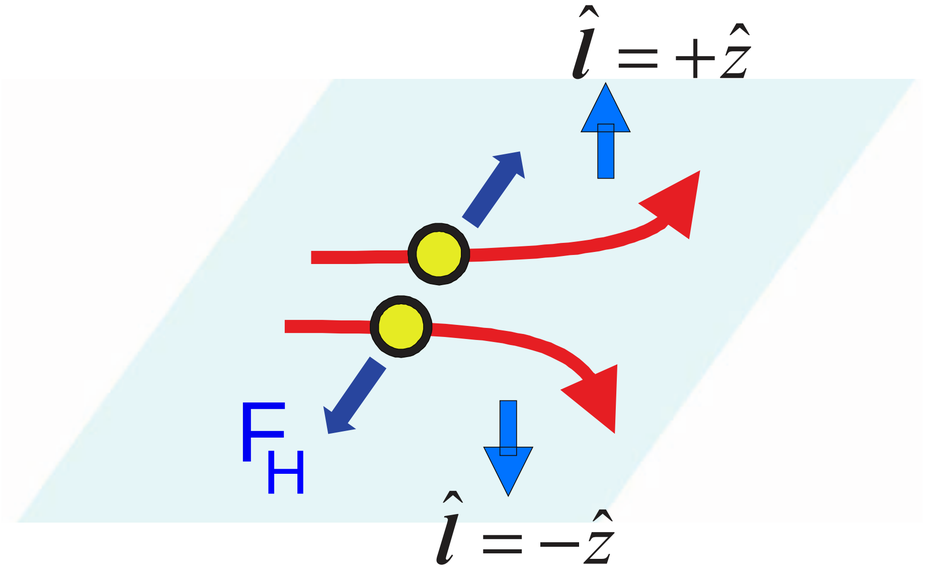
Anomalous Hall Effect for Electron Transport in ^{3}He-A





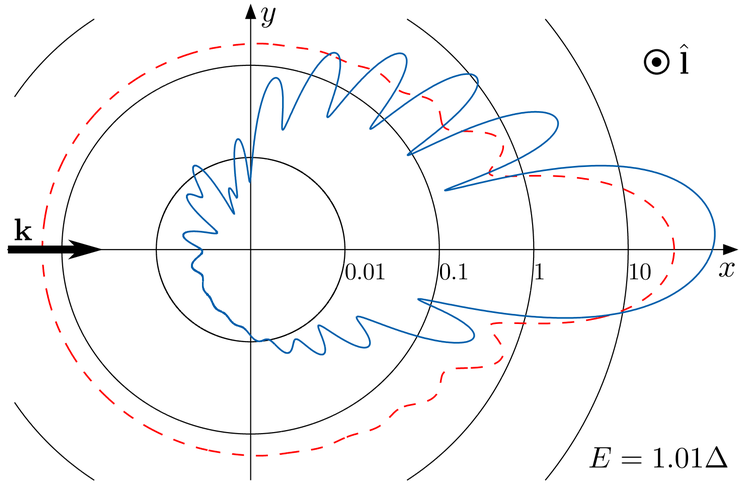
$$\vec{v} = \mu_{||}\,\vec{E} + \mu_{\mathrm{AH}}\vec{E}\times\hat{l}$$

R. Salmalin, M. Salomaa, V. Mineev, Phys. Rev. Lett.63, 868, (1989)



Ikegami, Tsutsumi & Kono, Science 341, 59 (2013)

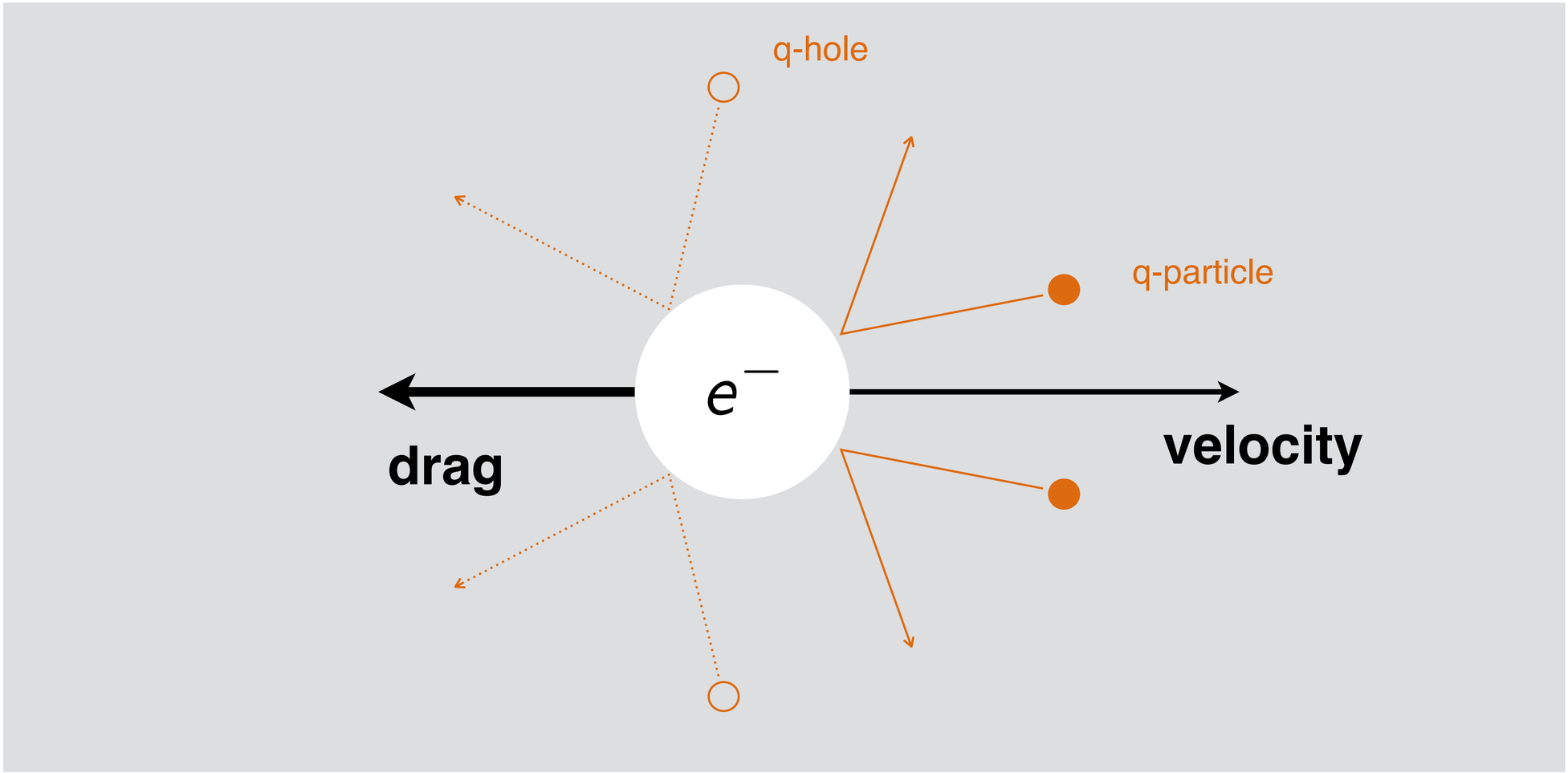
Resonant QP Skew Scattering by Chiral Edge States



O. Shevtsov & J. A. Sauls, Phys. Rev. B, 94, 064511, (2016)

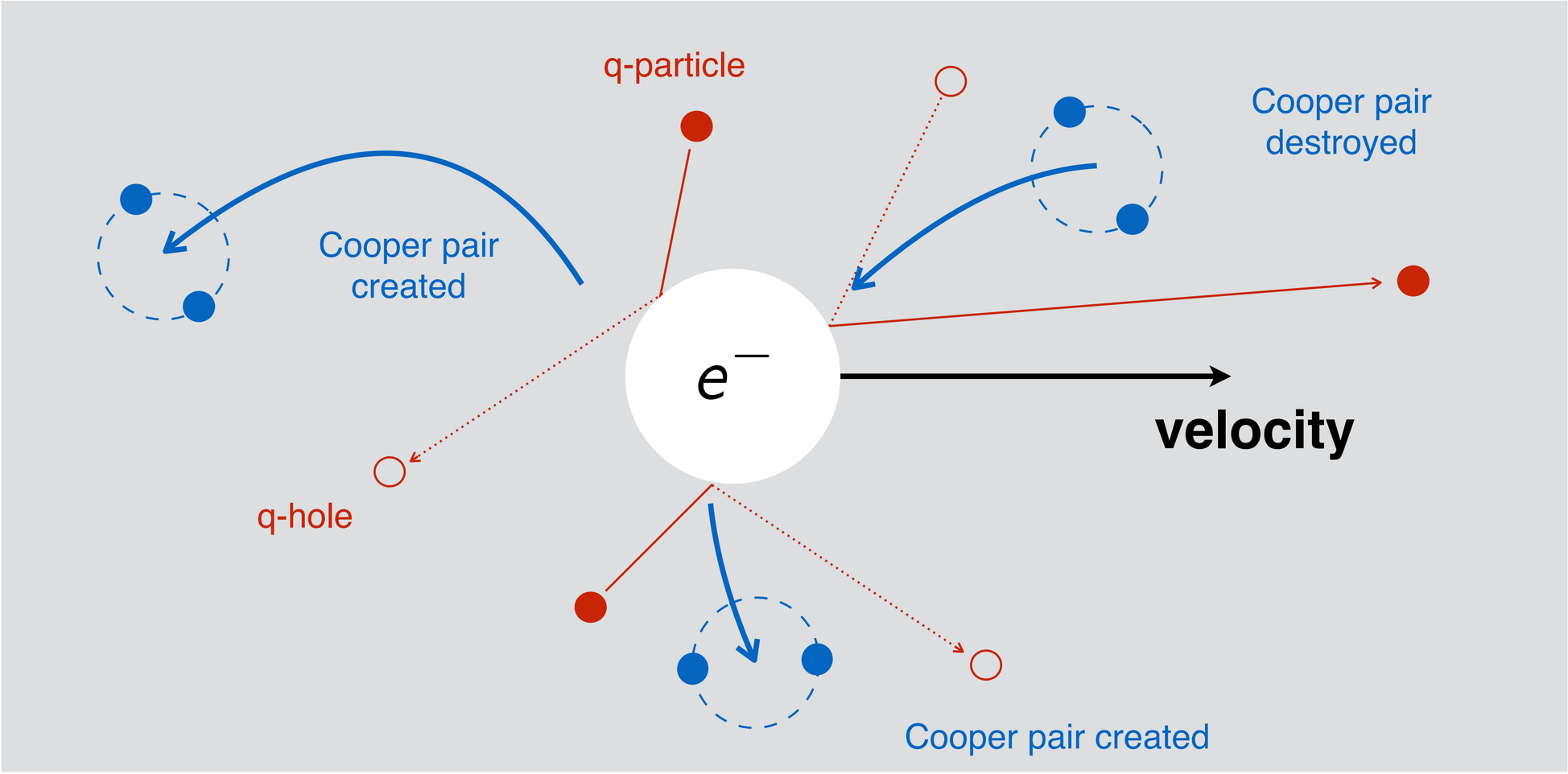
Scattering of quasiparticles off an e^- bubble in normal ^{3}He generates a ***Drag Force***

^{3}He thick film (top view)



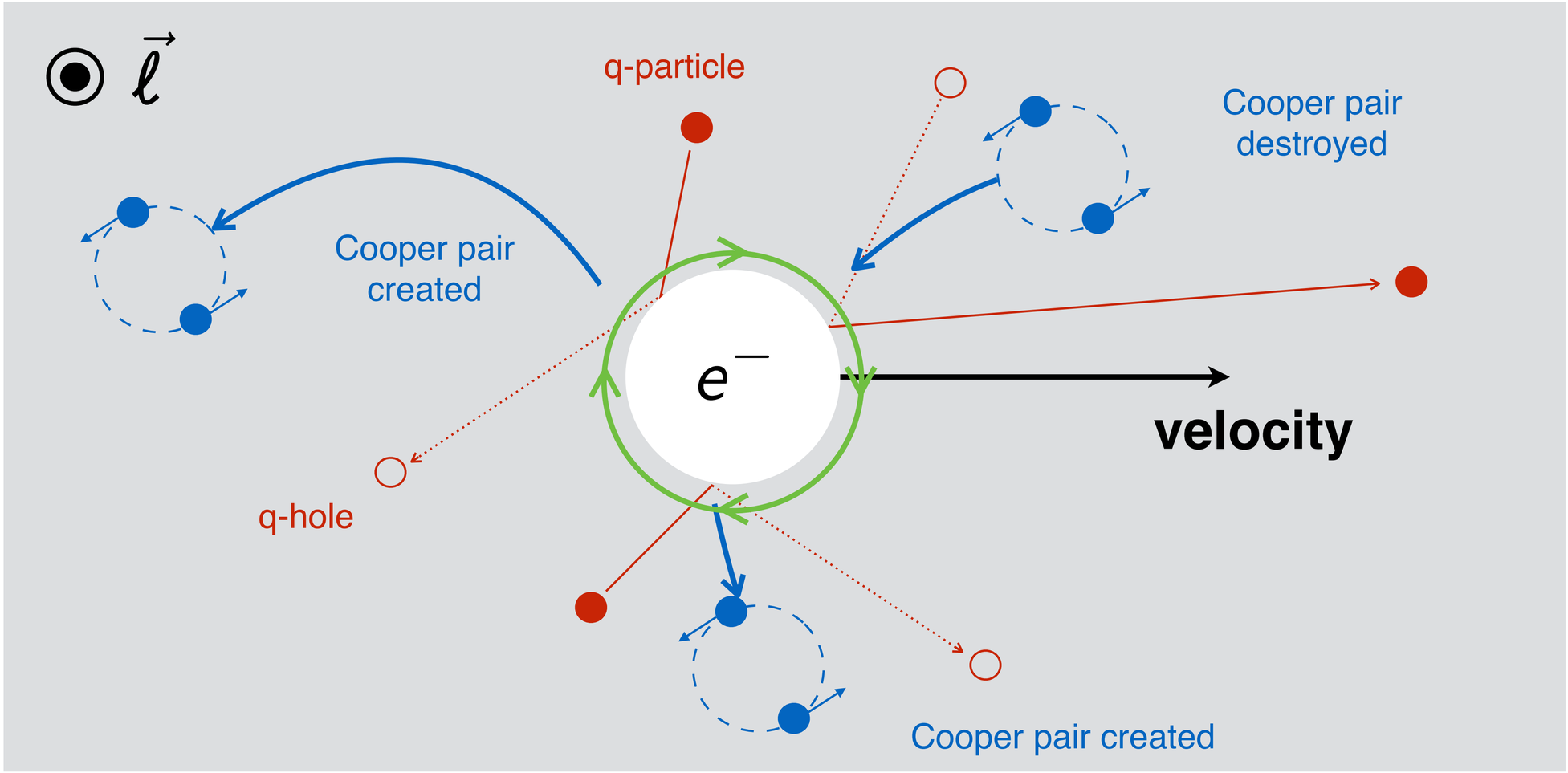
Andreev (branch conversion) scattering couples e^- bubbles to the condensate

Superfluid ^{3}He film (top view)



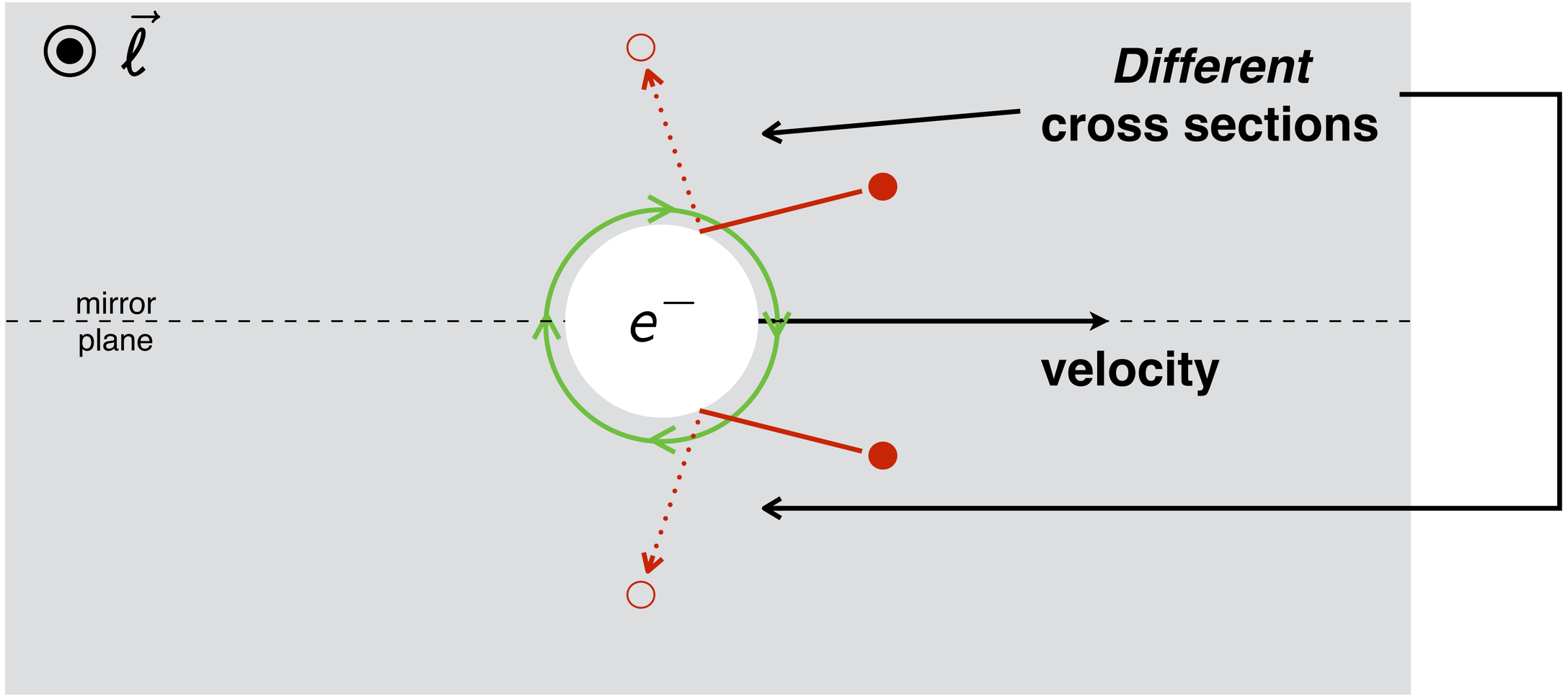
Coupling to a **Chiral Condensate** results in ***Chiral edge currents bound to the e- bubble***

^{3}He-A thick film (top view)



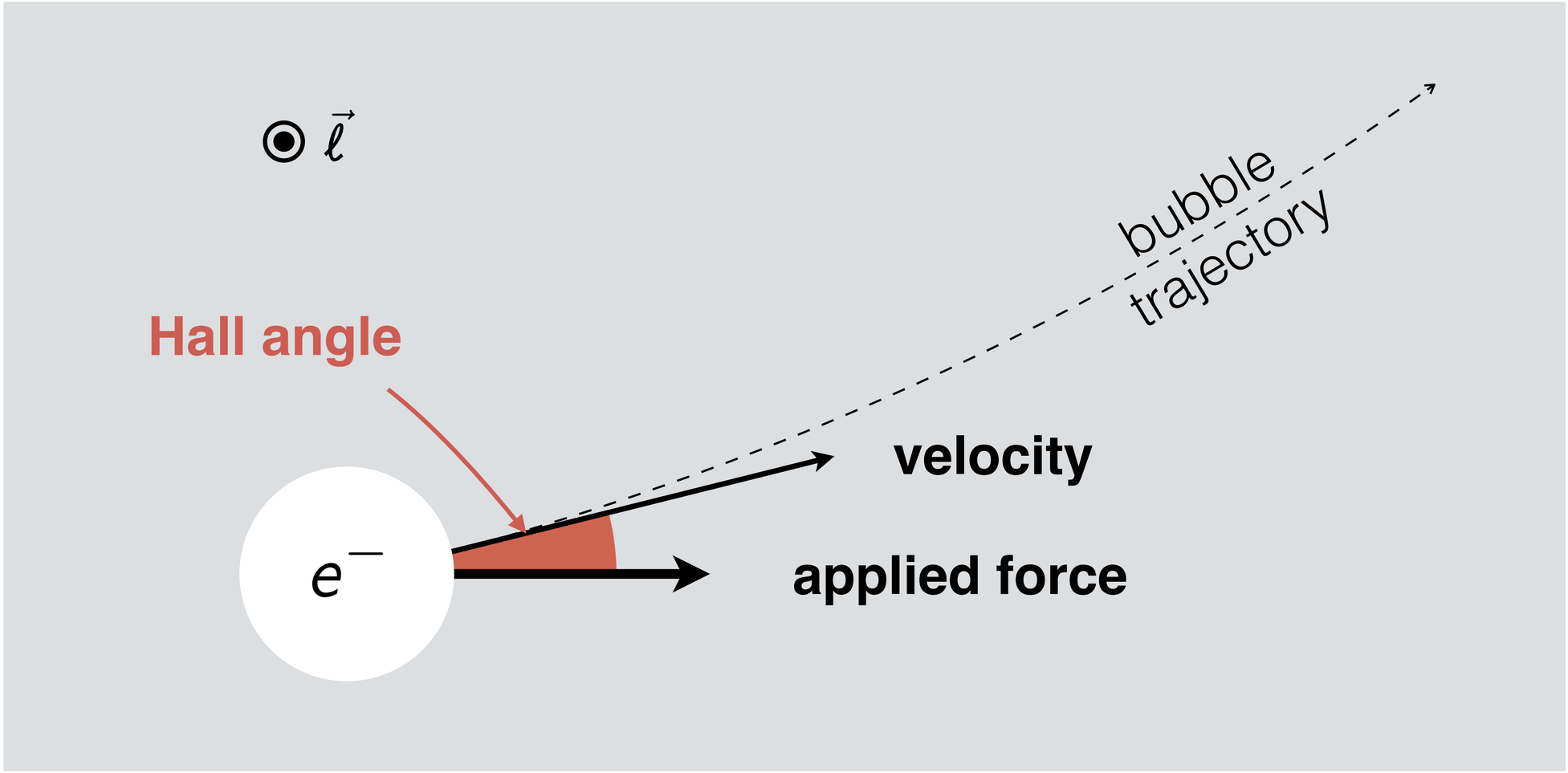
Edge current breaks mirror symmetry …

^{3}He-A thick film (top view)



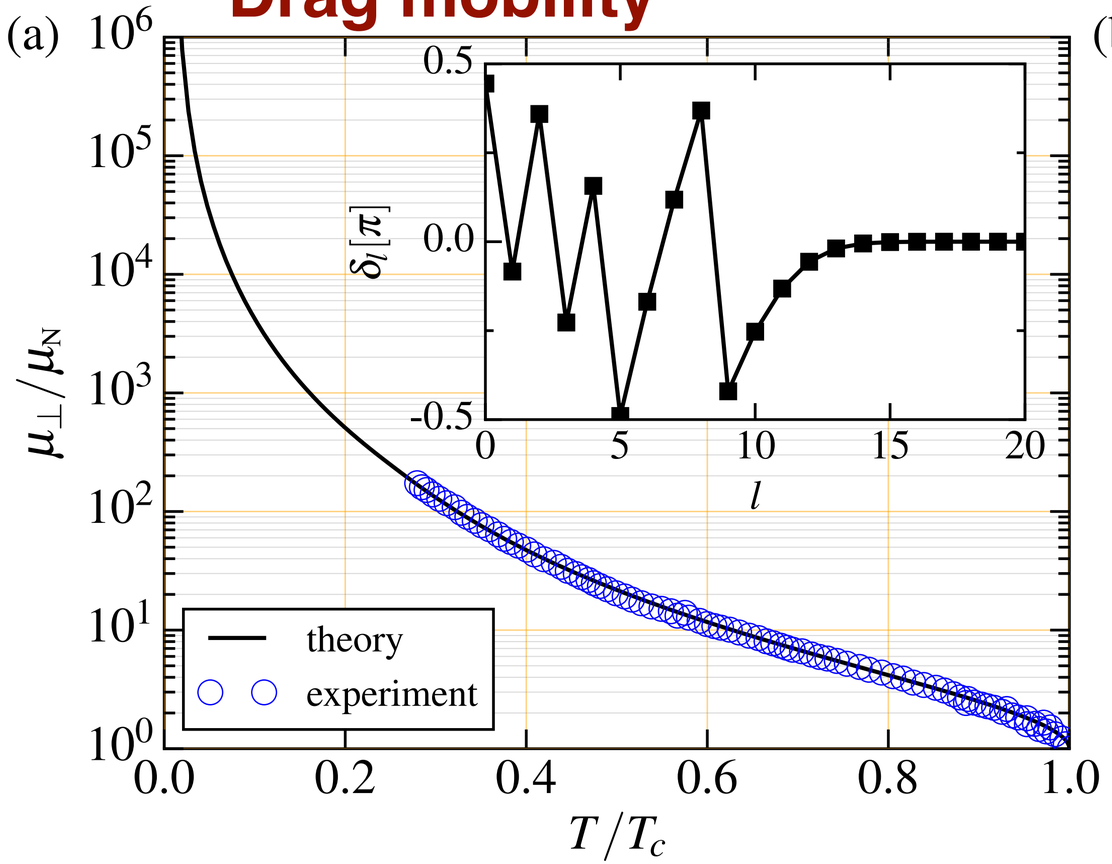
Scattering of quasiparticles by moving e- in ^{3}He-A generates a *Transverse Force*

^{3}He-A thick film (top view)

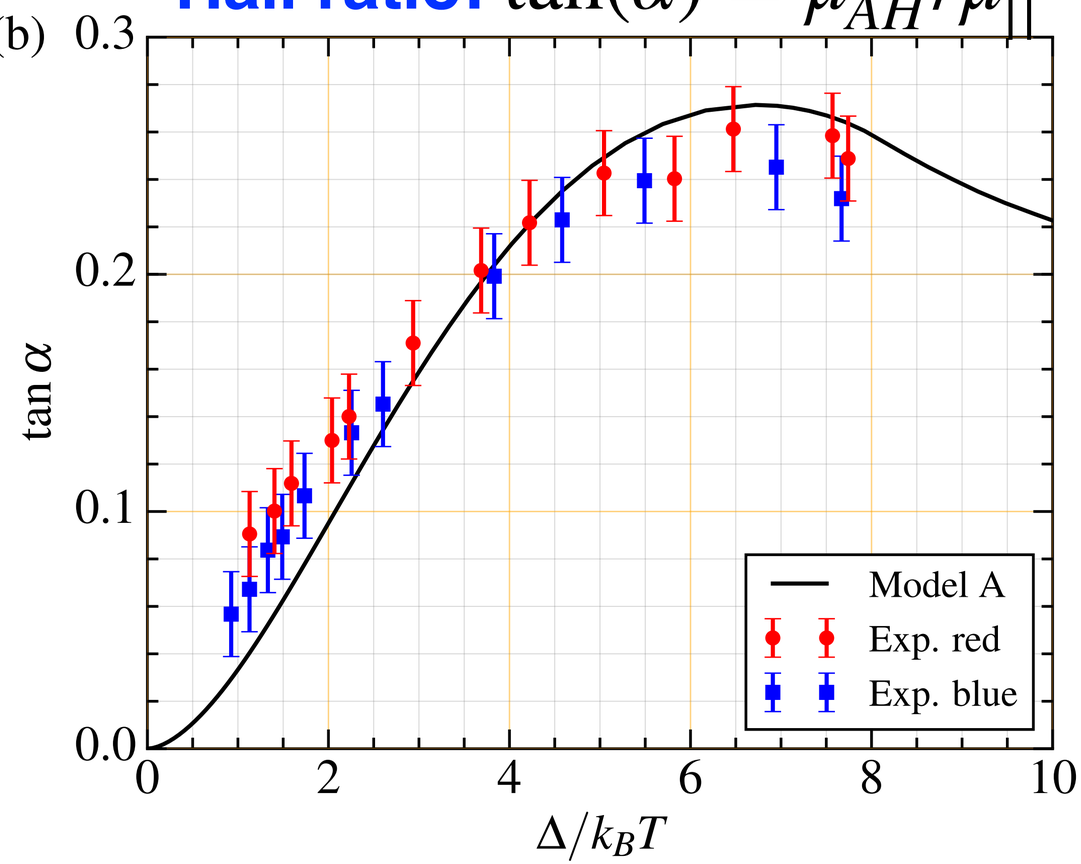


Scattering theory for e- bubbles moving through ^{3}He-A

Drag mobility



Hall ratio: $\tan(\alpha) = \mu_{AH}/\mu_{||}$

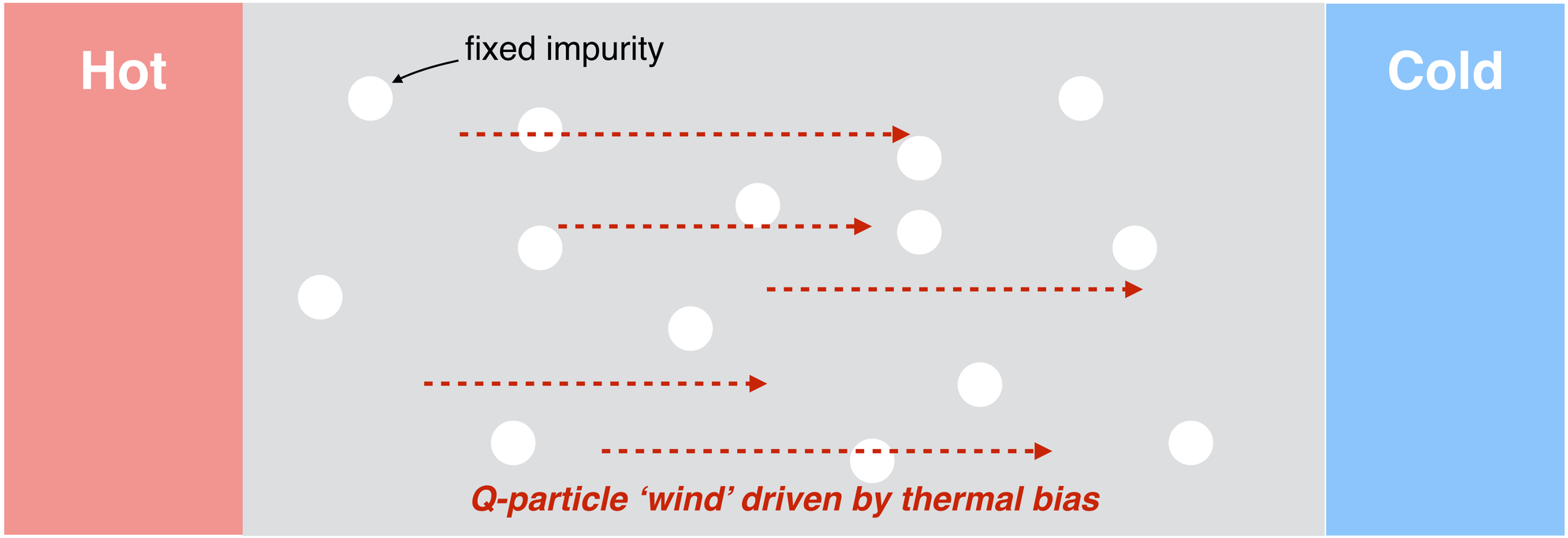


★ ***e⁻ bubble is a hard sphere to quasiparticles with ironic radius*** $k_fR = 11.17$

✓ **Experiment: Ikegami, Tsutsumi & Kono, Science (2013)**

✓ **Theory: Shevtsov & Sauls, PRB (2016)**

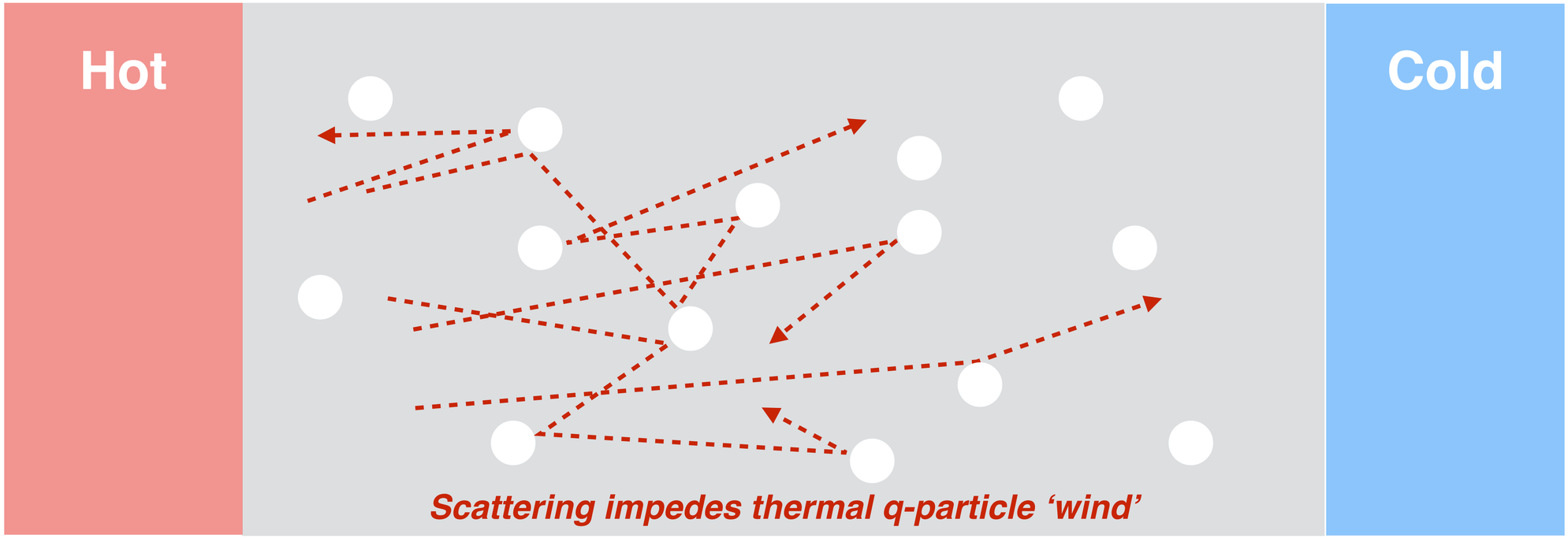
What happens when a quasiparticle ‘wind’ blows past fixed impurities?



Net heat current

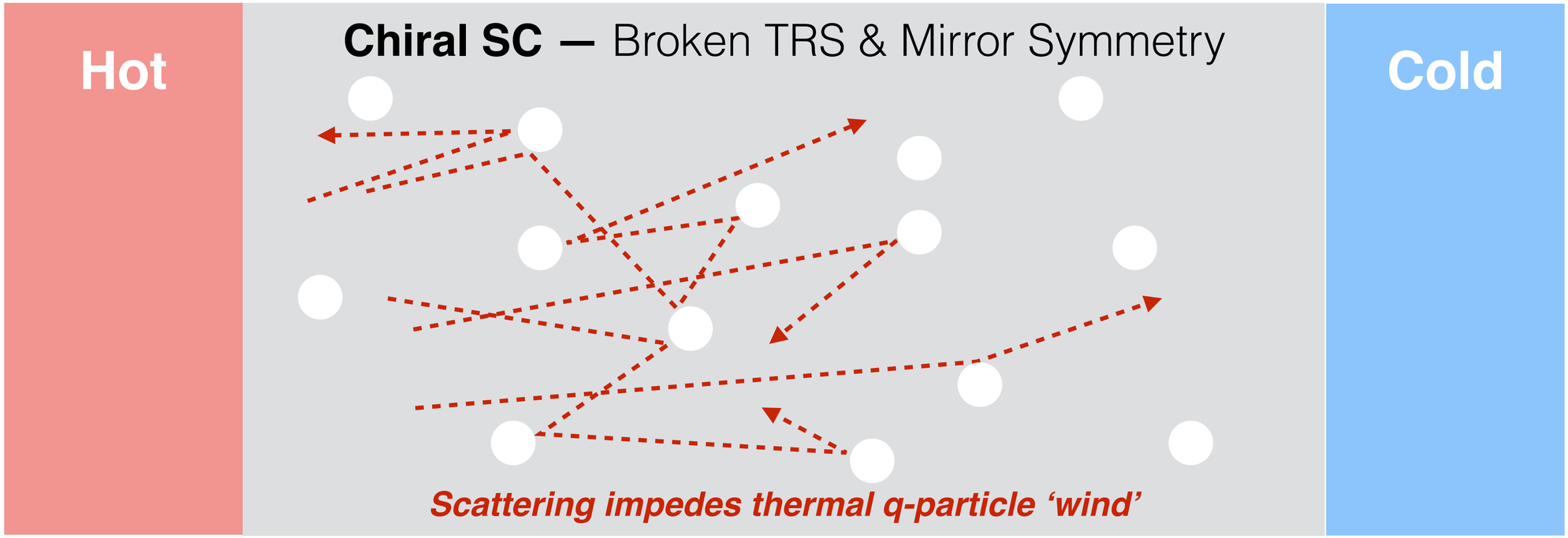
thermal bias

Scattering leads to diffusion & reduces the heat transport current

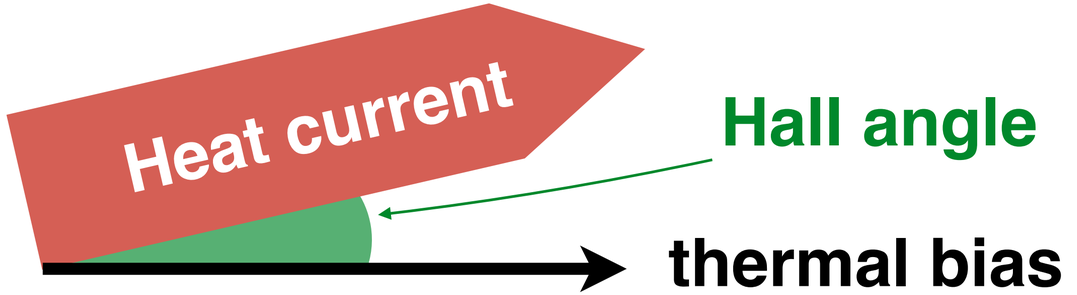


In Chiral Superconductors: **Asymmetry in the scattering cross section 'kicks' q-particles** $\perp (-\vec{\nabla} T)$

The ``reverse'' of e⁻ bubble transport in ³He-A



Impurity scattering induces
Anomalous Thermal Hall Effect!



Anomalous Thermal Hall effects arise from the ***edge*** and the ***bulk***

Edge Hall Effect

- For Chiral *p*-wave states
- Read & Green, PRB 61,10267 (2000)

$$K_{xy}^{\mathrm{edge}} = \nu \, \frac{\pi \, k_{\mathrm{B}}^2 \, T}{6\hbar}$$

Bulk Hall Effect

★ Induced by *impurity scattering*

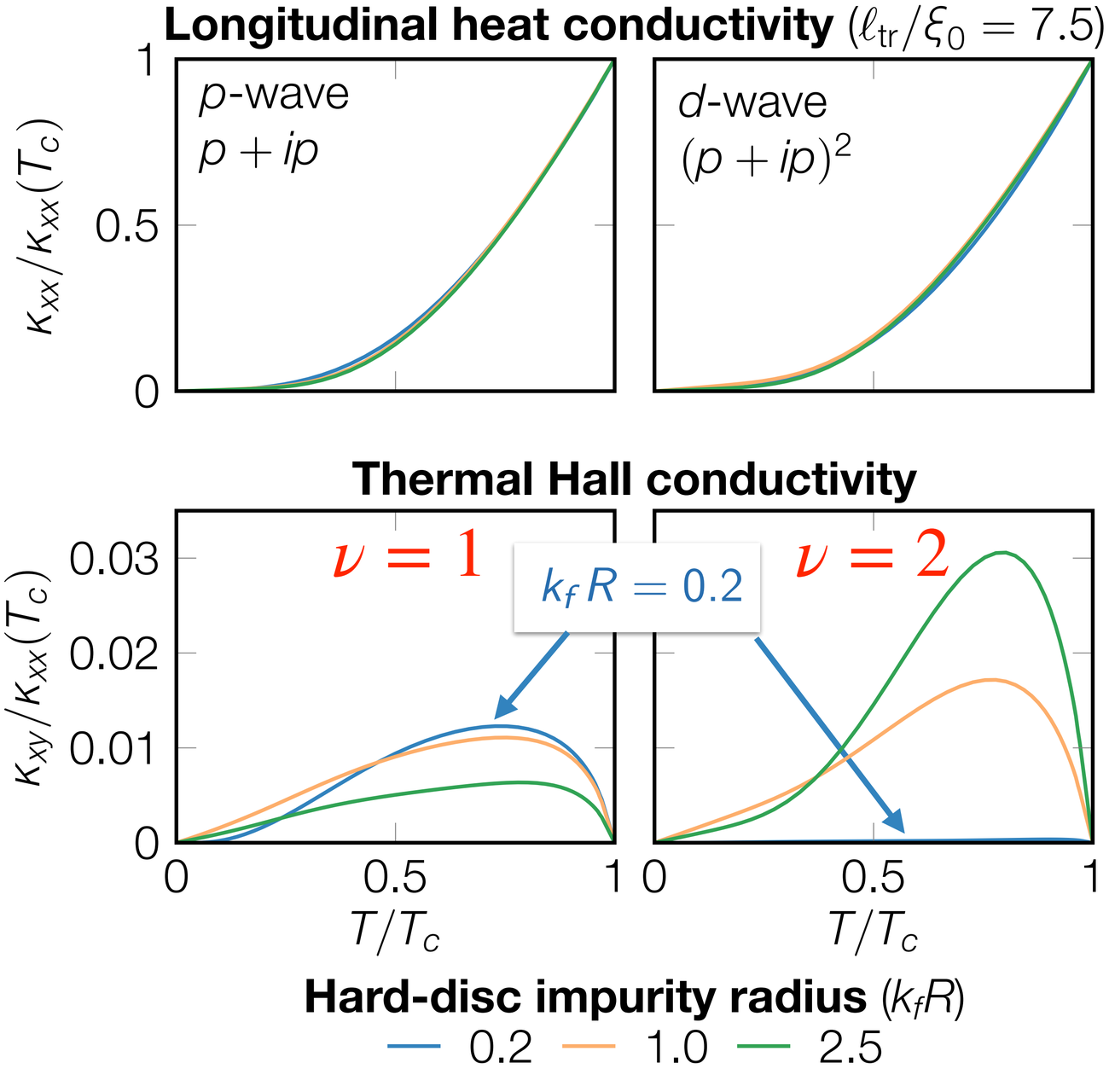
★ Typically dominates edge effect

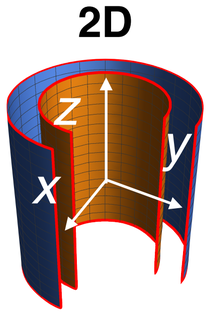
★ Ngampruetikorn & JAS, PRL **124**,157002 (2020)

This talk

✓ Both *require* **Broken TRS & Mirror Symmetry & non-trivial topology**

Thermal conductivity K_{xx} & Hall conductivity K_{xy}



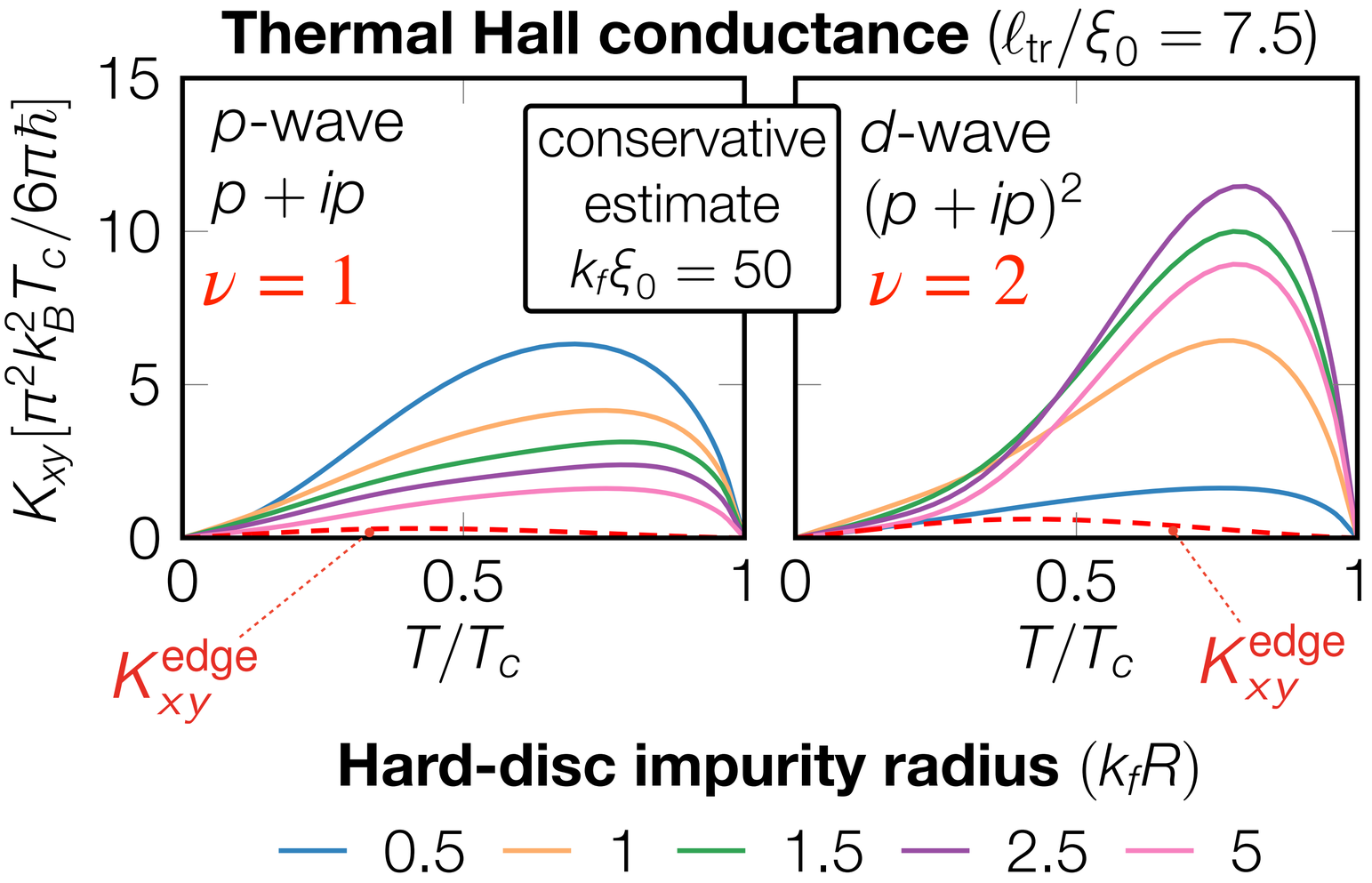


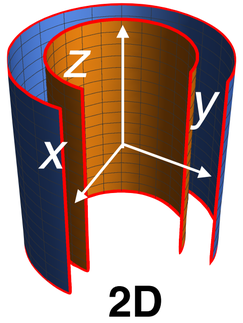
- **K_{xx} is insensitive** to impurity size and winding number ν
- **K_{xy} is sensitive** to impurity size and winding number ν

★ *Probe of symmetry & topology*

- **K_{xy} is supressed** for small impurity size, $k_f R < 1$, for chiral superconductors with $|\nu| \geq 2$.
- **Only for $|\nu| = 1$** is K_{xy} is finite for small impurity size, $k_f R < 1$.

Bulk Hall effect dominates the Edge Hall conductivity





Fully gapped Topological SC

Bulk Hall conductivity is ~2 orders of magnitude larger than the edge contribution!

$$K_{xy}^{bulk} \gg K_{xy}^{edge}$$

★ Edge Theory - Berry phase contribution: Qin, Niu & Shi, PRL 107, 236601 (2011)

Where to look for the Impurity-induced Anomalous Thermal Hall Effect?

- **^{3}He in ``anisotropic silica aerogel'' — Theory ⇝2 distinct Chiral phases**

  Priya Sharma poster group B **J. A. Sauls, PRB B 88, 214503 (2013)**

- **^{3}He-A confined in ~100 nm cavities - surface scattering ↔ impurity scattering**

- **UPt_3 — Thermal Hall would confirm broken PT in the low temperature B-phase**

- **Sr_2RuO_4 — ATHE would provide bulk probe for chiral pairing**

- **Test of recent proposals for broken PT in other materials: SrPtAr, $LaPt_3P$, UTe_2**