International Conference on Quantum Fluids and Solids, University of Tokyo, July 25, 2018

Frontiers in Quantum Matter
Symmetry, Topology & Strong Correlation Physics

J. A. Sauls

Northwestern University

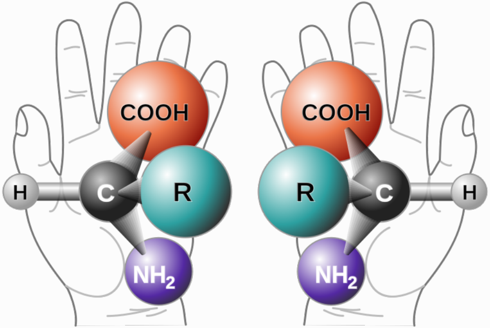
• Wave Ngampruetikorn • Takeshi Mizushima • Robert Regan • Oleksii Shevtsov • Joshua Wiman

- Chiral Fermions & Anomalous Hall Transport
- Quanta of a Superfluid Vacuum
- Strong Correlation Physics in ^{3}He
- Low Temperature Physics at 10^8 Kelvin

- Supported by National Science Foundation Grant DMR-1508730

Chiral Quantum Matter

Molecular Chiral Enantiomers



Handedness: Broken Mirror Symmetry

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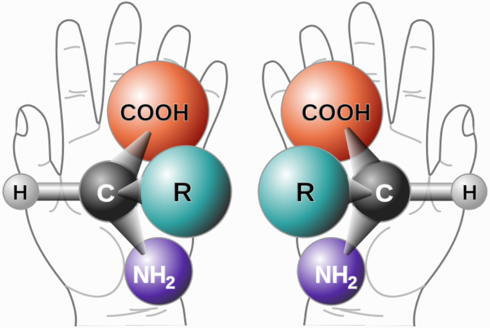

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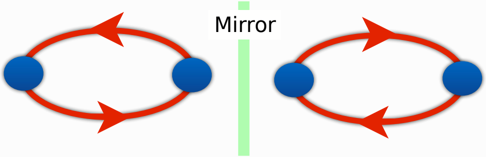
Chiral Diatomic Molecules

$$\Psi(\mathbf{r}) = f(r)\,(x + iy)$$



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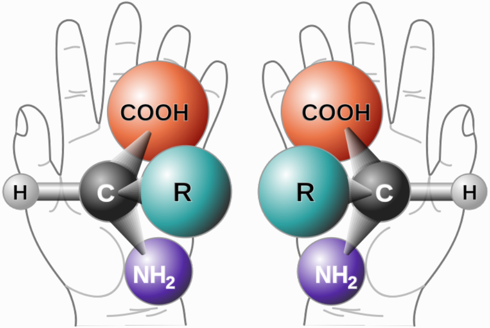

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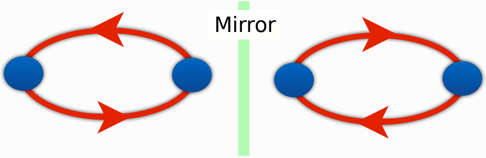

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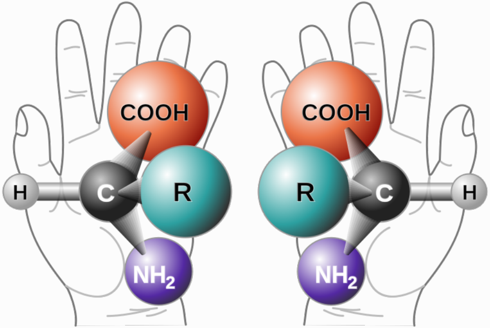

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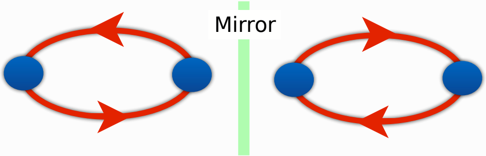

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Realized in Superfluid ^{3}He-A & possibly the ground states in unconventional superconductors

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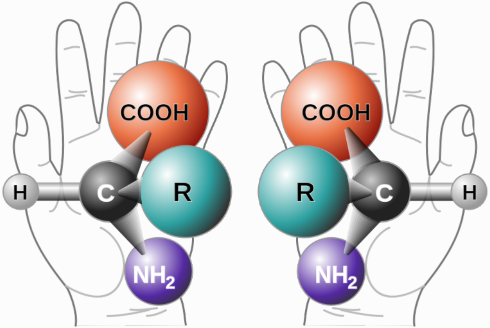

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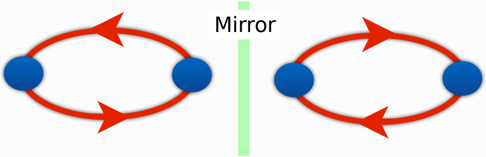

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Signatures: Chiral, Edge Fermions $\rightsquigarrow$ Anomalous Hall Transport

Chiral Superconductors

Ground states exhibiting:

- Emergent Topology of a Broken-Symmetry Vacuum of Cooper Pairs
- Weyl-Majorana excitations of the Vacuum
- Ground-State Edge Currents and Angular Momemtum
- Broken P and T $\rightsquigarrow$ Anomalous Hall-Type Transport

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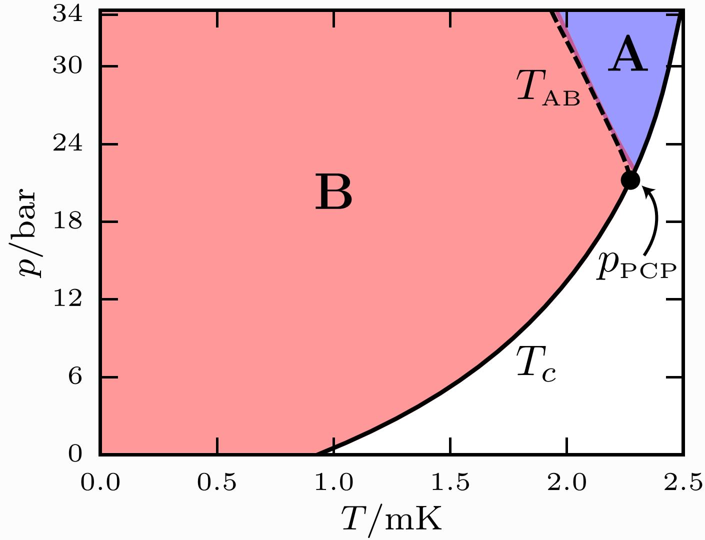
Where are They?

- ^{3}He-A: definitive chiral p-wave condensate; quantitative theory-experimental confirmation
- Sr_2RuO_4: proposed as the electronic analog of ^{3}He-A; evidence of chirality
- UPt_3: electronic analog to ^{3}He: Multiple Superconducting Phases; evidence of chirality
- Other candidates: URu_2Si_2; SrPtAs, doped graphene ...

The Pressure-Temperature Phase Diagram for Liquid ^{3}He

Maximal Symmetry: $\mathtt{G} = \mathtt{SO(3)}_{\mathsf{S}} \times \mathtt{SO(3)}_{\mathsf{L}} \times \mathtt{U(1)}_{\mathsf{N}} \times \mathtt{P} \times \mathtt{T} \rightarrow$ Superfluid Phases of ^{3}He



- J. Wiman & J. A. Sauls, PRB 92, 144515 (2015)

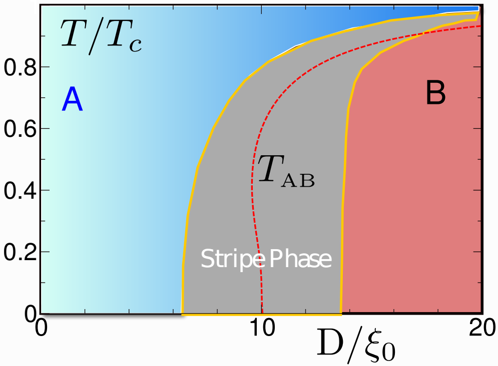
Realization of Broken Time-Reversal and Mirror Symmetry by the Vacuum State of ^{3}He Films

- Length Scale for Strong Confinement:

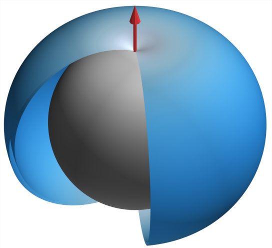
$$\xi_0 = \hbar v_f / 2\pi k_B T_c \approx 20 - 80\,\text{nm}$$

  - L. Levitov et al., Science 340, 6134 (2013)
- A. Vorontsov & J. A. Sauls, PRL 98, 045301 (2007)

$$\mathrm{SO(3)_S} \times \mathrm{SO(3)_L} \times \mathrm{U(1)_N} \times \mathrm{T} \times \mathrm{P}$$
$$\Downarrow$$
$$\mathrm{SO(2)_S} \times \mathrm{U(1)_{N\text{-}L_z}} \times \mathrm{Z_2}$$



Chiral ABM State $\vec{l} = \hat{\mathbf{z}}$



$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix}_{ABM} = \begin{pmatrix} p_x + ip_y \sim e^{+i\phi} & 0 \\ 0 & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

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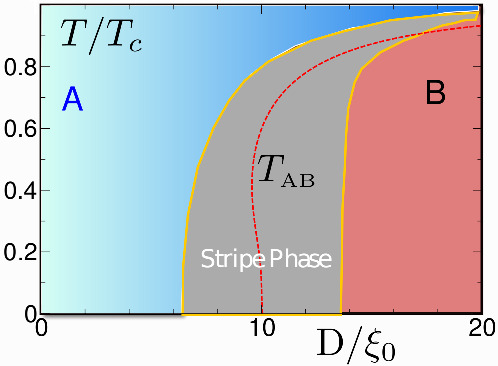

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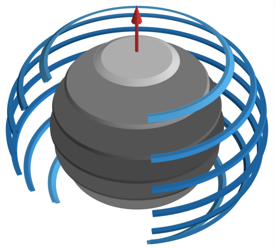

$L_z = 1,\ S_z = 0$

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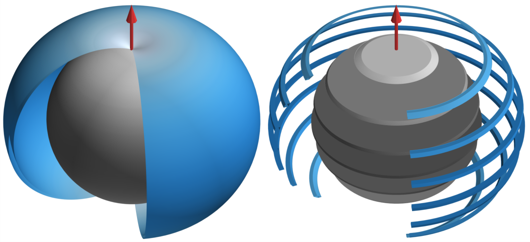
Ground-State Angular Momentum

$$\langle \widehat{L}_z \rangle = \frac{N}{2}\hbar \ ?$$

Open Question

Topology in Momentum Space

$$\Psi(\mathbf{p}) = \Delta(p_x \pm i p_y) \sim e^{\pm i \varphi_{\mathbf{p}}}$$



Winding Number of the Phase:

$$L_z = \pm 1$$

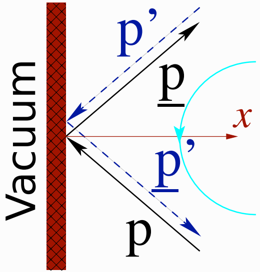
$$N_{2D} = \frac{1}{2\pi} \oint d\mathbf{p} \cdot \frac{1}{|\Psi(\mathbf{p})|} \mathrm{Im}[\boldsymbol{\nabla}_{\mathbf{p}} \Psi(\mathbf{p})] = L_z$$

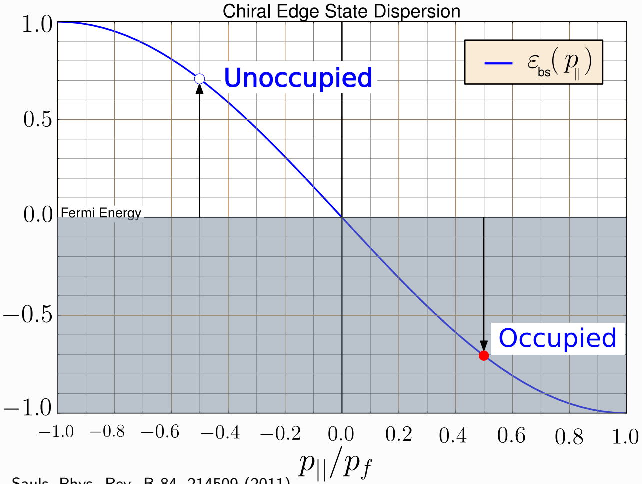
- Massless Chiral Fermions
- Nodal Fermions in 3D
- Edge Fermions in 2D

Massless Chiral Fermions in the 2D ^{3}He-A Films

Edge Fermions: $G^{R}_{\text{edge}}(\mathbf{p},\varepsilon;x) = \dfrac{\pi\Delta|\mathbf{p}_x|}{\varepsilon + i\gamma - \varepsilon_{\text{bs}}(\mathbf{p}_{||})}\; e^{-x/\xi_\Delta}$ $\qquad \xi_\Delta = \hbar v_f/2\Delta \approx 10^2\,\mathring{A} \gg \hbar/p_f$

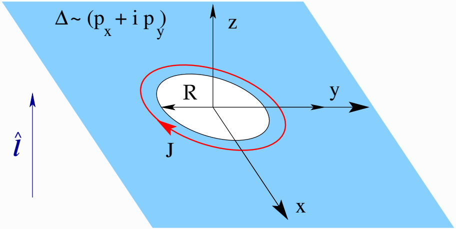
- $\varepsilon_{\text{bs}} = -c\,p_{||}$ with $c = \Delta/p_f \ll v_f$
- Broken P & T $\rightsquigarrow$ Edge Current





- M. Stone, R. Roy, PRB 69, 184511 (2004)
- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

Chiral Edge Current Circulating a Hole or Defect in a Chiral Superfluid



- $R \gg \xi_0 \approx 100\,\text{nm}$
- Sheet Current:

  $J \equiv \int dx\, J_\varphi(x)$

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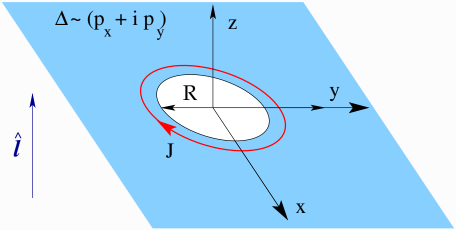

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- Quantized Sheet Current: $\frac{1}{4}\, n\, \hbar$ ($n = N/V = {}^3$He density)
- Edge Current *Counter*-Circulates: $J = -\frac{1}{4}\, n\, \hbar$ w.r.t. Chirality: $\hat{\mathbf{l}} = +\mathbf{z}$

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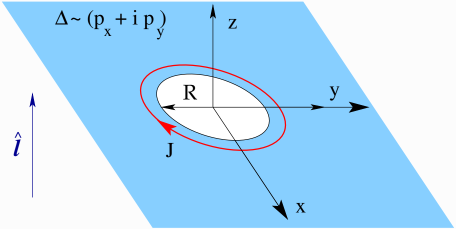

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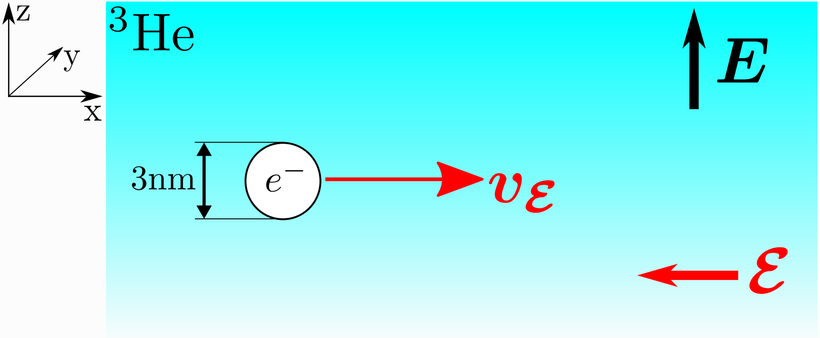
- Angular Momentum: $L_z = 2\pi\, h\, R^2 \times (-\frac{1}{4}\, n\, \hbar) = -(N_{\text{hole}}/2)\, \hbar$

$N_{\text{hole}}/2 =$ Number of ^{3}He Cooper Pairs excluded from the Hole

∴ An object in ^{3}He-A *inherits* angular momentum from the Condensate of Chiral Pairs!

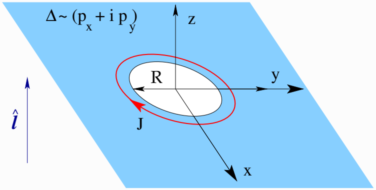
Electron bubbles in the Normal Fermi liquid phase of ^{3}He



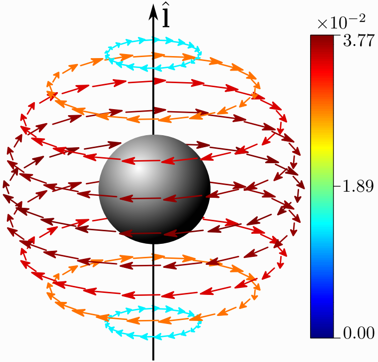
- Bubble with $R \simeq 1.5$ nm,
  $0.1\,\text{nm} \simeq \lambda_f \ll R \ll \xi_0 \simeq 80\,\text{nm}$
- Effective mass $M \simeq 100 m_3$
  (m_3 – atomic mass of ^{3}He)
- QPs mean free path $l \gg R$
- Mobility of ^{3}He is *independent of* T for $T_c < T < 50$ mK

  B. Josephson and J. Leckner, PRL 23, 111 (1969)

Current bound to an electron bubble ($k_f R = 11.17$)

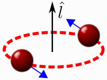


$\Longrightarrow$

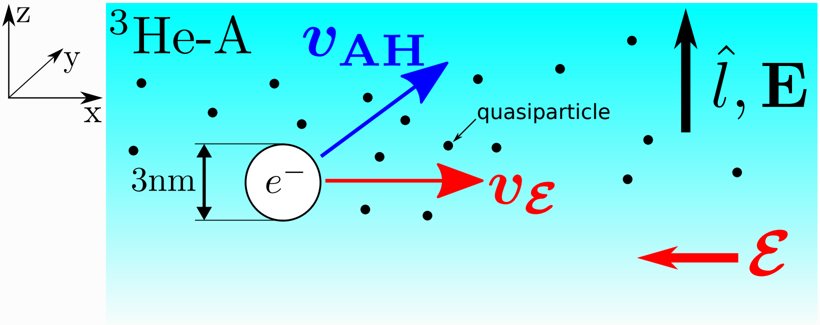


$$\mathbf{L}(T \to 0) \approx -\hbar N_{\mathrm{bubble}}/2\,\hat{\mathbf{l}} \approx -100\,\hbar\,\hat{\mathbf{l}}$$

Electron bubbles in chiral superfluid ^{3}He-A



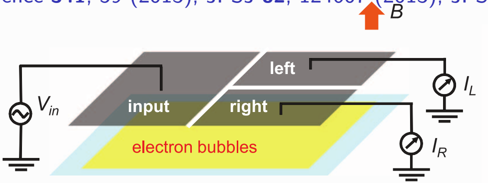
$\Delta(\hat{k}) = \Delta(\hat{k}_x + i\hat{k}_y) = \Delta\, e^{i\phi_{\mathbf{k}}}$

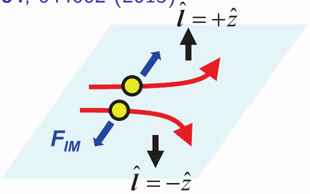


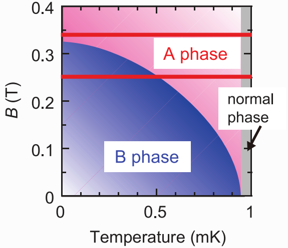
- Current: $\mathbf{v} = \overbrace{\mu_\perp \boldsymbol{\mathcal{E}}}^{\mathbf{v}_{\mathcal{E}}} + \overbrace{\mu_{\text{AH}} \boldsymbol{\mathcal{E}} \times \hat{\mathbf{l}}}^{\mathbf{v}_{\text{AH}}}$ R. Salmelin, M. Salomaa & V. Mineev, PRL **63**, 868 (1989)
- Hall ratio: $\tan\alpha = v_{\text{AH}}/v_{\mathcal{E}} = |\mu_{\text{AH}}/\mu_\perp|$

Mobility of e-bubbles in ^{3}He-A (Ikegami, et al., RIKEN)

Science **341**, 59 (2013); JPSJ **82**, 124607 (2013); JPSJ **84**, 044602 (2015)







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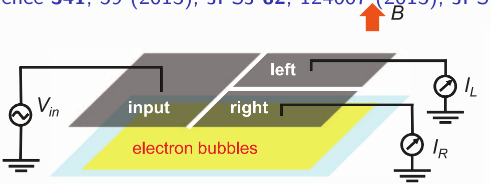

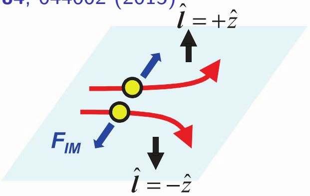

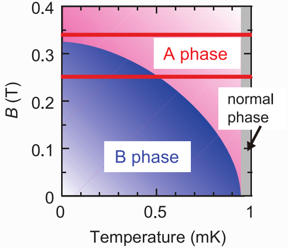

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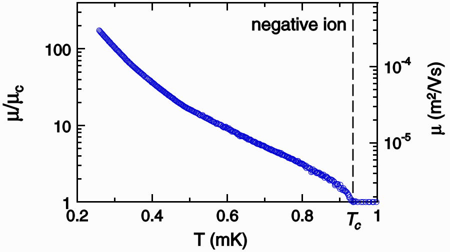

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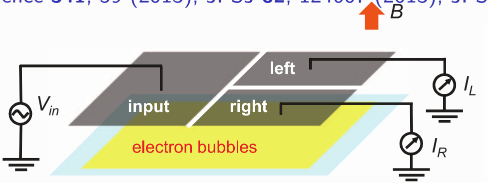

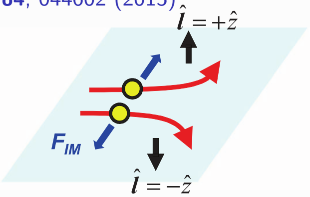

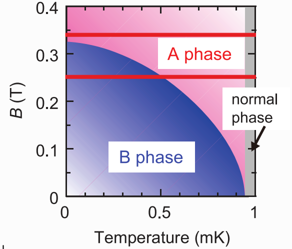

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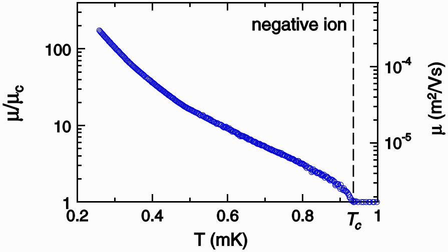

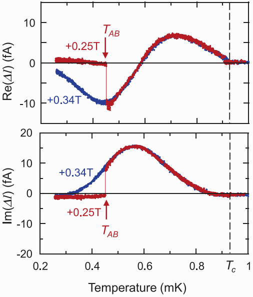

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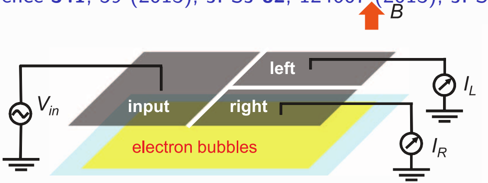

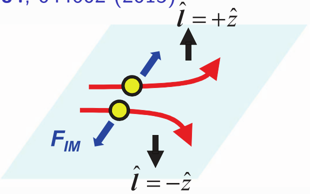

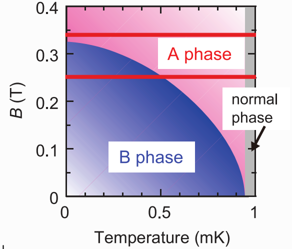

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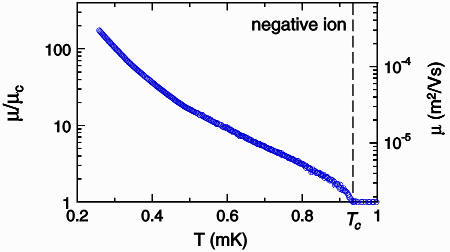

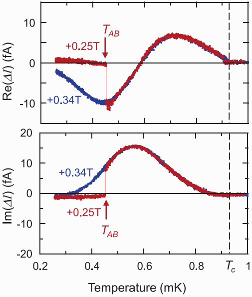

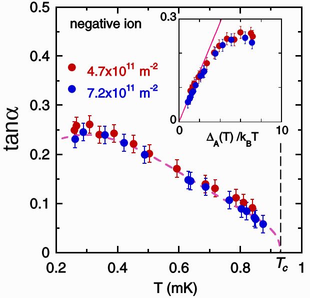

Forces on the Electron bubble in ^{3}He-A:

- $M\dfrac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} + \mathbf{F}_{\text{QP}}, \quad \mathbf{F}_{QP}$ – force from quasiparticle collisions

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- $\mathbf{F}_{QP} = -\overset{\leftrightarrow}{\eta}\cdot\mathbf{v}$, $\overset{\leftrightarrow}{\eta}$ – generalized Stokes tensor
- $\overset{\leftrightarrow}{\eta} = \begin{pmatrix} \eta_\perp & \eta_{\sf AH} & 0 \\ -\eta_{\sf AH} & \eta_\perp & 0 \\ 0 & 0 & \eta_\parallel \end{pmatrix}$ for broken PT symmetry with $\hat{\mathbf{l}} \parallel \mathbf{e}_z$

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- $\mathbf{B}_{\mathrm{eff}} = -\frac{c}{e}\eta_{\mathsf{AH}}\hat{\mathbf{l}}$ $B_{\mathrm{eff}} \simeq 10^3 - 10^4\ \mathrm{T}$!!!

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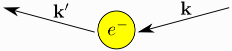
- $\mathbf{B}_{\mathrm{eff}} = -\frac{c}{e}\eta_{\mathsf{AH}}\hat{\mathbf{l}}$ $\quad B_{\mathrm{eff}} \simeq 10^3 - 10^4\ \mathrm{T}$!!!
- Mobility: $\frac{d\mathbf{v}}{dt} = 0 \rightsquigarrow \mathbf{v} = \overleftrightarrow{\mu}\boldsymbol{\mathcal{E}}$, where $\overleftrightarrow{\mu} = e\overleftrightarrow{\eta}^{-1}$

- O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

T-matrix description of Quasiparticle-Ion scattering



- Lippmann-Schwinger equation for the T-matrix ($\varepsilon = E + i\eta\,;\ \eta \to 0^+$):

$$\hat{T}^R_S(\mathbf{k}', \mathbf{k}, E) = \hat{T}^R_N(\mathbf{k}', \mathbf{k}) + \int \frac{d^3k''}{(2\pi)^3} \hat{T}^R_N(\mathbf{k}', \mathbf{k}'') \left[\hat{G}^R_S(\mathbf{k}'', E) - \hat{G}^R_N(\mathbf{k}'', E)\right] \hat{T}^R_S(\mathbf{k}'', \mathbf{k}, E)$$

$$\hat{G}^R_S(\mathbf{k}, E) = \frac{1}{\varepsilon^2 - E^2_{\mathbf{k}}} \begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad E_{\mathbf{k}} = \sqrt{\xi_k^2 + |\Delta(\hat{\mathbf{k}})|^2}, \ \ \xi_k = \frac{\hbar^2 k^2}{2m^*} - \mu$$

- Normal-state T-matrix:

$$\hat{T}^R_N(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = \begin{pmatrix} t^R_N(\hat{\mathbf{k}}', \hat{\mathbf{k}}) & 0 \\ 0 & -[t^R_N(-\hat{\mathbf{k}}', -\hat{\mathbf{k}})]^\dagger \end{pmatrix} \quad \text{in p-h (Nambu) space}$$

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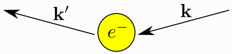

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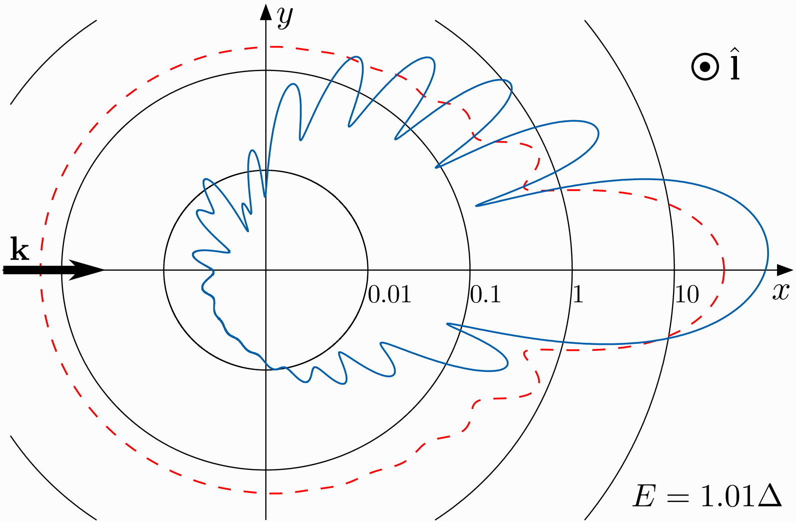
$$t_N^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = -\frac{1}{\pi N_f} \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin\delta_l P_l(\hat{\mathbf{k}}' \cdot \hat{\mathbf{k}}), \quad P_l(x) \text{ – Legendre function}$$

- Hard-sphere potential $\rightsquigarrow \tan\delta_l = j_l(k_f R)/n_l(k_f R)$ – spherical Bessel functions

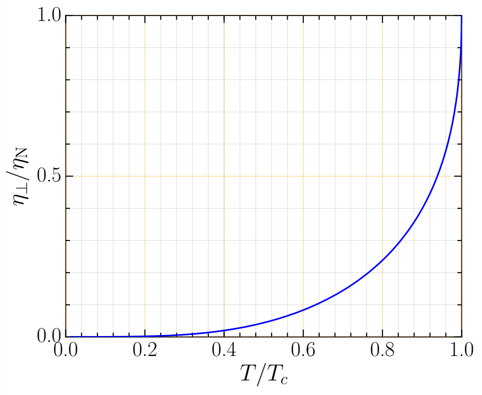
  - $k_f R$ – determined by the Normal-State Mobility $\rightsquigarrow$ $k_f R = 11.17$ ($R = 1.42\,\text{nm}$)

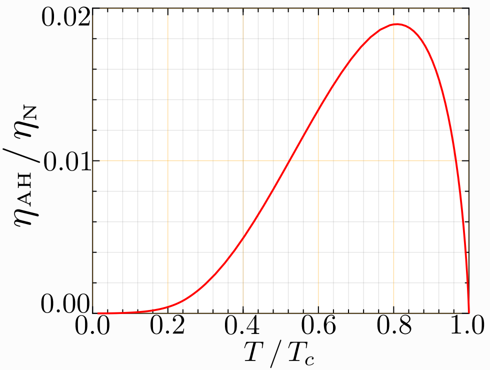
Differential cross section for Bogoliubov QP-Ion Scattering $k_f R = 11.17$



▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Theoretical Results for the Drag and Transverse Forces





- $\Delta p_x \approx p_f \qquad \sigma^{tr}_{xx} \approx \sigma^{tr}_N \approx \pi R^2$
- $F_x \approx n\, v_x\, \Delta p_x\, \sigma^{tr}_{xx}$
  $\approx n\, v_x\, p_f\, \sigma^{tr}_N$

$$|F_y/F_x| \approx \frac{\hbar}{p_f R}\,(\Delta(T)/k_B T_c)^2$$

$k_f R = 11.17$

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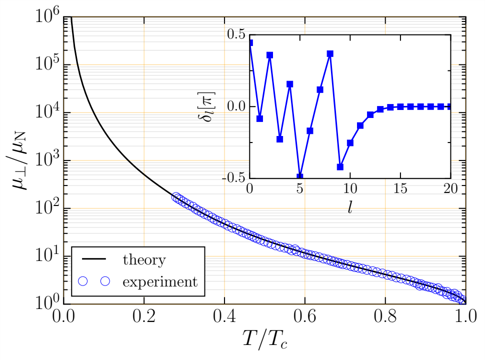
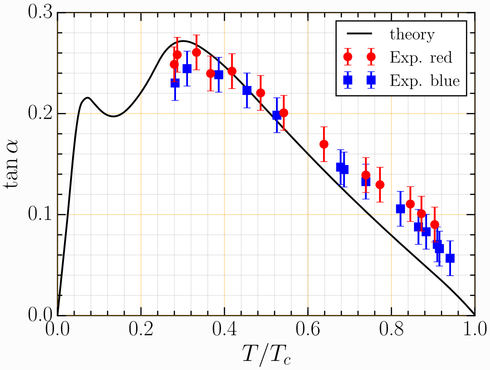
Branch Conversion Scattering in a Chiral Condensate

- O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Comparison between Theory and Experiment for the Drag and Transverse Forces





- $\mu_\perp = e \dfrac{\eta_\perp}{\eta_\perp^2 + \eta_{\mathrm{AH}}^2}$
- $\mu_{\mathrm{AH}} = -e \dfrac{\eta_{\mathrm{AH}}}{\eta_\perp^2 + \eta_{\mathrm{AH}}^2}$

- $\tan\alpha = \left|\dfrac{\mu_{\mathrm{AH}}}{\mu_\perp}\right| = \dfrac{\eta_{\mathrm{AH}}}{\eta_\perp}$
- Hard-Sphere Model: $k_f R = 11.17$

- O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)
- O. Shevtsov and JAS, JLTP 187, 340353 (2017)

Summary

- Electrons in ^{3}He-A are "dressed" by a spectrum of Chiral Fermions
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  • Drag Force $(-\eta_{\perp}\mathbf{v})$ • Transverse Force $(\frac{e}{c}\mathbf{v}\times\mathbf{B}_{\text{eff}})$

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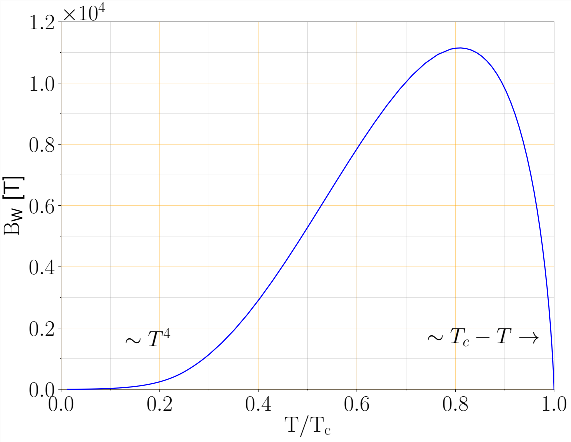
This theory fails as $T \to 0$

Radiation Dominated Motion of Electrons in a Chiral Vacuum

Vanishing of the Effective Magnetic Field for $T \to 0$

$$B_W = 5.9 \times 10^5 \, \text{T} \left(\frac{\eta_{xy}}{\eta_N} \right)$$



$$\eta_{xy}/\eta_N|_{T=0.8\,T_c} \approx \frac{\hbar}{p_f R}$$

Breakdown of Laminar Flow

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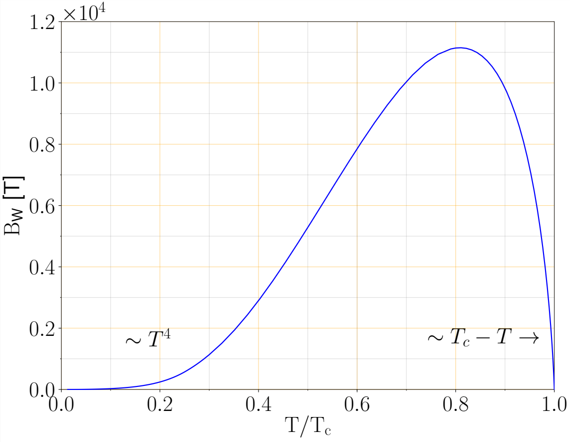
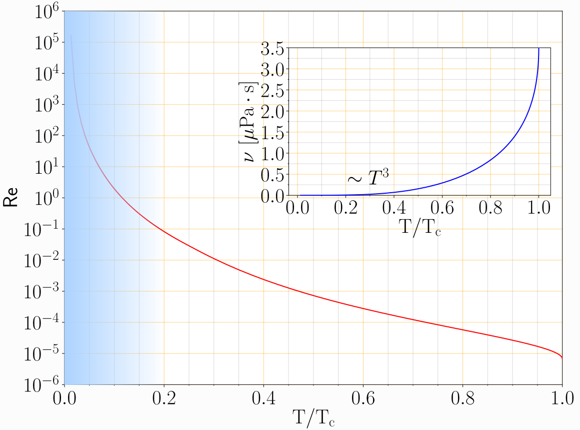


$$\eta_{xy}/\eta_{\mathsf{N}}|_{T=0.8\,T_c} \approx \frac{\hbar}{p_f R}$$

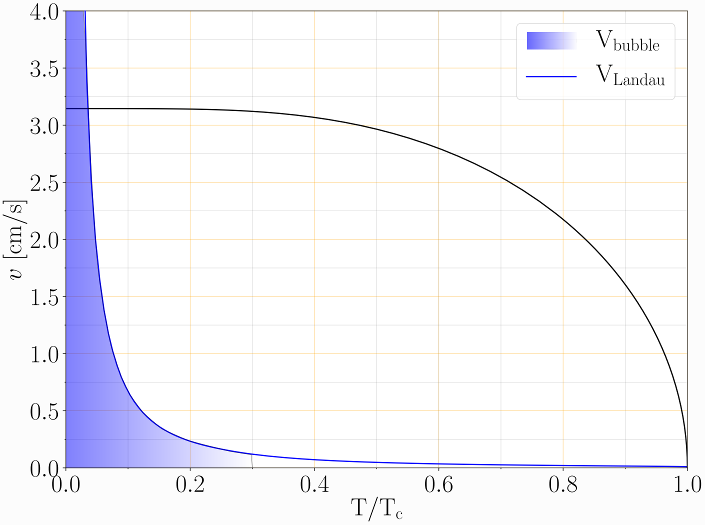
Breakdown of Laminar Flow

$$\mathsf{R}e = \mathsf{R}e_{\mathsf{N}} \left(\frac{\eta_{\mathsf{N}}}{\eta}\right)^{3/2} \xrightarrow[T\to 0]{} \sim \left(\frac{T_c}{T}\right)^{9/2}$$



$$\mathsf{R}e_{\mathsf{N}} = 6.7 \times 10^{-6}$$

Breakdown of Scattering Theory for $T \to 0$



Electron Bubble Velocity

- $V_{\mathsf{N}} = \mu_{\mathsf{N}} E_{\mathsf{N}} = 1.01 \times 10^{-4}$ m/s
- $V = \mu_{\mathsf{N}} E_{\mathsf{N}} \sqrt{\frac{\eta_{\mathsf{N}}}{\eta}}$

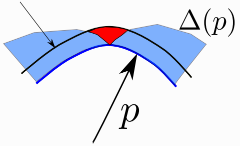
Maximum Landau critical velocity

- $V_c^{\text{max}} \approx 155 \times 10^{-4}$ m/s $\frac{\Delta_{\mathsf{A}}(T)}{k_b T_c}$

Nodal Superfluids:

- $V_c = \Delta(p)/p_f \to 0$ for $p \to p_{\text{node}}$

$$\varepsilon_{\text{Doppler}} = p_f V$$

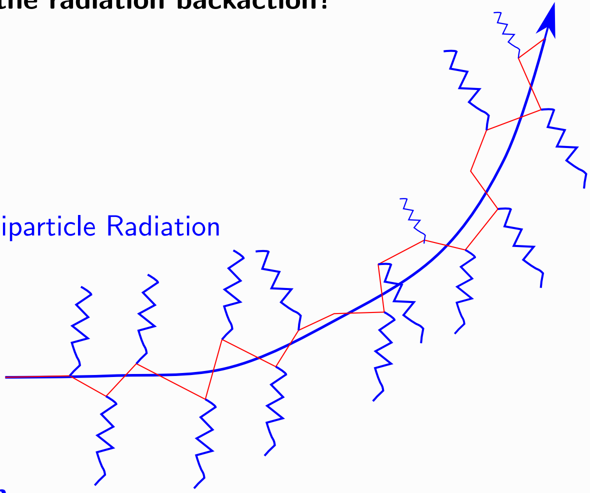


- Radiation Dominated Damping for $T \lesssim 0.1 T_c$

Is their a transverse component of the radiation backaction?

Quasiparticle Radiation

Stochastic Radiative Dynamics



Fluctuations of the Chiral Vacuum

- Mesoscopic Ion coupled and driven through a Chiral “Bath”

Frontier Topic: Low Temperature Transport in Chiral Superconductors

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- Anomalous Thermal Hall Conductivity (κ_{xy}) could provide detection of Broken Time-Reversal and Mirror Symmetries in the Bulk

$$\Downarrow$$

Introduce non-magnetic impurity disorder into a Chiral Superconductor

$$\Delta(\mathbf{p}) = \Delta\,(p_x \pm i p_y)^{\nu}$$

$$J_i^Q = -\kappa_{ij} \nabla_j T \rightsquigarrow \kappa_{xy} \neq 0$$

Edge and *Bulk* Hall Effects

Edge Hall Effect

- Thermal Hall conductance for Chiral *p*-wave states [Read&Green (2000)]

$$K_{xy}^{\text{edge}} = \frac{\pi^2 k_B^2 T}{6\pi\hbar}$$

- Could be sensitive to surface quality

Bulk Hall Effect

★ Induced by impurity scattering in the bulk

★ Often dominant when present

★ Both indicate **Broken TRS & Mirror Symmetry** ★

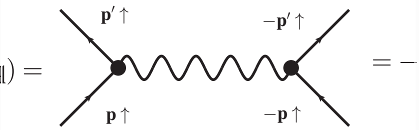
Strong Correlation Physics and the Low-Temperature Phases of ^{3}He

- Strong Interactions in ^{3}He
  - Spin-Fluctuation-Mediated Pairing in ^{3}He
  - Nearly Ferromagnetic or Nearly Localized?
- Strong-Coupling Theory of Superfluid ^{3}He
  - Beyond Weak-coupling BCS pairing
  - The Stabilization of the A phase - circa 2018

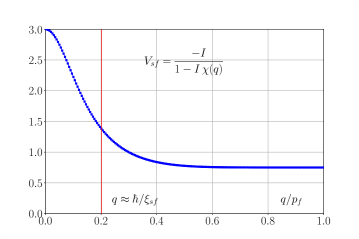
Paramagnon Exchange: Ferromagnetic Spin Fluctuations $\rightsquigarrow$ Odd-Parity, Spin-Triplet Pairing for ^{3}He

- A. Layzer and D. Fay, Int. J. Magn. 1, 135 (1971)

$V_{\text{sf}}(\mathbf{q}) =$



$= -\dfrac{I}{1 - I\,\chi(\mathbf{q})}$



$$-g_l = (2l+1)\int \frac{d\Omega_{\hat{p}}}{4\pi}\int \frac{d\Omega_{\hat{p}'}}{4\pi}\, V_{\text{sf}}(\mathbf{p}-\mathbf{p}')\, P_l(\hat{p}\cdot\hat{p}')$$

- $-g_l$ is a function of $I \approx 0.75$ & $\xi_{\text{sf}} \approx 5\,\hbar/p_f$
- $S = 1$, $S_z = 0$, ± 1 Cooper Pairs:
  $|\uparrow\downarrow + \downarrow\uparrow\rangle$, $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$
- $l = 1$ (p-wave) is dominant pairing channel
  - $\hat{p}_x + i\hat{p}_y \sim \sin\theta_{\hat{p}}\, e^{+i\phi_{\hat{p}}} \rightsquigarrow l_z = +1$
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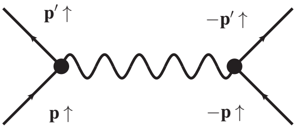

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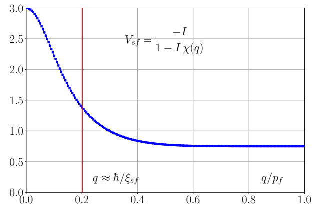

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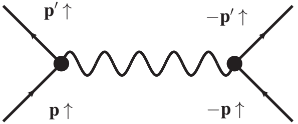
$V_{\text{sf}}(\mathbf{q}) =$ 

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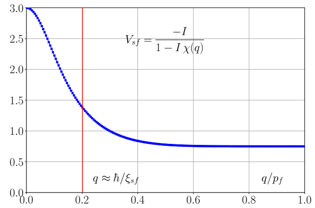

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- Weak-Coupling BCS Theory based on V_{sf}:
- $\rightsquigarrow$ a unique ground state: $\hat{\Delta}(\mathbf{p}) = (i\vec{\sigma}\sigma_y)\cdot\vec{d}(\mathbf{p})$ with $\vec{d}(\mathbf{p}) = \Delta\,\hat{p} \rightsquigarrow L = 1,\ S = 1$ and $J = 0$.
- $\rightsquigarrow$ Fully gapped excitations: $E_{\mathbf{p}} = \sqrt{\xi_{\mathbf{p}}^2 + \Delta^2}$
- $\rightsquigarrow$ BW order parameter for all p, T.
  - R. Balian and N. Werthamer, PR 131, 1553 (1963)

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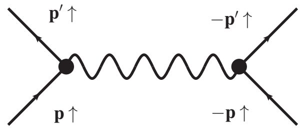
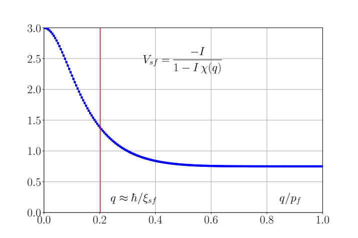


$$= -\frac{I}{1 - I\,\chi(\mathbf{q})}$$



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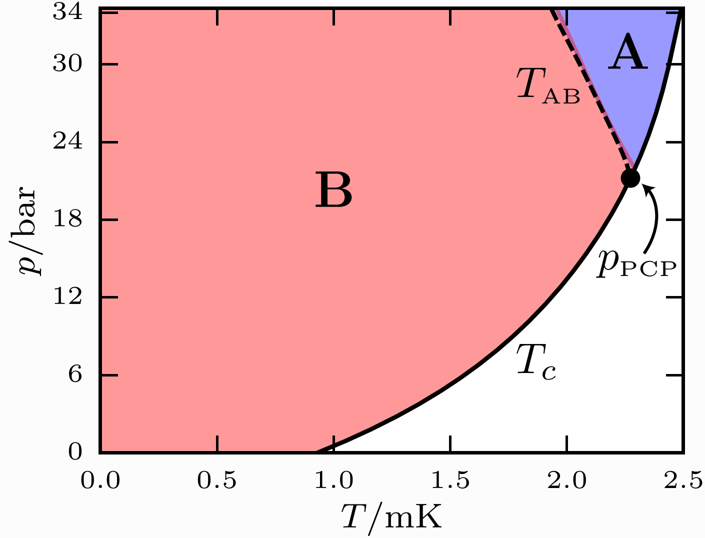
  - $\hat{p}_x + i\hat{p}_y \sim \sin\theta_{\hat{p}}\; e^{+i\phi_{\hat{p}}} \rightsquigarrow l_z = +1$
  - $\hat{p}_z \sim \cos\theta_{\hat{p}} \rightsquigarrow l_z = 0$
  - $\hat{p}_x - i\hat{p}_y \sim \sin\theta_{\hat{p}}\; e^{-i\phi_{\hat{p}}} \rightsquigarrow l_z = -1$
- $l = 3$ (f-wave) is attractive, but sub-dominant to the p-wave channel

- Weak-Coupling BCS Theory based on V_{sf}:
- ⇝ a unique ground state: $\hat{\Delta}(\mathbf{p}) = (i\vec{\sigma}\sigma_y)\cdot\vec{d}(\mathbf{p})$
  with $\vec{d}(\mathbf{p}) = \Delta\,\hat{p} \rightsquigarrow L = 1,\ S = 1$ and $J = 0$.
- ⇝ Fully gapped excitations: $E_{\mathbf{p}} = \sqrt{\xi_{\mathbf{p}}^2 + \Delta^2}$
- ⇝ BW order parameter for all p, T.
  - R. Balian and N. Werthamer, PR 131, 1553 (1963)

Not the Whole Story

The Pressure-Temperature Phase Diagram for Liquid ^{3}He

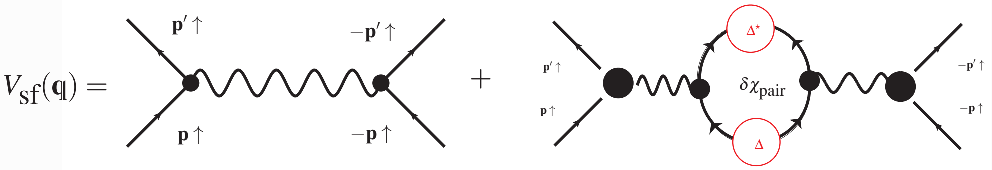
Maximal Symmetry: $\mathtt{G} = \mathtt{SO(3)}_\mathsf{S} \times \mathtt{SO(3)}_\mathsf{L} \times \mathtt{U(1)}_\mathsf{N} \times \mathtt{P} \times \mathtt{T} \rightarrow$ Superfluid Phases of ^{3}He



▶ J. Wiman & J. A. Sauls, PRB 92, 144515 (2015)

Spin Fluctuation Exchange: Feedback Effect $\rightsquigarrow$ Stabilization of ^{3}He-A

Spin-Triplet Pairing Fluctuations *modify* the Spin-Fluctuation Pairing Interaction



- $S = 1$ pairing fluctuations modify V_{sf}:

$$\delta V_{sf} \propto \delta\chi_{\text{pair}} \propto -\chi_N \left(\Delta\Delta^\dagger\right)$$

$$|\,A\,\rangle \sim (\hat{p}_x + i\hat{p}_y)(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) \rightsquigarrow \delta\chi^{\text{A}}_{\text{pair}} = 0$$

$$|\,B\,\rangle \sim (\hat{p}_x + i\hat{p}_y)|\downarrow\downarrow\rangle + (\hat{p}_x + i\hat{p}_y)|\uparrow\uparrow\rangle + \hat{p}_z\,|\uparrow\downarrow + \downarrow\uparrow\rangle) \rightsquigarrow \delta\chi^{\text{B}}_{\text{pair}} \sim -\chi_N\left(|\Delta|/\pi T_c\right)^2$$

"Feedback" Stabilization of ^{3}He-A

- P. W. Anderson and W. Brinkman, PRL 30, 1108 (1973)
- W. Brinkman, J. Serene, and P. Anderson, PRA 10, 2386 (1974)

^{3}He: Nearly Ferromagnetic vs. Almost Localized

Paramagnon Theory (Levin and Valls, Phys. Rep. 1 1983):

- Spin Susceptibility in Paramagnon Theory: $\chi/\chi_{\mathsf{P}} = \dfrac{1}{1-I} \gg 1$

$$\Downarrow$$

^{3}He is near to a ferromagnetic instability

$$\Downarrow$$

finite, but long-lived FM spin fluctuations.

- Effective Mass: $m^*/m - 1 = \ln(1/(1-I))$

- Fermi Liquid Theory: $\chi/\chi_{\mathsf{P}} = \dfrac{m^*/m}{1+F_0^a} \gg 1$
- Exchange Interaction: $F_0^a = -0.70$ to -0.75 is nearly constant
- $\therefore\ \chi/\chi_{\mathsf{P}}$ increases with pressure mainly due m^*/m

$$\Downarrow$$

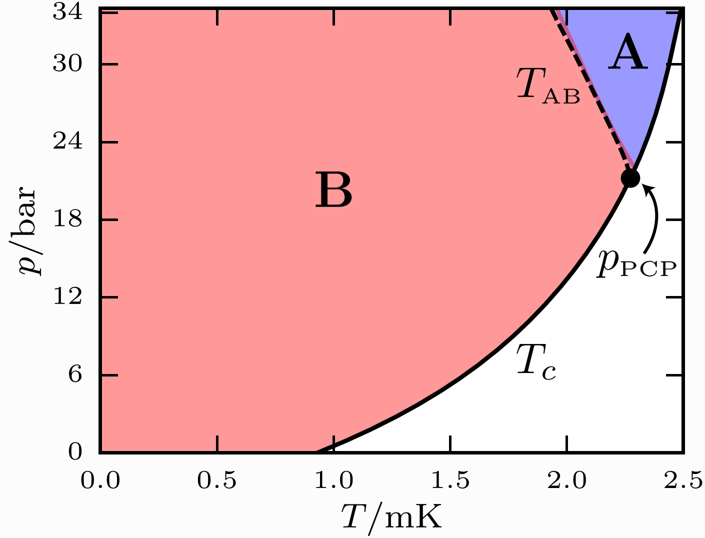
^{3}He is nearly localized (à la Mott) due to short-range repulsive interactions
P. W. Anderson, W. Brinkman, Scottish Summer School, St. Andrews (1975).

- ^{3}He is very incompressible: $F_0^s \approx 10$ to 100 at $p = 34$ bar

- D. Vollhardt, RMP 56, 101 (1984)

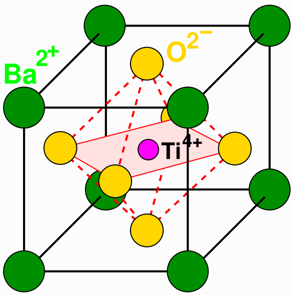
The Pressure-Temperature Phase Diagram for Liquid ^{3}He

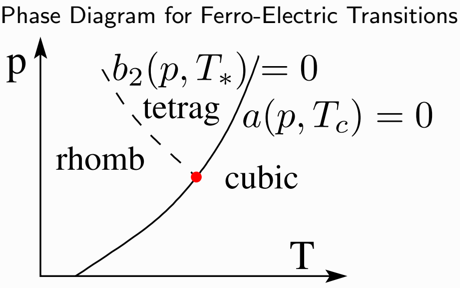
Maximal Symmetry: $\mathtt{G} = \mathtt{SO(3)_S} \times \mathtt{SO(3)_L} \times \mathtt{U(1)_N} \times \mathtt{P} \times \mathtt{T} \rightarrow$ Superfluid Phases of ^{3}He



▶ J. Wiman & J. A. Sauls, PRB 92, 144515 (2015)

Tri-Critical Points in Landau Theory: $\rightarrow$ Ferro-electric Transition in $BaTiO_3$





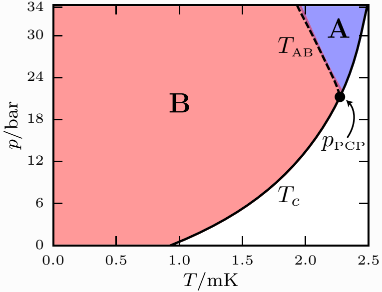
- Electric Polarization: $\mathbf{P} = P_x\hat{x} + P_y\hat{y} + P_z\hat{z}$
- Landau Free Energy Functional:

$$\begin{aligned}\Omega[\mathbf{P}] &= a(p,T)(P_x^2 + P_y^2 + P_z^2) \\ &+ b_1(p,T)(P_x^4 + P_y^4 + P_z^4) \\ &+ b_2(p,T)(P_x^2P_y^2 + P_y^2P_z^2 + P_z^2P_x^2)\end{aligned}$$

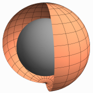
- $a(p,T_c) = 0 \rightsquigarrow$ Ferro-electric transition
- $b_1(p,T_c) > 0,\ b_2(p,T_c) > 0 \rightsquigarrow$ Tetragonal FE
- $b_1(p,T_c) > 0,\ b_2(p,T_c) < 0 \rightsquigarrow$ Rhombohedral FE
- $b_2(p,T_*) = 0 \rightsquigarrow 1^{st}$ Order Transition Line
- $T_*(p_c) = T_c(p_c) \rightsquigarrow$ tri-critical point

Maximal Symmetry: $G = SO(3)_S \times SO(3)_L \times U(1)_N \times P \times T \to$ Superfluid Phases of ^{3}He

J. Wiman & J. A. Sauls, PRB 92, 144515 (2015)

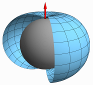


"Isotropic" BW State



$$A^{B}_{\alpha i} = \frac{\Delta}{\sqrt{3}}\,\delta_{\alpha i}$$

Chiral AM State



$$A^{A}_{\alpha i} = \frac{\Delta}{\sqrt{2}}\,\hat{d}_\alpha\,(\hat{m}_i + i\hat{n}_i)$$

- Ginzburg-Landau Free Energy Functional:

$$\Omega[A] = \alpha(p,T)\,\mathrm{Tr}\left\{AA^\dagger\right\} + \sum_{i=1}^{5} \beta_i(p,T)\, I_i^{(4)}[A]$$

  - $\alpha(T_c, p) = 0 \rightsquigarrow T_c(p)$
  - $\Omega_{A,B}(p,T) = -\dfrac{\alpha(p,T)^2}{4\beta_{A,B}(p,T)}$
  - $\beta_A = \beta_{245}$ and $\beta_B = \beta_{12} + \frac{1}{3}\beta_{345}$
  - $\Delta\beta(p, T_{AB}) = 0 \rightsquigarrow T_{AB}(p)$

- Microscopic Theory:
  - $\alpha(p,T) = \frac{1}{3}N_f(T - T_c)$
  - Weak-Coupling:

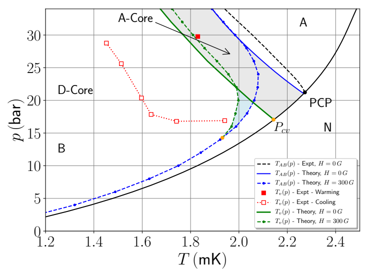
$$2\beta_1^{wc} = -\beta_2^{wc} = -\beta_3^{wc} = -\beta_4^{wc} = \beta_5^{wc} = -\frac{7\zeta(3)N_f}{240(\pi k_B T_c)^2}$$

  - Leading Order Strong-Coupling:
  - $\beta_i^{sc}(p,T) \approx \beta^{wc}(p) \times \langle w_i |T|^2\rangle_{FS} \times \left(\dfrac{T}{E_f}\right)$

$$\beta_i(p,T) = \beta_i^{wc}(p) + \left(\frac{T}{T_c}\right)\beta_i^{sc}(p, T_c(p))$$

Strong-Coupling GL Theory: Inhomogeneous Phases of Superfluid ^{3}He in Confined Geometries

- R. Regan et al., QFS 2018 (Poster P28.3)



- Rotating ^{3}He-B - P. Hakonen et al. 1983

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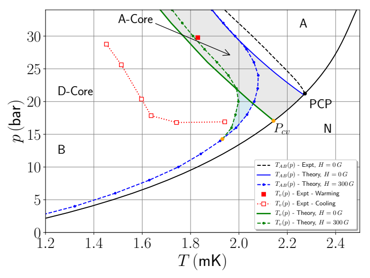

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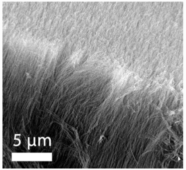
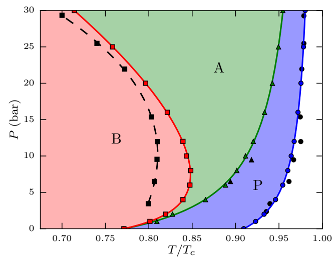
- V. Dmitriev et al.,PRL 115, 165304 (2015)



- ^{3}He Confined in Nematic Aerogel

- J.Wiman, S. Laine, E. Thuneberg & JAS, (2018).



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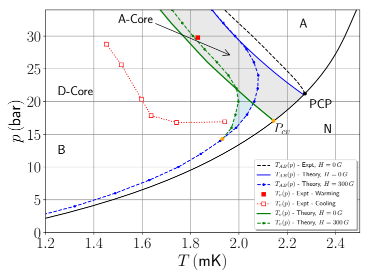

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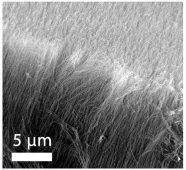

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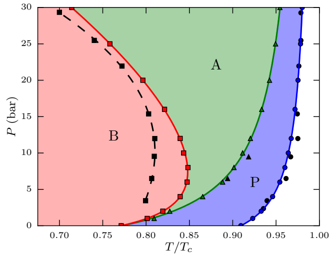

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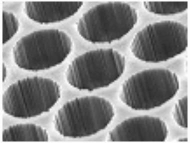

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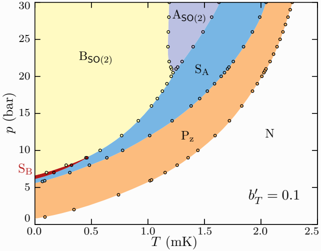

- A. Zimmerman (NU ULT)
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  - Nematic P_z Phase
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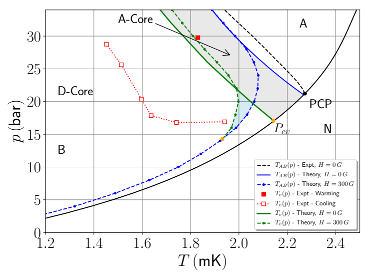

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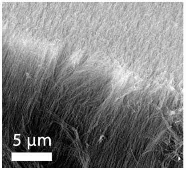

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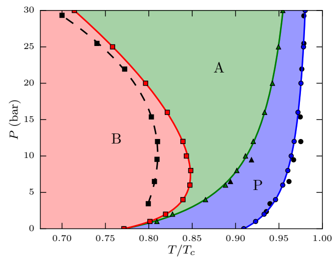

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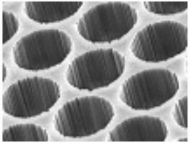

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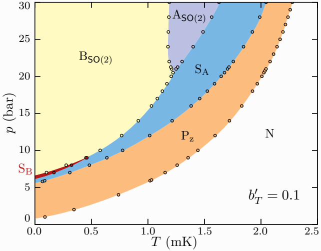

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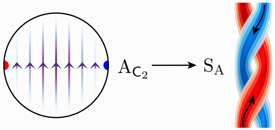
S_A - Double Helix Phase



The ^{3}He Paradigm: Strongly Correlated Fermi-Liquid Superconductors

- Microscopic extension of strong-coupling GL theory to all T

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Quasiclassical reduction of the Luttinger-Ward functional

$$\Omega = -\frac{T}{2}\sum_{\epsilon_n}\int \frac{d^3p}{(2\pi)^3}\mathrm{Tr}_4\Big\{\Delta\widehat{\Sigma}\,\widehat{G}$$

$$+\ln[-\widehat{G}_N^{-1} + \Delta\widehat{\Sigma}] - \ln[-\widehat{G}_N^{-1})]\Big\} + \Delta\Phi[\Delta\widehat{G}]$$

$$\widehat{G}(\vec{p},\,\epsilon_n) = \begin{pmatrix} \hat{G}(\vec{p},\,\epsilon_n) & \hat{F}(\vec{p},\,\epsilon_n) \\ \hat{F}^{\dagger}(\vec{p},\,-\epsilon_n) & -\hat{G}^{\mathrm{tr}}(-\vec{p},\,-\epsilon_n) \end{pmatrix}$$

$$\widehat{\Sigma}(\vec{p},\,\epsilon_n) = \begin{pmatrix} \hat{\Sigma}(\vec{p},\,\epsilon_n) & \hat{\Delta}(\vec{p},\,\epsilon_n) \\ \hat{\Delta}^{\dagger}(\vec{p},\,-\epsilon_n) & -\hat{\Sigma}^{\mathrm{tr}}(-\vec{p},\,-\epsilon_n) \end{pmatrix}$$

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$\delta\Omega[\widehat{\Sigma}]/\delta\widehat{\Sigma}^{\mathrm{tr}}(\vec{p},\,\epsilon_n) = 0$ and $\delta\Omega[\widehat{\Sigma}]/\delta\widehat{G}^{\mathrm{tr}}(\vec{p},\,\epsilon_n) = 0$

$\widehat{\Sigma} = \widehat{\Sigma}_{\mathrm{skel}} = 2\,\delta\Phi[\widehat{G}]/\delta\widehat{G}^{\mathrm{tr}}$ Expansion in $\dfrac{T}{E_f},\ \dfrac{\hbar}{p_f\xi_0},\ldots$

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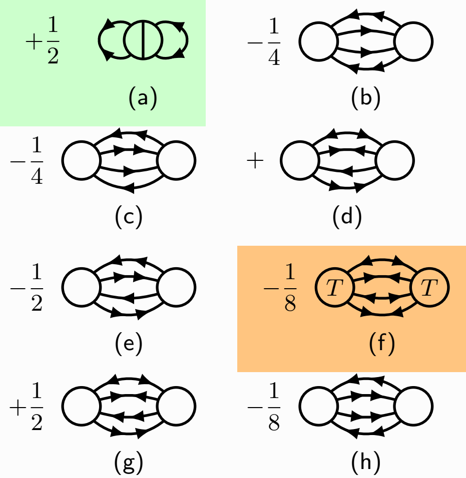
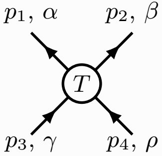


D. Rainer & J. Serene, Phys. Rev. B 13, 4745 (1976)   JAS & J. Serene, Phys. Rev. B 24, 181 (1981)   J. Wiman & JAS, (2018) [QFS: P26.16]

Strong-Correlations in ^{3}He: Effective Interactions J. Wiman, (2018) [QFS: P26.16]



$$= \delta_{\alpha\gamma}\delta_{\beta\rho}\ v(q_1) + \vec{\sigma}_{\alpha\gamma}\cdot\vec{\sigma}_{\beta\rho}\ j(q_1) - \delta_{\alpha\rho}\delta_{\beta\gamma}\ v(q_2) - \vec{\sigma}_{\alpha\rho}\cdot\vec{\sigma}_{\beta\gamma}\ j(q_2)$$

- $v(q)$ - spin-independent potential
- $j(q)$ - exchange interaction

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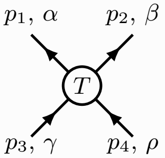

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- To leading order in ε/E_f:
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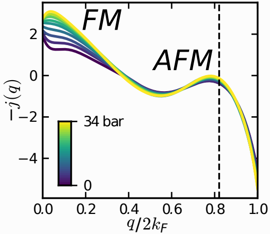

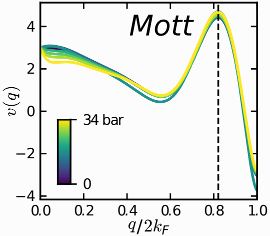

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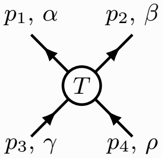

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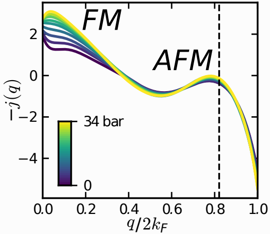

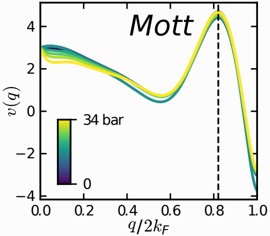

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- $v(q)$ and $j(q)$ determine:
  - Landau Interactions: Forward scattering $(\mathbf{p}_3 = \mathbf{p}_1)$
  - Thermodynamics: C_v/T, m^*/m, χ/χ_{Pauli}, c_1, c_0, $\ldots$
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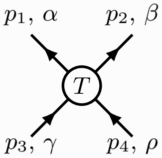
Strong-Correlations in ^{3}He: Effective Interactions J. Wiman, (2018) [QFS: P26.16]



$$= \delta_{\alpha\gamma}\delta_{\beta\rho}\, v(q_1) + \vec{\sigma}_{\alpha\gamma}\cdot\vec{\sigma}_{\beta\rho}\, j(q_1) - \delta_{\alpha\rho}\delta_{\beta\gamma}\, v(q_2) - \vec{\sigma}_{\alpha\rho}\cdot\vec{\sigma}_{\beta\gamma}\, j(q_2)$$

- $v(q)$ - spin-independent potential
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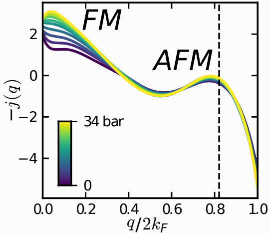

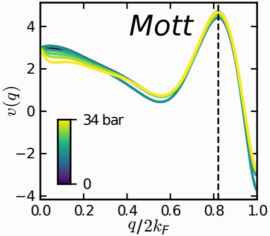

- To leading order in ε/E_f:
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D. S. Greywall, Phys. Rev. B 33, 7520 (1986)

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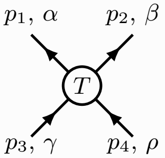

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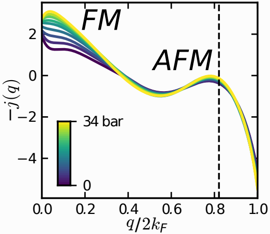

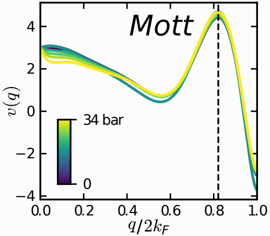

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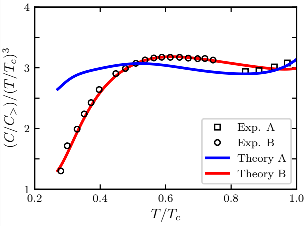

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- Frontier Topic: Inhomogeneous and non-equilibrium dynamics of Strong-Coupling ^{3}He

Thank You!

The End