From Spontaneous Symmetry Breaking to Topological Order

J. A. Sauls†

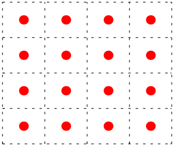
Northwestern University

- Hao Wu • Joshua Wiman
- Suk Bum Chung • Takeshi Mizushima
- Erkki Thuneberg • Samie Laine • Anton Vorontsov
- W. Halperin • J. Parpia • J. Saunders • Y. Lee

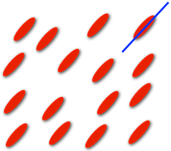
- Confinement: New Phases
- Dynamics: Bosonic Modes
- Topological Order: ^{3}He
- Signatures: Edge & Surface States

†NSF Grant DMR-1106315

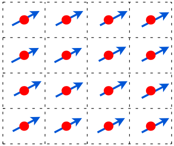
Broken Symmetry, Phase Transitions and Long-Range Order



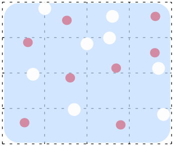
Solid



Nematic



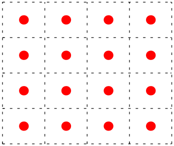
Ferromagnet



Super-liquid

Translations	Space Rotations	Spin Rotation	Gauge
$\mathsf{G}_{\mathsf{trans}}$	$\mathsf{SO(3)}_{\mathrm{L}}$	$\mathsf{SO(3)}_{\mathrm{S}}$	$\mathsf{U(1)}_{\mathrm{N}}$
$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r})$	$Q_{ij} = Q(T)\left(\mathbf{n}_i\mathbf{n}_j - \frac{1}{3}\delta_{ij}\right)$	$\mathbf{M} = \gamma \langle \mathbf{S} \rangle$	$\Psi = \langle \psi(\mathbf{r}) \rangle \simeq \sqrt{N/V}\, e^{i\vartheta}$

Broken Symmetry, Phase Transitions and Long-Range Order



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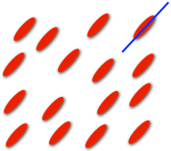

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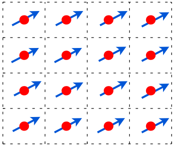

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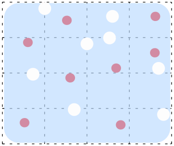


Super-liquid

Translations

G_{trans}

Space Rotations

$\mathrm{SO}(3)_{\mathrm{L}}$

Spin Rotation

$\mathrm{SO}(3)_{\mathrm{S}}$

Gauge

$\mathrm{U}(1)_{\mathrm{N}}$

$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r})$$

$$Q_{ij} = Q(T)\left(\mathbf{n}_i \mathbf{n}_j - \frac{1}{3}\delta_{ij}\right)$$

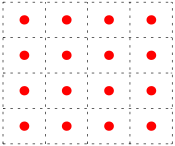
$$\mathbf{M} = \gamma \langle \mathbf{S} \rangle$$

$$\Psi = \langle \psi(\mathbf{r}) \rangle \simeq \sqrt{N/V}\, e^{i\vartheta}$$

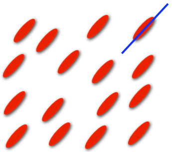
Unconventional Superconductivity

- Break one or more spin/space-group symmetries in conjunction with $\mathrm{U}(1)_{\mathrm{N}}$

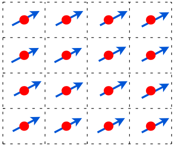
Broken Symmetry, Phase Transitions and Long-Range Order



Solid



Nematic



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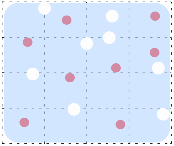

Super-liquid

Translations — G_{trans}

Space Rotations — $SO(3)_L$

Spin Rotation — $SO(3)_S$

Gauge — $U(1)_N$

$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r})$$

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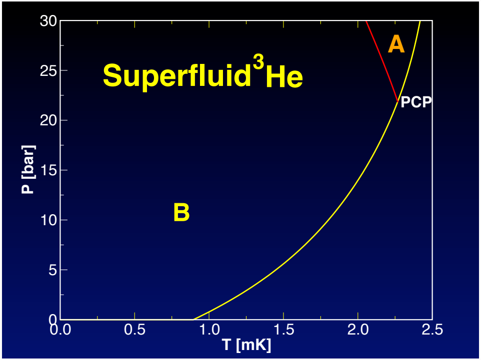
$$\Psi = \langle\psi(\mathbf{r})\rangle \simeq \sqrt{N/V}\, e^{i\vartheta}$$

Unconventional Superconductivity

- Break one or more spin/space-group symmetries in conjunction with $U(1)_N$
  - Phase of ^{3}He exhibit *all* of these broken symmetries!

Symmetry of Normal ^{3}*He* : $\mathtt{G} = \mathtt{SO(3)_S} \times \mathtt{SO(3)_L} \times \mathtt{U(1)_N} \times \mathtt{P} \times \mathtt{T}$

Phase Diagram of Bulk ^{3}He



Spin-Triplet, P-wave Order Parameter :

$$\Delta_{\alpha\beta}(\mathbf{p}) = \vec{\mathbf{d}}(\mathbf{p}) \cdot (i\vec{\boldsymbol{\sigma}}\sigma_y)_{\alpha\beta} \rightsquigarrow \mathbf{d}_\mu(\mathbf{p}) = \mathcal{A}_{\mu i}\,\mathbf{p}_i$$

Symmetry of Normal ^{3}*He*: $\mathtt{G} = \mathtt{SO(3)_S} \times \mathtt{SO(3)_L} \times \mathtt{U(1)_N} \times \mathtt{P} \times \mathtt{T}$

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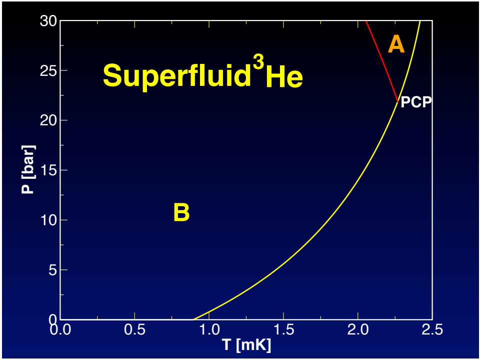
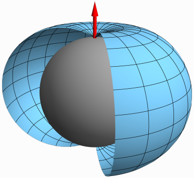


Chiral ABM State $\vec{l} = \hat{\mathbf{m}} \times \hat{\mathbf{n}}$



$$\mathcal{A}_{\mu i} = \Delta\, \hat{\mathbf{d}}_\mu\, (\hat{\mathbf{m}} + i\hat{\mathbf{n}})_i$$

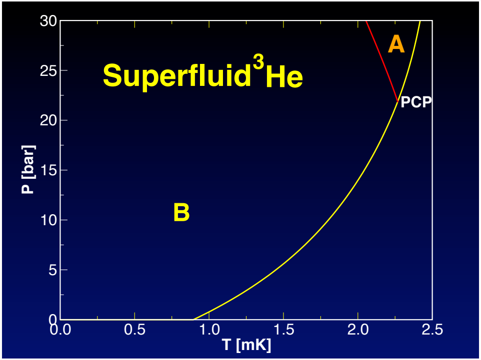
$$L_z = 1,\ S_z = 0$$

Spin-Triplet, P-wave Order Parameter:

$$\Delta_{\alpha\beta}(\mathbf{p}) = \vec{\mathbf{d}}(\mathbf{p}) \cdot (i\vec{\boldsymbol{\sigma}}\sigma_y)_{\alpha\beta} \rightsquigarrow \mathbf{d}_\mu(\mathbf{p}) = \mathcal{A}_{\mu i}\, \mathbf{p}_i$$

Symmetry of Normal ^{3}*He*: $\mathtt{G} = \mathtt{SO(3)_S} \times \mathtt{SO(3)_L} \times \mathtt{U(1)_N} \times \mathtt{P} \times \mathtt{T}$

Phase Diagram of Bulk ^{3}He



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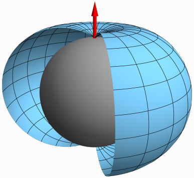
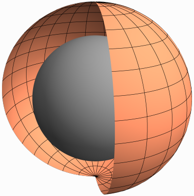


$$\mathcal{A}_{\mu i} = \Delta\, \hat{\mathbf{d}}_\mu\, (\hat{\mathbf{m}} + i\hat{\mathbf{n}})_i$$

$L_z = 1$, $S_z = 0$

"Isotropic" BW State



$$\mathcal{A}_{\mu i} = \Delta\, \delta_{\mu i}$$

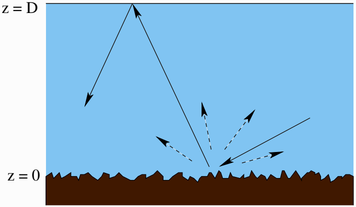
$J = 0$, $J_z = 0$

Spin-Triplet, P-wave Order Parameter:

$$\Delta_{\alpha\beta}(\mathbf{p}) = \vec{\mathbf{d}}(\mathbf{p}) \cdot (i\vec{\boldsymbol{\sigma}}\sigma_y)_{\alpha\beta} \rightsquigarrow \mathbf{d}_\mu(\mathbf{p}) = \mathcal{A}_{\mu i}\, \mathbf{p}_i$$

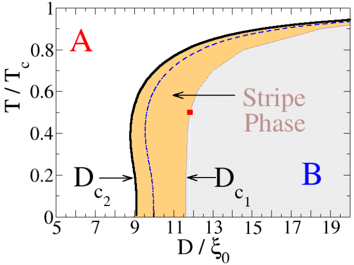
New Chiral Phase with Spontaneously Broken *Translational* Symmetry and BTRS

100nm-10μm Films & Cavities



A. Vorontsov & JAS, PRL (2007)

▶ J. J. Wiman, Poster - Monday-Wednesday



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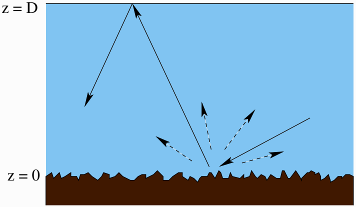

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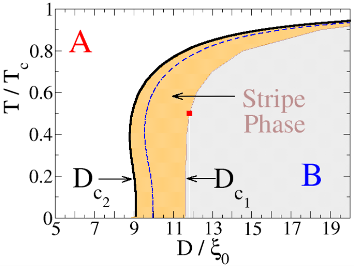
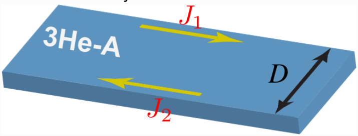
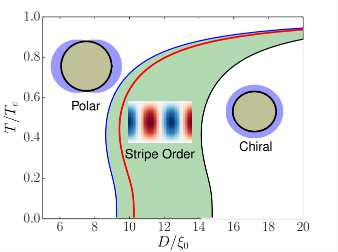


▶ Hao Wu - Saturday, 9:20 – 9:40

Laterally confined films of ^{3}He



Overlapping Chiral Edge States ⟶



Ginzburg-Landau Functional for Superfluid ^{3}He

Maximal Symmetry of ^{3}He: $\mathtt{G} = \mathtt{SO(3)_L} \times \mathtt{SO(3)_S} \times \mathtt{U(1)_N} \times \mathtt{P} \times \mathtt{T}$

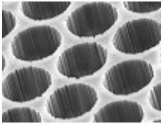
Order Parameter for P-wave ($L = 1$), Spin-Triplet ($S = 1$) Pairing

$$\widehat{\Psi}(\hat{p}) = \overbrace{\begin{pmatrix} S_{\mathbf{x}} & S_{\mathbf{y}} & S_{\mathbf{z}} \end{pmatrix}}^{\text{Spin Basis}} \times \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix} \times \overbrace{\begin{pmatrix} \hat{p}_x \\ \hat{p}_y \\ \hat{p}_z \end{pmatrix}}^{\text{Orbital Basis}}$$

- GL Functional: $A_{\alpha i}$ $\rightsquigarrow$ vector under both $\mathtt{SO(3)_S}$ $[\alpha]$ and $\mathtt{SO(3)_L}$ $[i]$

$$\begin{aligned}
\mathcal{F}[A] \;=\; & \int d^3r \Big[\alpha(T)\mathrm{Tr}\left\{AA^{\dagger}\right\} + \beta_1 \left|\mathrm{Tr}\left\{AA^{\mathrm{tr}}\right\}\right|^2 + \beta_2 \left(\mathrm{Tr}\left\{AA^{\dagger}\right\}\right)^2 \\
+ \; & \beta_3 \mathrm{Tr}\left\{AA^{\mathrm{tr}}(AA^{\mathrm{tr}})^*\right\} + \beta_4 \mathrm{Tr}\left\{(AA^{\dagger})^2\right\} + \beta_5 \mathrm{Tr}\left\{AA^{\dagger}(AA^{\dagger})^*\right\} \\
+ \; & \kappa_1 \partial_i A_{\alpha j}\, \partial_i A^*_{\alpha j} + \kappa_2 \partial_i A_{\alpha i}\, \partial_j A^*_{\alpha j} + \kappa_3 \partial_i A_{\alpha j}\, \partial_j A^*_{\alpha i} \Big]
\end{aligned}$$

100nm Pores

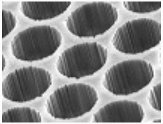


1D B-stripe phase

- K. Aoyama, Fri. 9:00

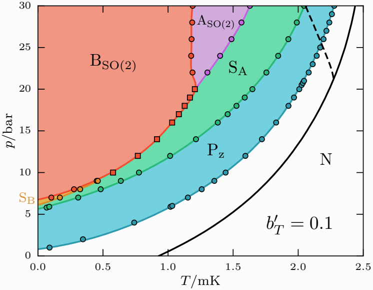
New Phases of Superfluid ^{3}He Under Strong Confinement

100nm Pores

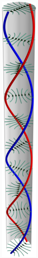


1D B-stripe phase
▶ K. Aoyama, Fri. 9:00

▶ J.J. Wiman, Poster, Mon-Wed



S_A - Spiral Phase



New Phases of Superfluid ^{3}He Under Strong Confinement

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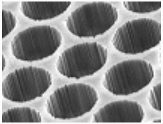

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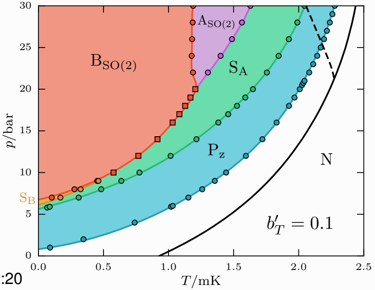
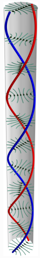


S_A - Spiral Phase



▶ V. Dmitriev, Fri. 9:20
Nematic Aerogel

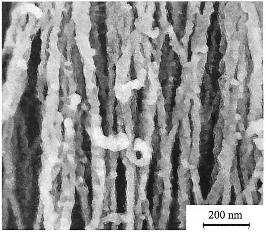
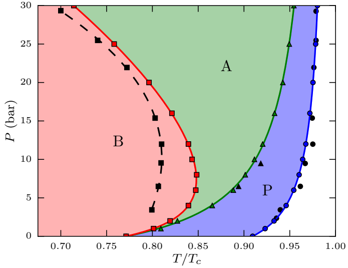


Fig. 1. The SEM-photo of "nematically ordered" aerogel

Ashkadullin et al. JETPL (2012)

▶ J.J. Wiman, S. Laine, E. Thuneberg & JAS



New Bosonic Excitations

New Bosonic Excitations

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN)

CERN-PH-EP/2012-220
2013/01/29

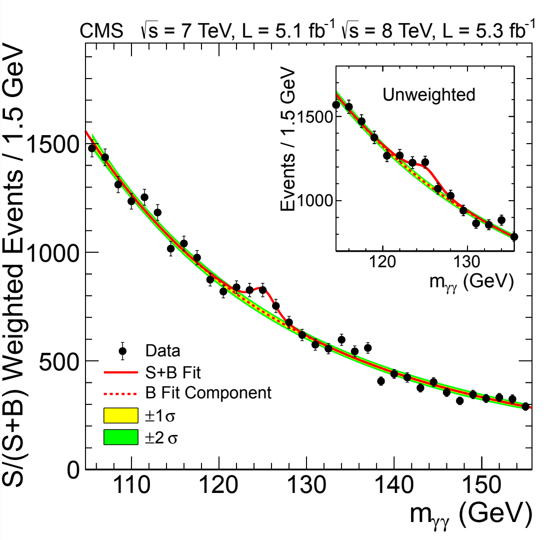
CMS-HIG-12-028

Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC

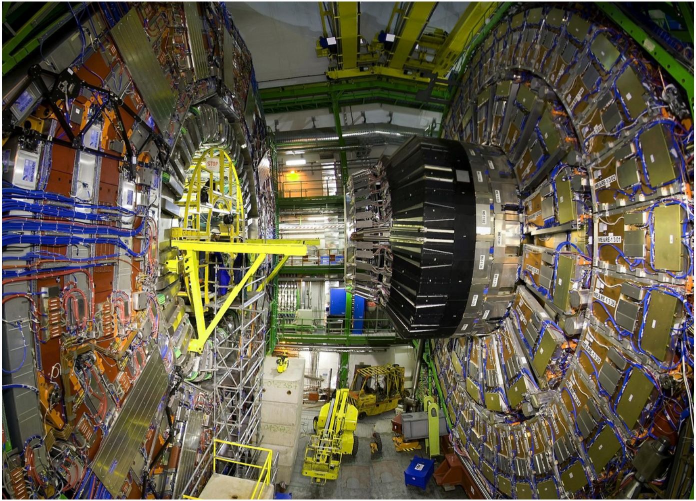
The CMS Collaboration

2013

Higgs Boson with mass $M = 125$ GeV



Emergence of New Bosonic Excitations



CMS Detector at the LHC

“Scalar” Higgs Boson (spin $J = 0$) [P. Higgs, PRL 13, 508 1964]

Energy Functional

$$E[\Delta] = \int dV \left\{ \alpha\, |\Delta|^2 + \beta\, |\Delta|^4 + \tfrac{1}{2}c^2\, |\boldsymbol{\nabla}\Delta|^2 \right\}$$

"Scalar" Higgs Boson (spin $J = 0$) [P. Higgs, PRL 13, 508 1964]

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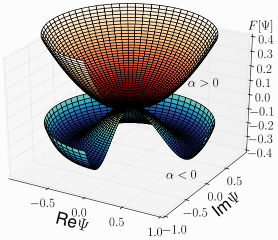
- Broken Symmetry State: $\Delta = \sqrt{|\alpha|}/2\beta$



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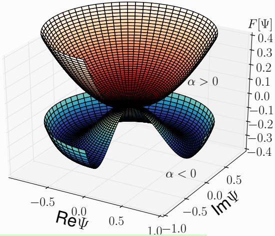

Space-Time Fluctuations about the Condensate Vacuum

$\Delta(\mathbf{r}, t) = \Delta + D(\mathbf{r}, t)$

- Eigenmodes: $D^{(\pm)} = D \pm D^*$ (Charge Conjugation Parity)

$$\mathcal{L} = \int d^3r \left\{ \frac{1}{2}[(\dot{D}^{(+)})^2 + (\dot{D}^{(-)})^2] - 2\Delta^2 \, (D^{(+)})^2 - \frac{1}{2}[c^2(\nabla D^{(+)})^2 + c^2(\nabla D^{(-)})^2] \right\}$$

"Scalar" Higgs Boson (spin $J = 0$) [P. Higgs, PRL 13, 508 1964]

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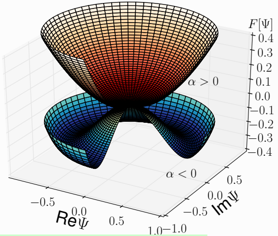

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- $\partial_t^2 D^{(-)} - c^2\boldsymbol{\nabla}^2\, D^{(-)} = 0$

Massless Nambu-Goldstone Mode

- $\partial_t^2 D^{(+)} + 4\Delta^2\, D^{(+)} - c^2\boldsymbol{\nabla}^2\, D^{(+)} = 0$

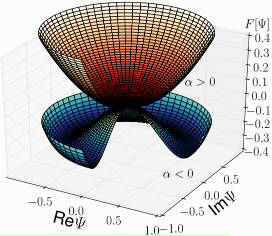
Massive Higgs Mode: $M = 2\Delta$

BCS Condensation of Spin-Singlet ($S = 0$), S-wave ($L = 0$) "Scalar" Cooper Pairs

Ginzburg-Landau Functional

$$F[\Delta] = \int dV \left\{ \alpha\,|\Delta|^2 + \beta\,|\Delta|^4 + \kappa\,|\boldsymbol{\nabla}\Delta|^2 \right\}$$

- Order Parameter: $\Delta = \sqrt{|\alpha|/2\beta}$



Space-Time Fluctuations of the Condensate Order Parameter

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$$\mathcal{L} = \int d^3r \left\{ \frac{1}{2}[(\dot{D}^{(+)})^2 + (\dot{D}^{(-)})^2] - 2\Delta^2\,(D^{(+)})^2 - \frac{1}{2}[v^2(\boldsymbol{\nabla}D^{(+)})^2 + v^2(\boldsymbol{\nabla}D^{(-)})^2] \right\}$$

- $\partial_t^2 D^{(-)} - v^2\boldsymbol{\nabla}^2\,D^{(-)} = 0$

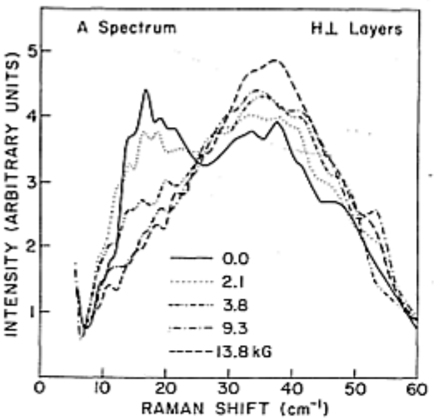
  Anderson-Bogoliubov Mode

- $\partial_t^2 D^{(+)} + 4\Delta^2\; D^{(+)} - v^2\boldsymbol{\nabla}^2\,D^{(+)} = 0$

  Amplitude Higgs Mode: $M = 2\Delta$

Higgs Mode with mass: $M = 3$ meV and spin $J = 0$ at Univ. Illinois-Urbana

Raman Absorption in $NbSe_2$



- R. Sooyakumar and M. V. Klein, Phys. Rev. Lett. 45, 660 (1980).
- Theory: P. Littlewood and C. M. Varma, Phys. Rev. Lett. 47, 811 (1981).

First Reported Observations of Higgs Modes in BCS Condensates

VOLUME 45, NUMBER 4 PHYSICAL REVIEW LETTERS 28 JULY 1980

Measurements of High-Frequency Sound Propagation in ^{3}He-B

D. B. Mast, Bimal K. Sarma, J. R. Owers-Bradley, I. D. Calder,
J. B. Ketterson, and W. P. Halperin
*Department of Physics and Astronomy and Materials Research Center, Northwestern University,
Evanston, Illinois 60201*
(Received 10 April 1980)

Measurements of the attenuation and velocity of pulsed high-frequency sound have been performed up to 133 MHz in superfluid ^{3}He-B. A new collective mode of the order parameter was discovered at a frequency extrapolated to T_c of $\omega = (1.165 \pm 0.05)\Delta_{BCS}(T_c)$, where $\Delta_{BCS}(T)$ is the energy gap in the weak-coupling BCS theory. The group velocity has been observed to decrease by as much as $\frac{2}{3}$ of the zero-sound velocity.

Observation of a New Sound-Attenuation Peak in Superfluid ^{3}He-B

R. W. Giannetta,[a] A. Ahonen,[b] E. Polturak, J. Saunders,
E. K. Zeise, R. C. Richardson, and D. M. Lee
*Laboratory of Atomic and Solid State Physics and Materials Science Center, Cornell University,
Ithaca, New York 14853*
(Received 25 March 1980)

Results of zero-sound attenuation measurements in ^{3}He-B, at frequencies up to 60 MHz and pressures between 0 and 20 bars, are reported. At frequencies of 30 MHz and above, a new attenuation feature is observed which bears the signature of a collective mode of the superfluid.

VOLUME 45, NUMBER 8 PHYSICAL REVIEW LETTERS 25 AUGUST 1980

Raman Scattering by Superconducting-Gap Excitations and Their Coupling to Charge-Density Waves

R. Sooryakumar and M. V. Klein
*Department of Physics and Materials Research Laboratory, University of Illinois at Urbana-Champaign,
Urbana, Illinois 61801*
(Received 24 March 1980)

2H-$NbSe_2$ undergoes a charge-density-wave (CDW) distortion at 33 K which induces A and E Raman-active phonon modes. These are joined in the superconducting state at 2 K by new A and E Raman modes close in energy to the BCS gap 2Δ. Magnetic fields suppress the intensity of the new modes and enhance that of the CDW-induced modes, thus providing evidence of coupling between the superconducting-gap excitations and the CDW.

$$^3\text{He-B:}\quad B_{\alpha i} = \frac{1}{\sqrt{3}}\Delta\,\delta_{\alpha i} \quad L=1\,,\; S=1 \rightsquigarrow J=0$$

- Symmetry of ^{3}He-B: $\mathtt{H} = \mathtt{SO(3)}_\mathtt{J} \times \mathtt{T}$
- Fluctuations: $\mathcal{D}_{\alpha i}(\mathbf{r},t) = A_{\alpha i}(\mathbf{r},t) - B_{\alpha i} = \sum_{J,m} D_{J,m}(\mathbf{r},t)\, t^{(J,m)}_{\alpha i}$
- Lagrangian:

$$\mathcal{L} = \int d^3r \left\{ \tau\,\text{Tr}\left\{\dot{\mathcal{D}}\dot{\mathcal{D}}^\dagger\right\} - \alpha\,\text{Tr}\left\{\mathcal{D}\mathcal{D}^\dagger\right\} - \sum_{p=1}^{5}\beta_p\,u_p(\mathcal{D}) - \sum_{l=1}^{3} K_l\,v_l(\partial\mathcal{D}) \right\}$$

$$\partial_t^2 D^{(\mathtt{C})}_{J,m} + E^{(\mathtt{C})}_{J,m}(\mathbf{q})^2\,D^{(\mathtt{C})}_{J,m} = \frac{1}{\tau}\eta^{(\mathtt{C})}_{J,m}$$

with $\quad J = \{0,1,2\}\,,\, m = -J\ldots+J\,,\, \mathtt{C} = \pm 1$

- 4 Nambu-Goldstone Modes and 14 Higgs modes

$$E^{(\mathtt{C})}_{J,m}(\mathbf{q}) = \sqrt{M^2_{J,\mathtt{C}} + \left(c^{(\mathtt{C})}_{J,|m|}|\mathbf{q}|\right)^2}$$

Mode	Symmetry	Mass	Name
$D^{(+)}_{0,m}$	$J=0,\ \mathtt{C}=+1$	2Δ	Amplitude Higgs
$D^{(-)}_{0,m}$	$J=0,\ \mathtt{C}=-1$	0	Phase Mode
$D^{(+)}_{1,m}$	$J=1,\ \mathtt{C}=+1$	0	NG Spin-Orbit Modes
$D^{(-)}_{1,m}$	$J=1,\ \mathtt{C}=-1$	2Δ	AH Spin-Orbit Modes
$D^{(+)}_{2,m}$	$J=2,\ \mathtt{C}=+1$	$\sqrt{\frac{8}{5}}\Delta$	2^+ AH Modes
$D^{(-)}_{2,m}$	$J=2,\ \mathtt{C}=-1$	$\sqrt{\frac{12}{5}}\Delta$	2^- AH Modes

- Vdovin, Maki, Wölfle, Serene, Volovik, Schopohl, McKenzie, JAS ...

Bosonic Excitations of ^{3}He-B

Goldstone Mode w/ J=0⁻ → $D^{(-)}_{00} = i|\Delta|\,\varphi(q,\omega)$

phase mode

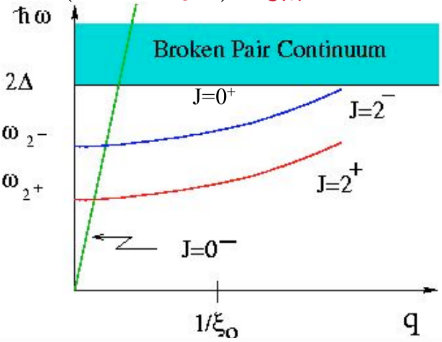
$$\left(\partial_t^2 - c_{00}^2 \nabla^2\right) D^{(-)}_{00} = \ldots$$

Pair Excitons w/ J=2⁺/⁻

$$\left(\partial_t^2 + \Omega_{JC}^2\right) D^{(C)}_{JM} = \ldots$$

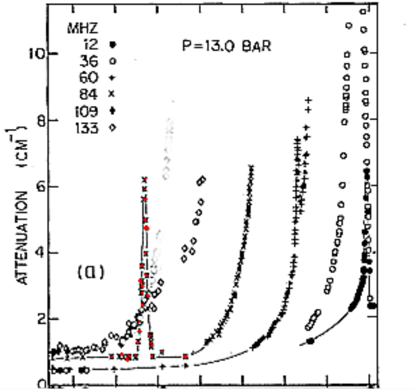
Anderson-Higgs Modes

coupling to internal & external fields

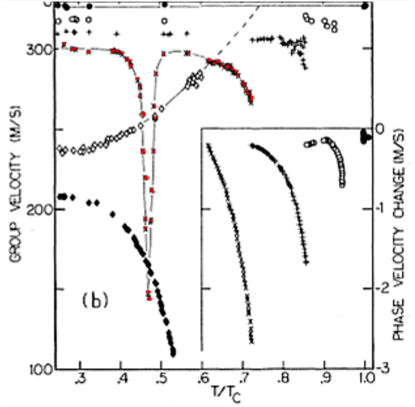


Higgs Mode with mass: $M = 500$ neV and spin $J = 2$ at ULT-Northwestern

Group Velocity

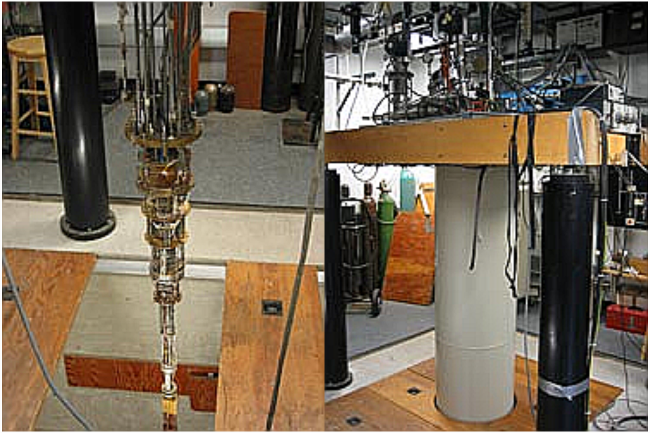


T/T_c



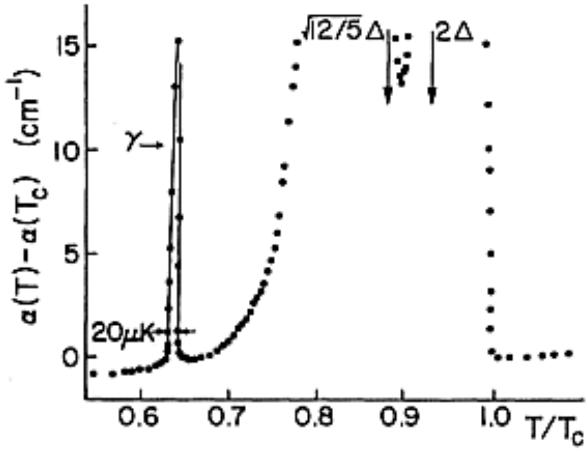
- D. Mast et al. Phys. Rev. Lett. 45, 266 (1980).

Higgs Mode with mass $M = 500$ neV and spin $J = 2$ at ULT-Northwestern



Superfluid ^{3}He Higgs Detector at ULT-Northwestern

Higgs Mode with mass: $M = 500$ neV and spin $J = 2$ at LASSP-Cornell



- R. Giannetta et al., PRL 45, 262 (1980)

Bosonic Excitations of ^{3}He-B

Goldstone Mode w/ J=0⁻ → $D^{(-)}_{00} = i|\Delta|\,\varphi(q,\omega)$

phase mode

$$(\partial_t^2 - c_{00}^2\nabla^2)\, D^{(-)}_{00} = \dots$$

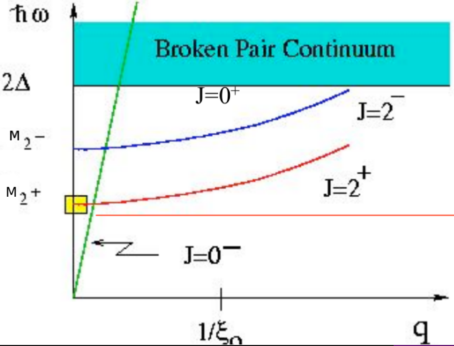
Pair Excitons w/ J=2⁺/⁻

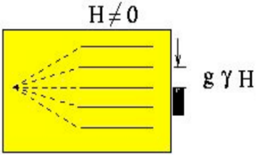
$$(\partial_t^2 + \Omega_{JC}^2)\, D^{(C)}_{JM} = \dots$$

Anderson-Higgs Modes

coupling to internal & external fields

Nuclear Zeeman levels





JAS & J. Serene, PRL 1982

Coupling of $J = 2^-$ Modes to Transverse Currents

- $J = 2^- \; m = \pm 1$ Modes transport mass (Transverse Sound)

$$C_t(\omega)^2 = \frac{F_1}{15}\,\rho_n(\omega) + \frac{2F_1}{75}\rho_s(\omega)\,\underbrace{\left\{\frac{\omega^2}{(\omega+i\Gamma)^2 - \frac{12}{5}\Delta^2 - \frac{2}{5}(q^2 v_f^2)}\right\}}_{D^{(-)}_{2,\pm 1}}$$

- G. Moores and JAS, JLTP (1993)

Coupling of $J = 2^-$ Modes to Transverse Currents

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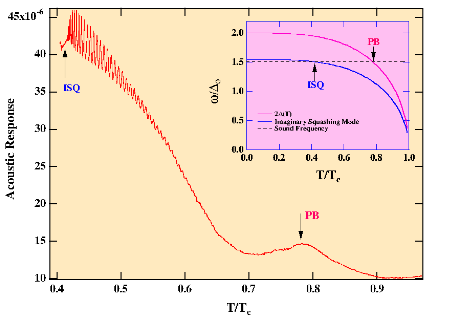
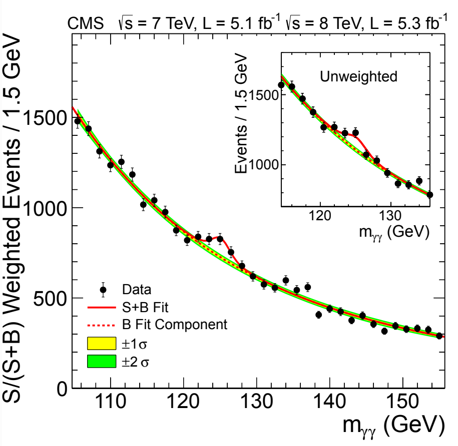
- G. Moores and JAS, JLTP (1993)

Transverse Zero Sound Propagation in Superfluid ^{3}He-B



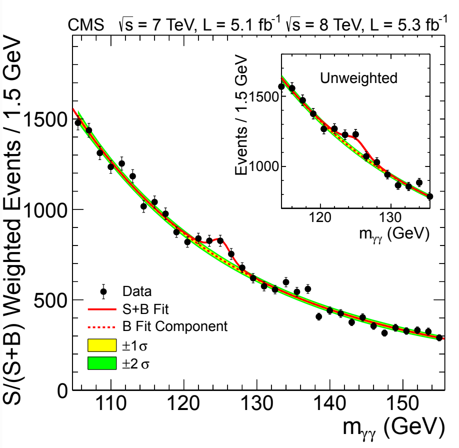
- Y. Lee et al. Nature 400 (1999)

Higgs Boson with mass $M = 125$ GeV



Is this all there is?

Higgs Boson with mass $M = 125$ GeV



Is this all there is?

Higgs Bosons in Particle Physics and in Condensed Matter,
G.E. Volovik & M. Zubkov, JLTP 175, 486-497 (2014).

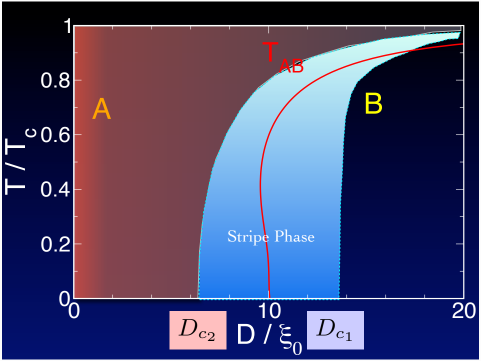
- Majorana excitations, spin and mass currents on the surface of topological superfluid ^{3}He-B,
  Hao Wu , JAS, Phys. Rev. B 88, 18 184506 (2013)
  [arXiv:1308.4436]

- Surface states, edge currents, and the angular momentum of chiral p-wave superfluids,
  JAS, Phys. Rev. B 84, 214509 (2011) [arXiv:1209.5501]

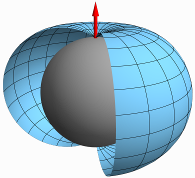
- Symmetry Protected Topological Superfluids and Superconductors — From the Basics to ^{3}He,
  T. Mizushima, Y. Tsutsumi , T. Kawakami, M. Sato, M. Ichioka, K. Machida [arXiv:1508.00787]

Symmetry or Normal ^{3}*He*: $G = SO(3)_S \times SO(3)_L \times U(1)_N \times P \times T$

A. Vorontsov & JAS, PRL, 2007



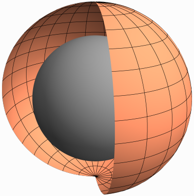
Chiral ABM State $\vec{l} = \hat{\mathbf{m}} \times \hat{\mathbf{n}}$



$$\mathcal{A}_{\mu i} = \Delta\, \hat{\mathbf{d}}_\mu\, (\hat{\mathbf{m}} + i\hat{\mathbf{n}})_i$$

$L_z = 1$, $S_z = 0$

"Isotropic" BW State



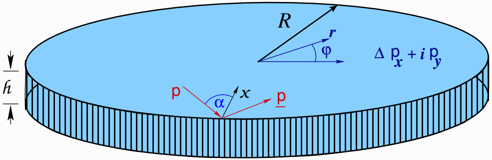
Spin-Triplet, P-wave Order Parameter:

$$\Delta_{\alpha\beta}(\mathbf{p}) = \vec{\mathbf{d}}(\mathbf{p}) \cdot (i\vec{\boldsymbol{\sigma}}\sigma_y)_{\alpha\beta} \rightsquigarrow \mathbf{d}_\mu(\mathbf{p}) = \mathcal{A}_{\mu i}\, \mathbf{p}_i$$

$$\mathcal{A}_{\mu i} = \Delta\, \delta_{\mu i}$$

$J = 0$, $J_z = 0$

^{3}He-A confined in a cylindrical cavity with $h \ll \xi_0$ and $R \gg \xi_0$.



2D Chiral ABM State:

$$\vec{\mathbf{d}}(\mathbf{p}) = \Delta\,\hat{\mathbf{z}}\,(p_x + ip_y)/p_f \sim e^{+i\varphi_{\mathbf{p}}}$$

Fully Gapped: $|\vec{\mathbf{d}}(\mathbf{p})|^2 = \Delta^2$

Bogoliubov Equations for Fermionic Excitations: $\mathbf{p} \rightarrow \dfrac{\hbar}{i}\boldsymbol{\nabla}$

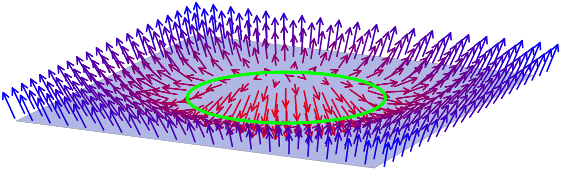
$$\begin{pmatrix} |\mathbf{p}|^2/2m^* - \mu & \Delta\,(p_x + ip_y)/p_f \\ \Delta\,(p_x - ip_y)/p_f & -|\mathbf{p}|^2/2m^* + \mu \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \varepsilon \begin{pmatrix} u \\ v \end{pmatrix}$$

Nambu Representation with particle-hole matrices $\widehat{\vec{\boldsymbol{\tau}}} = (\widehat{\tau}_1, \widehat{\tau}_2, \widehat{\tau}_3)$

$$\widehat{H} = \left(|\mathbf{p}|^2/2m - \mu\right)\widehat{\tau}_3 + \left[\Delta\,p_x\,\widehat{\tau}_1 \mp \Delta\,p_y\,\widehat{\tau}_2\right]/p_f = \vec{\mathbf{m}}(\mathbf{p}) \cdot \widehat{\vec{\boldsymbol{\tau}}}$$

Hamiltonian for 2D ^{3}He-A : $\widehat{H} = \begin{pmatrix} \xi(\mathbf{p}) & c(p_x + ip_y) \\ c(p_x - ip_y) & -\xi(\mathbf{p}) \end{pmatrix} = \vec{\mathbf{m}}(\mathbf{p}) \cdot \widehat{\vec{\boldsymbol{\tau}}}$

$\vec{\mathbf{m}} = (\, cp_x \,, \mp cp_y \,, \xi(\mathbf{p}))$ with $|\vec{\mathbf{m}}(\mathbf{p})|^2 = \left(|\mathbf{p}|^2/2m - \mu\right)^2 + c^2|\mathbf{p}|^2 > 0 \,, \mu \neq 0$



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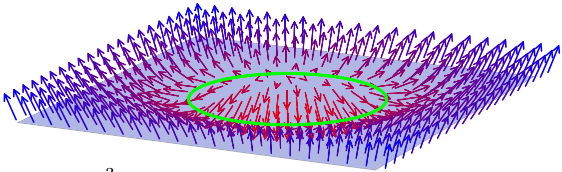
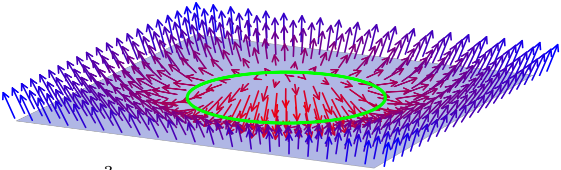


Topological Invariant for 2D ^{3}He-A [G.E. Volovik, JETP 1988]:

$$N_{\text{2D}} = \pi \int \frac{d^2p}{(2\pi)^2}\, \hat{\mathbf{m}}(\mathbf{p}) \cdot \left(\frac{\partial \hat{\mathbf{m}}}{\partial p_x} \times \frac{\partial \hat{\mathbf{m}}}{\partial p_y}\right) = \begin{cases} \pm 1 \,; & \mu > 0 \text{ and } \Delta \neq 0 \\ 0 \,; & \mu < 0 \text{ or } \Delta = 0 \end{cases}$$

Hamiltonian for 2D ^{3}He-A : $\widehat{H} = \begin{pmatrix} \xi(\mathbf{p}) & c(p_x + ip_y) \\ c(p_x - ip_y) & -\xi(\mathbf{p}) \end{pmatrix} = \vec{\mathbf{m}}(\mathbf{p}) \cdot \widehat{\vec{\tau}}$

$\vec{\mathbf{m}} = (\, cp_x \,, \mp cp_y \,, \xi(\mathbf{p}))$ with $|\vec{\mathbf{m}}(\mathbf{p})|^2 = (|\mathbf{p}|^2/2m - \mu)^2 + c^2|\mathbf{p}|^2 > 0 \,, \mu \neq 0$



Topological Invariant for 2D ^{3}He-A [G.E. Volovik, JETP 1988]:

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“Vacuum” $(\Delta = 0)$ with $N_{2D} = 0$ | ^{3}He-A$(\Delta \neq 0)$ with $N_{2D} = 1$

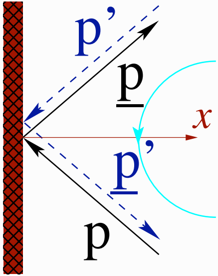
Zero Energy Fermions ↑ Confined on the Edge

Edge Fermions: $\mathfrak{g}^{R}_{\text{edge}}(\mathbf{p}, \varepsilon; x) = \dfrac{\pi\Delta|\mathbf{p}_x|}{\varepsilon + i\gamma - \varepsilon_{\text{bs}}(\mathbf{p}_{||})}\, e^{-x/\xi_\Delta}$

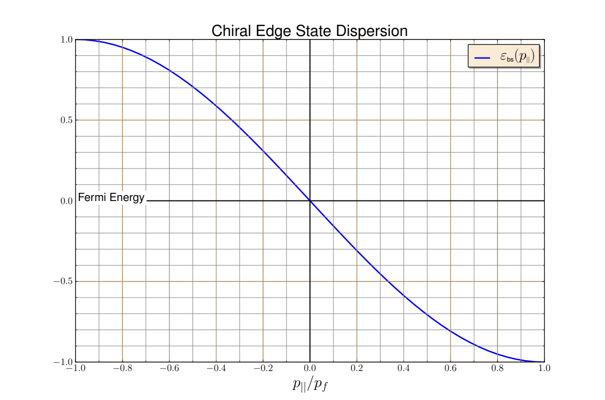
Confinement: $\xi_\Delta = \hbar v_f / 2\Delta \approx 10^3\, \mathring{A} \gg \hbar / p_f$

$\varepsilon_{\text{bs}} = -c\, p_{||}$ with

$c = \Delta / p_f \ll v_f$



Broken P & T $\rightsquigarrow$ Edge Current



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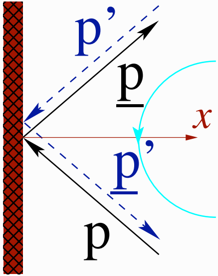
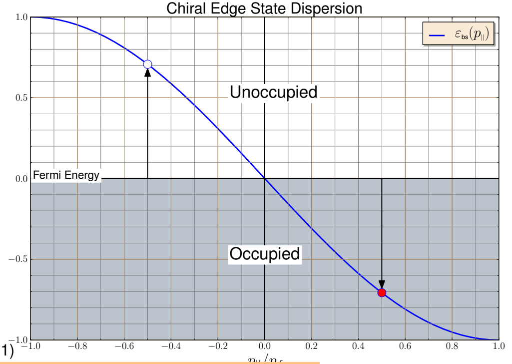
Broken P & T $\rightsquigarrow$ Edge Current

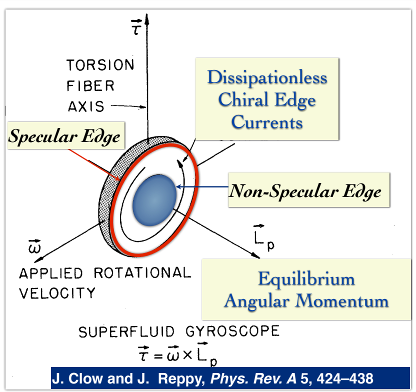




- M. Stone, R. Roy, PRB 69, 184511 (2004)
- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

▶ Hao Wu, Saturday 9:20

Possible Gyroscopic Experiment to Measure of $L_z(T)$



Thermal Signature of Chiral Edge States

- Power Law for $T \lesssim 0.5 T_c$

$$L_z = (N/2)\hbar \, (1 - c\,(T/\Delta)^2)$$

Toroidal Geometry with Engineered Surfaces

- Incomplete Screening

$$L_z > (N/2)\hbar$$

Direct Signature of Edge Currents

- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)
- Y. Tsutsumi, K. Machida, JPSJ 81, 074607 (2012)

Topological Invariant for 3D Time-Reversal Invariant ^{3}He-B

- Nambu-Bogoliubov Hamiltonian for Bulk ^{3}He-B:

$$\widehat{H}_{\text{B}} = \xi(\mathbf{p})\hat{\tau}_3 + c\,\mathbf{p}\cdot\vec{\boldsymbol{\sigma}}\,\hat{\tau}_1$$

  - $E(\mathbf{p}) = \sqrt{\xi(\mathbf{p})^2 + c^2|\mathbf{p}|^2} \geq \Delta = c\,p_f$ (Gapped)
  - Emergent *spin-orbit* coupling $\rightsquigarrow$ Helicity eigenstates
  - Emergent Topology of the B-phase

- Nambu-Bogoliubov Hamiltonian for Bulk ^{3}He-B:

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  - $E(\mathbf{p}) = \sqrt{\xi(\mathbf{p})^2 + c^2|\mathbf{p}|^2} \geq \Delta = c\,p_f$ (Gapped)
  - Emergent *spin-orbit* coupling $\rightsquigarrow$ Helicity eigenstates
  - Emergent Topology of the B-phase

- Topology of the B-phase Bogoliubov Hamiltonian:

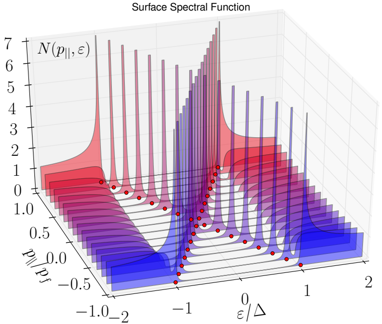
$$N_{\text{3D}} = \frac{\pi}{4}\int\frac{d^3p}{(2\pi)^3}\,\epsilon_{ijk}\text{Tr}\Big\{\Gamma(\widehat{H}_{\text{B}}^{-1}\partial_{p_i}\widehat{H}_{\text{B}})(\widehat{H}_{\text{B}}^{-1}\partial_{p_j}\widehat{H}_{\text{B}})(\widehat{H}_{\text{B}}^{-1}\partial_{p_k}\widehat{H}_{\text{B}})\Big\} = \begin{cases} 0, & \Gamma = 1 \\ 2, & \Gamma = \mathtt{CT} \end{cases}$$

Zero Energy Fermions Confined on a 2D Surface $\uparrow$

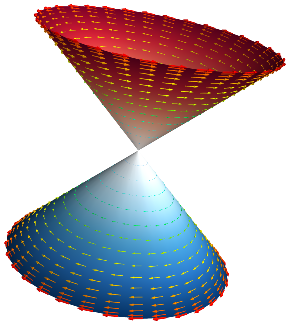
Helical Majorana Modes

Protected by $\Gamma = \mathtt{CT}$ symmetry: $\Gamma\widehat{H}_{\text{B}}\Gamma^{\dagger} = -\widehat{H}_{\text{B}}$

- Schnyder et al., PRB 78, 195125 (2008); Volovik, JETP Lett. 90, 587 (2009)



- **Surface Majorana Modes**



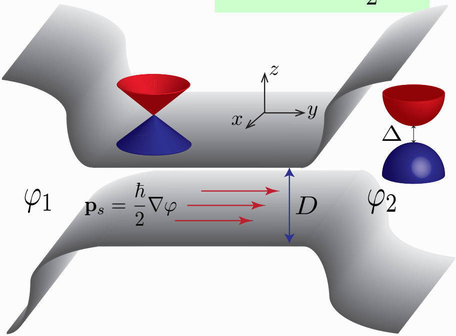
- Surface Spectrum:

$$N_b(\mathbf{p}, z; \varepsilon) = \frac{\pi}{2}\,\Delta_\perp \hat{p}_z\, e^{-2\Delta_\perp z/v_f}$$
$$\times[\delta(\varepsilon - c|\mathbf{p}_{||}|) + \delta(\varepsilon + c|\mathbf{p}_{||}|)]$$

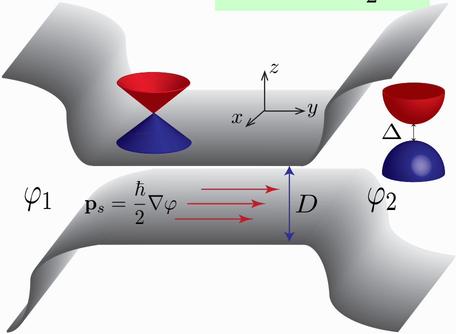
- Y. Tsutsumi, K. Machida, JPSJ 81, 074607 (2012)
- Hao Wu, JAS, Phys. Rev. B 88, 18 184506 (2013)

- $\varepsilon_b^{\pm} = \pm c|\mathbf{p}_{||}|, \quad c = \Delta_{||}/p_f \ll v_f$
- Helical Spin-Orbit Locking: $\vec{s} \perp \mathbf{p}$
- $\varepsilon_b^- < 0 \rightsquigarrow$ Helical Spin Current at $T = 0$
- $K_{xy} = \dfrac{1}{6}\; n_{\text{2D}}\, v_f\, \dfrac{\hbar}{2} \times (1 - a\, T^3)$

Condensate Flow: $\mathbf{p}_s \equiv m\mathbf{v}_s = \frac{\hbar}{2}\boldsymbol{\nabla}\varphi$



Condensate Flow: $\mathbf{p}_s \equiv m\mathbf{v}_s = \frac{\hbar}{2}\boldsymbol{\nabla}\varphi$



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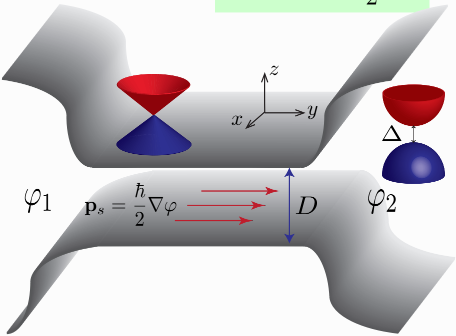

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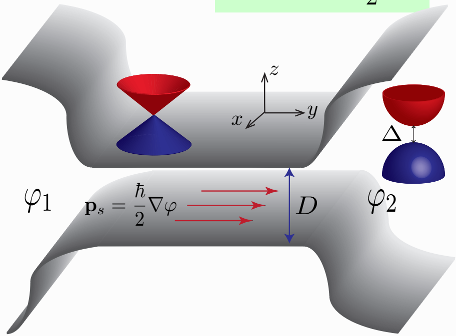

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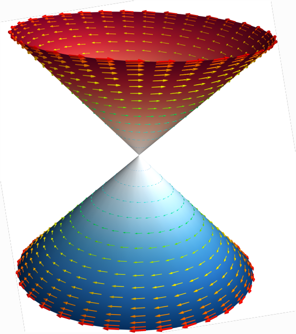
- Doppler Shifted Majorana Spectrum: $\varepsilon \rightarrow \varepsilon = c|\mathbf{p}_{||}| + \mathbf{p}_{||}\cdot\mathbf{v}_s$
- Thermal Signature: $\vec{J} = n\,\mathbf{p}_s \times \left(1 - \frac{27\pi\zeta(3)}{2}\frac{\xi_\Delta}{D}\frac{\Delta_\perp}{\Delta_{||}}\frac{m^*}{m_3}\left(\frac{T}{\Delta_{||}}\right)^3\right)$

- T. Mizushima et al., Phys. Rev. Lett. 109, 165301, (2012)
- Hao Wu, JAS, Phys. Rev. B 88, 18 184506 (2013)

- Helical Majorana Exciations:

$$\vec{\mathbf{s}} \perp \vec{\mathbf{p}}_{||}$$



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$$J_{xy}(0) = \frac{1}{6}\, n_{\text{2D}}\, v_f\, \frac{\hbar}{2}$$

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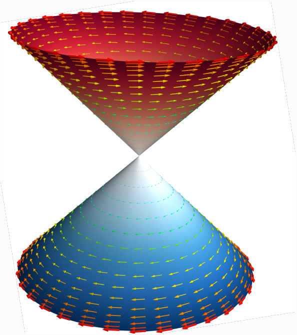

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$$\mathcal{D}^{(\pm)}_{\alpha i}(\mathbf{q},\omega) \sim \left(\mathbf{e}^{(\pm)}_{\alpha}\,\mathbf{q}_i + \mathbf{q}_{\alpha}\,\mathbf{e}^{(\pm)}_{i}\right)$$

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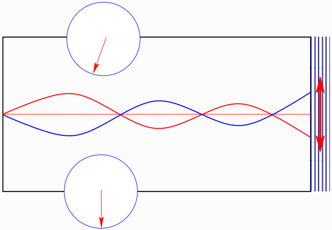

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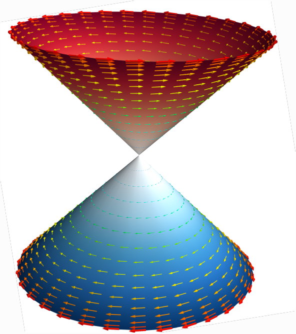

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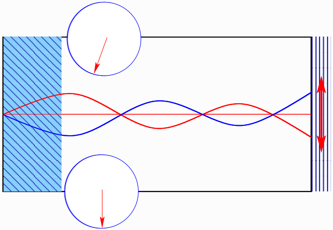

- T. Mizushima talk: Friday, 9:40

Summary

- Spontaneous Symmetry Breaking
- Confinement: New Phases
- Dynamics: Bosonic Modes
- Topological Order: ^{3}He
- Signatures: Edge & Surface States
- Towards a Spectroscopy

Research Supported by NSF Grant DMR-1106315