# Electron bubbles and Weyl fermions in chiral superfluid $^3$He-A


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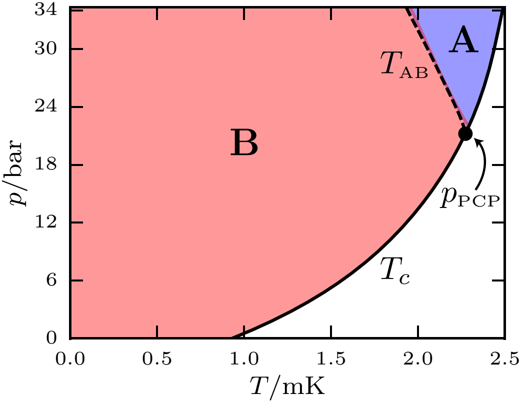
(1) Introduction to superfluid $^3$He and *e*-bubbles

(2) Transport and AHE of *e*-bubbles in $^3$He-A: Experiments

(3) Structure and transport of *e*-bubbles in $^3$He-A: Theory [O. Shevtsov and J. A. Sauls, PRB 94, 064511 (2016)]


DMR-1508730

Symmetry group of normal-state $^3$He: $G = SO(3)_S \times SO(3)_L \times U(1)_N \times P \times T$



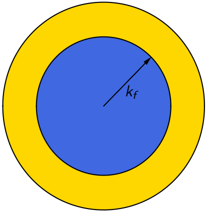

J. J. Wiman and J. A. Sauls PRB **92**, 144515 (2015)

Spin-triplet $p$-wave order parameter:

$$\Delta_{\alpha\beta}(\mathbf{k}) = \vec{d}(\mathbf{k}) \cdot (i\vec{\sigma}\sigma_y)_{\alpha\beta},\ d_\mu(\mathbf{k}) = A_{\mu j}\mathbf{k}_j$$

BW state (B-phase):

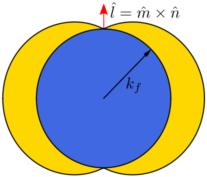
$J = 0, J_z = 0$



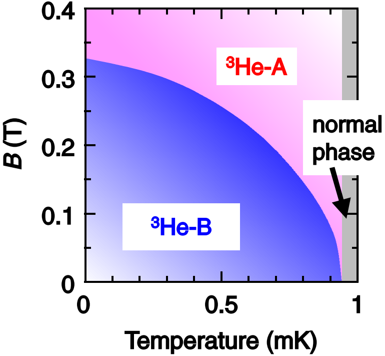

$$A_{\mu j} = \Delta\delta_{\mu j}$$

ABM state (A-phase):

$L_z = 1, S_z = 0$



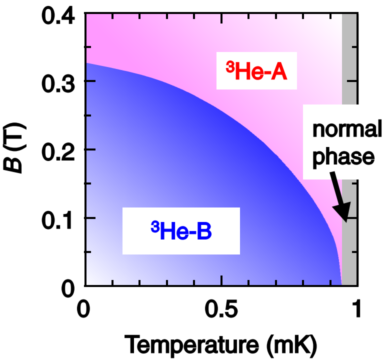

$$A_{\mu j} = \Delta\hat{d}_\mu(\hat{m} + i\hat{n})_j$$




Magnetic field **B**:

- suppresses the $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ spin component in the order parameter
- makes the A-phase ($|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle$) more favorable at critical temperature $T_c$

Ikegami et al. J. Phys. Soc. Jpn. **84**, 044602 (2015)

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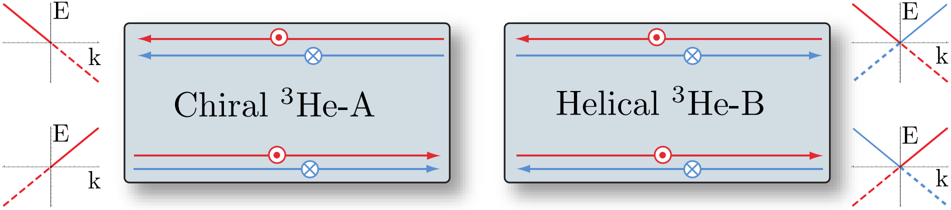
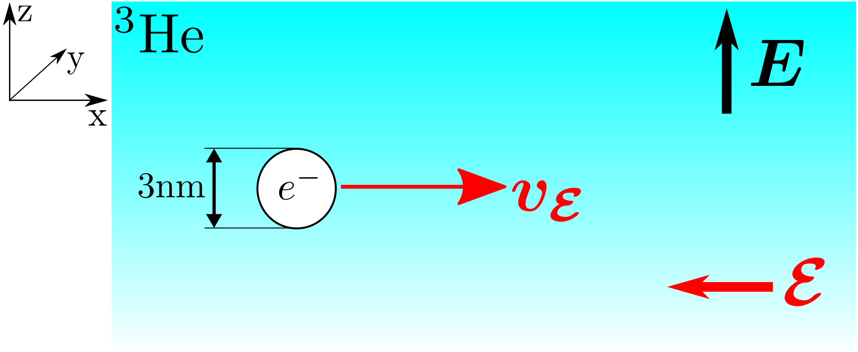
Edge states spectrum:

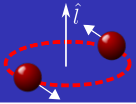


# Electron bubbles in liquid $^3$He

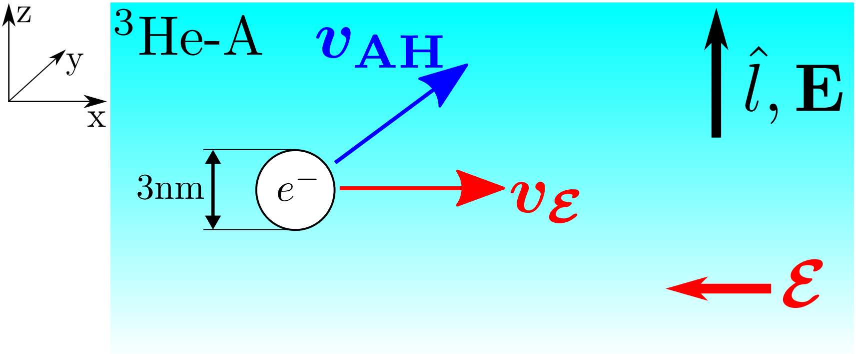



- Bubble with $R \simeq 1.5$ nm, $\lambda_f \ll R \ll \xi_0$
- Effective mass $M \simeq 100 m_3$ ($m_3$ – atomic mass of $^3$He)
- QPs mean free path $l \gg R$, Knudsen limit
- Normal-state mobility is const below $T = 50$ mK

# Electron bubbles in chiral superfluid $^3$He-A




$$\Delta_\mathrm{A}(\hat{\mathbf{k}}) = \Delta \frac{k_x + i k_y}{k_f}$$

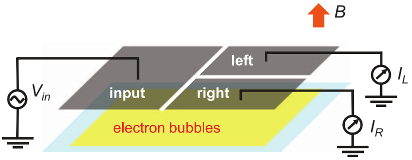


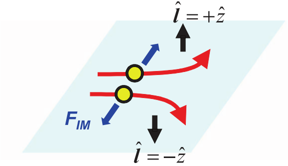

Electric current: $\mathbf{v} = \overbrace{\mu_\perp \boldsymbol{\mathcal{E}}}^{\mathbf{v}_{\mathcal{E}}} + \overbrace{\mu_\mathrm{AH} \boldsymbol{\mathcal{E}} \times \hat{\mathbf{l}}}^{\mathbf{v}_\mathrm{AH}}$ Salmelin et al. PRL **63**, 868 (1989)

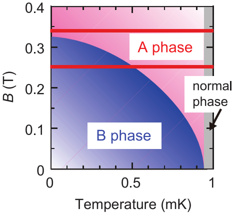
Hall ratio: $\tan\alpha = v_\mathrm{AH}/v_{\mathcal{E}} = |\mu_\mathrm{AH}/\mu_\perp|$

# Mobility of $e$-bubbles in $^3$He-A (Ikegami et al., RIKEN)

Science **341**, 59 (2013); JPSJ **82**, 124607 (2013); JPSJ **84**, 044602 (2015)

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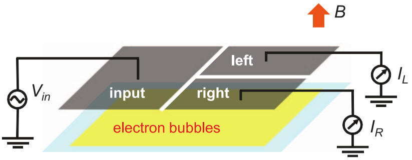


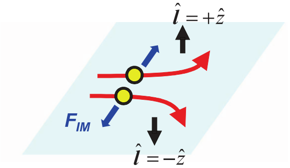


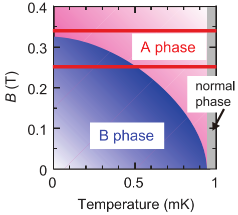


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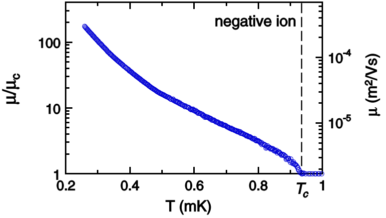

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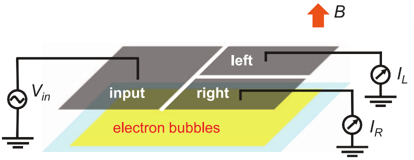


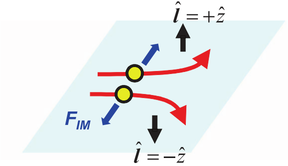


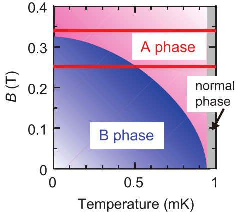


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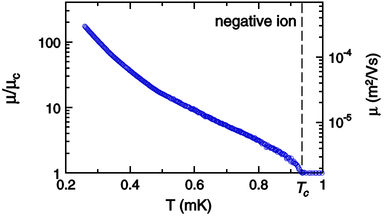


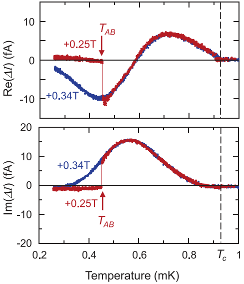

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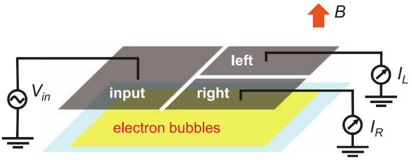


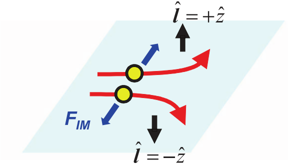


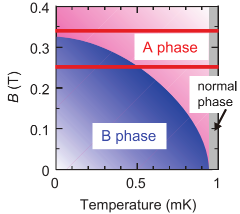


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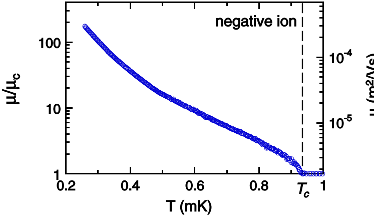


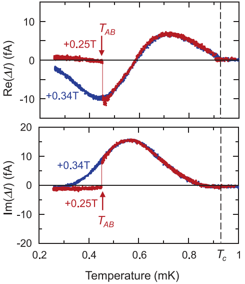


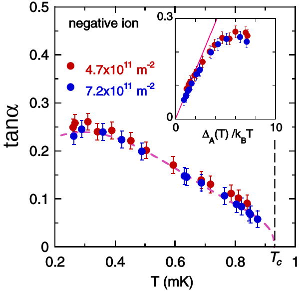

# Equation of motion for *e*-bubbles in $^3$He-A:

(i) $M\frac{d\mathbf{v}}{dt} = e\mathcal{E} + \mathbf{F}_{\rm QP},\quad \mathbf{F}_{QP}$ – force due to quasiparticle scattering

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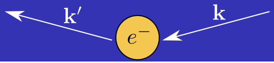
(vi) $\frac{d\mathbf{v}}{dt} = 0 \rightsquigarrow \mathbf{v} = \overleftrightarrow{\mu}\mathcal{E}$, where $\overleftrightarrow{\mu} = e\overleftrightarrow{\eta}^{-1}$

$$\mu_\parallel = \frac{e}{\eta_\parallel}, \quad \mu_\perp = e\frac{\eta_\perp}{\eta_\perp^2 + \eta_{\mathrm{AH}}^2}, \quad \mu_{\mathrm{AH}} = -e\frac{\eta_{\mathrm{AH}}}{\eta_\perp^2 + \eta_{\mathrm{AH}}^2}$$

# Scattering of Bogoliubov QPs off the negative ion



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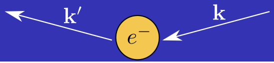


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$$\hat{T}_S^R(\mathbf{k}', \mathbf{k}, E) = \hat{T}_N^R(\mathbf{k}', \mathbf{k}) + \int \frac{d^3k''}{(2\pi)^3} \hat{T}_N^R(\mathbf{k}', \mathbf{k}'') \Big[ \hat{G}_S^R(\mathbf{k}'', E) - \hat{G}_N^R(\mathbf{k}'', E) \Big] \hat{T}_S^R(\mathbf{k}'', \mathbf{k}, E)$$

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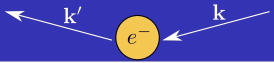


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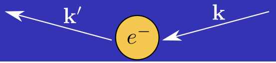


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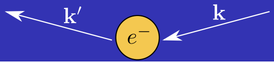


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$$t^R_N(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = -\frac{1}{\pi N_f} \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin\delta_l \, P_l(\hat{\mathbf{k}}' \cdot \hat{\mathbf{k}}), \quad P_l \text{ – Legendre polynomial}$$

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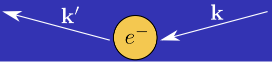


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$$\hat{G}_S^R(\mathbf{k}, E) = \frac{1}{\varepsilon^2 - E_{\mathbf{k}}^2} \begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad E_{\mathbf{k}} = \sqrt{\xi_k^2 + |\Delta(\hat{\mathbf{k}})|^2}, \ \xi_k = \frac{\hbar^2 k^2}{2m^*} - \mu$$

(ii) Normal-state $T$-matrix:

$$\hat{T}_N^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = \begin{pmatrix} t_N^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}) & 0 \\ 0 & -[t_N^R(-\hat{\mathbf{k}}', -\hat{\mathbf{k}})]^\dagger \end{pmatrix}$$

in p-h (Nambu) space, where

$$t_N^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = -\frac{1}{\pi N_f} \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin\delta_l P_l(\hat{\mathbf{k}}' \cdot \hat{\mathbf{k}}),$$

$P_l$ – Legendre polynomial

Hard-sphere model $\rightsquigarrow$ $\tan\delta_l = j_l(k_f R)/n_l(k_f R)$, $j_l, n_l$ – spherical Bessel fn-s

$k_f R$ – the only adjustable parameter (to be determined)!

# From T-matrix to drag force

(i) QP scattering rate – Fermi's golden rule:

$$\Gamma(\mathbf{k}', \mathbf{k}) = \frac{2\pi}{\hbar} W(\hat{\mathbf{k}}', \hat{\mathbf{k}}) \delta(E_{\mathbf{k}'} - E_{\mathbf{k}})$$

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$$\mathbf{F}_{\mathrm{QP}} = -\sum_{\mathbf{k},\mathbf{k}'} \hbar(\mathbf{k}' - \mathbf{k}) \left[ \hbar \mathbf{k}' \mathbf{v} f_{\mathbf{k}} \left( -\frac{\partial f_{\mathbf{k}'}}{\partial E} \right) - \hbar \mathbf{k} \mathbf{v} (1 - f_{\mathbf{k}'}) \left( -\frac{\partial f_{\mathbf{k}}}{\partial E} \right) \right] \Gamma(\mathbf{k}',\mathbf{k})$$

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$n_3 = \frac{k_f^3}{3\pi^2}$ – $^3$He particle density, $\sigma_{ij}(E)$ – transport scattering cross section,

$f(E) = [\exp(E/k_B T) + 1]^{-1}$ – Fermi-Dirac f-n

# Mirror-symmetric scattering ⇒ linear drag

$$\mathbf{F}_{QP} = -\overleftrightarrow{\eta} \cdot \mathbf{v}, \quad \eta_{ij} = n_3 p_f \int_0^\infty dE \left( -2 \frac{\partial f}{\partial E} \right) \sigma_{ij}(E)$$

Subdivide by mirror symmetry:

$$W(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = \boxed{W^{(+)}(\hat{\mathbf{k}}', \hat{\mathbf{k}})} + W^{(-)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}),$$

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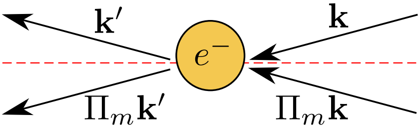

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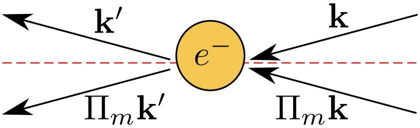


$$\sigma_{ij}^{(+)}(E) = \frac{3}{4}\int_{E\geq|\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'}\int_{E\geq|\Delta(\hat{\mathbf{k}})|}\frac{d\Omega_{\mathbf{k}}}{4\pi}\left[(\hat{\mathbf{k}}'_i-\hat{\mathbf{k}}_i)(\hat{\mathbf{k}}'_j-\hat{\mathbf{k}}_j)\right]\frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E)$$

Mirror-symmetric diff. cross section: $W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W(\hat{\mathbf{k}},\hat{\mathbf{k}}')]/2$

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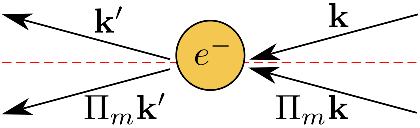


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No transverse force! $\left[\eta_{ij}^{(+)}\right]_{i\neq j} = 0, \quad \eta_{xx}^{(+)} = \eta_{yy}^{(+)} \equiv \eta_\perp, \quad \eta_{zz}^{(+)} \equiv \eta_\parallel$

# Mirror-antisymmetric scattering ⇒ transverse force

$$\mathbf{F}_{QP} = -\overleftrightarrow{\eta} \cdot \mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^\infty dE \left(-2\frac{\partial f}{\partial E}\right) \sigma_{ij}(E)$$

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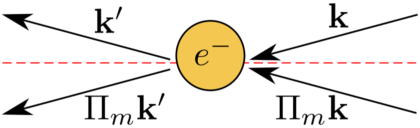

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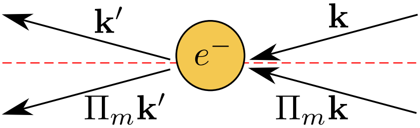


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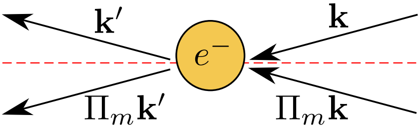


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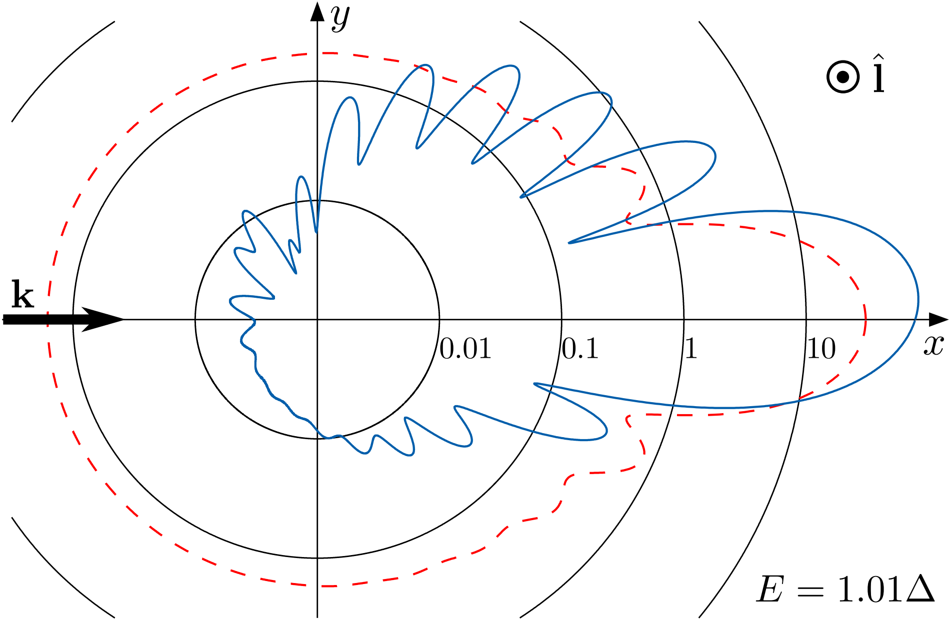
Transverse force! $\eta_{xy}^{(-)} = -\eta_{yx}^{(-)} \equiv \eta_{\mathrm{AH}}$ ⇒ anomalous Hall effect!

# Differential scattering cross section for Bigoliubov QPs



# Local Density of States (LDOS) around an *e*-bubble

$$\mu_N = \frac{e}{n_3 p_f \sigma_N} \quad \Rightarrow \quad \mu_N^{\mathrm{exp}} = 1.7 \times 10^{-6}\, \frac{m^2}{V\, s}$$

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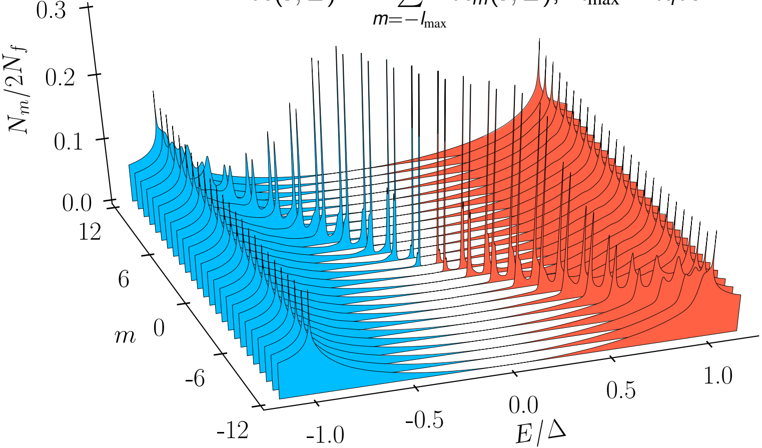

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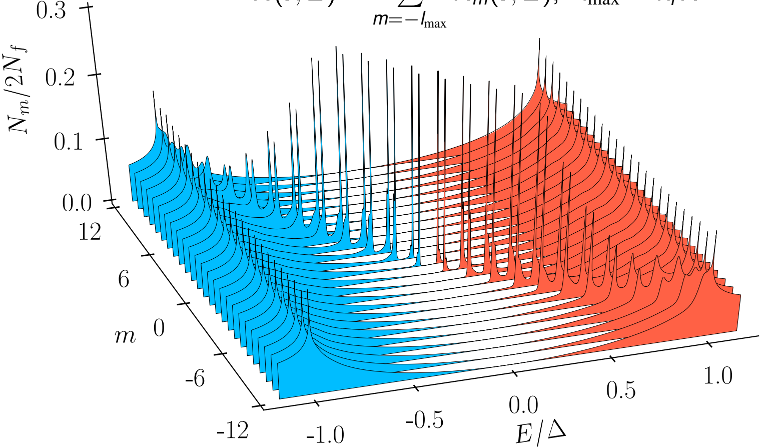
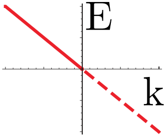
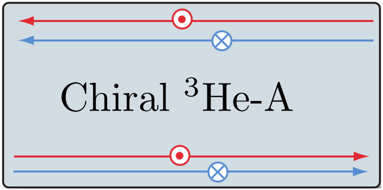


# Current density around an $e$-bubble ($k_f R = 11.17$)



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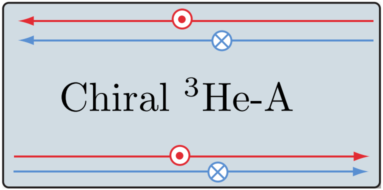
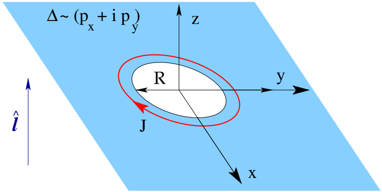



$\Longrightarrow$



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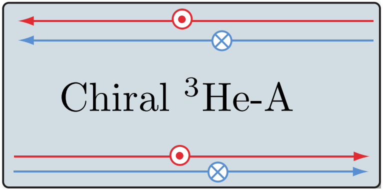


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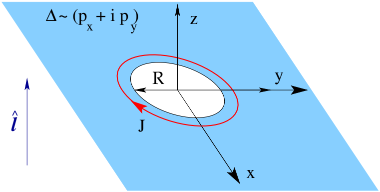


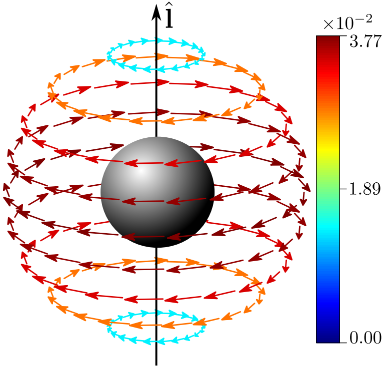


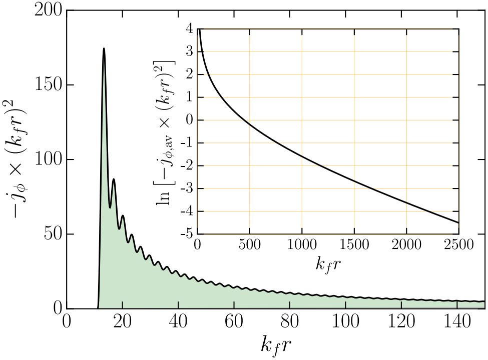


$$\mathbf{j}(\mathbf{r})/v_f N_f k_B T_c = j_\phi(\mathbf{r})\hat{\mathbf{e}}_\phi$$

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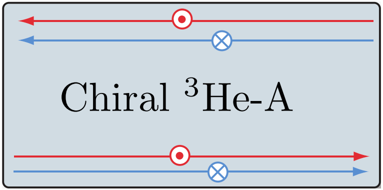


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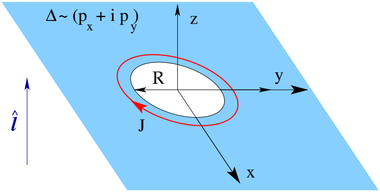
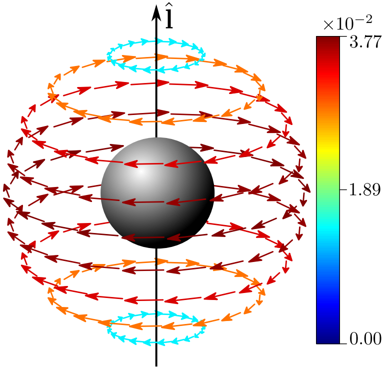
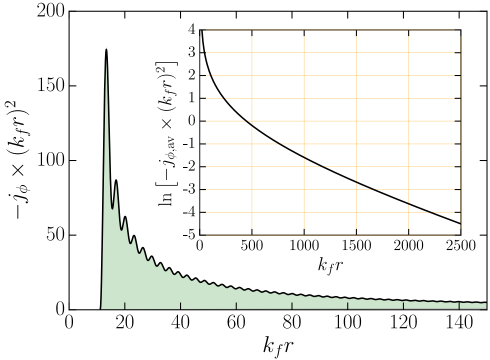


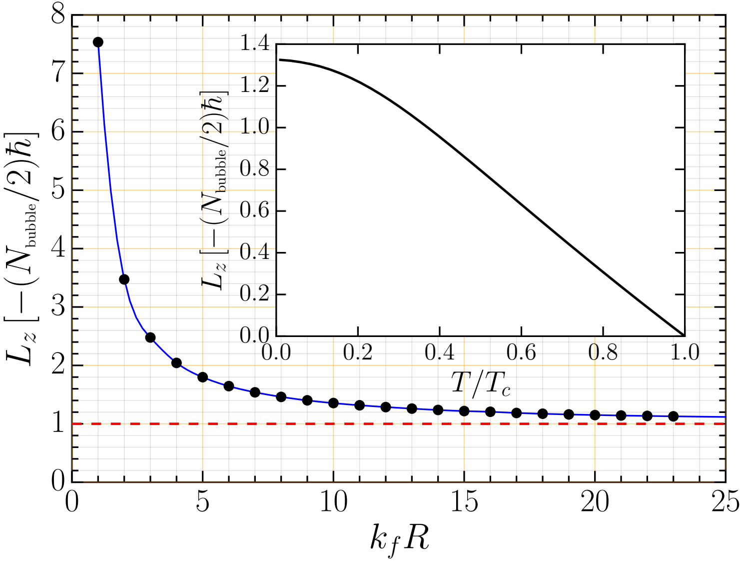




$\mathbf{j}(\mathbf{r})/v_f N_f k_B T_c = j_\phi(\mathbf{r})\hat{\mathbf{e}}_\phi \quad \Longrightarrow \quad \mathbf{L}(T \to 0) \approx -\hbar N_{\mathrm{bubble}} \hat{\mathbf{l}}/2$

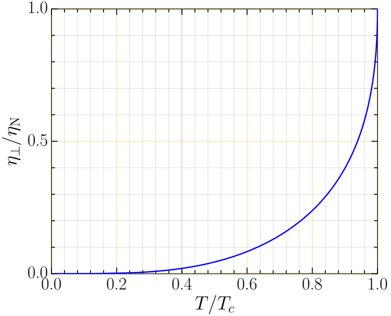
J. A. Sauls PRB **84**, 214509 (2011)

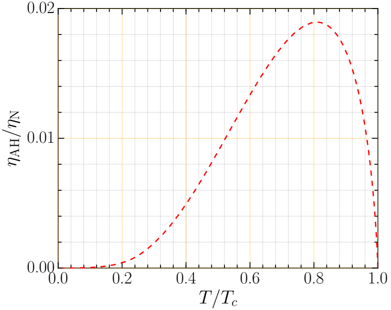
# Angular momentum around an $e$-bubble ($k_f R = 11.17$)

$$\mathbf{L}(T \to 0) \approx -\hbar N_{\text{bubble}} \hat{\mathbf{l}}/2$$

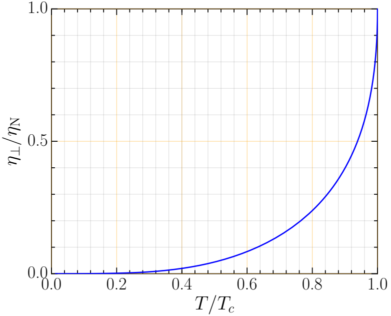


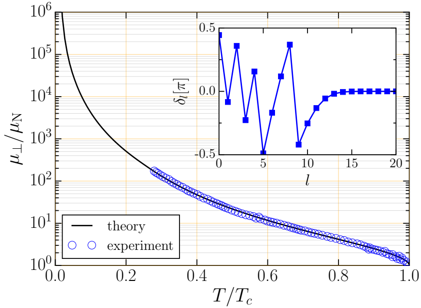
$$\mu_\perp = e\,\frac{\eta_\perp}{\eta_\perp^2 + \eta_{\rm AH}^2}, \quad \mu_{\rm AH} = -e\,\frac{\eta_{\rm AH}}{\eta_\perp^2 + \eta_{\rm AH}^2}, \quad \tan\alpha = \left|\frac{\mu_{\rm AH}}{\mu_\perp}\right| = \frac{\eta_{\rm AH}}{\eta_\perp}, \quad k_f R = 11.17$$

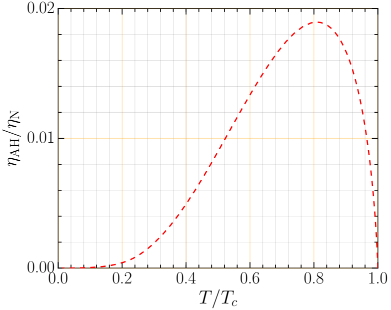


$$\mu_\perp = e\,\frac{\eta_\perp}{\eta_\perp^2 + \eta_{\rm AH}^2}, \quad \mu_{\rm AH} = -e\,\frac{\eta_{\rm AH}}{\eta_\perp^2 + \eta_{\rm AH}^2}, \quad \tan\alpha = \left|\frac{\mu_{\rm AH}}{\mu_\perp}\right| = \frac{\eta_{\rm AH}}{\eta_\perp}, \quad k_f R = 11.17$$

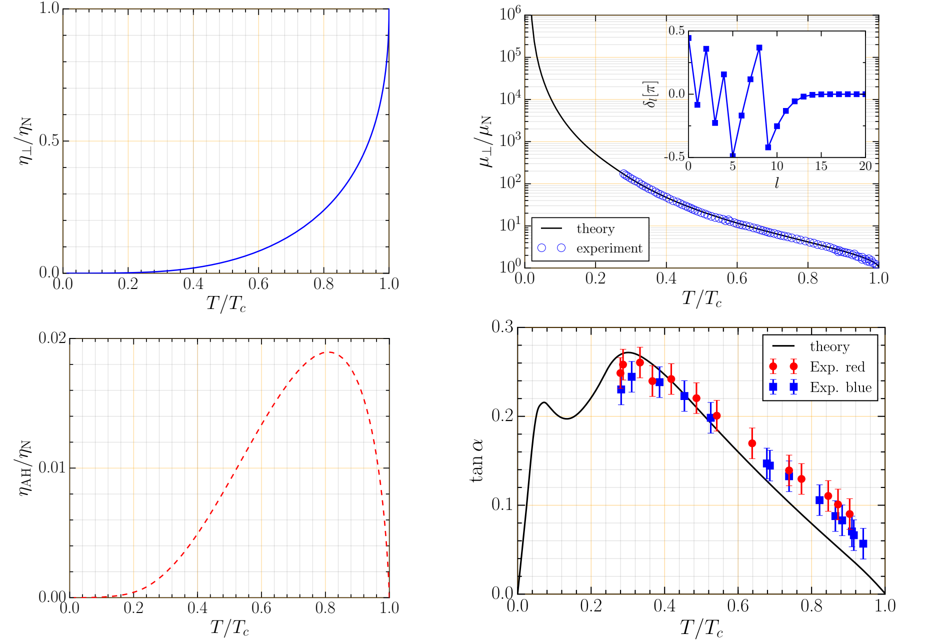


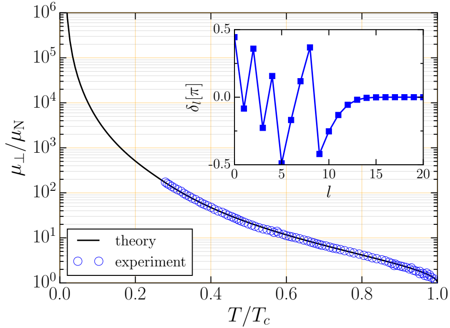
$$\mu_\perp = e\,\frac{\eta_\perp}{\eta_\perp^2 + \eta_{\mathrm{AH}}^2}, \quad \mu_{\mathrm{AH}} = -e\,\frac{\eta_{\mathrm{AH}}}{\eta_\perp^2 + \eta_{\mathrm{AH}}^2}, \quad \tan\alpha = \left|\frac{\mu_{\mathrm{AH}}}{\mu_\perp}\right| = \frac{\eta_{\mathrm{AH}}}{\eta_\perp},$$

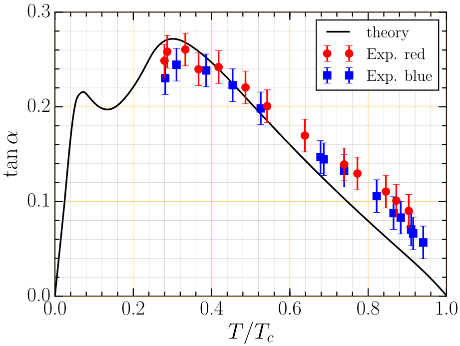
$k_f R = 11.17$



# Summary

- Electrons in 3He-A are "dressed" by a spectrum of Weyl fermions $\rightsquigarrow L_z \approx -(N_{bubble}/2)\hbar \approx -100\,\hbar$
- Scattering of Bogoliubov QPs by the dressed Ion $\rightsquigarrow$ Drag ($-\eta_\perp \mathbf{v}$) and Transverse ($\frac{e}{c}\mathbf{v} \times \mathbf{B}_{eff}$) forces on the Ion
- $\mathbf{B}_{eff} \approx \frac{\Phi_0}{3\pi^2}\, k_f^2\, (k_f R)^2\, \left(\frac{\eta_{AH}}{\eta_{\mathrm{N}}}\right) \hat{\mathbf{l}} \simeq 10^3 - 10^4$ ! $\rightsquigarrow$ *Anomalous Hall Effect*
- Mechanism:
  Skew/Andreev Scattering of Bogoliubov QPs by the dressed Ion
- Origin:
  Broken Mirror & Time-Reversal Symmetry $\rightsquigarrow W(\mathbf{k}, \mathbf{k}') \neq W(\mathbf{k}', \mathbf{k})$
- Input: Hard-sphere scattering with $k_f R = 11.17$
- Theory: $\rightsquigarrow$ Quantitative account of RIKEN mobility experiments
- Future: New directions for Ion, Heat & Spin Transport in 3He

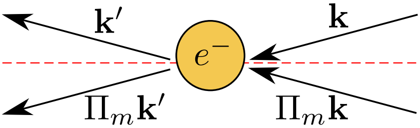

$\mu_\perp/\mu_N$
$T/T_c$
theory
experiment
$\delta_l[\pi]$
$l$



$\tan\alpha$
$T/T_c$
theory
Exp. red
Exp. blue

# Mirror-symmetric scattering $\Rightarrow$ linear drag

$$\mathbf{F}_{QP} = -\overleftrightarrow{\eta} \cdot \mathbf{v}, \quad \eta_{ij} = n_3 p_f \int_0^\infty dE \left(-2\frac{\partial f}{\partial E}\right) \sigma_{ij}(E)$$

Subdivide by mirror symmetry:




$$W(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = W^{(+)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}) + W^{(-)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}),$$

$$\sigma_{ij}(E) = \sigma_{ij}^{(+)}(E) + \sigma_{ij}^{(-)}(E),$$

$$\sigma_{ij}^{(+)}(E) = \frac{3}{4} \int_{E \geq |\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'} \int_{E \geq |\Delta(\hat{\mathbf{k}})|} \frac{d\Omega_{\mathbf{k}}}{4\pi} \left[(\hat{\mathbf{k}}'_i - \hat{\mathbf{k}}_i)(\hat{\mathbf{k}}'_j - \hat{\mathbf{k}}_j)\right] \frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E)$$

Mirror-symmetric diff. cross section: $W^{(+)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}', \hat{\mathbf{k}}) + W(\hat{\mathbf{k}}, \hat{\mathbf{k}}')]/2$

$$\frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2 \frac{E}{\sqrt{E^2 - |\Delta(\hat{\mathbf{k}}')|^2}} W^{(+)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}) \frac{E}{\sqrt{E^2 - |\Delta(\hat{\mathbf{k}})|^2}}$$

No transverse force! $\left[\eta_{ij}^{(+)}\right]_{i \neq j} = 0, \quad \eta_{xx}^{(+)} = \eta_{yy}^{(+)} \equiv \eta_\perp, \quad \eta_{zz}^{(+)} \equiv \eta_\parallel$

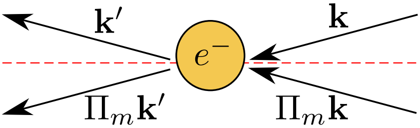
# Mirror-antisymmetric scattering ⇒ transverse force

$$\mathbf{F}_{QP} = -\overset{\leftrightarrow}{\eta} \cdot \mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^\infty dE \left(-2\frac{\partial f}{\partial E}\right) \sigma_{ij}(E)$$

Subdivide by mirror symmetry:

$$W(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = W^{(+)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}) + W^{(-)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}),$$

$$\sigma_{ij}(E) = \sigma_{ij}^{(+)}(E) + \sigma_{ij}^{(-)}(E),$$



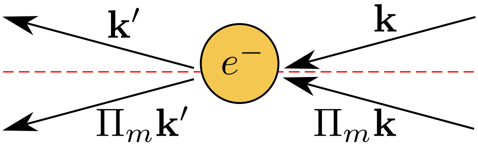

$$\sigma_{ij}^{(-)}(E) = \frac{3}{4}\int_{E\geq|\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'} \int_{E\geq|\Delta(\hat{\mathbf{k}})|} \frac{d\Omega_{\mathbf{k}}}{4\pi} [\epsilon_{ijk}(\hat{\mathbf{k}}' \times \hat{\mathbf{k}})_k] \frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E)\left[f(E) - \frac{1}{2}\right]$$

Mirror-antisymmetric diff. cross section: $W^{(-)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}', \hat{\mathbf{k}}) - W(\hat{\mathbf{k}}, \hat{\mathbf{k}}')]/2$

$$\frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2 \frac{E}{\sqrt{E^2 - |\Delta(\hat{\mathbf{k}}')|^2}} W^{(-)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}) \frac{E}{\sqrt{E^2 - |\Delta(\hat{\mathbf{k}})|^2}}$$

Transverse force! $\eta_{xy}^{(-)} = -\eta_{yx}^{(-)} \equiv \eta_{\mathrm{AH}}$ ⇒ anomalous Hall effect!

# Broken TR and mirror symmetries in $^3$He-A




(1) Broken TRS: $\mathrm{T}\hat{\mathbf{l}} = -\hat{\mathbf{l}}$

(2) Broken mirror symmetry: $\Pi_m \hat{\mathbf{l}} = -\hat{\mathbf{l}}$

(3) Chiral symmetry: $\mathrm{C} = \mathrm{T} \times \Pi_m$

(4) Microscopic reversibility for $^3$He-A: $W(\hat{\mathbf{k}}', \hat{\mathbf{k}}; \hat{\mathbf{l}}) = W(\hat{\mathbf{k}}, \hat{\mathbf{k}}'; -\hat{\mathbf{l}})$

(5) If the chiral axis $\hat{\mathbf{l}}$ is fixed: $W(\hat{\mathbf{k}}', \hat{\mathbf{k}}) \neq W(\hat{\mathbf{k}}, \hat{\mathbf{k}}')$

# Alternative QP-ion scattering potential models

- $U(r) = \begin{cases} U_0, & r < R, \\ -U_1, & R < r < R', \\ 0, & r > R'. \end{cases}$

- $U(x) = U_0[1 - \tanh[(x-b)/c]], \quad x = k_f r$

- $U(x) = U_0/\cosh^2[\alpha x^n], \quad x = k_f r$ (Pöschl-Teller-like potential)

- random phase shifts model: $\{\delta_l,\ l = 1 \ldots l_{\max}\}$ are generated while $\delta_0$ is a tuning parameter

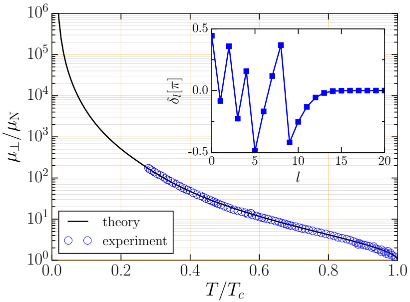
Parameters for each model are chosen to fit the experimental value of the normal-state mobility, $\mu_N^{\exp} = 1.7 \times 10^{-6}\ m^2/V \cdot s$

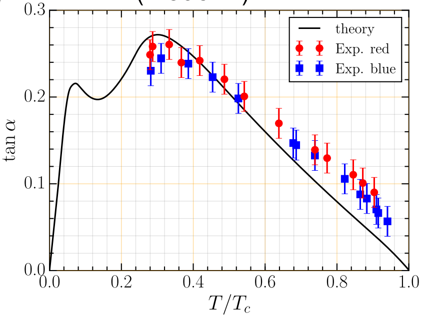
# Alternative QP-ion scattering potential models

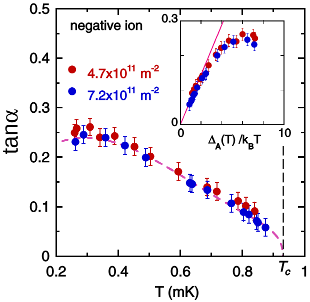
| Label | Potential | Parameters |
|---|---|---|
| Model A | hard sphere | $k_f R = 11.17$ |
| Model B | attractive well with a repulsive core | $U_0 = 100E_f, U_1 = 10E_f, k_f R' = 11, R/R' = 0.36$ |
| Model C | random phase shifts model 1 | $l_{\max} = 11$ |
| Model D | random phase shifts model 2 | $l_{\max} = 11$ |
| Model E | Pöschl-Teller-like | $U_0 = 1.01E_f, k_f R = 22.15, \alpha = 3 \times 10^{-5}, n = 4$ |
| Model F | Pöschl-Teller-like | $U_0 = 2E_f, k_f R = 19.28, \alpha = 6 \times 10^{-5}, n = 4$ |
| Model G | hyperbolic tangent | $U_0 = 1.01E_f, k_f R = 14.93, b = 12.47, c = 0.246$ |
| Model H | hyperbolic tangent | $U_0 = 2E_f, k_f R = 14.18, b = 11.92, c = 0.226$ |
| Model I | soft sphere 1 | $U_0 = 1.01E_f, k_f R = 12.48$ |
| Model J | soft sphere 2 | $U_0 = 2E_f, k_f R = 11.95$ |

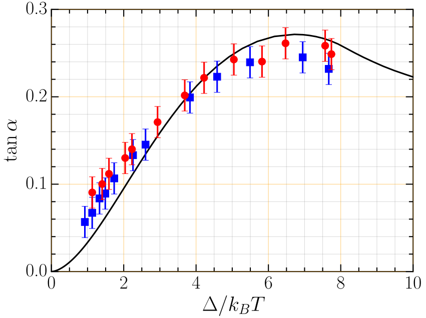
# Alternative QP-ion scattering potential models

Hard-sphere model with $k_f R = 11.17$ (Model A)



# Alternative QP-ion scattering potential models

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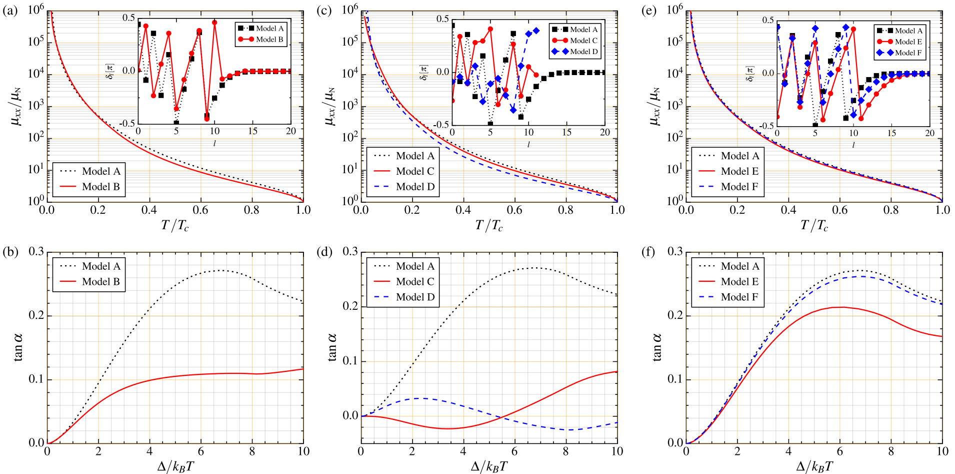


# Alternative way of determining R

(i) Energy required to create a bubble:
$$E(R,P) = E_0(U_0,R) + 4\pi R^2\gamma + \frac{4\pi}{3}R^3 P, \quad P \text{ – pressure}$$

(ii) $\gamma = 0.15\,\mathrm{erg/cm^2}$ is the surface tension of helium

(iii) For $U_0 \to \infty$:
$E_0 = -U_0 + \pi^2\hbar^2/2m_e R^2$ – ground state energy

(iv) Minimizing E wrt $R$: $P = \pi\hbar^2/4m_e R^5 - 2\gamma/R$

(v) For zero pressure, $P = 0$:
$$R = \left(\frac{\pi\hbar^2}{8m_e\gamma}\right)^{1/4} \approx 2.38\,\mathrm{nm} \quad \rightsquigarrow \quad k_f R = 18.67$$

# Normal-state mobility of an $e$-bubble

(i) $t_{\mathrm{N}}^{R}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E) = \sum_{l=0}^{\infty} (2l+1) t_l^R(E) P_l(\hat{\mathbf{k}}' \cdot \hat{\mathbf{k}})$

(ii) $t_l^R(E) = -\frac{1}{\pi N_f} e^{i\delta_l} \sin\delta_l$

(iii) $\frac{d\sigma}{d\Omega_{\mathbf{k}'}} = \left( \frac{m^*}{2\pi\hbar^2} \right)^2 |t_{\mathrm{N}}^{R}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E)|^2$

(iv) $\sigma_{\mathrm{N}} = \int \frac{d\Omega_{\mathbf{k}'}}{4\pi} (1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}') \frac{d\sigma}{d\Omega_{\mathbf{k}'}} = \frac{4\pi}{k_f^2} \sum_{l=0}^{\infty} (l+1) \sin^2(\delta_{l+1} - \delta_l)$

(v) $\mu_{\mathrm{N}} = \frac{e}{n_3 p_f \sigma_{\mathrm{N}}}, \quad p_f = \hbar k_f, \quad n_3 = \frac{k_f^3}{3\pi^2}$

# Calculation of LDOS and current density

(i) $\hat{\mathcal{G}}_S^R(\mathbf{r}', \mathbf{r}, E) = \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} e^{i\mathbf{k}'\mathbf{r}'} e^{-i\mathbf{k}\mathbf{r}} \hat{\mathcal{G}}_S^R(\mathbf{k}', \mathbf{k}, E)$

(ii) $\hat{\mathcal{G}}_S^R(\mathbf{k}', \mathbf{k}, E) = (2\pi)^3 \hat{G}_S^R(\mathbf{k}, E)\delta_{\mathbf{k}',\mathbf{k}} + \hat{G}_S^R(\mathbf{k}', E)\hat{T}_S(\mathbf{k}', \mathbf{k}, E)\hat{G}_S^R(\mathbf{k}, E)$

(iii) $\hat{G}_S^R(\mathbf{k}, E) = \frac{1}{\varepsilon^2 - E_{\mathbf{k}}^2} \begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad \varepsilon = E + i\eta, \quad \eta \to 0^+$

(iv) $N(\mathbf{r}, E) = -\frac{1}{2\pi} \mathrm{Im} \left\{ \mathrm{Tr} \left[ \hat{\mathcal{G}}_S^R(\mathbf{r}, \mathbf{r}, E) \right] \right\}$

(v) $\mathbf{j}(\mathbf{r}) = \frac{\hbar}{4mi} k_B T \sum_{n=-\infty}^{\infty} \lim_{\mathbf{r}\to\mathbf{r}'} \mathrm{Tr} \left[ (\nabla_{\mathbf{r}'} - \nabla_{\mathbf{r}}) \hat{\mathcal{G}}^M(\mathbf{r}', \mathbf{r}, \epsilon_n) \right]$

(vi) $\hat{\mathcal{G}}_S^R(\mathbf{r}', \mathbf{r}, E) = \hat{\mathcal{G}}_S^M(\mathbf{r}', \mathbf{r}, \epsilon_n)\Big|_{i\epsilon_n \to \varepsilon}$, for $n \geq 0$

(vii) $\hat{\mathcal{G}}_S^M(\mathbf{k}, \mathbf{k}', -\epsilon_n) = \left[ \hat{\mathcal{G}}_S^M(\mathbf{k}', \mathbf{k}, \epsilon_n) \right]^\dagger$

# Temperature scaling of the Stokes tensor components

- For $1 - \frac{T}{T_c} \to 0^+$:

$$\frac{\eta_\perp}{\eta_N} - 1 \propto -\Delta(T) \propto \sqrt{1 - \frac{T}{T_c}}$$

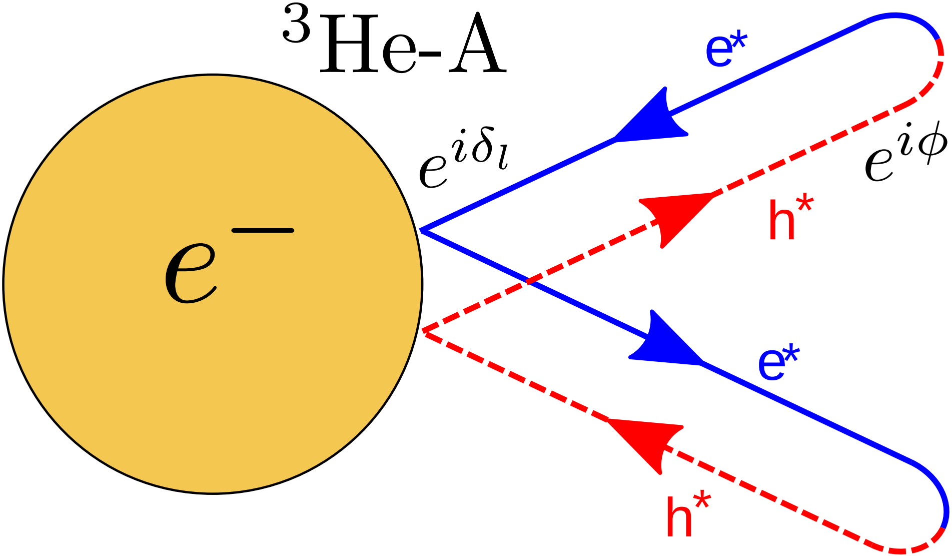
$$\frac{\eta_{\mathrm{AH}}}{\eta_N} \propto \Delta^2(T) \propto 1 - \frac{T}{T_c}$$

- For $\frac{T}{T_c} \to 0^+$:

$$\frac{\eta_\perp}{\eta_N} \propto \left(\frac{T}{T_c}\right)^2$$

$$\frac{\eta_{\mathrm{AH}}}{\eta_N} \propto \left(\frac{T}{T_c}\right)^3$$

# Formation of Weyl fermions on *e*-bubbles




$$\Delta(\hat{\mathbf{k}}) = \Delta \sin\theta e^{i\phi}$$