# Excitations & Structure of Topological Defects in Exotic Superfluids

J. A. Sauls

Northwestern University

Overview:

- Vortex States in Superfluid Mixtures
- Spin-Triplet, Chiral Superfluids: $^3$He-A, $Sr_2RuO_4$, $UPt_3$

Recent Results:

- Chiral p-wave Vortices, JAS and ME, New. J. Phys. 11, 075008 (2009)
- Vortex Dynamics, ME & JAS, New. J. Phys. 11, 075009 (2009)
- Chiral p-wave Lattices, M. Ichioka & JAS, (unpublished)

Matthias Eschrig, Royal Holloway University of London

Mansanori Ichioka, Okayama University

# Superfluid Mixtures

➡Babaev, Sudbo, Moschalkov et al.

n-p Matter

$^3$He-$^4$He

$^6$Li - $^7$Li

"2-Band" SC

2D $^3$He-A

$Sr_2RuO_4$

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$$\left.\begin{array}{l}\text{n-p Matter}\\ {}^3\text{He-}{}^4\text{He}\\ {}^6\text{Li - }{}^7\text{Li}\end{array}\right\} \longrightarrow U(1)_1 \times U(1)_2$$

$\Psi_1$ $\Psi_2$

independent gauge symmetries

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independent gauge symmetries

$\Psi_1$ $\Psi_2$

~~$\Psi_1 \Psi_2^*$~~

$$f_{\text{GL}} = \Big\{ \alpha_1|\Psi_1|^2 + \alpha_2|\Psi_2|^2 + \frac{1}{2}\beta_1|\Psi_1|^4 + \frac{1}{2}\beta_2|\Psi_2|^4 + \beta_{12}|\Psi_2|^2|\Psi_2|^2$$

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$$+ \frac{1}{2m_1^*} \Big| \left( \frac{\hbar}{i} \boldsymbol{\nabla} - \frac{e_1^*}{c} \mathbf{A} \right) \Psi_1 \Big|^2 + \frac{1}{2m_2^*} \Big| \left( \frac{\hbar}{i} \boldsymbol{\nabla} - \frac{e_2^*}{c} \mathbf{A} \right) \Psi_2 \Big|^2$$

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$$+ \frac{1}{2m_1^*}\left| \left(\frac{\hbar}{i}\boldsymbol{\nabla} - \frac{e_1^*}{c}\mathbf{A}\right)\Psi_1\right|^2 + \frac{1}{2m_2^*}\left| \left(\frac{\hbar}{i}\boldsymbol{\nabla} - \frac{e_2^*}{c}\mathbf{A}\right)\Psi_2\right|^2$$

$$+\mu_{12}\left[\Psi_1^*\Psi_2\left(\frac{\hbar}{i}\boldsymbol{\nabla}\Psi_1 - \frac{e_1^*}{c}\mathbf{A}\Psi_1\right)\cdot\left(-\frac{\hbar}{i}\boldsymbol{\nabla}\Psi_2^* - \frac{e_2^*}{c}\mathbf{A}\Psi_2^*\right) + c.c.\right]$$

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$$+ \mu_{12} \left[ \Psi_1^* \Psi_2 \left( \frac{\hbar}{i} \boldsymbol{\nabla} \Psi_1 - \frac{e_1^*}{c} \mathbf{A} \Psi_1 \right) \cdot \left( -\frac{\hbar}{i} \boldsymbol{\nabla} \Psi_2^* - \frac{e_2^*}{c} \mathbf{A} \Psi_2^* \right) + c.c. \right]$$

$$+ \frac{1}{8\pi} \left| \boldsymbol{\nabla} \times \mathbf{A} \right|^2 \Big\}$$

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$$+\frac{1}{8\pi}\Big|\boldsymbol{\nabla}\times\mathbf{A}\Big|^2\Big\}$$

London Limit $\xi_i|\boldsymbol{\nabla}\Psi_i| \ll |\Psi_i|$

$$\Psi_{1,2} = \Psi_{1,2}^{\rm eq}\, e^{i\vartheta_{1,2}}$$

Andreev & Bashkin (1975)

Alpar & JAS (Ap. J. 1984)

Meyerovich (JETP 1984)

# Hydrodynamics of Superfluid Quantum Mixtures

London Limit $\xi_i|\boldsymbol{\nabla}\Psi_i| \ll |\Psi_i|$ $\Psi_{1,2} = \Psi_{1,2}^{\mathrm{eq}}\, e^{i\vartheta_{1,2}}$

Galilean Invariance $$\mathbf{v}_{1,2} = \frac{\hbar}{m_{1,2}}\left(\boldsymbol{\nabla}\vartheta_{1,2} - \frac{e^*_{1,2}}{\hbar c}\mathbf{A}\right)$$

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$$F[\mathbf{v}_1, \mathbf{v}_2, \mathbf{A}] = \int d^3r \left\{ \frac{1}{2}\rho_1|\mathbf{v}_1|^2 + \frac{1}{2}\rho_2|\mathbf{v}_2|^2 \right.$$

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$$\left. +\rho_{12}\mathbf{v}_1 \cdot \mathbf{v}_2 \right.$$

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$$\rho_{1,2} = \frac{1}{m_{1,2}^*} \left( \frac{m_{1,2}}{\hbar} \right)^2 |\Psi_{1,2}^{\text{eq}}|^2 \qquad \rho_{12} = \mu \left( \frac{m_1}{\hbar} \right) \left( \frac{m_2}{\hbar} \right) |\Psi_1^{\text{eq}}|^2 \, |\Psi_2^{\text{eq}}|^2$$

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Conserved Currents

$$\mathbf{J}_1 = \rho_1 \mathbf{v}_1 + \rho_{12}\mathbf{v}_2$$

$$\mathbf{J}_2 = \rho_2 \mathbf{v}_2 + \rho_{12}\mathbf{v}_1$$

Drag Currents

# Hydrodynamics of Superfluid Quantum Mixtures

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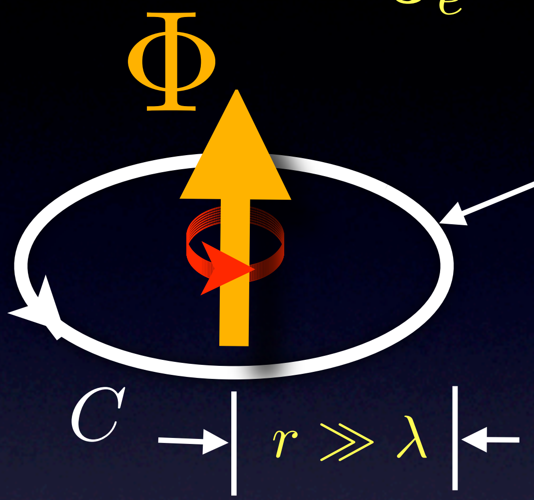
Drag Currents

External Fields & Rotation

$$F_H = -\frac{1}{4\pi}\int d^3r\left\{\mathbf{H}\cdot\boldsymbol{\nabla}\times\mathbf{A}\right\}$$
$$F_{\mathrm{rot}} = -\int d^3r\left\{\boldsymbol{\Omega}\cdot(\mathbf{r}\times\mathbf{J})\right\}$$

# Flux Quantization in Superconducting Mixtures




$$\mathbf{J}_e = \frac{e_1^*}{m_1}\left(\rho_1 \mathbf{v}_1 + \rho_{12}\mathbf{v}_2\right) + \frac{e_2^*}{m_2}\left(\rho_2 \mathbf{v}_2 + \rho_{12}\mathbf{v}_1\right)$$

$$\mathbf{J}_e = 0$$

$$\mathbf{v}_{1,2} = \frac{\hbar}{m_{1,2}}\left(\boldsymbol{\nabla}\vartheta_{1,2} - \frac{e_{1,2}^*}{\hbar c}\mathbf{A}\right)$$

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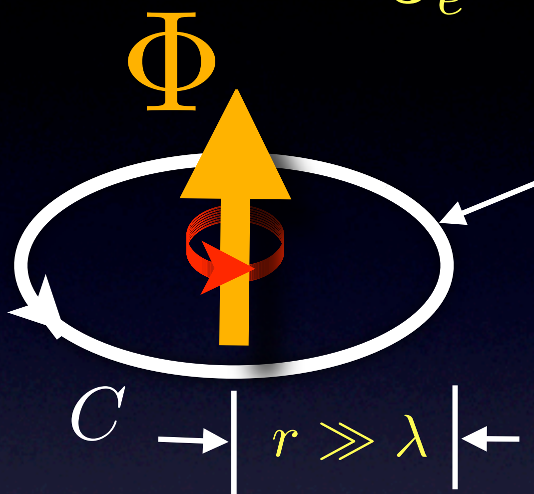


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$$\boxed{N_{1,2} = \frac{1}{2\pi}\oint_C d\boldsymbol{\ell}\cdot\boldsymbol{\nabla}\vartheta_{1,2} = 0, \pm 1, \pm 2, \ldots}$$

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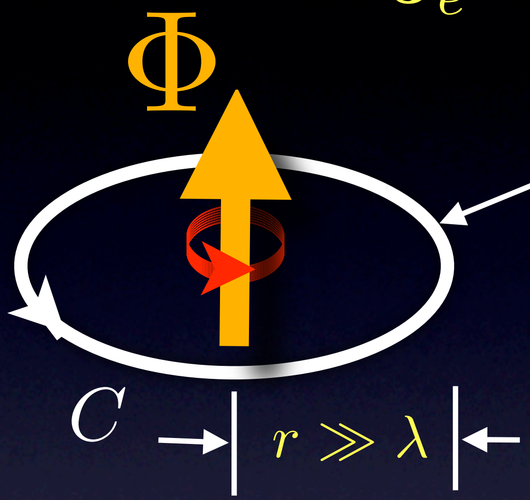


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$$\Phi = hc\left\{\frac{\frac{N_1 e_1^*}{m_1^2}\rho_1 + \frac{N_2 e_2^*}{m_2^2}\rho_2 + \frac{N_1 e_1^* + N_2 e_2^*}{m_1 m_2}\rho_{12}}{\left(\frac{e_1^*}{m_1}\right)^2\rho_1 + \left(\frac{e_2^*}{m_2}\right)^2\rho_2 + 2\left(\frac{e_1^* e_2^*}{m_1 m_2}\right)\rho_{12}}\right\}$$

# Flux Quantization in Superconducting Mixtures

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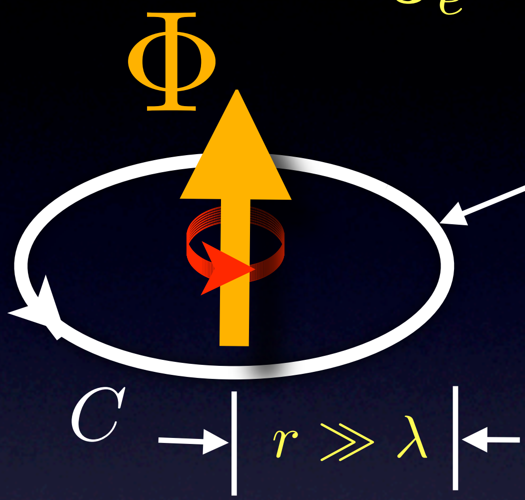


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$$N_{1,2} = \frac{1}{2\pi}\oint_C d\boldsymbol{\ell}\cdot\boldsymbol{\nabla}\vartheta_{1,2} = 0, \pm 1, \pm 2, \ldots$$

$$\Phi = \frac{hc}{2e}\left\{\frac{N_1\rho_1 + N_2\rho_2 + (N_1+N_2)\,\rho_{12}}{\rho_1+\rho_2+2\rho_{12}}\right\}$$

“Two Band” SC

$$e_1^* = e_2^* = 2e$$

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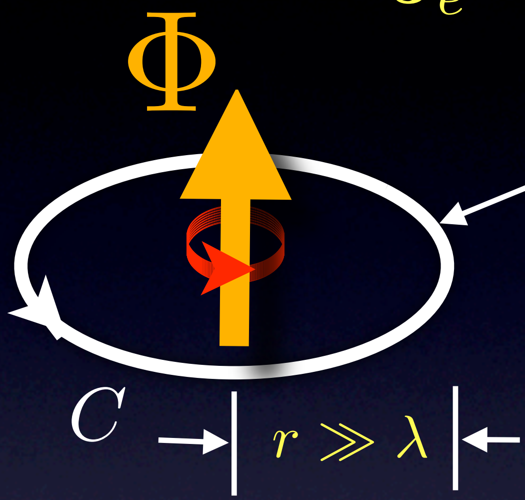


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"Two Band" SC $\quad N_1 = N_2 = N$

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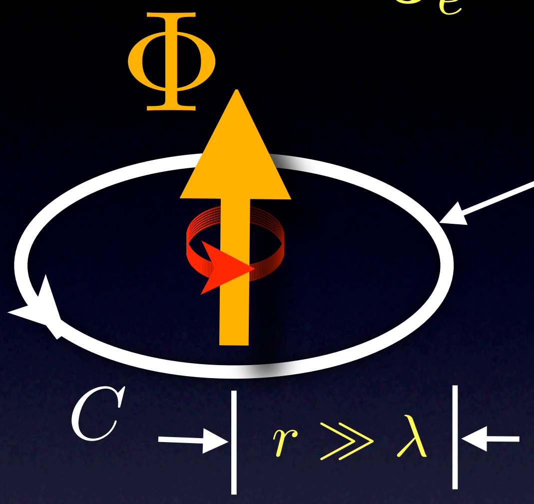


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"Two Band" SC

$$e_1^* = e_2^* = 2e$$

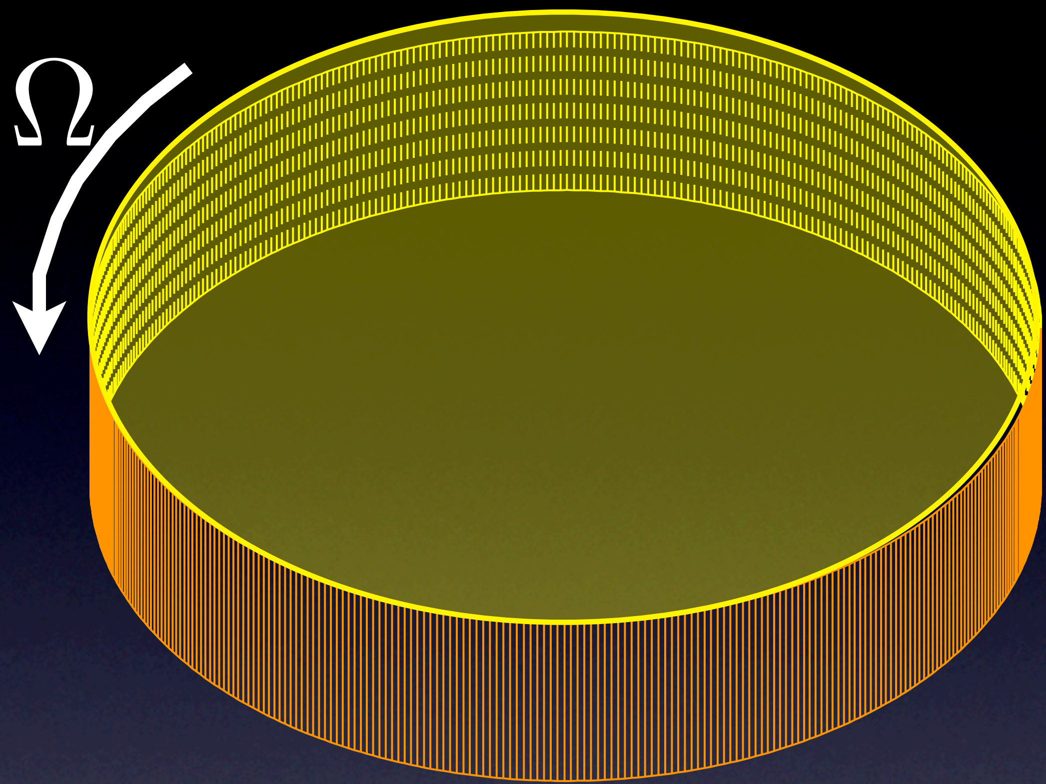
$$N_1 = N_2 = N$$

$$\Phi = N\frac{hc}{2e}$$

$$N_1 = 1, N_2 = 0$$

$$\Phi = \frac{hc}{2e}\left\{\frac{\rho_1 + \rho_{12}}{\rho_1 + \rho_2 + 2\rho_{12}}\right\} < \frac{hc}{2e}$$

# Rotating Equilibrium of a Neutral Superfluid




$$F_{\mathbf{\Omega}}[\mathbf{v}_4, \mathbf{v}_{\mathrm{N}}] = \int d^3r \left\{ \frac{1}{2}\rho_4 |\mathbf{v}_4 - \mathbf{\Omega} \times \mathbf{r}|^2 + \frac{1}{2}\rho_{\mathrm{N}} |\mathbf{v}_{\mathrm{N}} - \mathbf{\Omega} \times \mathbf{r}|^2 \right\}$$

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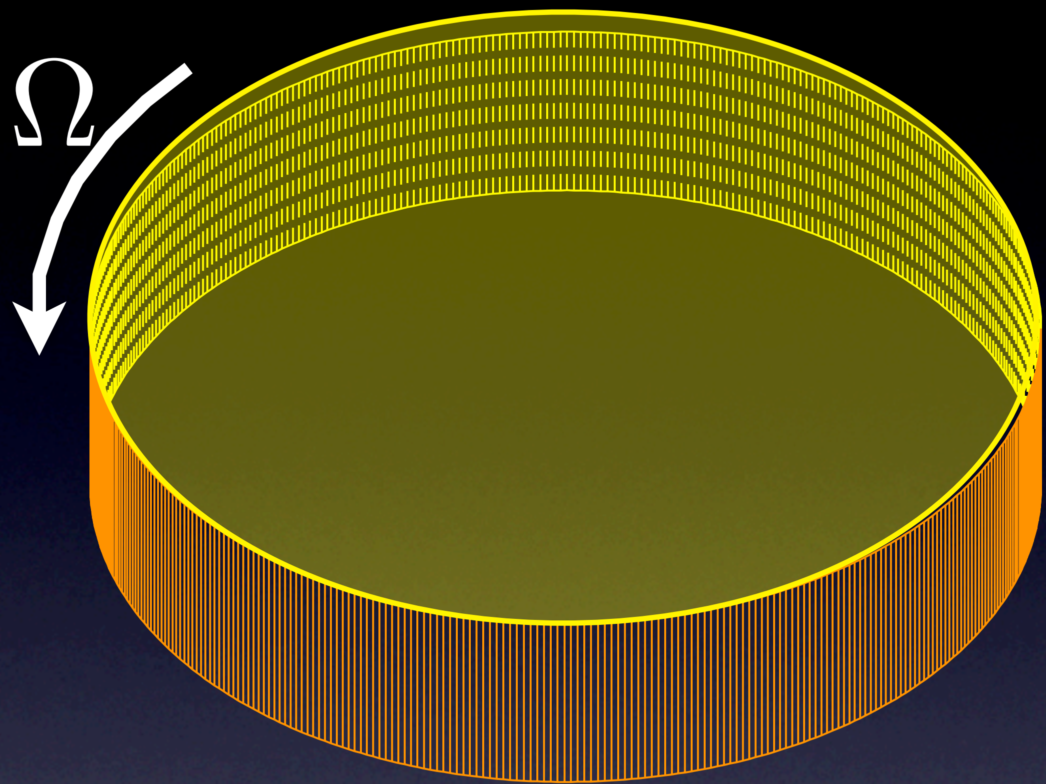


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Excitations - co-rotation

$$\mathbf{v}_{\mathrm{N}} = \boldsymbol{\Omega} \times \mathbf{r} \qquad \boldsymbol{\nabla} \times \mathbf{v}_{\mathrm{N}} = 2\,\boldsymbol{\Omega}$$

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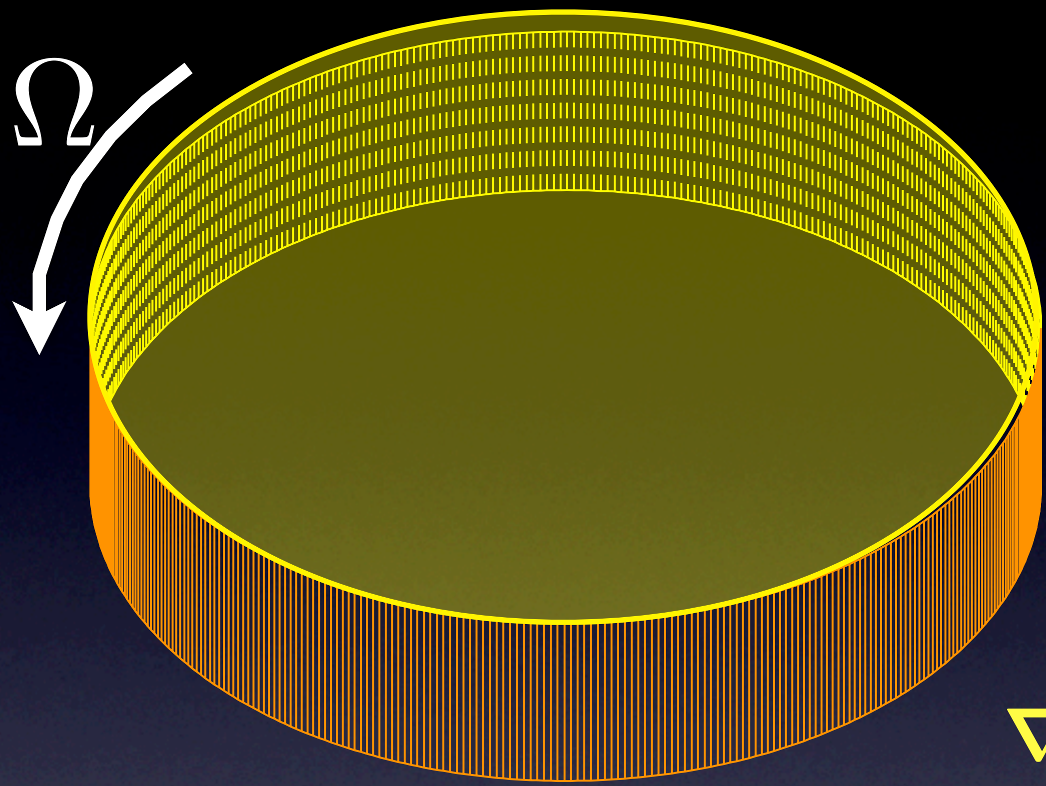


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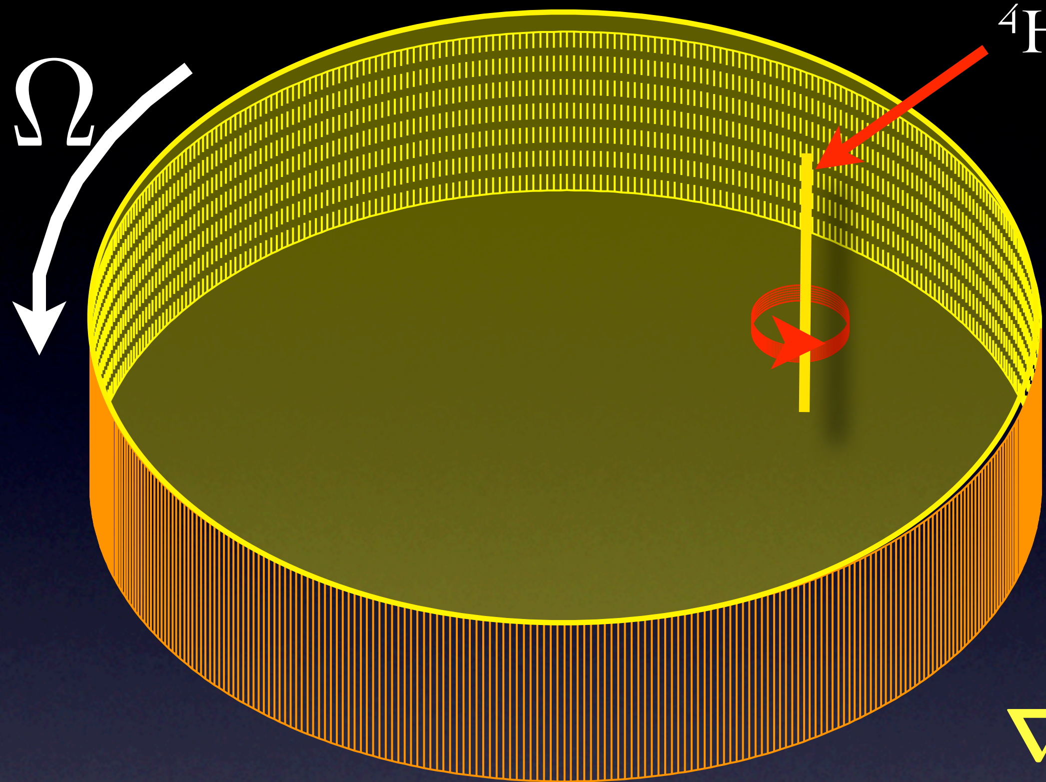
Excitations - co-rotation

$$\mathbf{v}_{\rm N} = \boldsymbol{\Omega}\times\mathbf{r} \qquad \boldsymbol{\nabla}\times\mathbf{v}_{\rm N} = 2\,\boldsymbol{\Omega}$$

Condensate ? $\quad \mathbf{v}_4 = \dfrac{\hbar}{m_4}\boldsymbol{\nabla}\vartheta_4$

$$\boldsymbol{\nabla}\times\mathbf{v}_4 = 0$$

# Rotating Equilibrium of a Neutral Superfluid




$^4$He vortex

$$F_{\mathbf{\Omega}}[\mathbf{v}_4, \mathbf{v}_{\text{N}}] = \int d^3r \left\{ \frac{1}{2}\rho_4 |\mathbf{v}_4 - \mathbf{\Omega} \times \mathbf{r}|^2 + \frac{1}{2}\rho_{\text{N}} |\mathbf{v}_{\text{N}} - \mathbf{\Omega} \times \mathbf{r}|^2 \right\}$$

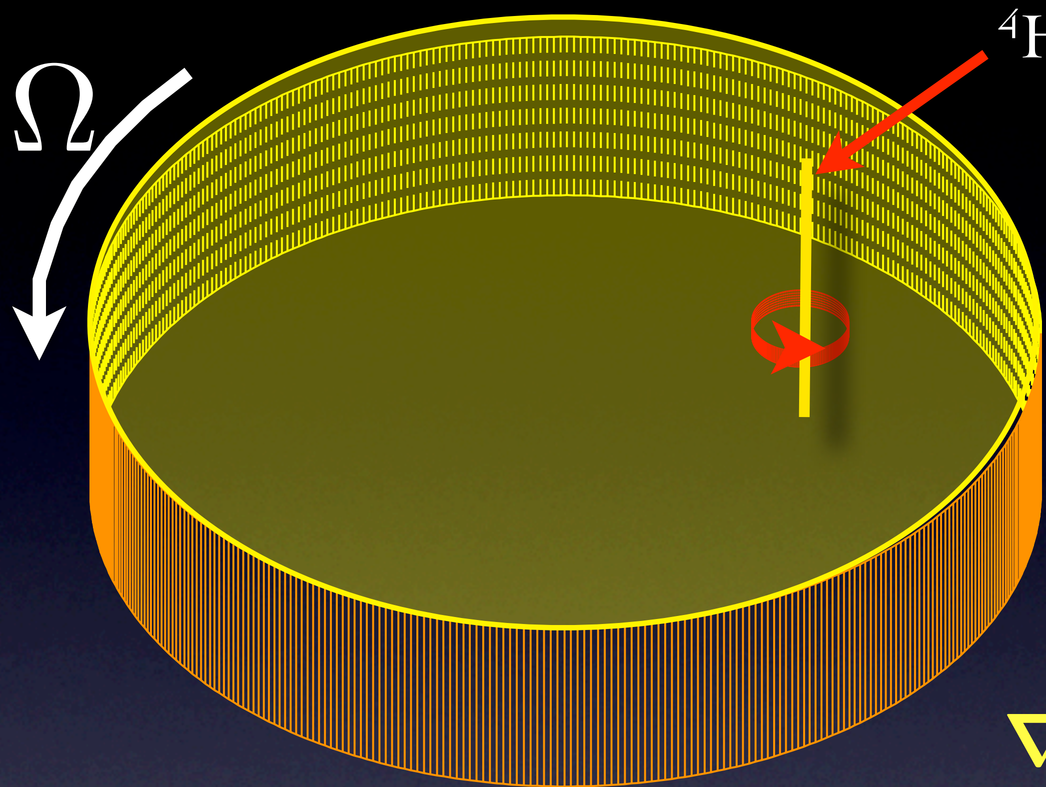
Excitations - co-rotation

$$\mathbf{v}_{\text{N}} = \mathbf{\Omega} \times \mathbf{r} \qquad \nabla \times \mathbf{v}_{\text{N}} = 2\,\mathbf{\Omega}$$

Condensate ? $\mathbf{v}_4 = \dfrac{\hbar}{m_4} \nabla \vartheta_4$

$$\nabla \times \mathbf{v}_4 = \qquad \kappa_i \, \delta^{(2)}(\mathbf{r} - \mathbf{r}_i)$$

# Rotating Equilibrium of a Neutral Superfluid




$$F_{\mathbf{\Omega}}[\mathbf{v}_4, \mathbf{v}_N] = \int d^3r \left\{ \frac{1}{2}\rho_4 |\mathbf{v}_4 - \mathbf{\Omega}\times\mathbf{r}|^2 + \frac{1}{2}\rho_N |\mathbf{v}_N - \mathbf{\Omega}\times\mathbf{r}|^2 \right\}$$

Excitations - co-rotation

$$\mathbf{v}_N = \mathbf{\Omega}\times\mathbf{r} \qquad \nabla\times\mathbf{v}_N = 2\,\mathbf{\Omega}$$

Condensate ? $\mathbf{v}_4 = \frac{\hbar}{m_4}\nabla\vartheta_4$

$$\nabla\times\mathbf{v}_4 = \kappa_i\,\delta^{(2)}(\mathbf{r}-\mathbf{r}_i)$$

1 vortex w/ N=1

$$\kappa = \oint \mathbf{v}_4\cdot d\boldsymbol{\ell} = \frac{h}{m_4}$$

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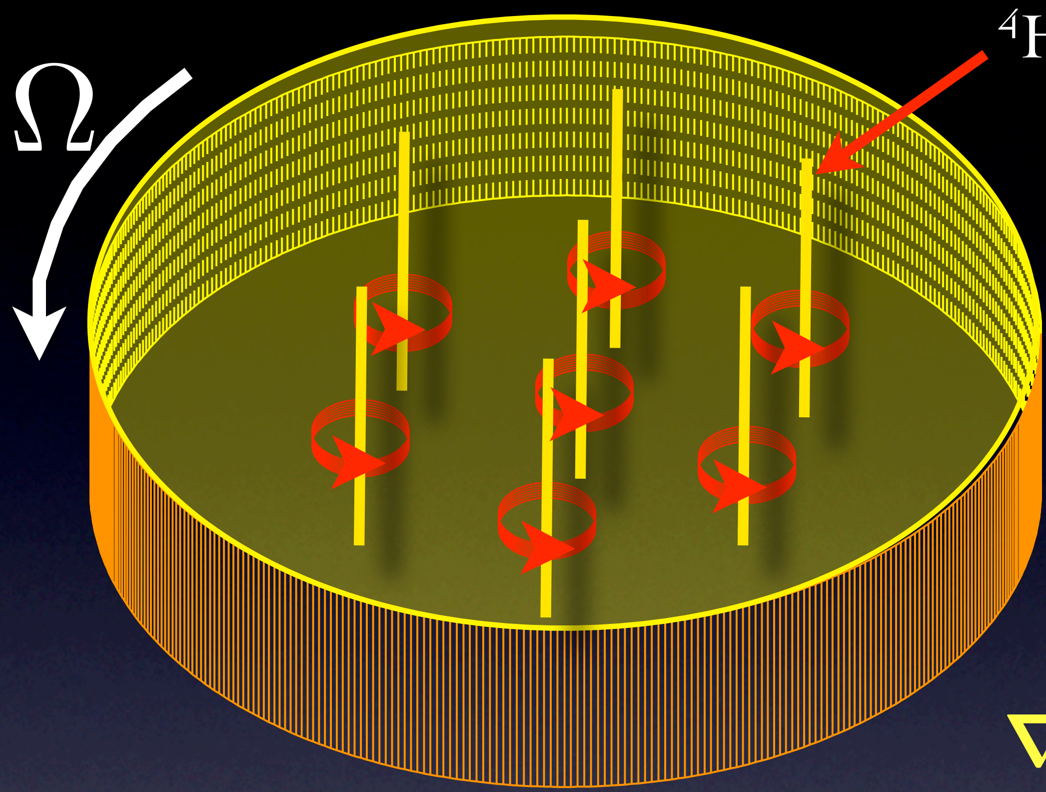


$^4$He vortex

$$F_{\mathbf{\Omega}}[\mathbf{v}_4, \mathbf{v}_{\rm N}] = \int d^3r \left\{ \frac{1}{2}\rho_4 |\mathbf{v}_4 - \mathbf{\Omega} \times \mathbf{r}|^2 + \frac{1}{2}\rho_{\rm N} |\mathbf{v}_{\rm N} - \mathbf{\Omega} \times \mathbf{r}|^2 \right\}$$

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$$\mathbf{v}_{\rm N} = \mathbf{\Omega} \times \mathbf{r} \qquad \nabla \times \mathbf{v}_{\rm N} = 2\,\mathbf{\Omega}$$

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$$\nabla \times \mathbf{v}_4 = \sum_{i=1}^{N_v} \kappa_i \, \delta^{(2)}(\mathbf{r} - \mathbf{r}_i)$$

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Many vortices

$$\oint_{\rm C} \mathbf{v}_4 \cdot d\boldsymbol{\ell} = N_v \, \frac{h}{m_4}$$

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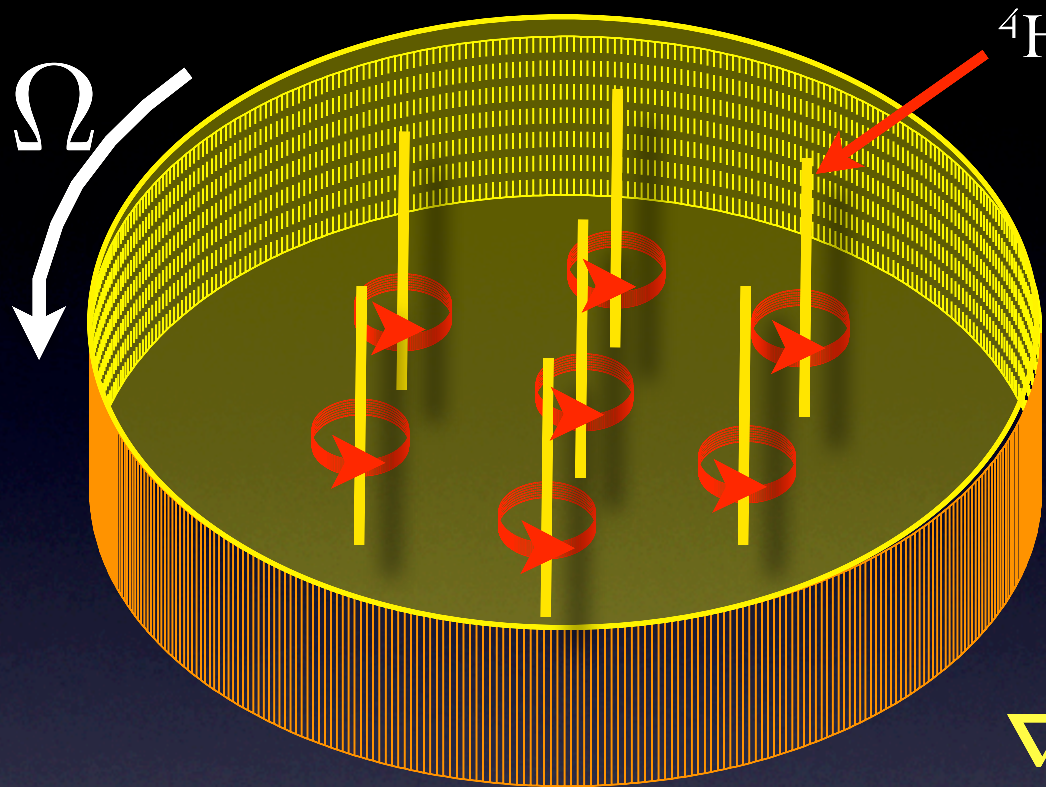


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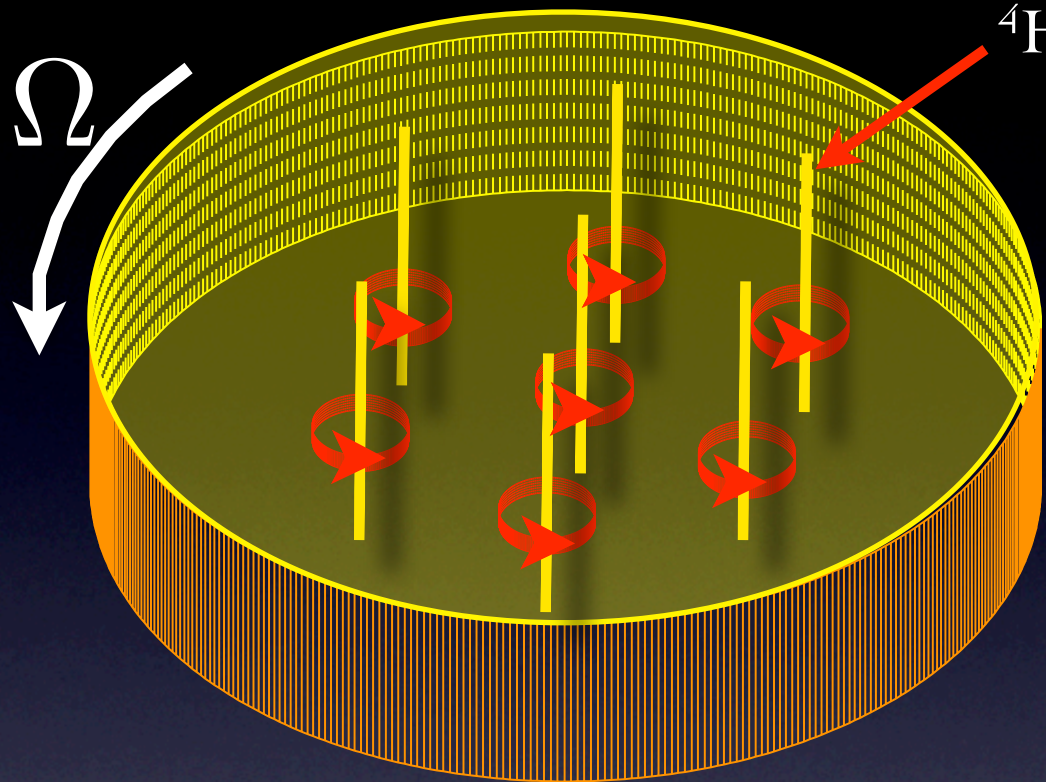


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Feynman-Onsager

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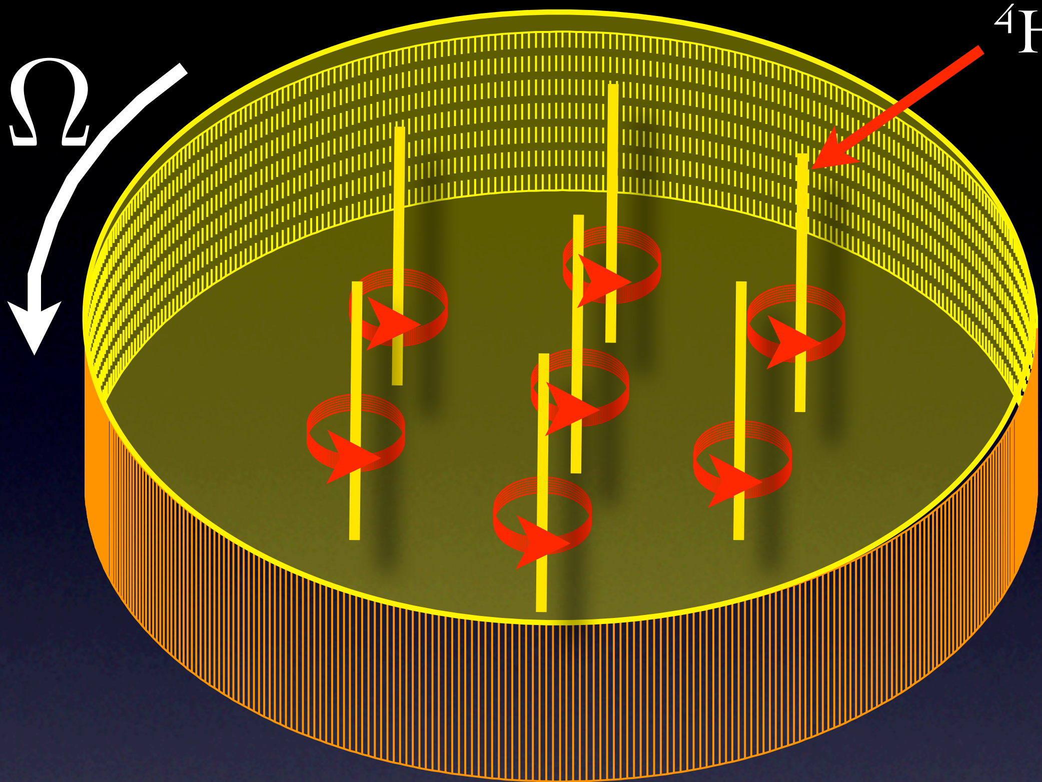


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Vortex-vortex repulsion

$$U(|\mathbf{r}_i - \mathbf{r}_j|) = 2\pi\rho_4 \left(\frac{\hbar}{m_4}\right)^2 \ln(R/|\mathbf{r}_i - \mathbf{r}_j|)$$

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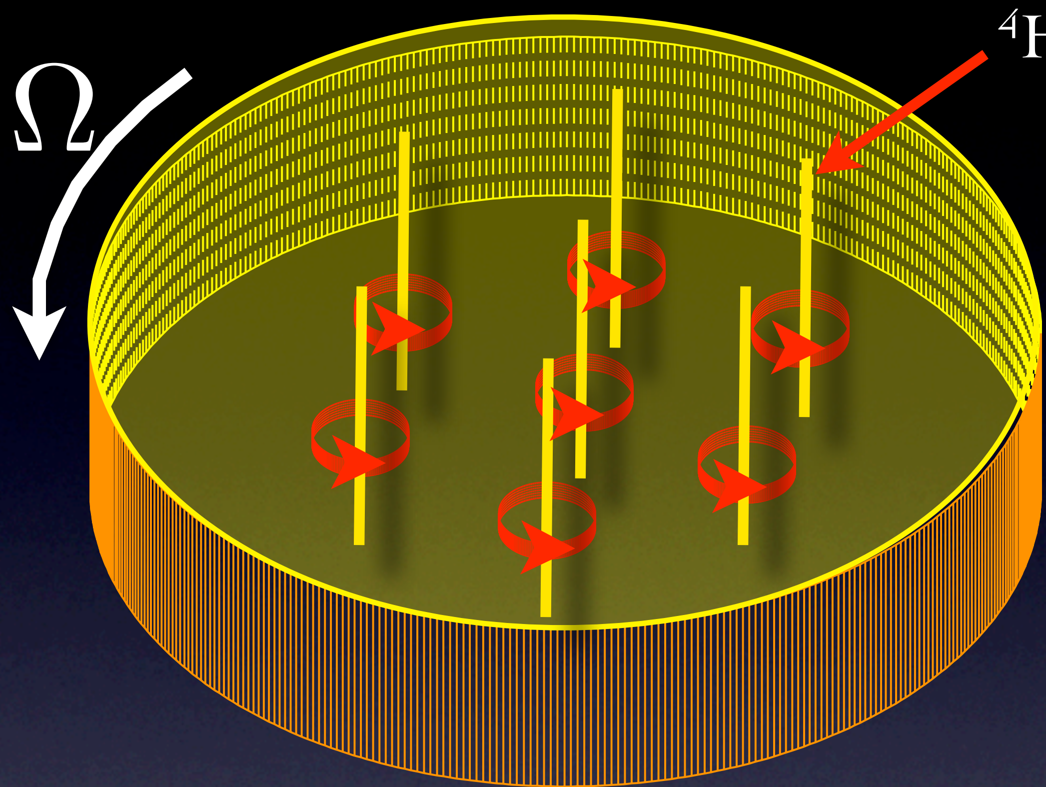


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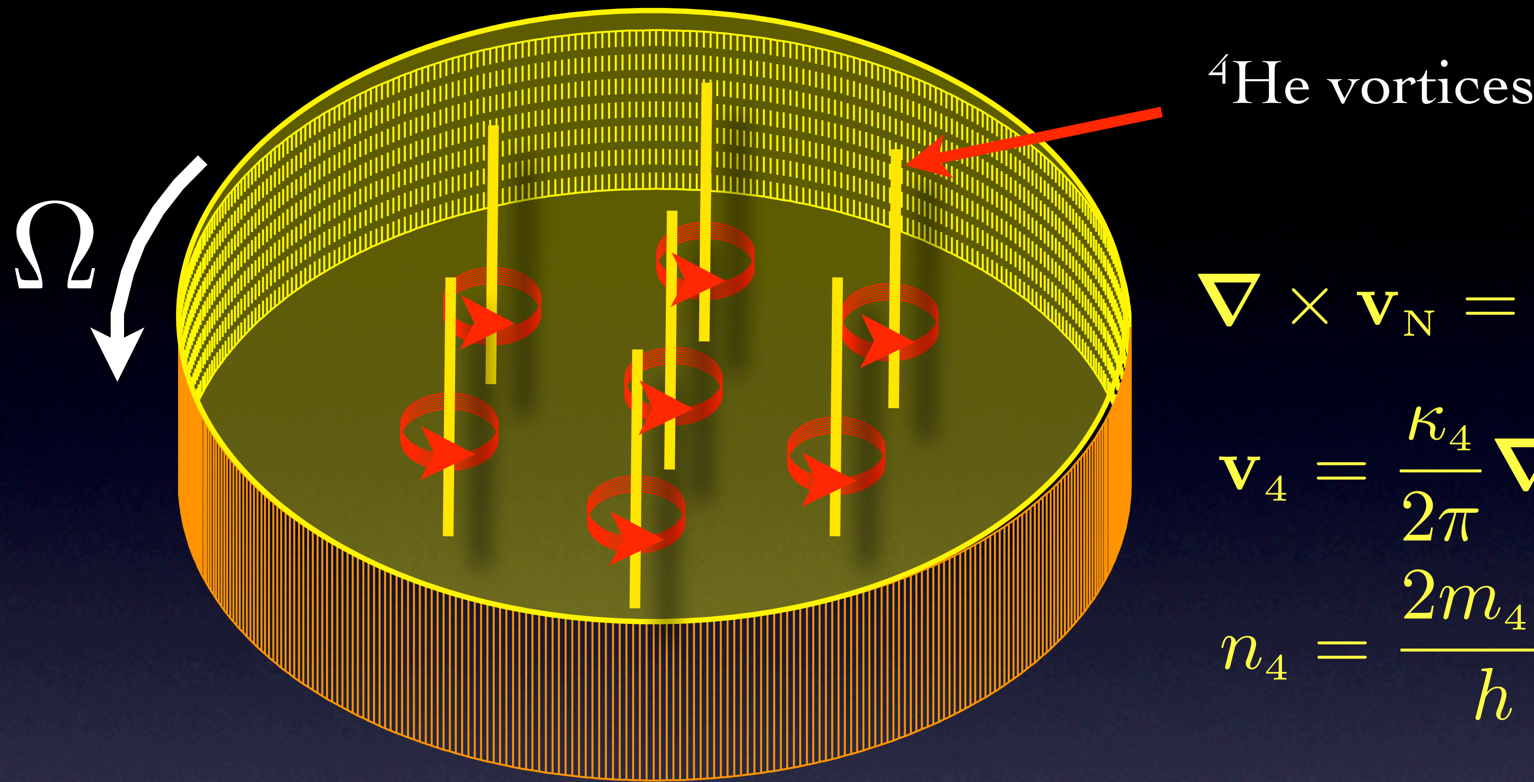
Triangular Vortex Lattice

# Superfluid $^3$He-$^4$He in Rotating Equilibrium




$$\nabla \times \mathbf{v}_N = 2\,\boldsymbol{\Omega}$$

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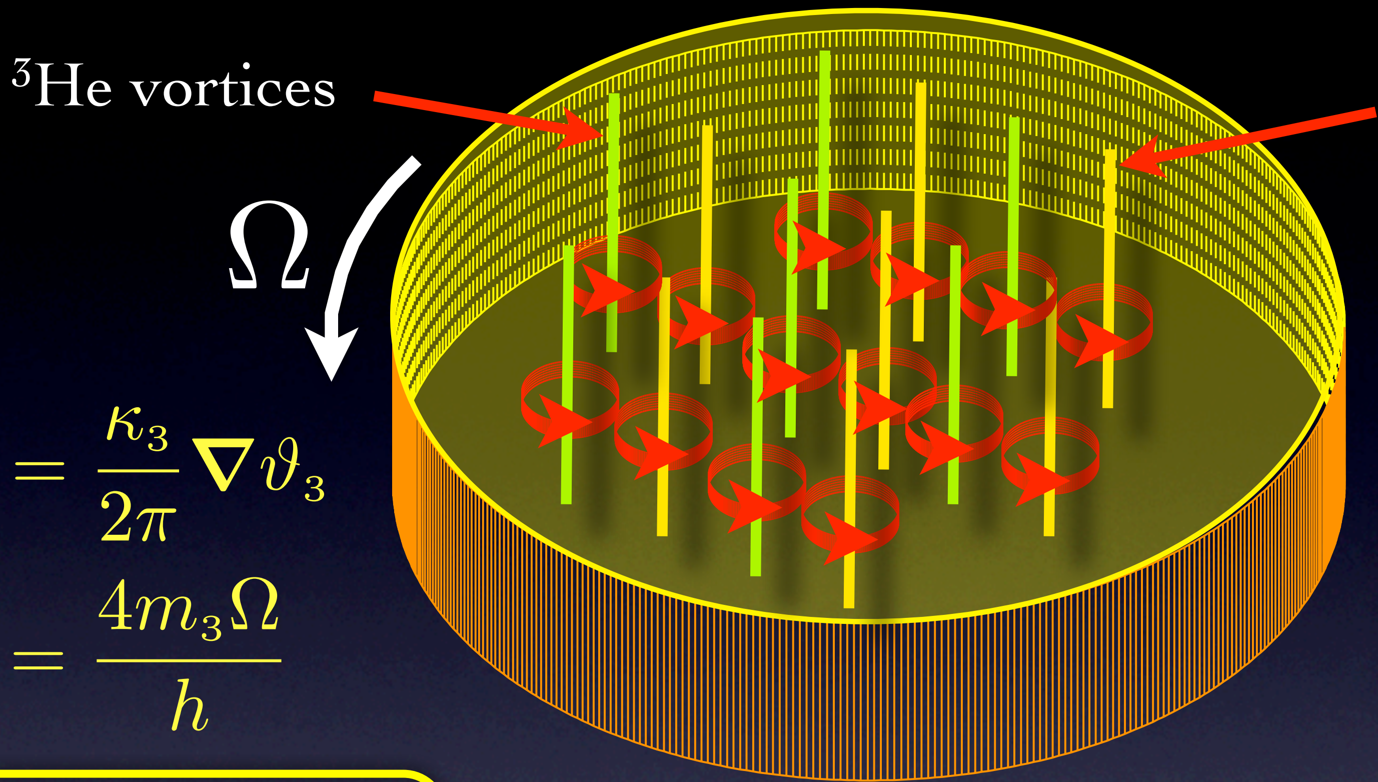


$$\nabla \times \mathbf{v}_N = 2\,\boldsymbol{\Omega}$$

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$$n_3 = \frac{4m_3\Omega}{h}$$

$$\mathbf{v}_4 = \frac{\kappa_4}{2\pi}\nabla\vartheta_4$$

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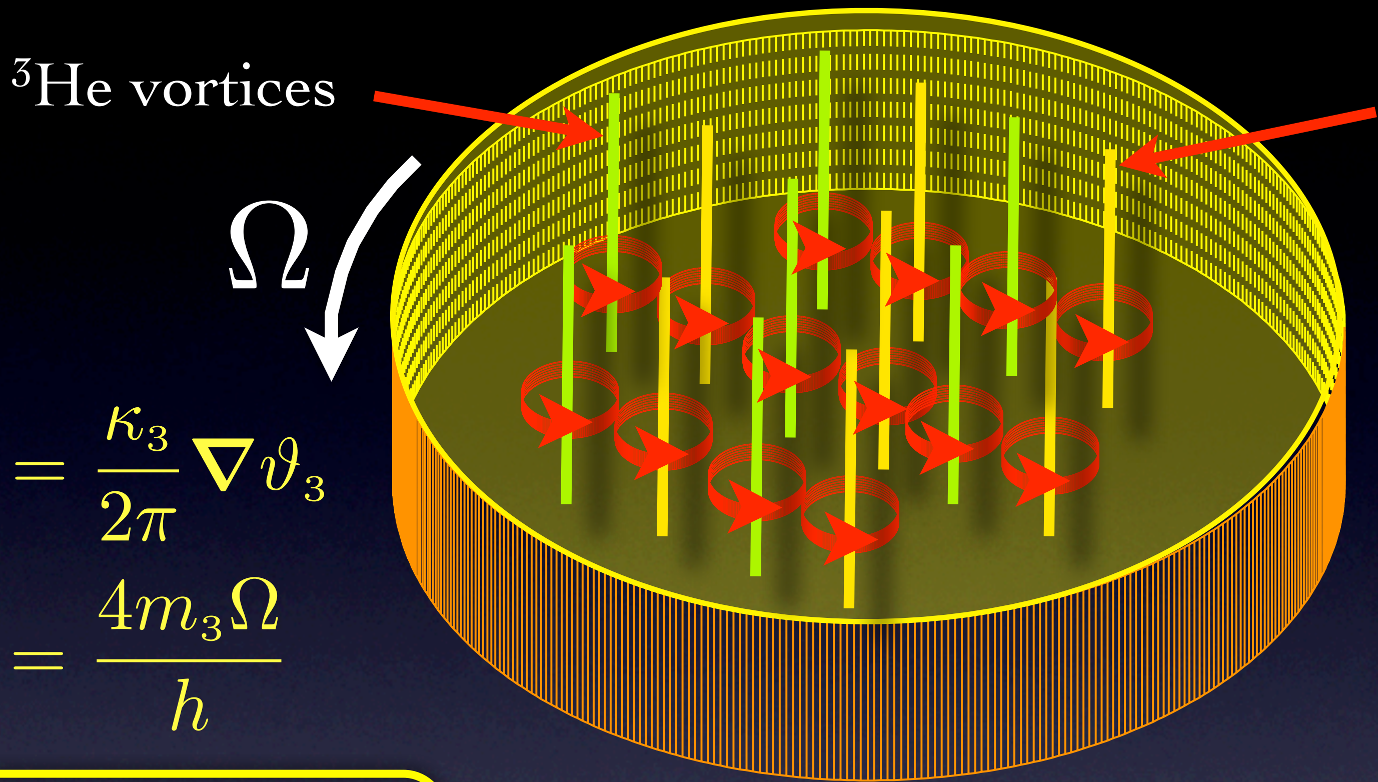
$$\frac{n_3}{n_4} = \frac{2m_3}{m_4} = \frac{3}{2}$$

# Superfluid $^3$He-$^4$He in Rotating Equilibrium

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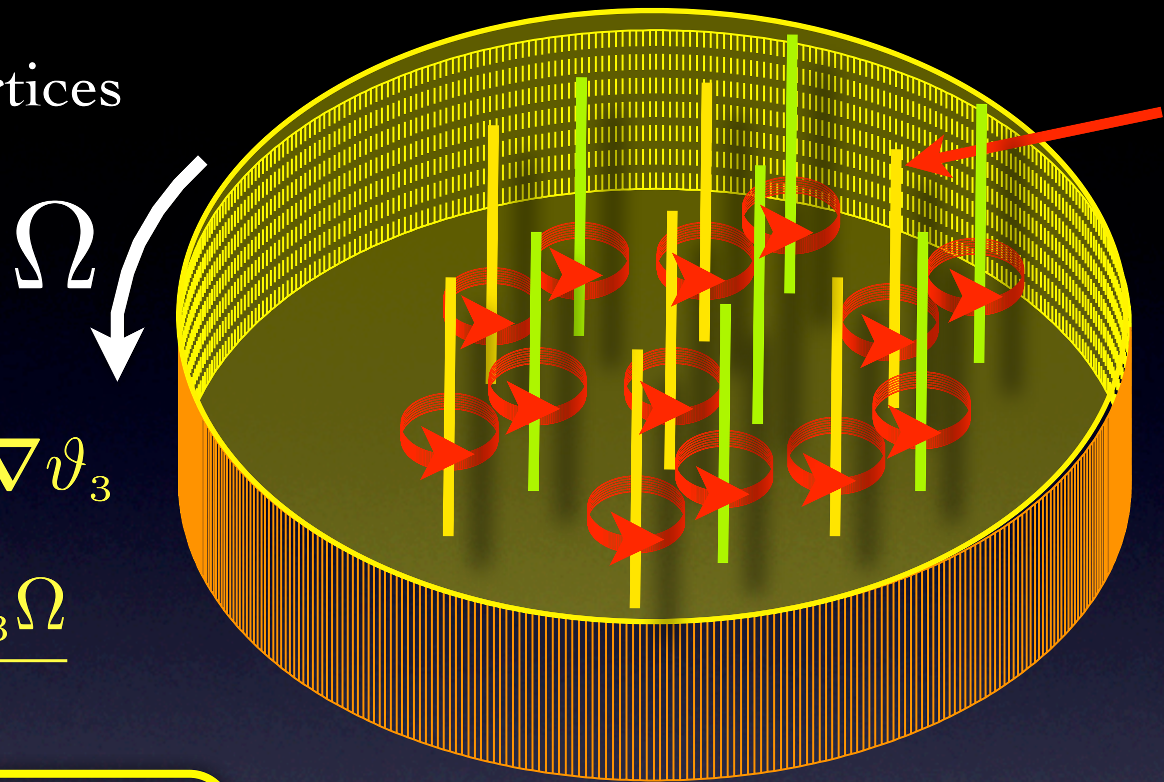
How does the $^3$He array adjust to the $^4$He lattice?

# Superfluid $^3$He-$^4$He in Rotating Equilibrium

$^3$He vortices

$^4$He vortices

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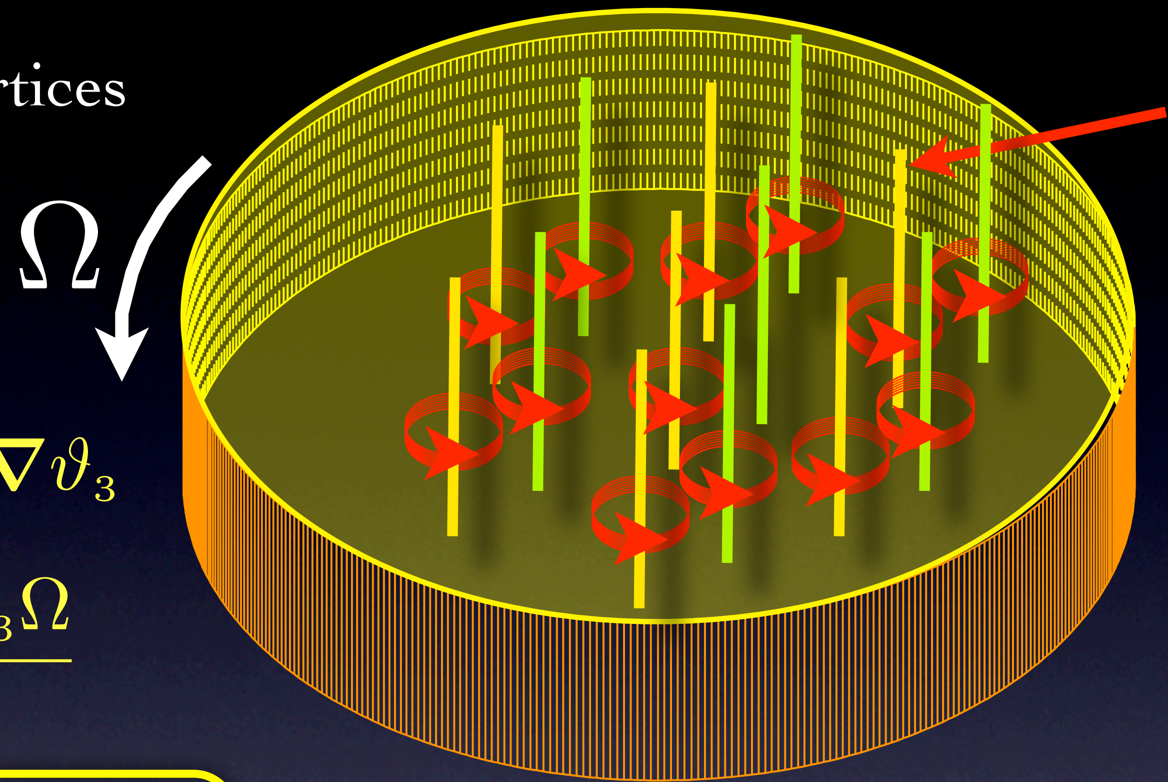


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- Feynman-Onsager constraint
- Superfluid Drag Interaction

$$U_{34} = 2\pi\rho_{34}\left(\frac{\kappa_3}{2\pi}\right)\left(\frac{\kappa_4}{2\pi}\right)\ln(R/|\mathbf{r}_3 - \mathbf{r}_4|)$$

- $^3$He anti-vortex - $^4$He vortex “Molecules”

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$$F[\mathbf{v}_3, \mathbf{v}_4] = \int d^3r \left\{ \frac{1}{2}\rho_4 |\mathbf{v}_4|^2 + \frac{1}{2}\rho_3 |\mathbf{v}_3|^2 + \rho_{34} \mathbf{v}_3 \cdot \mathbf{v}_4 \right\}$$

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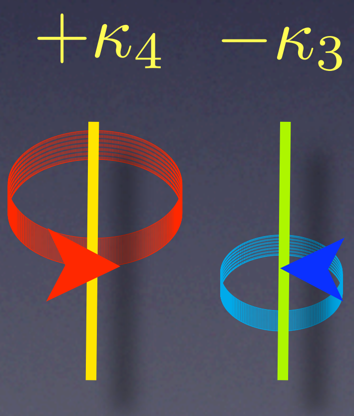

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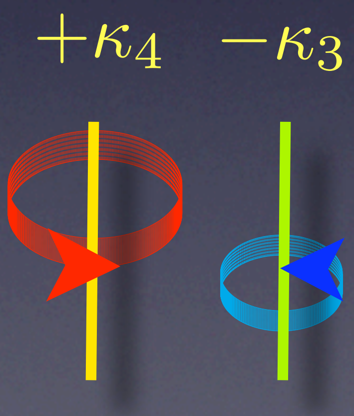

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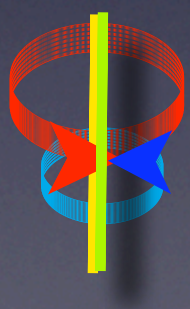

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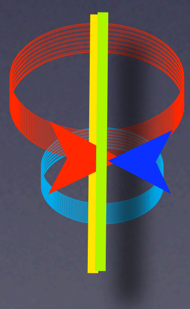

❖ Isolated $^4$He vortex + 2 x $^3$He *anti*-vortices

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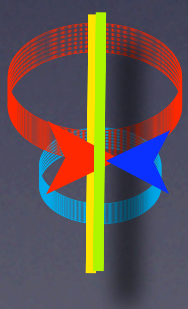

- Isolated $^4$He vortex + 2 x $^3$He *anti*-vortices
- Rotating Equilibrium  Feynman-Onsager

$$n_{3,-} = n_{4,+} \text{ bound} \qquad n_{3,+} = \frac{5}{2}n_{4,+} \quad \text{un-bound}$$

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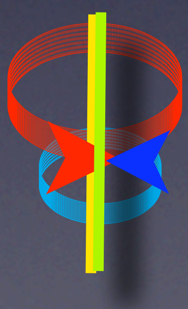

- Isolated $^4$He vortex + 2 x $^3$He *anti*-vortices
- Rotating Equilibrium Feynman-Onsager

  $n_{3,-} = n_{4,+}$ bound $\quad n_{3,+} = \frac{5}{2} n_{4,+}$ un-bound
- Renormalized Interactions

  $$U_{34} \rightarrow U_{3(4\bar{3})} \qquad U_{44} \rightarrow U_{(4\bar{3})(4\bar{3})}$$

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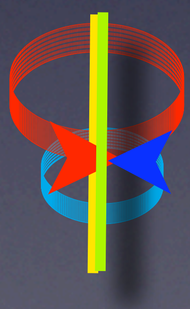

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- Renormalized Interactions

$$U_{34} \to U_{3(4\bar{3})} \qquad U_{44} \to U_{(4\bar{3})(4\bar{3})}$$

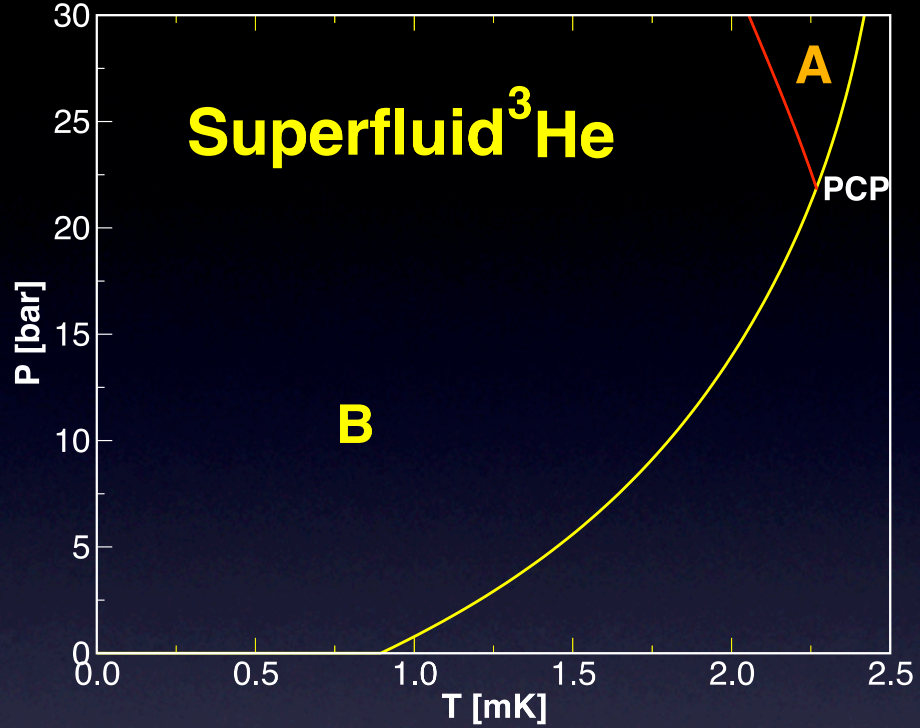
Novel Lattice Symmetry

# Exotic Pairing - Broken U(1) ... and More

- ❖ Broken Parity, Reflections, Time-Reversal
- ❖ Broken Orbital & Spin Rotational Symmetries

| | |
|---|---|
| Multiple SC Phases | $^3He$ $UPt_3$ |
| Unusual Transport | $Sr_2RuO_4$ |
| Exotic Defects and Vortices | UCoGe |

# $^3$He-A    Chiral Spin-Triplet Superfluids



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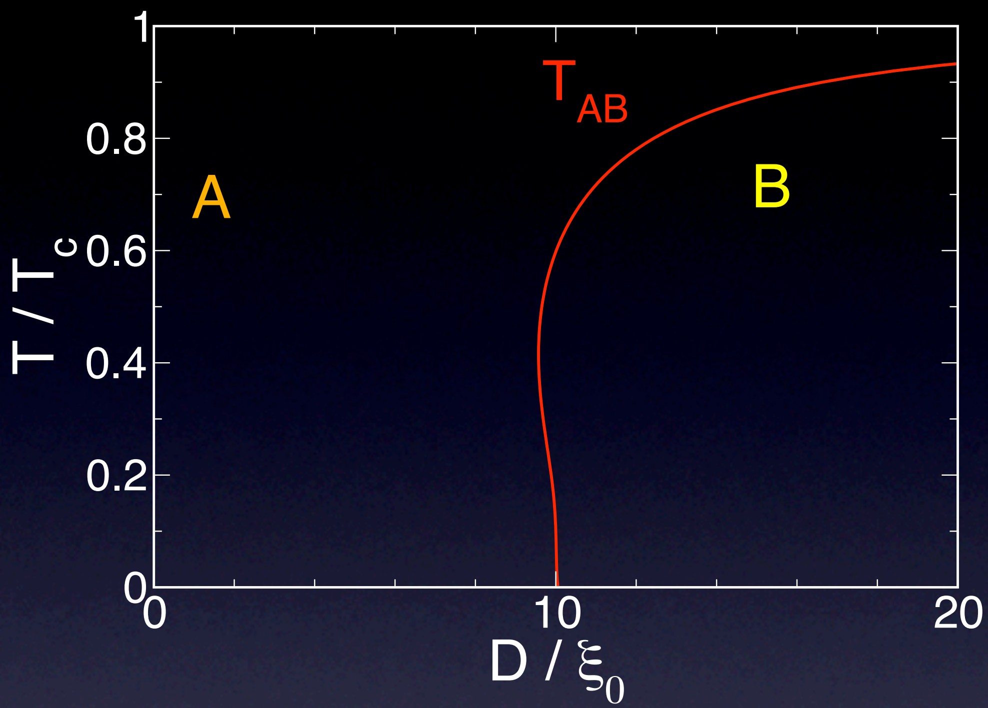

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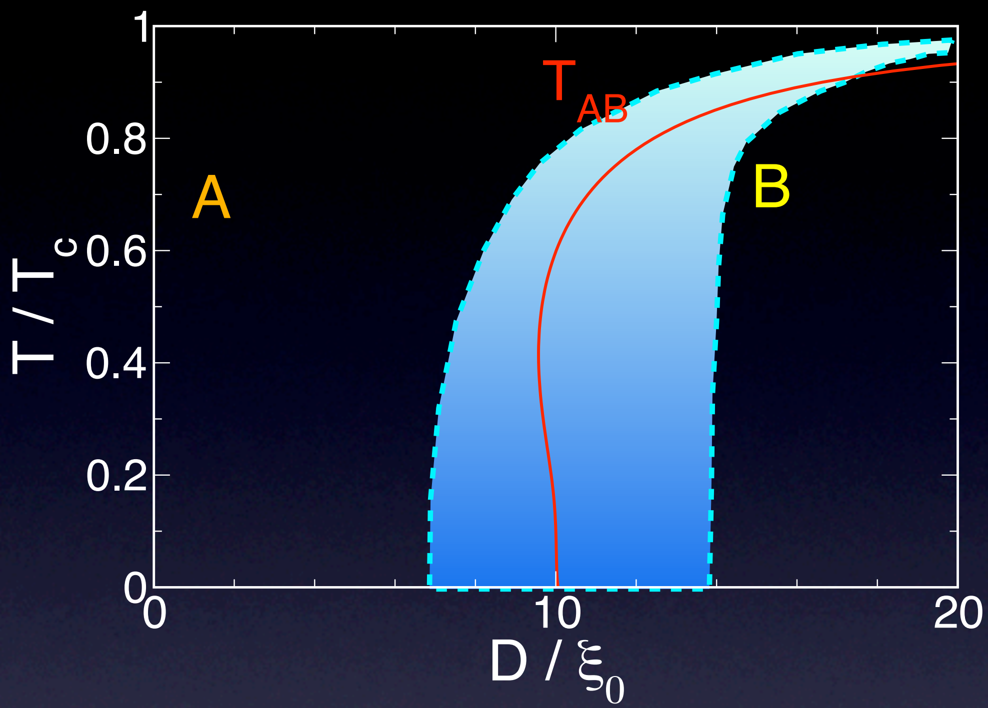
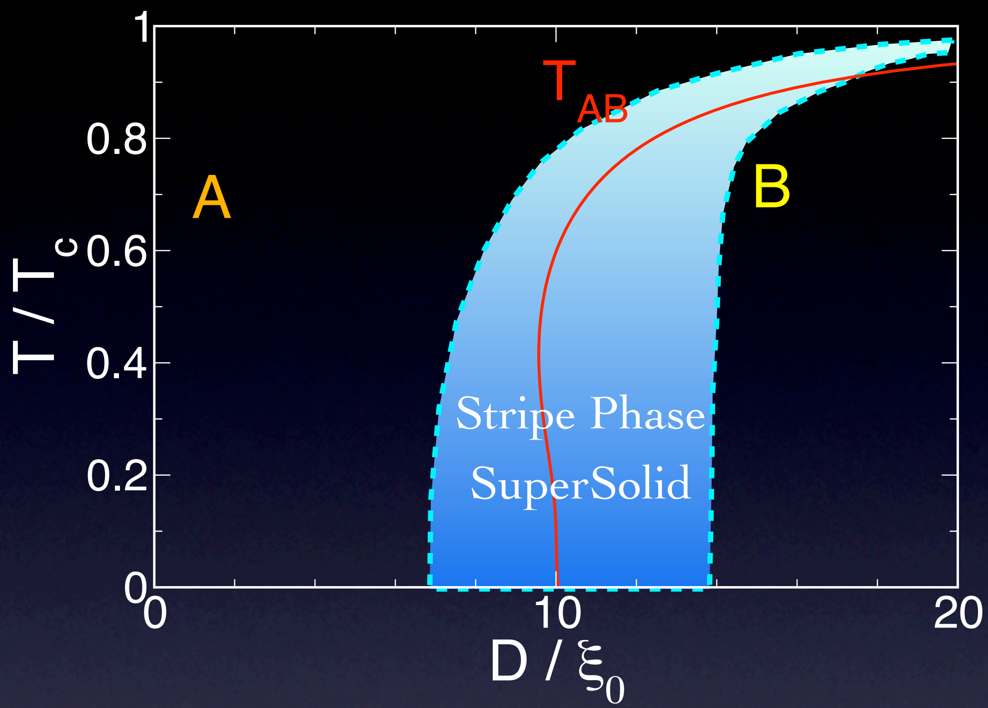
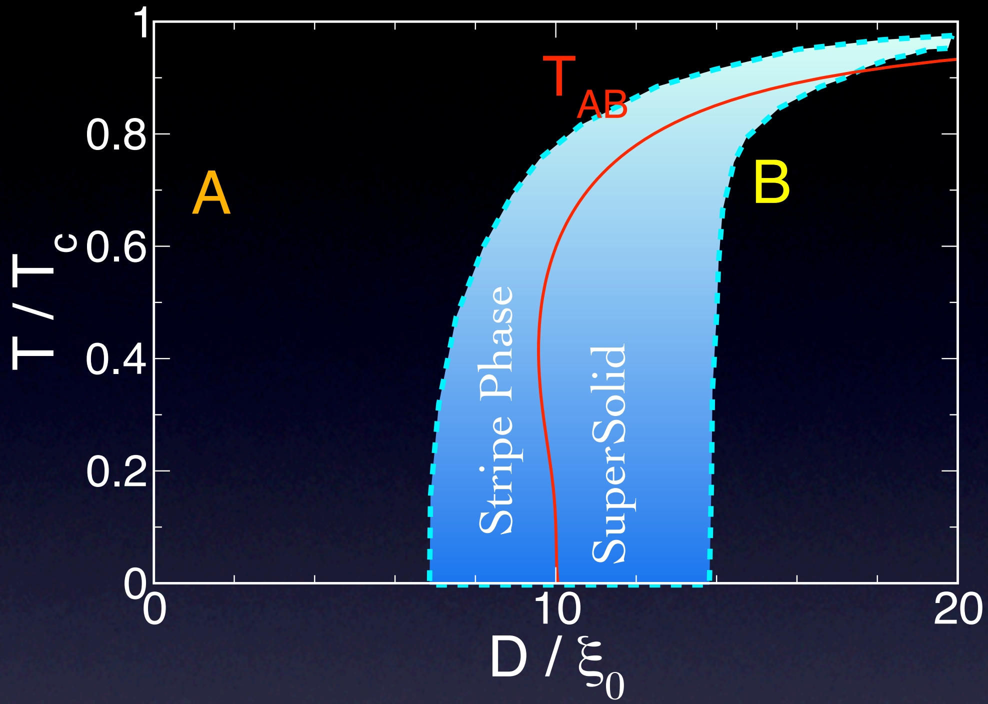


# $^3$He-A Chiral Spin-Triplet Superfluids



# $^3$He-A Chiral Spin-Triplet Superfluids

Vorontsov & JAS (PRL, 2007)



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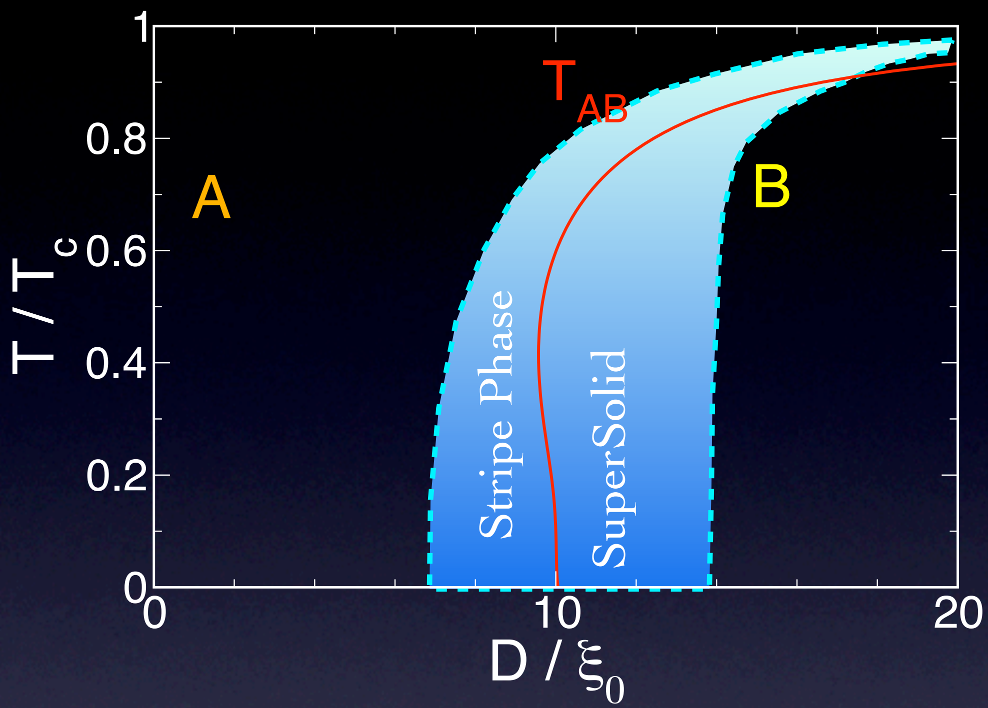


$$\Psi_{\alpha\beta}(\hat{\mathbf{r}}) = \Psi_0 \left[ (i\boldsymbol{\sigma}\sigma_y)_{\alpha\beta} \cdot \mathbf{d} \right] (\mathbf{m} + i\mathbf{n}) \cdot \hat{\mathbf{r}}$$

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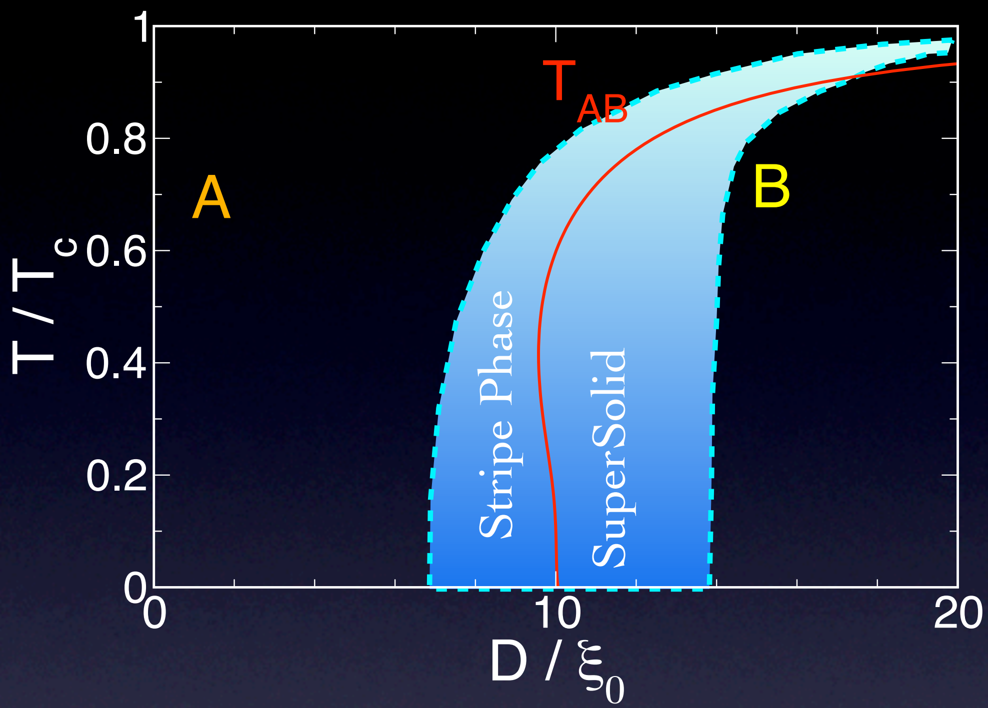


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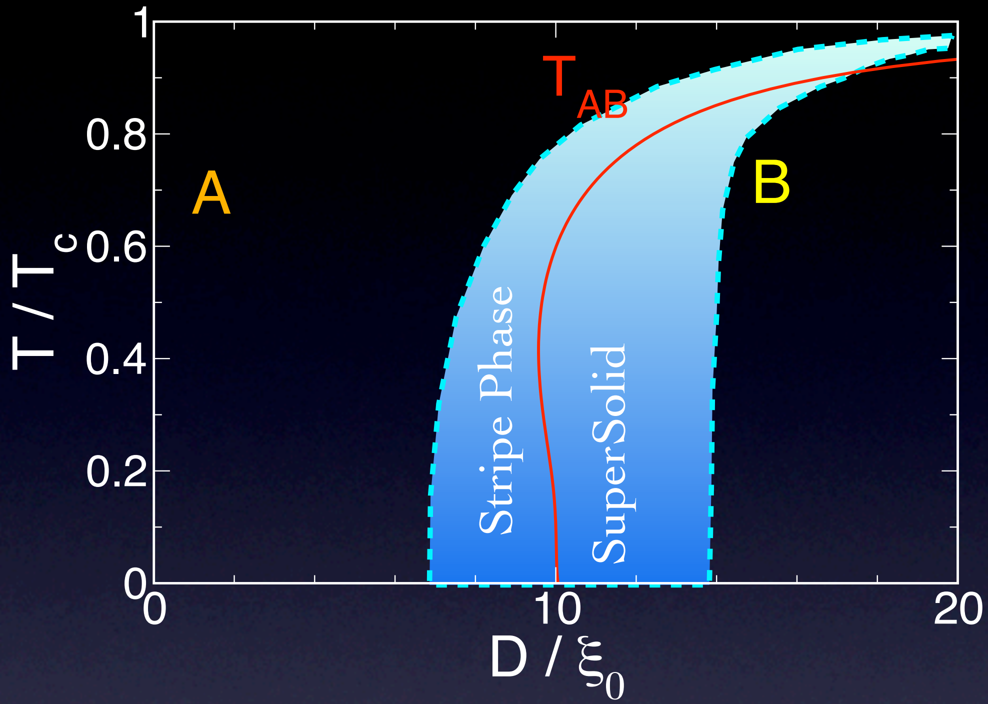


$$\Psi_{\alpha\beta}(\hat{\mathbf{r}}) = \Psi_0 \underbrace{\left[ (i\boldsymbol{\sigma}\sigma_y)_{\alpha\beta} \cdot \mathbf{d} \right]}_{S=1} \underbrace{(\mathbf{m} + i\mathbf{n}) \cdot \hat{\mathbf{r}}}_{L=1}$$

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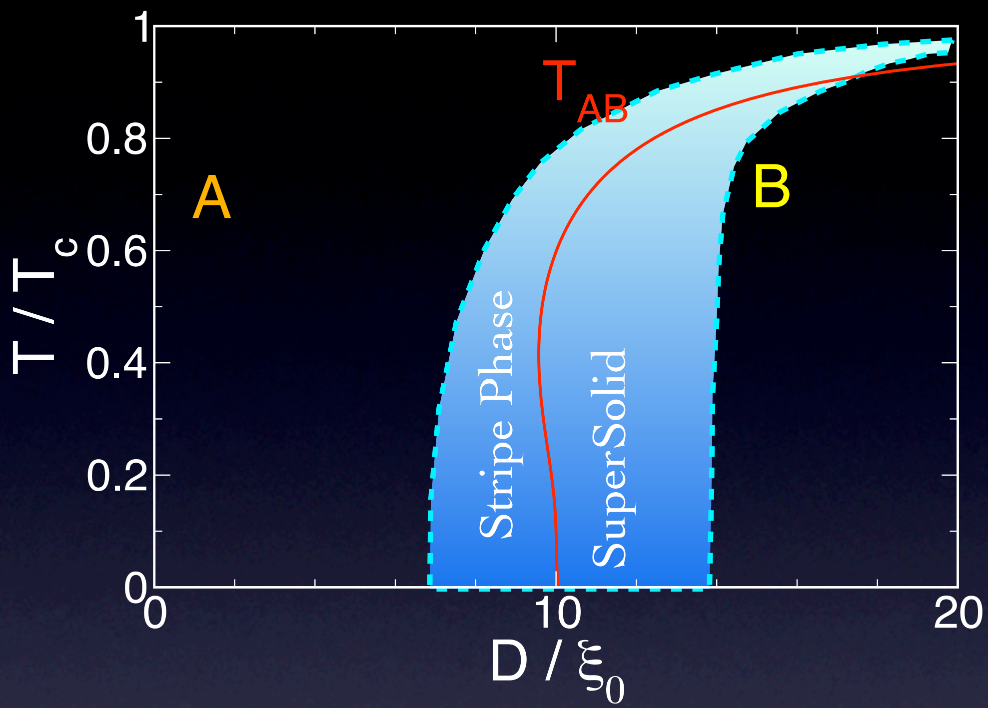


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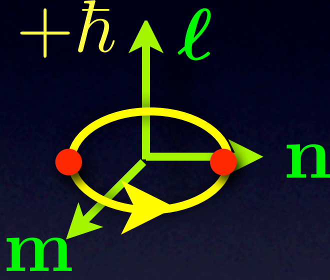


Orbital FM

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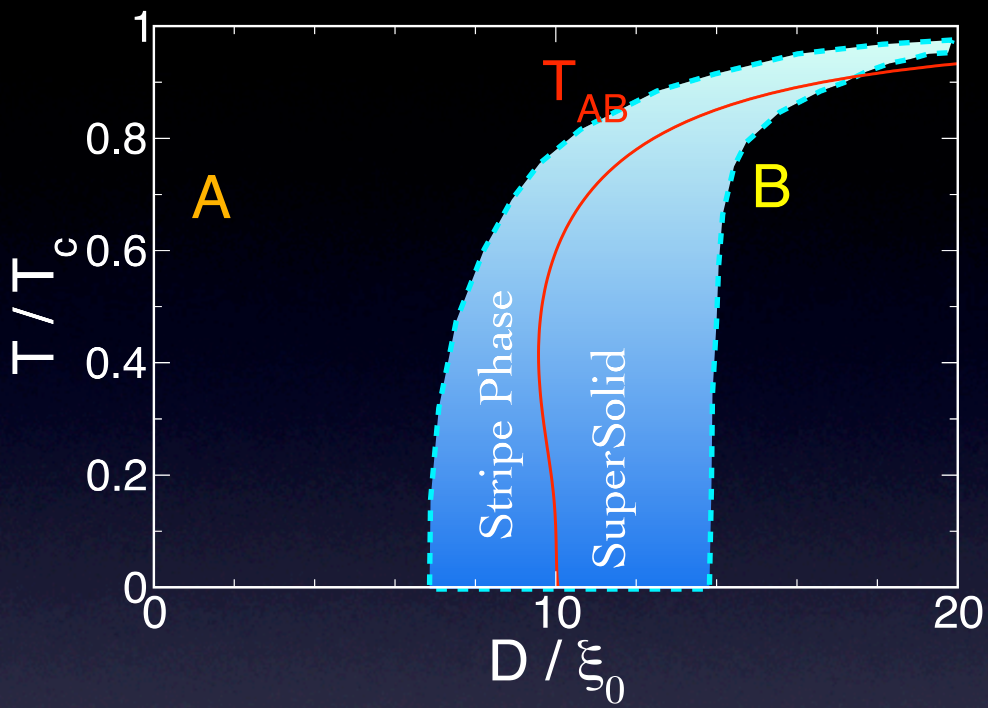
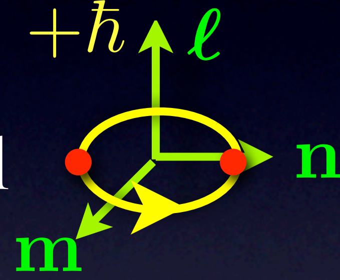



$$\Psi_{\alpha\beta}(\hat{\mathbf{r}}) = \Psi_0 \left[ \overbrace{(i\boldsymbol{\sigma}\sigma_y)_{\alpha\beta} \cdot \mathbf{d}}^{S=1} \right] \overbrace{(\mathbf{m} + i\mathbf{n}) \cdot \hat{\mathbf{r}}}^{L=1}$$

$|\uparrow\downarrow + \downarrow\uparrow\rangle$ $\quad$ $\mathbf{d}\cdot\mathbf{S} = 0$

Spin $\quad$ Orbital

AFM $\quad$ FM



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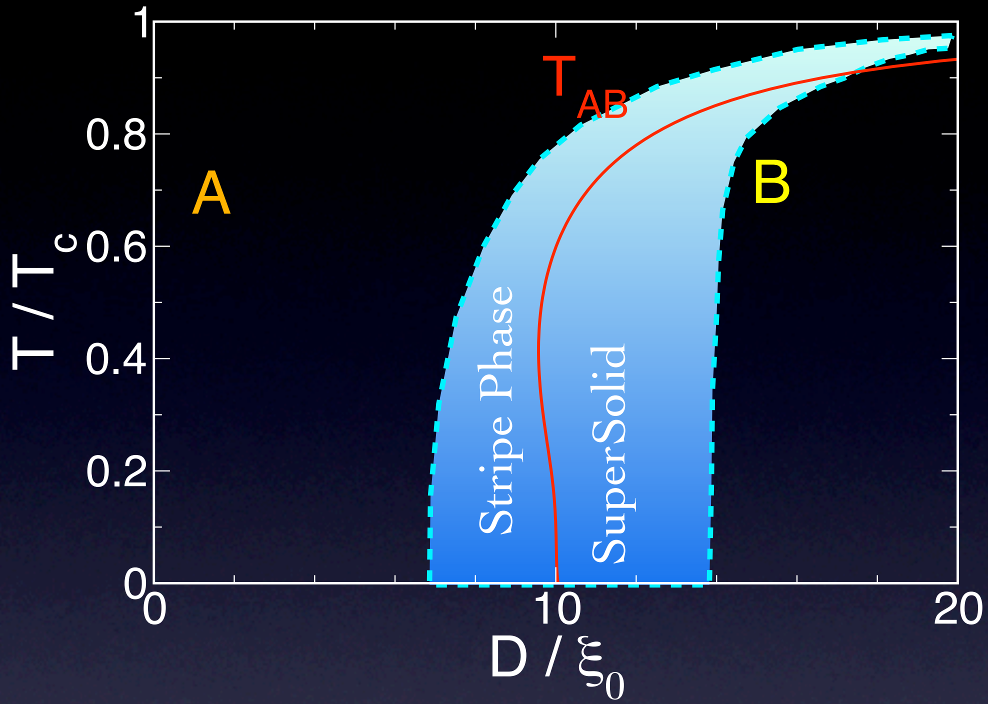


$S = 1$ $\quad$ $L = 1$

$$\Psi_{\alpha\beta}(\hat{\mathbf{r}}) = \Psi_0 \left[ (i\boldsymbol{\sigma}\sigma_y)_{\alpha\beta} \cdot \mathbf{d} \right] (\mathbf{m} + i\mathbf{n}) \cdot \hat{\mathbf{r}}$$

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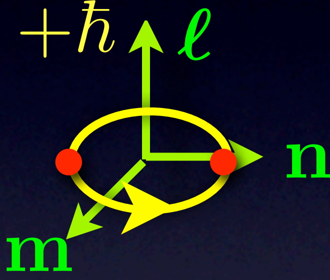
Spin $\quad$ Orbital

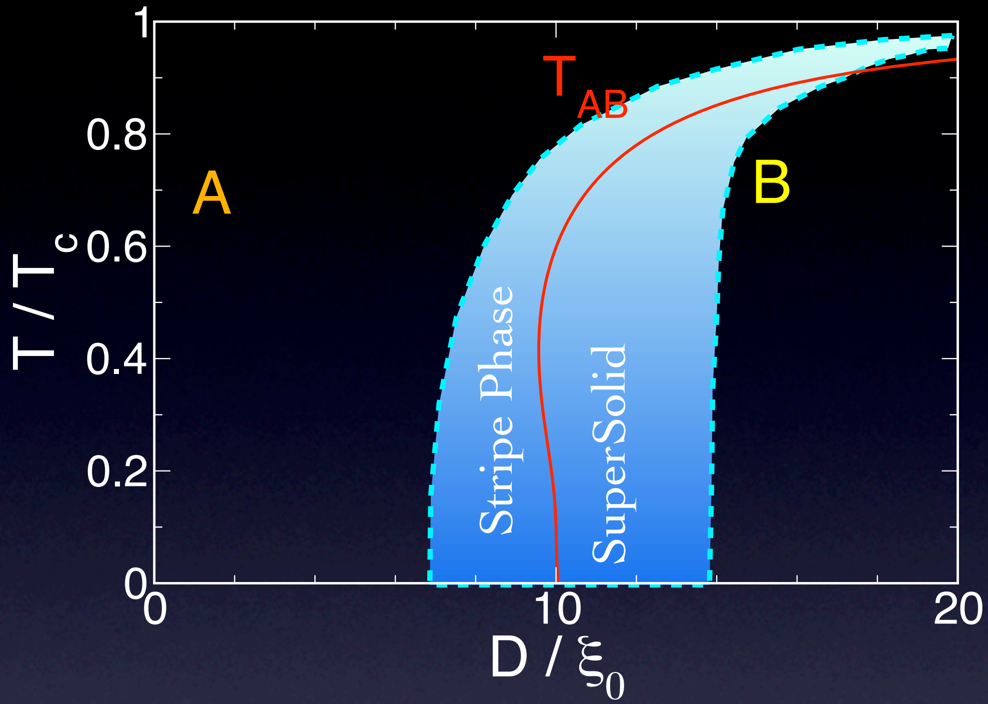
AFM $\quad$ FM




$$G = SO(3)_S \times SO(3)_L \times U(1) \times P \times T$$

# $^3$He-A    Chiral Spin-Triplet Superfluids

Vorontsov & JAS (PRL, 2007)

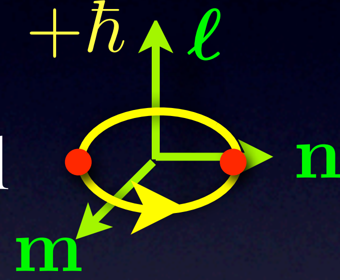



$$\Psi_{\alpha\beta}(\hat{\mathbf{r}}) = \Psi_0 \Big[ \underbrace{\overbrace{(i\boldsymbol{\sigma}\sigma_y)_{\alpha\beta} \cdot \mathbf{d}}^{S=1}}_{} \Big] \underbrace{\overbrace{(\mathbf{m} + i\mathbf{n}) \cdot \hat{\mathbf{r}}}^{L=1}}_{}$$

$|\uparrow\downarrow + \downarrow\uparrow\rangle$   $\mathbf{d}\cdot\mathbf{S} = 0$

Spin   Orbital

AFM   FM

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Friday, January 28, 2011

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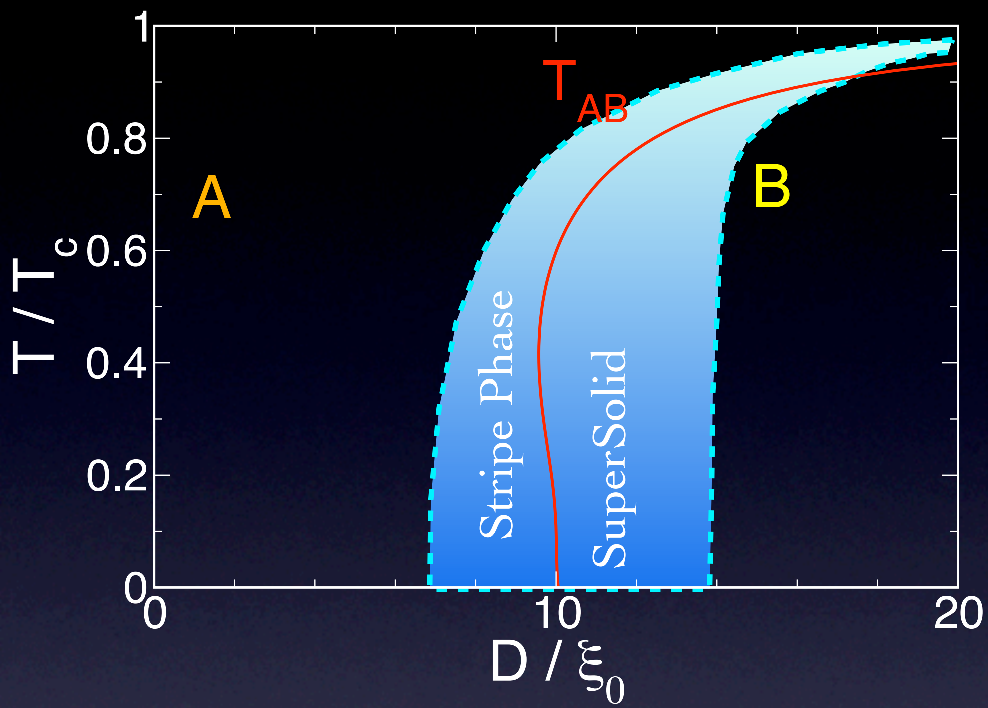


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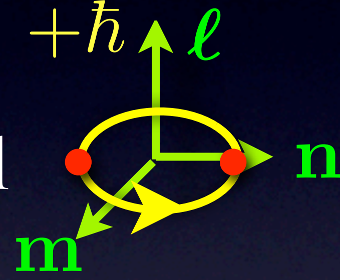
$|\uparrow\downarrow + \downarrow\uparrow\rangle$ $\mathbf{d} \cdot \mathbf{S} = 0$

Spin — AFM

Orbital — FM




$$G' = U(1)_{S_z} \times U(1)_{L_z+N}$$

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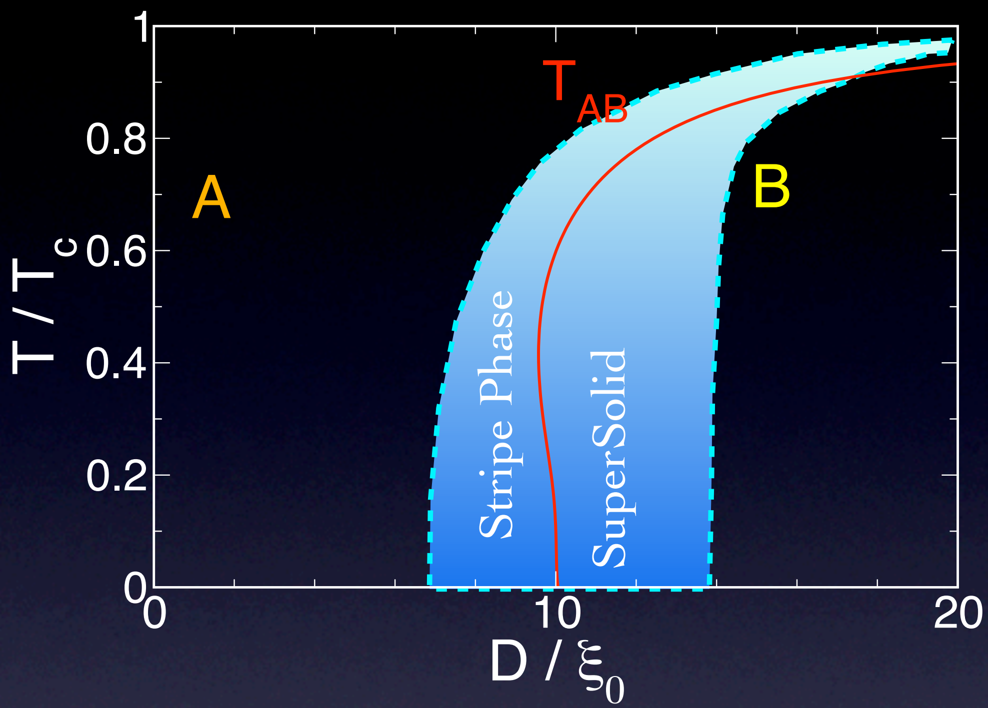


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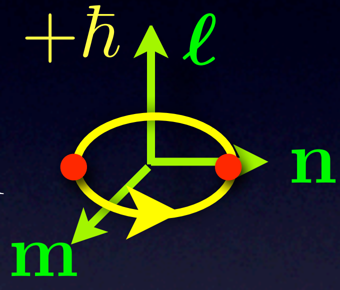
$|\uparrow\downarrow + \downarrow\uparrow\rangle$ $\quad \mathbf{d}\cdot\mathbf{S} = 0$

| Spin | Orbital |
|---|---|
| AFM | FM |




$$G' = U(1)_{S_z} \times U(1)_{L_z+N}$$

Exotic Vortices

- ❖ Single Vortices - Chirality
- ❖ Double Quantum Vortices
- ❖ 1/2 quantum vortices Volovik & Mineev (1980)

Y. Tsutsumi et al. PRL 101, 135302 (2008)

# 1/2 Quantum Vortex Chiral Spin-Triplet Superfluids

## $^3$He-A Films

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T. Kawakami, JPSPJ 79, 044607, 2010
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## HQV Pairs

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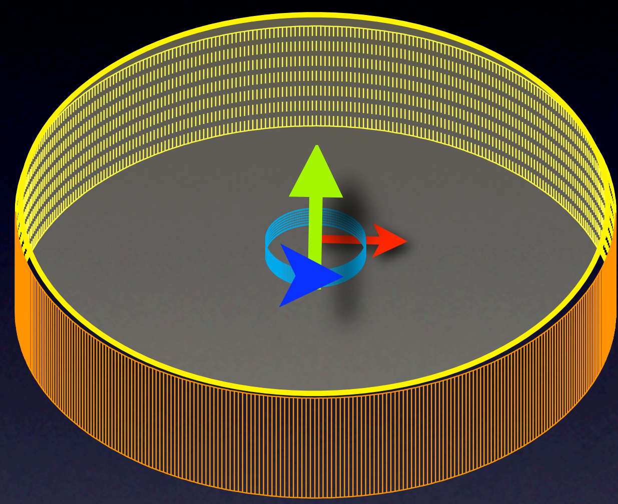

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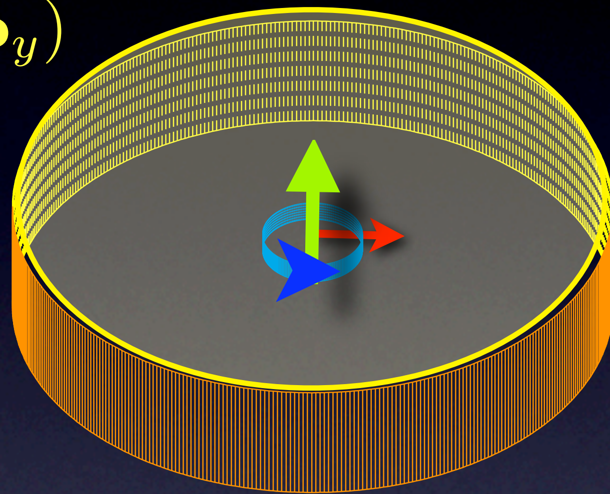

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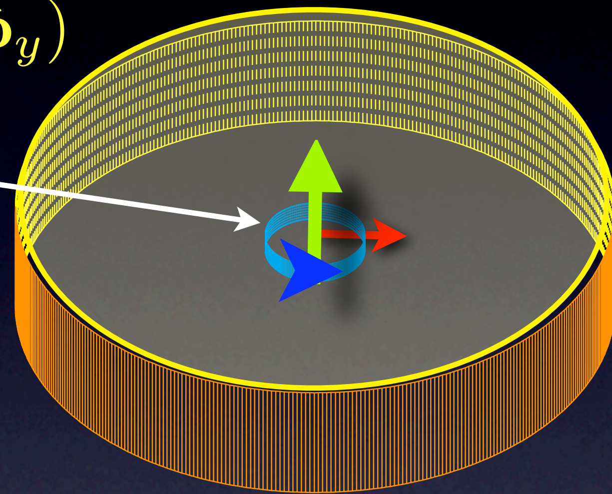
$$\Delta\vartheta = \pi$$



1/2 Phase Vortex

$$\vartheta = \frac{1}{2}\phi$$

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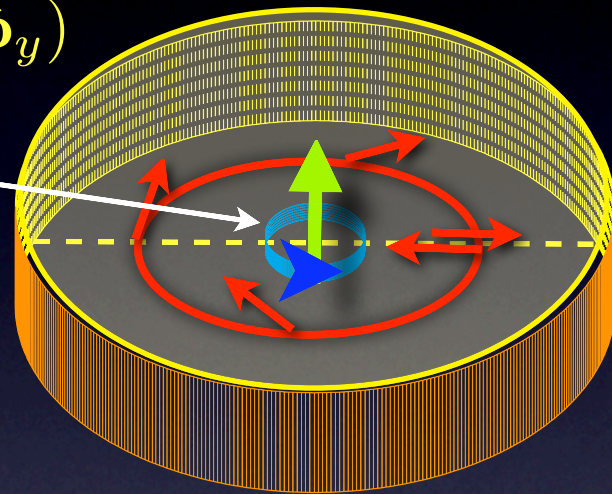
$$\mathbf{d} = \cos(\alpha)\hat{\mathbf{x}} + \sin(\alpha)\hat{\mathbf{y}} \qquad \Delta\vartheta = \pi$$



Spin disgyration $\qquad$ 1/2 Phase Vortex

$$\alpha = -\frac{1}{2}\phi \qquad \vartheta = \frac{1}{2}\phi$$

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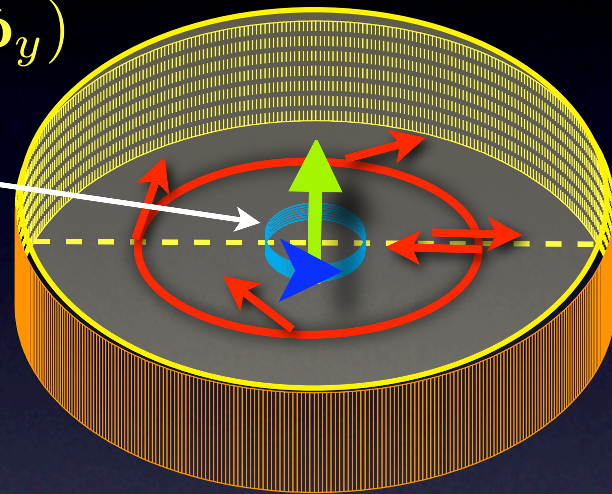

Spin disgyration

$$\alpha = -\frac{1}{2}\phi$$

1/2 Phase Vortex

$$\vartheta = \frac{1}{2}\phi$$

$$\Psi_{\alpha\beta}(\hat{\mathbf{p}},\mathbf{r}) \rightarrow \Psi_0 e^{i\vartheta}\begin{pmatrix} -\mathbf{d}_x + i\mathbf{d}_y & 0 \\ 0 & +\mathbf{d}_x + i\mathbf{d}_y \end{pmatrix}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$$

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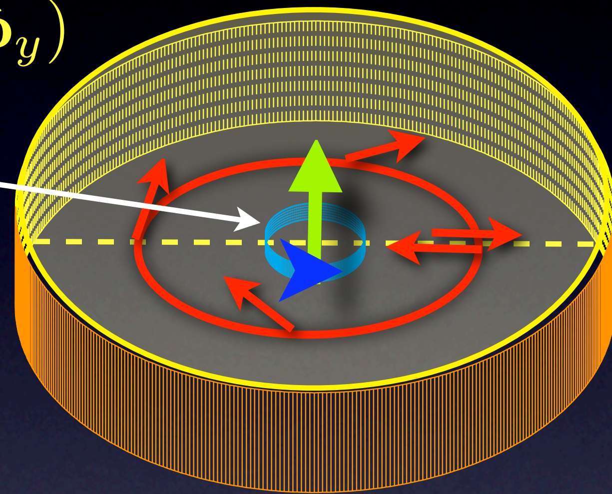

Spin disgyration    1/2 Phase Vortex

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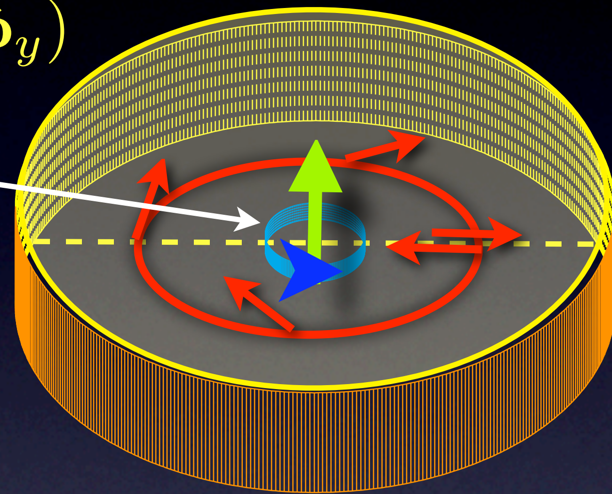

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Superfluid Mixture of ↑↑ and ↓↓ Cooper Pairs

$$\Psi_{\uparrow\uparrow} = \Psi_0\, e^{i\phi}$$

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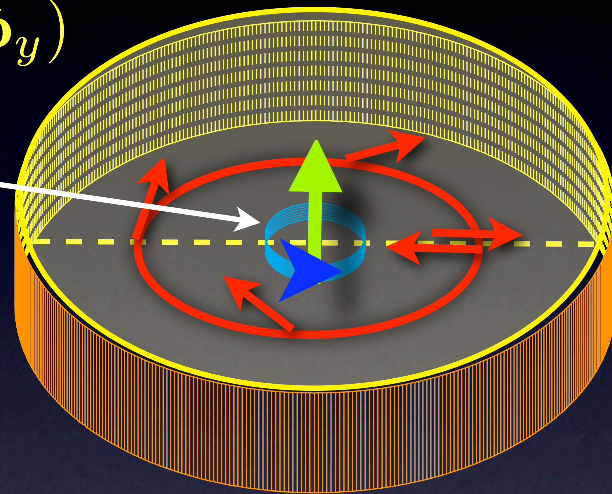

Spin disgyration 1/2 Phase Vortex

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Superfluid Mixture of ↑↑ and ↓↓ Cooper Pairs

$$\Psi_{\uparrow\uparrow} = \Psi_0\, e^{i\phi}$$

$$\Psi_{\downarrow\downarrow} = \Psi_0$$

$$\Phi = \frac{hc}{2e}\left\{\frac{|\Psi_{\uparrow\uparrow}|^2}{|\Psi_{\uparrow\uparrow}|^2 + |\Psi_{\downarrow\downarrow}|^2}\right\}$$

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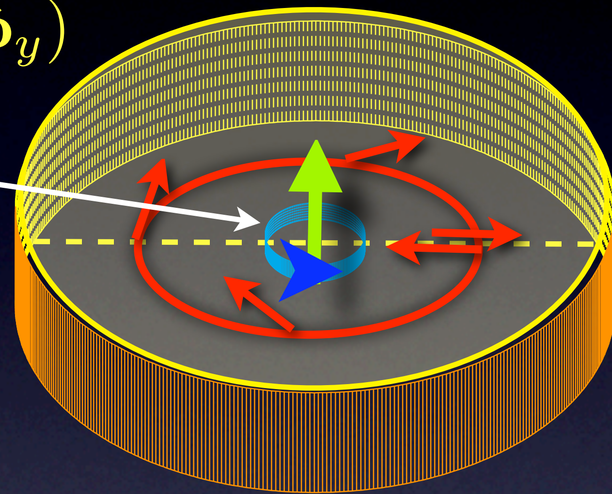

Spin disgyration 1/2 Phase Vortex

$$\alpha = -\frac{1}{2}\phi \qquad \vartheta = \frac{1}{2}\phi$$

$$\Psi_{\alpha\beta}(\hat{\mathbf{p}},\mathbf{r}) \rightarrow e^{i\frac{1}{2}\phi}\begin{pmatrix}\Psi_0 e^{i\frac{1}{2}\phi} & 0\\ 0 & \Psi_0 e^{-i\frac{1}{2}\phi}\end{pmatrix}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$$

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S.Chung et al PRL 2008

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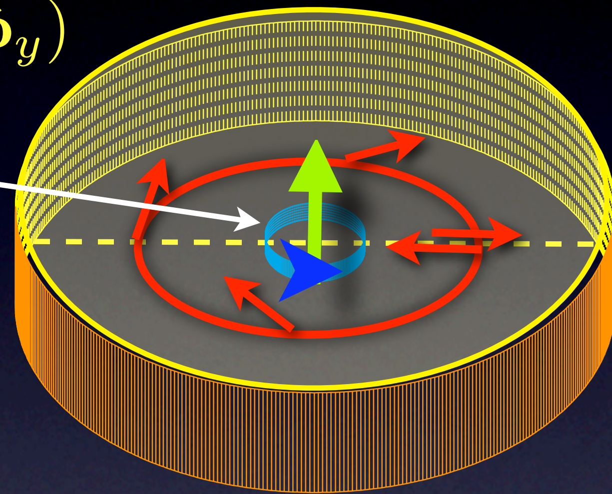
$$\Psi_{\alpha\beta}(\hat{\mathbf{p}},\mathbf{r}) = \Psi_0\, e^{i\vartheta(\mathbf{r})}\left[(i\boldsymbol{\sigma}\sigma_y)_{\alpha\beta}\cdot\mathbf{d}(\mathbf{r})\right](\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$$

$$\mathbf{d} = \cos(\alpha)\hat{\mathbf{x}} + \sin(\alpha)\hat{\mathbf{y}} \qquad \Delta\vartheta = \pi$$



Spin disgyration      1/2 Phase Vortex

$$\alpha = -\frac{1}{2}\phi \qquad \vartheta = \frac{1}{2}\phi$$

$$\Psi_{\alpha\beta}(\hat{\mathbf{p}},\mathbf{r}) \rightarrow e^{i\frac{1}{2}\phi}\begin{pmatrix}\Psi_0 e^{i\frac{1}{2}\phi} & 0 \\ 0 & \Psi_0 e^{-i\frac{1}{2}\phi}\end{pmatrix}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$$

Superfluid Mixture of ↑↑ and ↓↓ Cooper Pairs

$$\Psi_{\uparrow\uparrow} = \Psi_0\, e^{i\phi}$$

$$\Psi_{\downarrow\downarrow} = \Psi_0 \qquad \Phi = \frac{hc}{2e}\left\{\frac{|\Psi_{\uparrow\uparrow}|^2 + \mu|\Psi_{\uparrow\uparrow}|^2|\Psi_{\downarrow\downarrow}|^2}{|\Psi_{\uparrow\uparrow}|^2 + |\Psi_{\downarrow\downarrow}|^2 + 2\mu|\Psi_{\uparrow\uparrow}|^2|\Psi_{\downarrow\downarrow}|^2}\right\}$$

# 1/2 Quantum Vortex Chiral Spin-Triplet Superfluids    $Sr_2RuO_4$ ?

$^3$He-A Films

M. Salomaa & G. Volovik, RMP 1986
T. Kawakami, JPSPJ 79, 044607, 2010
V. Vakaryuk, A. Leggett PRL 2010

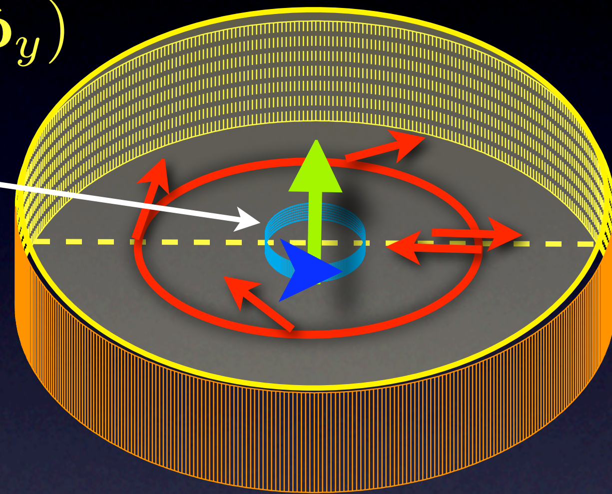
HQV Pairs

D. Ivanov PRL 2001
S.Chung et al PRL 2008

➡ NMR Search for HQV Pairs in Rotating $^3$He-A Films: M. Yamashita et al. PRL 101, 025302 (2008).

$$\Psi_{\alpha\beta}(\hat{\mathbf{p}},\mathbf{r}) = \Psi_0\, e^{i\vartheta(\mathbf{r})}\Big[(i\boldsymbol{\sigma}\sigma_y)_{\alpha\beta}\cdot\mathbf{d}(\mathbf{r})\Big]\,(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$$

$$\mathbf{d} = \cos(\alpha)\hat{\mathbf{x}} + \sin(\alpha)\hat{\mathbf{y}} \qquad \Delta\vartheta = \pi$$



Spin disgyration    1/2 Phase Vortex

$$\alpha = -\frac{1}{2}\phi \qquad \vartheta = \frac{1}{2}\phi$$

$$\Psi_{\alpha\beta}(\hat{\mathbf{p}},\mathbf{r}) \rightarrow e^{i\frac{1}{2}\phi}\begin{pmatrix}\Psi_0 e^{i\frac{1}{2}\phi} & 0 \\ 0 & \Psi_0 e^{-i\frac{1}{2}\phi}\end{pmatrix}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$$

Superfluid Mixture of $\uparrow\uparrow$ and $\downarrow\downarrow$ Cooper Pairs    $\mu \propto \frac{1}{3}F_1^a$

$$\Psi_{\uparrow\uparrow} = \Psi_0\, e^{i\phi}$$

$$\Psi_{\downarrow\downarrow} = \Psi_0 \qquad \Phi = \frac{hc}{2e}\left\{\frac{|\Psi_{\uparrow\uparrow}|^2 + \mu|\Psi_{\uparrow\uparrow}|^2|\Psi_{\downarrow\downarrow}|^2}{|\Psi_{\uparrow\uparrow}|^2 + |\Psi_{\downarrow\downarrow}|^2 + 2\mu|\Psi_{\uparrow\uparrow}|^2|\Psi_{\downarrow\downarrow}|^2}\right\} \neq \frac{hc}{4e}$$

# S = 1 Order Parameter Symmetry

$$\Delta_{\alpha\beta}(\mathbf{p}_f) = (i\vec{\boldsymbol{\sigma}}\sigma_y)_{\alpha\beta} \cdot \vec{\mathbf{d}}(\mathbf{p}_f)$$

$$\vec{\mathbf{d}}(\mathbf{p}_f) = \sum_{\Gamma\nu} \Delta_\nu^\Gamma \, \vec{\eta}_{\Gamma\nu}(\mathbf{p}_f)$$

$D_{4h}$ $Sr_2RuO_4$

| $\Gamma_u$ | $\vec{\eta}_{\Gamma\nu}$ |
|---|---|
| $A_{1u}$ | $\vec{d}\,\hat{\mathbf{p}}_z$ |
| $A_{2u}$ | $\vec{d}\,\hat{\mathbf{p}}_z\hat{\mathbf{p}}_x\hat{\mathbf{p}}_y(\hat{\mathbf{p}}_x^2 - \hat{\mathbf{p}}_y^2)$ |
| $B_{1u}$ | $\vec{d}\,\hat{\mathbf{p}}_z\hat{\mathbf{p}}_x\hat{\mathbf{p}}_y$ |
| $B_{2u}$ | $\vec{d}\,\hat{\mathbf{p}}_z\,(\hat{\mathbf{p}}_x^2 - \hat{\mathbf{p}}_y^2)$ |
| $E_u$ | $\vec{d}\begin{pmatrix} \hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y \\ \hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y \end{pmatrix}$ |

$\vec{d} \,||\, \hat{c}$ ?

$D_{6h}$ $UPt_3$ $\vec{d} \,||\, \hat{c}$

| $\Gamma_u$ | $\vec{\eta}_{\Gamma\nu}$ |
|---|---|
| $A_{1u}$ | $\vec{d}\,\hat{\mathbf{p}}_z$ |
| $A_{2u}$ | $\vec{d}\,\hat{\mathbf{p}}_z\mathfrak{Im}\,(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)^6$ |
| $B_{1u}$ | $\vec{d}\,\mathfrak{Im}\,(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)^3$ |
| $B_{2u}$ | $\vec{d}\,\Re(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)^3$ |
| $E_{1u}$ | $\vec{d}\begin{pmatrix} (\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y) \\ (\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y) \end{pmatrix}$ |
| $E_{2u}$ | $\vec{d}\,\hat{\mathbf{p}}_z\begin{pmatrix} (\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)^2 \\ (\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)^2 \end{pmatrix}$ |

# S = 1 Order Parameter Symmetry

$$\Delta_{\alpha\beta}(\mathbf{p}_f) = (i\vec{\boldsymbol{\sigma}}\sigma_y)_{\alpha\beta} \cdot \vec{\mathbf{d}}(\mathbf{p}_f)$$

$$\vec{\mathbf{d}}(\mathbf{p}_f) = \sum_{\Gamma\nu} \Delta_\nu^\Gamma \, \vec{\eta}_{\Gamma\nu}(\mathbf{p}_f)$$

$D_{4h}$ *$Sr_2RuO_4$*

| $\Gamma_u$ | $\vec{\eta}_{\Gamma\nu}$ |
|---|---|
| $A_{1u}$ | $\vec{d}\,\hat{\mathbf{p}}_z$ |
| $A_{2u}$ | $\vec{d}\,\hat{\mathbf{p}}_z\hat{\mathbf{p}}_x\hat{\mathbf{p}}_y(\hat{\mathbf{p}}_x^2 - \hat{\mathbf{p}}_y^2)$ |
| $B_{1u}$ | $\vec{d}\,\hat{\mathbf{p}}_z\hat{\mathbf{p}}_x\hat{\mathbf{p}}_y$ |
| $B_{2u}$ | $\vec{d}\,\hat{\mathbf{p}}_z\,(\hat{\mathbf{p}}_x^2 - \hat{\mathbf{p}}_y^2)$ |
| $E_u$ | $\vec{d}\begin{pmatrix}\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y \\ \hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y\end{pmatrix}$ |

$\vec{d}\,||\,\hat{c}$ **?**

$D_{6h}$ *$UPt_3$* $\vec{d}\,||\,\hat{c}$

| $\Gamma_u$ | $\vec{\eta}_{\Gamma\nu}$ |
|---|---|
| $A_{1u}$ | $\vec{d}\,\hat{\mathbf{p}}_z$ |
| $A_{2u}$ | $\vec{d}\,\hat{\mathbf{p}}_z\,\mathfrak{Im}\,(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)^6$ |
| $B_{1u}$ | $\vec{d}\,\mathfrak{Im}\,(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)^3$ |
| $B_{2u}$ | $\vec{d}\,\Re(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)^3$ |
| $E_{1u}$ | $\vec{d}\begin{pmatrix}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y) \\ (\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)\end{pmatrix}$ |
| $E_{2u}$ | $\vec{d}\,\hat{\mathbf{p}}_z\begin{pmatrix}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)^2 \\ (\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)^2\end{pmatrix}$ |

# S = 1 Order Parameter Symmetry

$$\Delta_{\alpha\beta}(\mathbf{p}_f) = (i\vec{\boldsymbol{\sigma}}\sigma_y)_{\alpha\beta} \cdot \vec{\mathbf{d}}(\mathbf{p}_f)$$

$$\vec{\mathbf{d}}(\mathbf{p}_f) = \sum_{\Gamma\nu} \Delta^{\Gamma}_{\nu}\, \vec{\eta}_{\Gamma\nu}(\mathbf{p}_f)$$

$D_{4h}$ $\quad$ $Sr_2RuO_4$

| $\Gamma_u$ | $\vec{\eta}_{\Gamma\nu}$ |
|---|---|
| $A_{1u}$ | $\vec{d}\,\hat{\mathbf{p}}_z$ |
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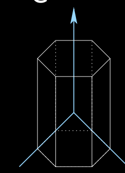
| $B_{2u}$ | $\vec{d}\,\hat{\mathbf{p}}_z\,(\hat{\mathbf{p}}_x^2 - \hat{\mathbf{p}}_y^2)$ |
| $E_u$ | $\vec{d}\begin{pmatrix}\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y \\ \hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y\end{pmatrix}$ |

$\vec{d}\,\|\,\hat{c}$ ?

$D_{6h}$ $\quad$ $UPt_3$ $\quad$ $\vec{d}\,\|\,\hat{c}$

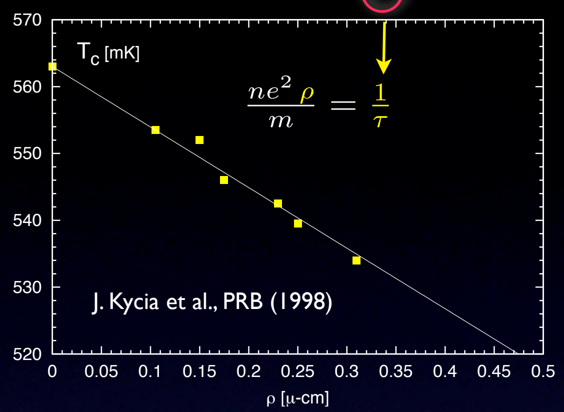
| $\Gamma_u$ | $\vec{\eta}_{\Gamma\nu}$ |
|---|---|
| $A_{1u}$ | $\vec{d}\,\hat{\mathbf{p}}_z$ |
| $A_{2u}$ | $\vec{d}\,\hat{\mathbf{p}}_z\,\mathfrak{Im}\,(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)^6$ |
| $B_{1u}$ | $\vec{d}\,\mathfrak{Im}\,(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)^3$ |
| $B_{2u}$ | $\vec{d}\,\Re(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)^3$ |
| $E_{1u}$ | $\vec{d}\begin{pmatrix}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y) \\ (\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)\end{pmatrix}$ |
| $E_{2u}$ | $\vec{d}\,\hat{\mathbf{p}}_z\begin{pmatrix}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)^2 \\ (\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)^2\end{pmatrix}$ |

# Unconventional Pairing in $UPt_3$



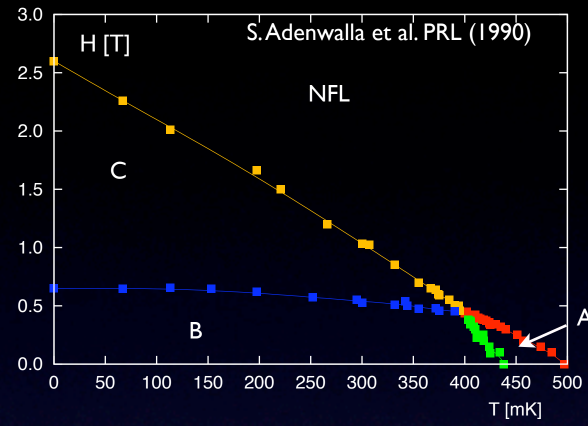

L. Gorkov (1987)

$$T_c = T_{c0} - \frac{\pi}{8}\frac{\hbar/k_B}{\tau}$$



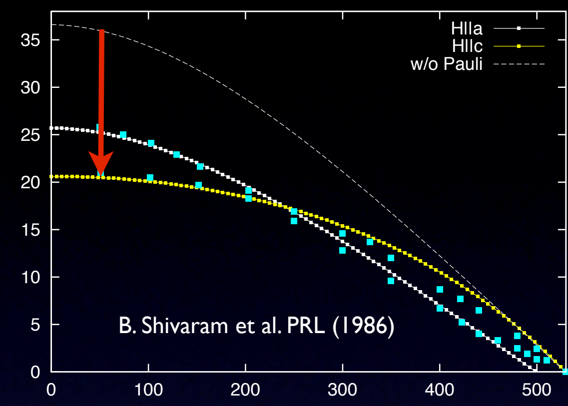

J.A. Sauls, Adv. Phys. 43, 113 (1994).

H-T phase diagram - Tetracritical point - E2u




C. Choi & JAS, PRL (1991)

*Anisotropic* Pauli limiting




$\vec{d}||\hat{c}$ $\quad |\uparrow\downarrow + \downarrow\uparrow\rangle$ $\mathbf{H}||\vec{d}$ *pair breaking*

$|\rightrightarrows\rangle + |\leftleftarrows\rangle$ $\mathbf{H} \perp \vec{d}$ *No pair breaking*

$$(1+\eta H)|\rightrightarrows\rangle + (1-\eta H)|\leftleftarrows\rangle$$

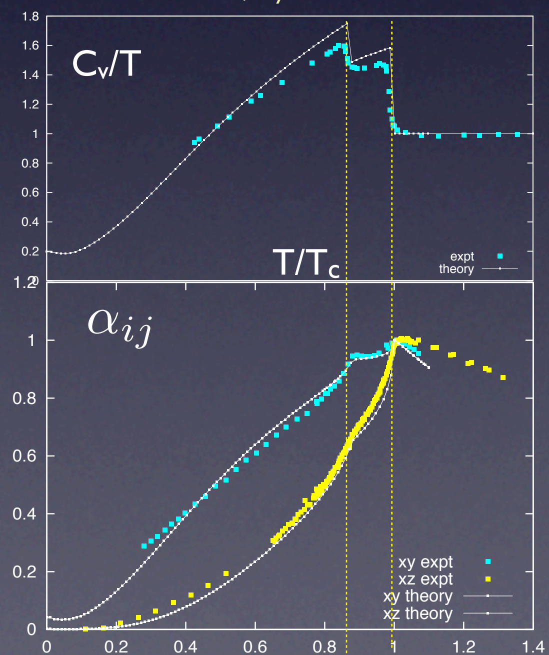
➡ Spin-Triplet, $E_{2u}$, w/ strong Spin-Orbit Coupling

Weak Symmetry Breaking - AFM order

R. Joynt, Sup. Sci. Tech. 1 1988

D. Hess, et al., J. Phys.: Cond. Mat. 1, 8135 (1989).

R.A. Fisher et al., Phys. Rev. Lett. 1989.

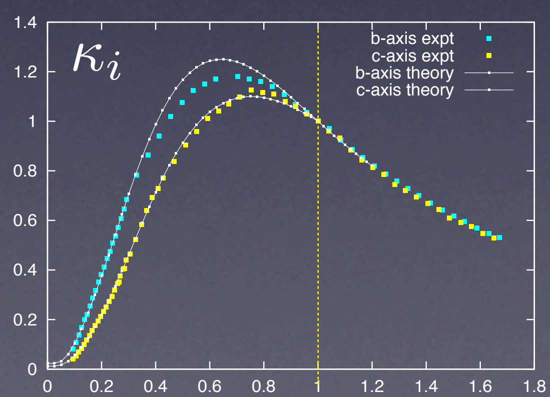



B. Ellman et al.., Phys. Rev. B 54 (1996)

M. Graf, S.K. Yip & JAS, PRB (1996)

✓Heat Capacity Anomalies
✓Anisotropy Transverse Sound
✓Anistropic Thermal Conductivity

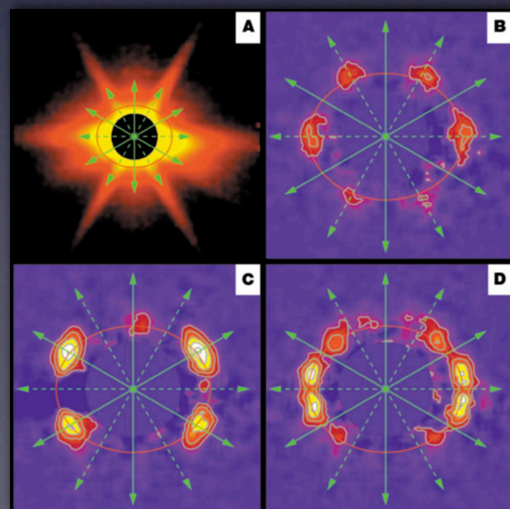
➡ $E_{2u}$ orbital symmetry



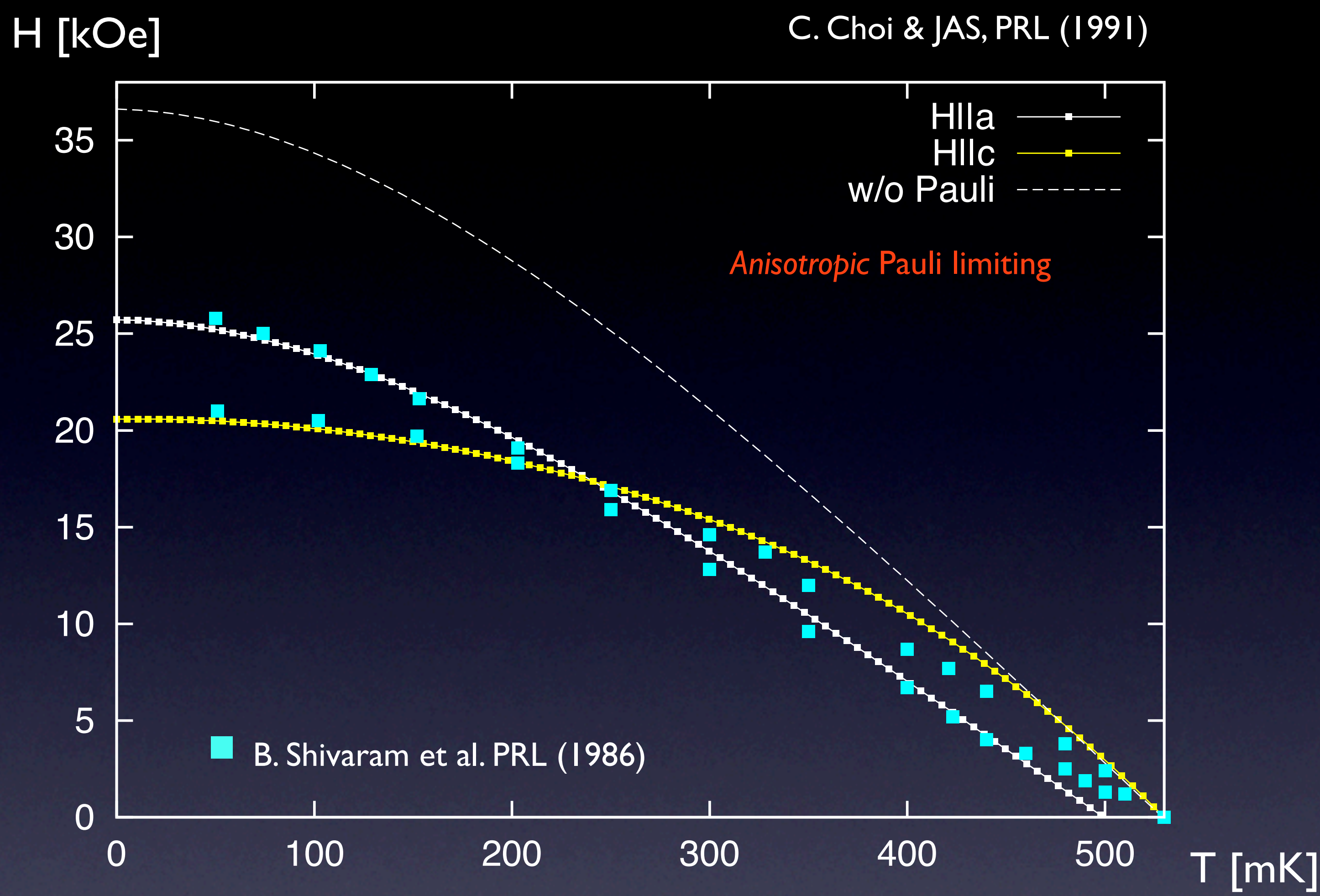

B. Lussier et al.., Phys. Rev. B 53 (1996)

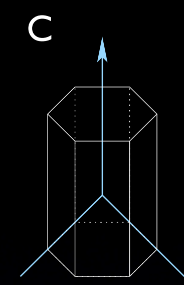
Realignment of the flux-line lattice in $UPt_3$ ➡ $E_{2u}$ orbital symmetry

Andrew Huxley et al. Nature (2000). T. Champel & V. Mineev (2001)



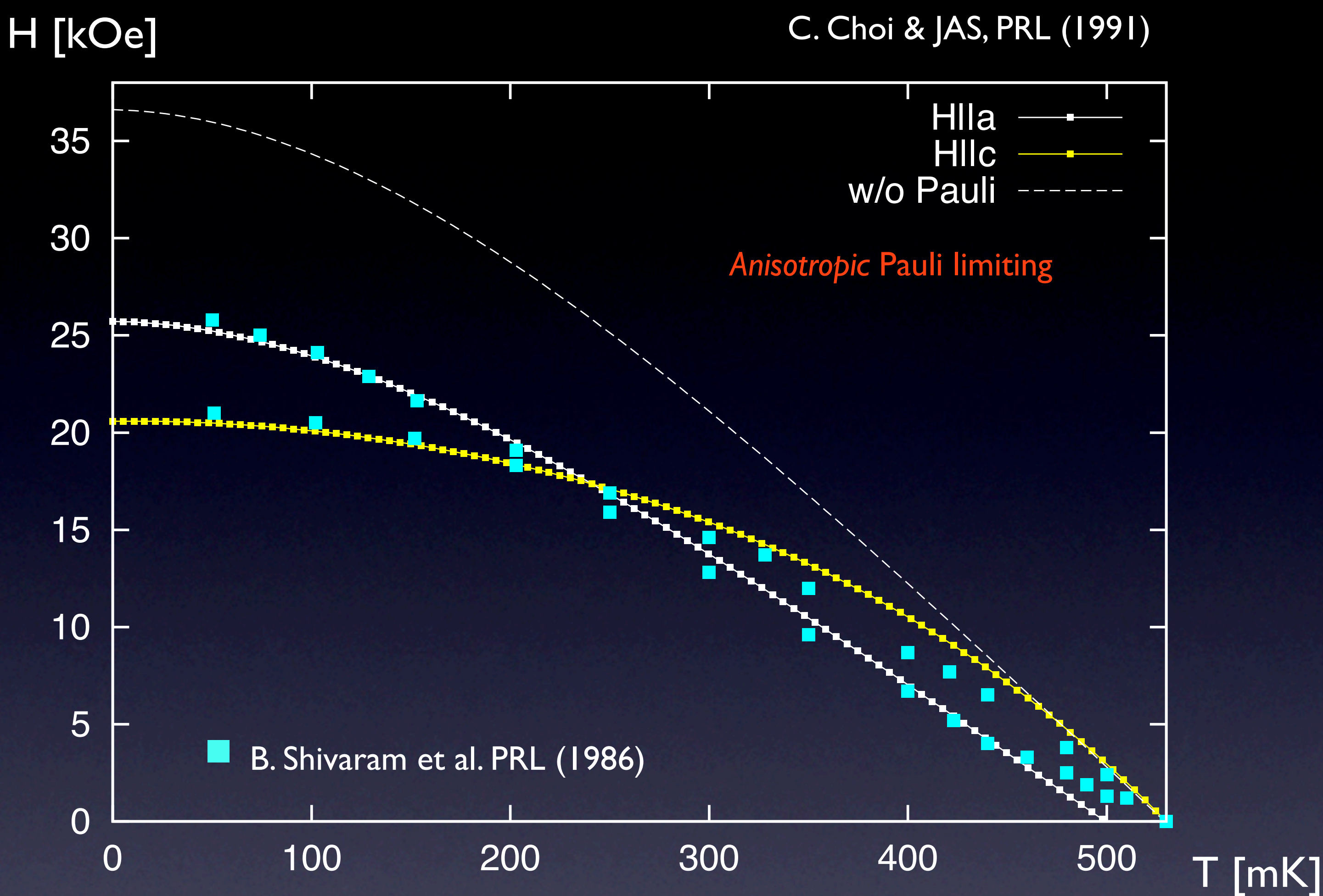
# Spin Triplet Pairing in $UPt_3$

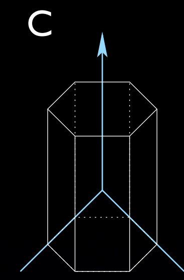



$\vec{\mathbf{d}}||\hat{\mathbf{c}}$ $\quad |\uparrow\downarrow + \downarrow\uparrow\rangle$

# Spin Triplet Pairing in $UPt_3$



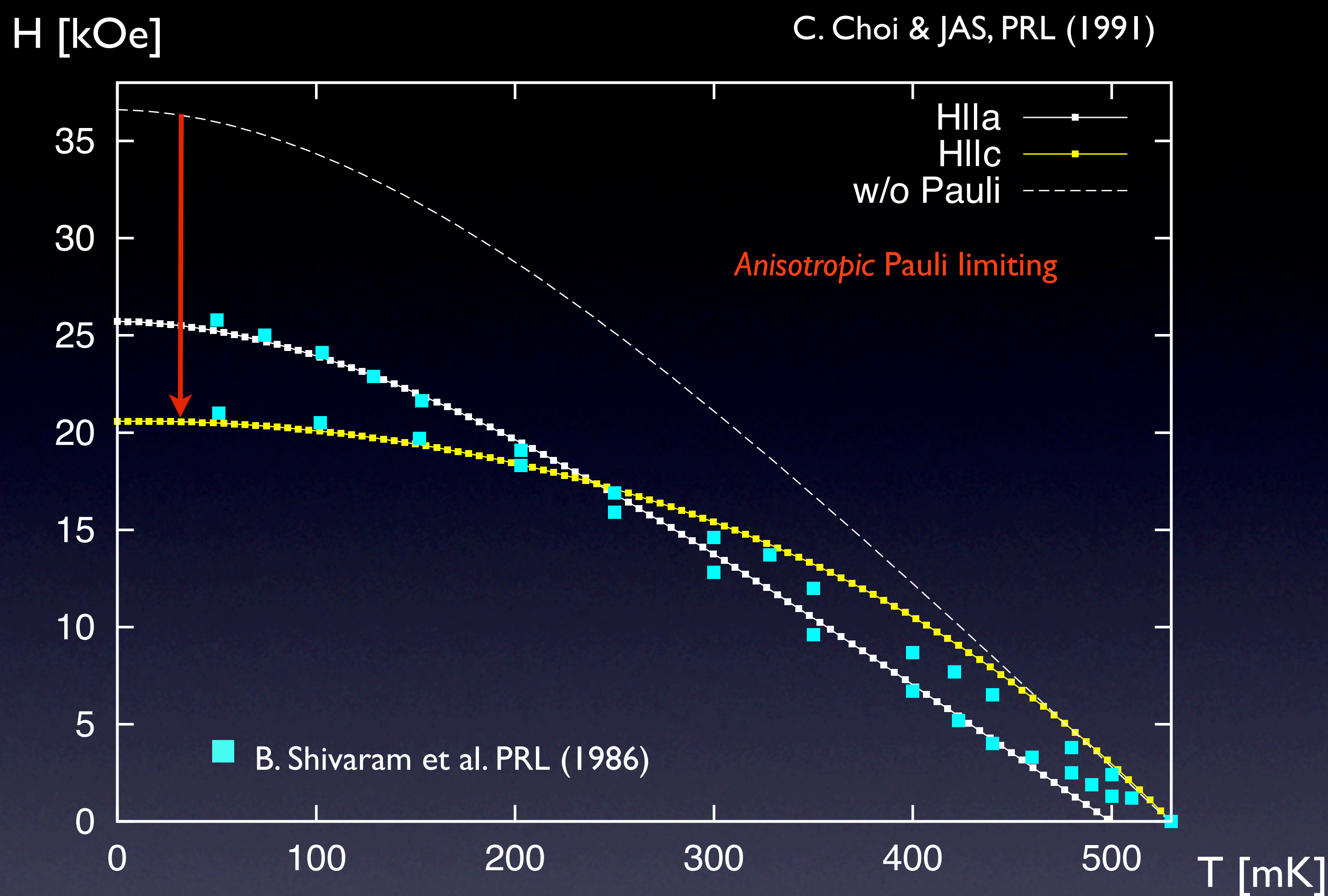

$$\vec{\mathbf{d}} || \hat{\mathbf{c}} \qquad | \uparrow\downarrow + \downarrow\uparrow \rangle$$

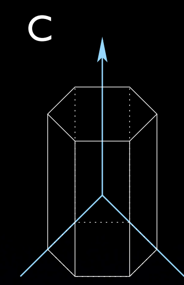
$$= \; |\rightrightarrows\rangle + |\leftleftarrows\rangle \qquad \mathbf{H} \perp \vec{\mathbf{d}}$$

*No pair breaking*

$$\longrightarrow (1 + \eta H)|\rightrightarrows\rangle + (1 - \eta H)|\leftleftarrows\rangle$$

# Spin Triplet Pairing in $UPt_3$




$\vec{\mathbf{d}}||\hat{\mathbf{c}}$ $\quad | \uparrow\downarrow + \downarrow\uparrow \rangle$ $\quad \mathbf{H}||\vec{\mathbf{d}}$ *pair breaking*

$= |\rightrightarrows\rangle + |\leftleftarrows\rangle$ $\quad \mathbf{H} \perp \vec{\mathbf{d}}$ *No pair breaking*

$$\longrightarrow (1+\eta H)|\rightrightarrows\rangle + (1-\eta H)|\leftleftarrows\rangle$$

➡ Spin-Triplet, w/ strong Spin-Orbit Coupling - $E_{2u}$

PRL 103, 197002 (2009) PHYSICAL REVIEW LETTERS week ending 6 NOVEMBER 2009

# Evidence for Complex Superconducting Order Parameter Symmetry in the Low-Temperature Phase of $UPt_3$ from Josephson Interferometry

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*Department of Physics and Frederick Seitz Materials Research Laboratory, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*

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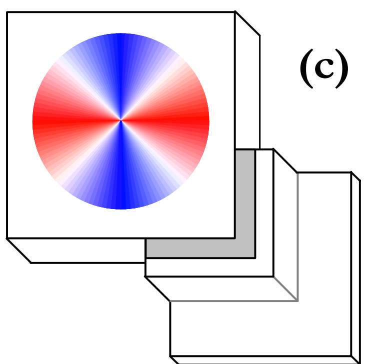
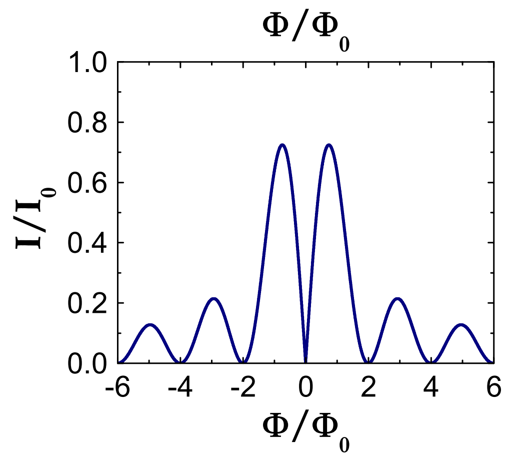
(Received 30 June 2009; published 4 November 2009)

We present data on the modulation of the critical current with applied magnetic field in $UPt_3$–Cu–Pb Josephson junctions and SQUIDs. The junctions were fabricated on polished surfaces of $UPt_3$ single crystals. The shape of the resulting diffraction patterns provides phase-sensitive information on the superconducting order parameter. Our corner junction data show asymmetric patterns with respect to magnetic field, indicating a complex order parameter, and both our junction and SQUID measurements point to a phase shift of $\pi$, supporting the $E_{2u}$ representation of the order parameter.

DOI: 10.1103/PhysRevLett.103.197002 PACS numbers: 74.70.Tx, 74.20.Rp, 74.50.+r



(c)




$$\Delta_{E_{2u}}(\mathbf{p}_f) \sim p_z(p_x + ip_y)^2 \sim e^{i2\vartheta}$$

# Vortex Structure in Chiral Spin-Triplet Superfluids

2D $^3$He-A

UPt$_3$

Sr$_2$RuO$_4$

UCoGe

Local Equilibrium OP

$$\vec{\Psi}(\hat{\mathbf{p}},\mathbf{r}) = \vec{\mathbf{d}}\left[\overbrace{|\Psi_+|e^{ip\phi}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)} + |\Psi_-|e^{im\phi}(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)\right]$$

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UCoGe

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$$\xrightarrow[|\mathbf{r}| \gg \xi_0]{} \vec{\mathbf{d}}\,\Psi_0 e^{ip\phi}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y) + \frac{c}{r^n}\,e^{im\phi}(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$$

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Sr$_2$RuO$_4$

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Vortex Core OP

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$$L_z^{\text{orbit}}(\hat{\mathbf{p}}_x \pm i\hat{\mathbf{p}}_y) = \pm\hbar(\hat{\mathbf{p}}_x \pm i\hat{\mathbf{p}}_y)$$

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Sr$_2$RuO$_4$

UCoGe

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Vortex Core OP

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$$L_z^{\text{orbit}}(\hat{\mathbf{p}}_x \pm i\hat{\mathbf{p}}_y) = \pm\hbar(\hat{\mathbf{p}}_x \pm i\hat{\mathbf{p}}_y)$$

$$L_z^{\text{cm}}|\Psi|e^{ip\phi} = p\hbar\,|\Psi|e^{ip\phi}$$

# Vortex Structure in Chiral Spin-Triplet Superfluids

2D $^3$He-A

UPt$_3$

Sr$_2$RuO$_4$

UCoGe

Local Equilibrium OP

Vortex Core OP

$$\vec{\Psi}(\hat{\mathbf{p}},\mathbf{r}) = \vec{\mathbf{d}}\left[\overbrace{|\Psi_+|e^{ip\phi}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)}^{\text{Local Equilibrium OP}} + \overbrace{|\Psi_-|e^{im\phi}(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)}^{\text{Vortex Core OP}}\right]$$

$$\xrightarrow[|\mathbf{r}| \gg \xi_0]{} \vec{\mathbf{d}}\,\Psi_0 e^{ip\phi}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y) + \frac{c}{r^n}\,e^{im\phi}(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$$

$$L_z^{\text{orbit}}(\hat{\mathbf{p}}_x \pm i\hat{\mathbf{p}}_y) = \pm\hbar(\hat{\mathbf{p}}_x \pm i\hat{\mathbf{p}}_y)$$

$$L_z^{\text{cm}}|\Psi|e^{ip\phi} = p\hbar\,|\Psi|e^{ip\phi}$$

Axial Symmetry $(r \gg \xi_0)$ $L_z^{\text{tot}}\vec{\Psi} = l\hbar\,\vec{\Psi} \longrightarrow p + 1 = m - 1$

2D $^3$He-A

# Vortex Structure in Chiral Spin-Triplet Superfluids

$UPt_3$

$Sr_2RuO_4$

UCoGe

Local Equilibrium OP — Vortex Core OP

$$\vec{\Psi}(\hat{\mathbf{p}},\mathbf{r}) = \vec{\mathbf{d}}\left[\overbrace{|\Psi_+|e^{ip\phi}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)}^{\text{Local Equilibrium OP}} + \overbrace{|\Psi_-|e^{im\phi}(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)}^{\text{Vortex Core OP}}\right]$$

$$\xrightarrow[|\mathbf{r}| \gg \xi_0]{} \vec{\mathbf{d}}\,\Psi_0 e^{ip\phi}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y) + \frac{c}{r^n}\,e^{im\phi}(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$$

$$L_z^{\mathrm{orbit}}(\hat{\mathbf{p}}_x \pm i\hat{\mathbf{p}}_y) = \pm\hbar(\hat{\mathbf{p}}_x \pm i\hat{\mathbf{p}}_y)$$

$$L_z^{\mathrm{cm}}|\Psi|e^{ip\phi} = p\hbar\,|\Psi|e^{ip\phi}$$

Axial Symmetry $(r \gg \xi_0)$ $L_z^{\mathrm{tot}}\vec{\Psi} = l\hbar\,\vec{\Psi} \longrightarrow p+1 = m-1$

| p | m |
|---|---|
| +1 | +3 |
| -1 | +1 |
| +2 | +4 |
| -2 | 0 |
| 0 | +2 |

# Vortex Structure in Chiral Spin-Triplet Superfluids

2D $^3$He-A

$UPt_3$

$Sr_2RuO_4$

UCoGe

Local Equilibrium OP — Vortex Core OP

$$\vec{\Psi}(\hat{\mathbf{p}},\mathbf{r}) = \vec{\mathbf{d}}\left[\overbrace{|\Psi_+|e^{ip\phi}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)}^{\text{Local Equilibrium OP}} + \overbrace{|\Psi_-|e^{im\phi}(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)}^{\text{Vortex Core OP}}\right]$$

$$\xrightarrow{|\mathbf{r}| \gg \xi_0} \vec{\mathbf{d}}\,\Psi_0 e^{ip\phi}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y) + \frac{c}{r^n}\, e^{im\phi}(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$$

$$L_z^{\text{orbit}}(\hat{\mathbf{p}}_x \pm i\hat{\mathbf{p}}_y) = \pm\hbar(\hat{\mathbf{p}}_x \pm i\hat{\mathbf{p}}_y)$$

$$L_z^{\text{cm}}|\Psi|e^{ip\phi} = p\hbar\,|\Psi|e^{ip\phi}$$

Axial Symmetry $(r \gg \xi_0)$ $L_z^{\text{tot}}\vec{\Psi} = l\hbar\,\vec{\Psi} \longrightarrow p + 1 = m - 1$

| p | m |
|---|---|
| +1 | +3 |
| -1 | +1 |
| +2 | +4 |
| -2 | 0 |
| 0 | +2 |

# Vortex Structure in Chiral Spin-Triplet Superfluids

2D $^3$He-A

UPt$_3$

Sr$_2$RuO$_4$

UCoGe

Local Equilibrium OP — Vortex Core OP

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|---|---|
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| -1 | +1 |
| +2 | +4 |
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| 0 | +2 |

2D $^3$He-A

# Vortex Structure in Chiral Spin-Triplet Superfluids

$UPt_3$
$Sr_2RuO_4$
UCoGe

Local Equilibrium OP / Vortex Core OP

$$\vec{\Psi}(\hat{\mathbf{p}},\mathbf{r}) = \vec{\mathbf{d}}\left[\overbrace{|\Psi_+|e^{ip\phi}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)}^{\text{Local Equilibrium OP}} + \overbrace{|\Psi_-|e^{im\phi}(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)}^{\text{Vortex Core OP}}\right]$$

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|---|---|
| +1 | +3 |
| -1 | +1 |
| +2 | +4 |
| -2 | 0 |
| 0 | +2 |

→ ❖ Broken T-symmetry

# Vortex Structure in Chiral Spin-Triplet Superfluids

2D $^3$He-A

UPt$_3$

Sr$_2$RuO$_4$

UCoGe

Local Equilibrium OP

Vortex Core OP

$$\vec{\Psi}(\hat{\mathbf{p}},\mathbf{r}) = \vec{\mathbf{d}}\left[\overbrace{|\Psi_+|e^{ip\phi}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)}^{\text{Local Equilibrium OP}} + \overbrace{|\Psi_-|e^{im\phi}(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)}^{\text{Vortex Core OP}}\right]$$

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| p | m |
|---|---|
| +1 | +3 |
| -1 | +1 |
| +2 | +4 |
| -2 | 0 |
| 0 | +2 |

$\longrightarrow$ ❖ Broken T-symmetry

# Vortex Structure in Chiral Spin-Triplet Superfluids

2D $^3$He-A

UPt$_3$

Sr$_2$RuO$_4$

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Axial Symmetry $(r \gg \xi_0)$ $L_z^{\text{tot}}\vec{\Psi} = l\hbar\,\vec{\Psi} \longrightarrow p+1 = m-1$

| p | m |
|---|---|
| +1 | +3 |
| -1 | +1 |
| +2 | +4 |
| -2 | 0 |
| 0 | +2 |

(rows +1/+3 and −1/+1) ⟶ ❖ Broken T-symmetry

# Vortex Structure in Chiral Spin-Triplet Superfluids

2D $^3$He-A

UPt$_3$

Sr$_2$RuO$_4$

UCoGe

Local Equilibrium OP

Vortex Core OP

$$\vec{\Psi}(\hat{\mathbf{p}},\mathbf{r}) = \vec{\mathbf{d}}\left[\overbrace{|\Psi_+|e^{ip\phi}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)}^{\text{Local Equilibrium OP}} + \overbrace{|\Psi_-|e^{im\phi}(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)}^{\text{Vortex Core OP}}\right]$$

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| p | m |
|---|---|
| +1 | +3 |
| -1 | +1 |
| +2 | +4 |
| -2 | 0 |
| 0 | +2 |

❖ Broken T-symmetry

❖ "Coreless" 2 Quantum Vortex

2D $^3$He-A

# Vortex Structure in Chiral Spin-Triplet Superfluids

UPt$_3$

Sr$_2$RuO$_4$

UCoGe

Local Equilibrium OP — Vortex Core OP

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Axial Symmetry ($r \gg \xi_0$) $L_z^{\text{tot}}\vec{\Psi} = l\hbar\,\vec{\Psi} \longrightarrow p + 1 = m - 1$

| p | m |
|---|---|
| +1 | +3 |
| -1 | +1 |
| +2 | +4 |
| -2 | 0 |
| 0 | +2 |

❖ Broken T-symmetry

❖ "Coreless" 2 Quantum Vortex

# Vortex Structure in Chiral Spin-Triplet Superfluids

2D $^3He$-A

$UPt_3$

$Sr_2RuO_4$

Local Equilibrium OP — Vortex Core OP

$$\vec{\Psi}(\hat{\mathbf{p}},\mathbf{r}) = \vec{\mathbf{d}}\left[\overbrace{|\Psi_+|e^{ip\phi}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)}^{\text{Local Equilibrium OP}} + \overbrace{|\Psi_-|e^{im\phi}(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)}^{\text{Vortex Core OP}}\right]$$

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Axial Symmetry ($r \gg \xi_0$) $L_z^{\mathrm{tot}}\vec{\Psi} = l\hbar\,\vec{\Psi} \longrightarrow p+1 = m-1$

| p | m |
|---|---|
| +1 | +3 |
| -1 | +1 |
| +2 | +4 |
| -2 | 0 |
| 0 | +2 |

- ❖ Broken T-symmetry (p = +1, m = +3; p = -1, m = +1)
- ❖ "Coreless" 2 Quantum Vortex (p = -2, m = 0)
- ❖ "No-Vortex" Vortex (p = 0, m = +2)

# Vortex Structure in Chiral Spin-Triplet Superfluids

2D $^3$He-A

UPt$_3$

Sr$_2$RuO$_4$

Local Equilibrium OP — Vortex Core OP

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~~Axial~~ Symmetry $(r \gg \xi_0)$ $L_z^{\text{tot}}\vec{\Psi} = l\hbar\,\vec{\Psi} \longrightarrow p + 1 = m - 1$

$$SO(2) \rightarrow C_4$$

| p | m |
|---|---|
| +1 | +3 |
| -1 | +1 |
| +2 | +4 |
| -2 | 0 |
| 0 | +2 |

- ❖ Broken T-symmetry
- ❖ "Coreless" 2 Quantum Vortex
- ❖ "No-Vortex" Vortex

# Vortex Structure in Chiral Spin-Triplet Superfluids

2D $^3$He-A

$UPt_3$

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$$L_z^{\text{cm}}|\Psi|e^{ip\phi} = p\hbar\,|\Psi|e^{ip\phi}$$

Axial Symmetry ($r \gg \xi_0$) $L_z^{\text{tot}}\vec{\Psi} = l\hbar\,\vec{\Psi} \longrightarrow p + 1 = m - 1 + 4n$

$SO(2) \rightarrow C_4$

| p | m |
|---|---|
| +1 | +3 |
| -1 | +1 |
| +2 | +4 |
| -2 | 0 |
| 0 | +2 |

- ❖ Broken T-symmetry
- ❖ "Coreless" 2 Quantum Vortex
- ❖ "No-Vortex" Vortex

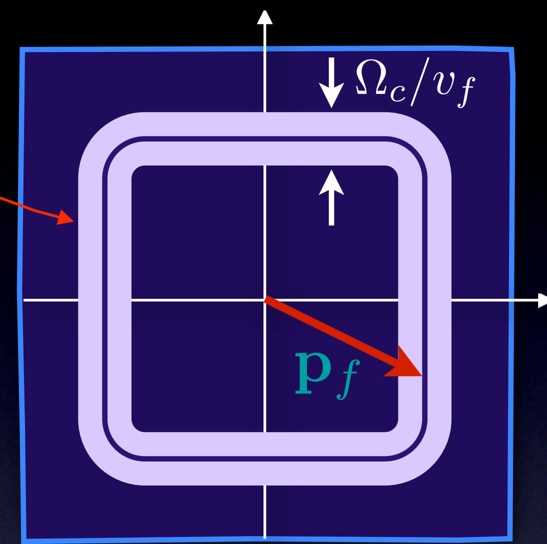
# Theoretical Methods

Gorkov-Nambu

$$\widehat{\mathfrak{G}}(\mathbf{p},\mathbf{R};\epsilon_n) = -\int_0^\beta d\tau\, e^{i\epsilon_n\tau}\int d^3r\, e^{-i\mathbf{p}\cdot\mathbf{r}}\,\langle \mathsf{T}_\tau\hat{\psi}(\mathbf{r}_1,\tau)\hat{\bar{\psi}}(\mathbf{r}_2,0)\rangle\,, \qquad \hat{\psi} = (\psi_\uparrow\,,\,\psi_\downarrow\,,\,\psi_\uparrow^\dagger\,,\,\psi_\downarrow^\dagger)$$

## Quasiclassical Green's Functions

$$\widehat{g}(\mathbf{p}_f,\mathbf{R};\epsilon_n) = \frac{1}{a}\int_{-\Omega_c}^{+\Omega_c} d\xi_{\mathbf{p}}\,\widehat{\tau}_3\widehat{G}(\mathbf{p},\mathbf{R};\epsilon_n) = \begin{pmatrix} \hat{\mathfrak{g}} & \hat{\mathfrak{f}} \\ \hat{\underline{\mathfrak{f}}} & \hat{\underline{\mathfrak{g}}} \end{pmatrix}$$



# Theoretical Methods

Gorkov-Nambu

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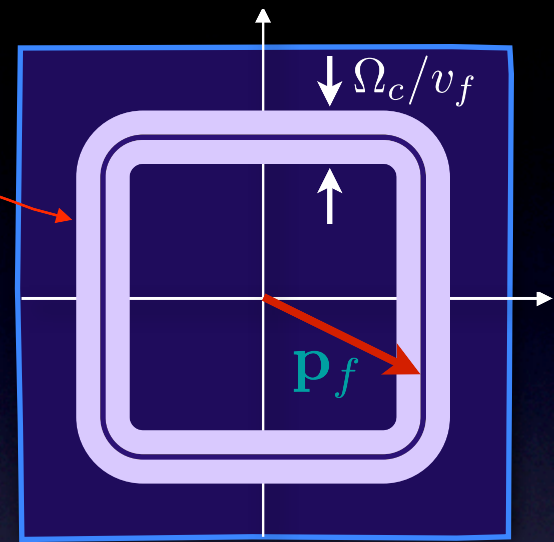


## Eilenberger, Larkin & Ovchinnikov Equations

$$i\mathbf{v}_f\cdot\boldsymbol{\nabla}_{\mathbf{R}}\,\widehat{g} + \left[i\epsilon_n\hat{\tau}_3 - \widehat{\Delta} - \widehat{\boldsymbol{\Sigma}}\,,\,\widehat{g}\right] = 0$$

$$\epsilon_n = (2n+1)\pi T$$

# Theoretical Methods

Gorkov-Nambu

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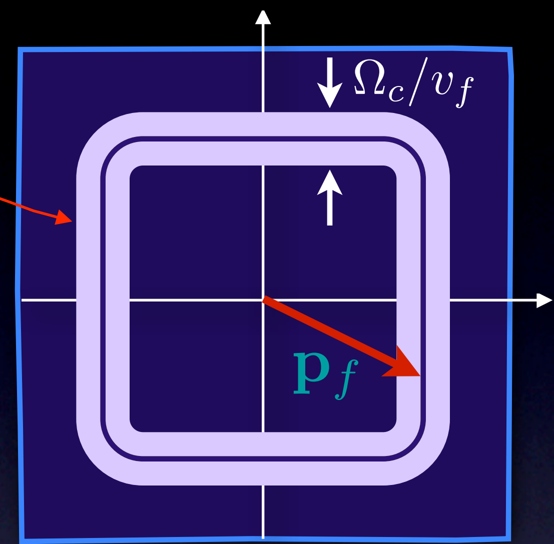


## Eilenberger, Larkin & Ovchinnikov Equations

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Pairing Self Energy

$$\Delta_{\alpha\beta}(\mathbf{p}_f\mathbf{R}) \equiv$$


$$\lambda(\mathbf{p}_f,\mathbf{p}'_f) = \sum_{\Gamma\nu}\lambda_\Gamma\,\eta_{\Gamma\nu}(\mathbf{p}_f)^*\,\eta_{\Gamma\nu}(\mathbf{p}'_f)$$

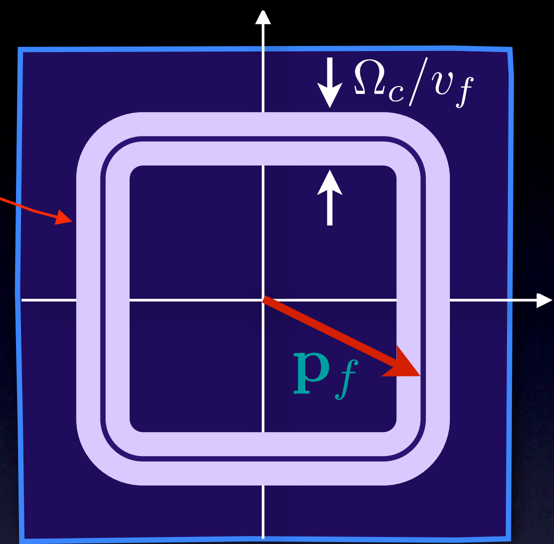
$$= N_f\int d^2\mathbf{p}'_f\,\lambda_{\alpha\beta,\gamma\rho}(\mathbf{p}_f,\mathbf{p}'_f)\,T\sum_{\epsilon'_n}^{|\epsilon'_n|\le\Omega_c}\mathfrak{f}_{\gamma\rho}(\mathbf{p}'_f;\epsilon'_n)$$

# Theoretical Methods

$$\widehat{\mathfrak{G}}(\mathbf{p},\mathbf{R};\epsilon_n) = -\int_0^\beta d\tau\, e^{i\epsilon_n\tau}\int d^3r\, e^{-i\mathbf{p}\cdot\mathbf{r}}\,\langle \mathsf{T}_\tau \hat\psi(\mathbf{r}_1,\tau)\hat{\bar\psi}(\mathbf{r}_2,0)\rangle\,,$$

Gorkov-Nambu

$$\hat\psi = (\psi_\uparrow\,,\,\psi_\downarrow\,,\,\psi_\uparrow^\dagger\,,\,\psi_\downarrow^\dagger)$$

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## Eilenberger, Larkin & Ovchinnikov Equations

$$i\mathbf{v}_f\cdot\boldsymbol{\nabla}_{\mathbf{R}}\,\widehat{g} + \left[i\epsilon_n\hat{\tau}_3 - \widehat{\Delta} - \widehat{\boldsymbol{\Sigma}}\,,\,\widehat{g}\right] = 0 \qquad \epsilon_n = (2n+1)\pi T$$

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$\Delta_{\alpha\beta}(\mathbf{p}_f\mathbf{R}) \equiv$


$$\lambda(\mathbf{p}_f,\mathbf{p}'_f) = \sum_{\Gamma\nu}\lambda_\Gamma\,\eta_{\Gamma\nu}(\mathbf{p}_f)^*\,\eta_{\Gamma\nu}(\mathbf{p}'_f)$$

$$= N_f\int d^2\mathbf{p}'_f\,\lambda_{\alpha\beta,\gamma\rho}(\mathbf{p}_f,\mathbf{p}'_f)\,T\sum_{\epsilon'_n}^{|\epsilon'_n|\le\Omega_c}\mathfrak{f}_{\gamma\rho}(\mathbf{p}'_f;\epsilon'_n)$$

### Impurity Self Energy

$$\widehat{\boldsymbol{\Sigma}}(\epsilon_n) = n_s\,\widehat{\mathfrak{t}}$$

$\widehat{\mathfrak{t}}(\mathbf{p}_f,\mathbf{p}'_f;\epsilon_n) \equiv$

$$\ell_{\text{imp}} = 10\,\xi_0$$

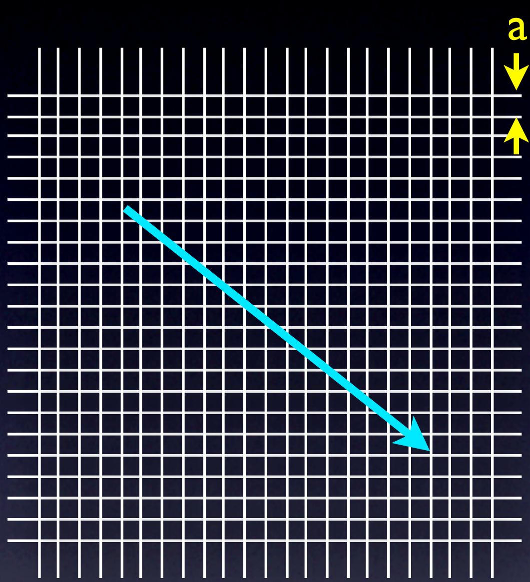
$$\Delta^2/E_f \ll \hbar/\tau \ll \Delta$$

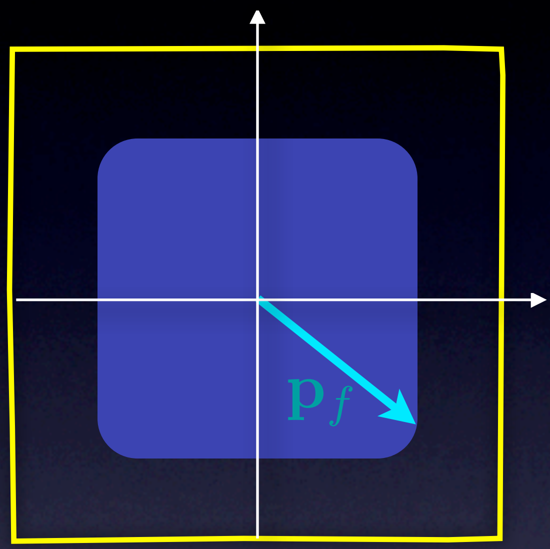
### T-matrix

$$= \widehat{\mathfrak{u}}(\mathbf{p}_f,\mathbf{p}'_f;\epsilon_n) + N_f\int d^2\mathbf{p}''_f\,\widehat{\mathfrak{u}}(\mathbf{p}_f,\mathbf{p}''_f;\epsilon_n)\,\widehat{\mathfrak{g}}(\mathbf{p}''_f;\epsilon_n)\widehat{\mathfrak{t}}(\mathbf{p}''_f,\mathbf{p}'_f;\epsilon_n)$$

Trajectories in Real Space $\mathbf{v}(\mathbf{p}_f)$

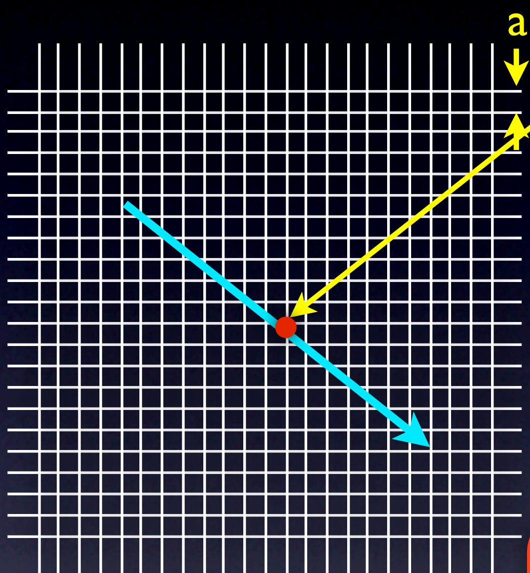
Momentum Space



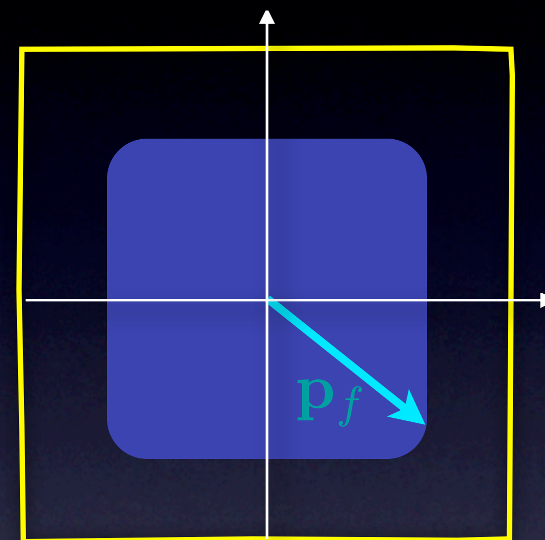



Trajectories $\mathbf{v}(\mathbf{p}_f)$
in Real Space

Momentum Space




$$\mathbf{r} = a\,(m\,\hat{\mathbf{x}} + n\,\hat{\mathbf{y}})$$



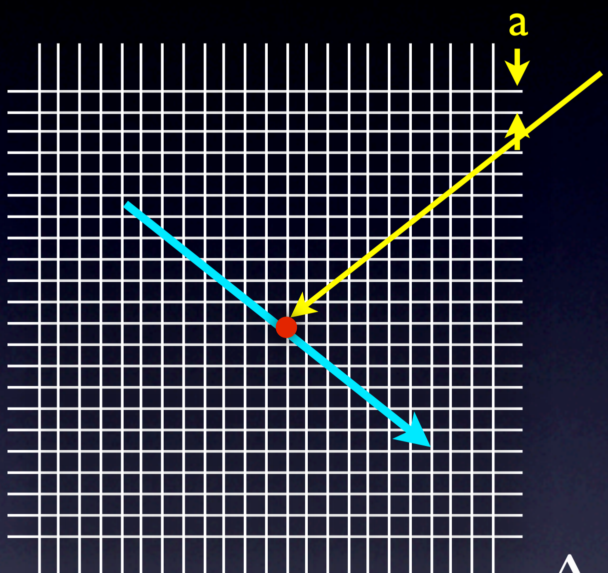

$$\Delta(\mathbf{p}_f, \mathbf{r}) = \sum_{\Gamma} \Delta_{\Gamma}(\mathbf{r})\,\eta_{\Gamma}(\mathbf{p}_f)$$

$$\left[ i\epsilon_n \hat{\tau}_3 - \hat{\Delta} - \hat{\Sigma} \,,\, \hat{g} \right] + i\mathbf{v}_f \cdot \boldsymbol{\nabla}_{\mathbf{r}} \hat{g} = 0$$

$$\hat{\Sigma}\,[\hat{g}]\,(\mathbf{p}_f, \mathbf{r}, \epsilon_n) \qquad \hat{\Delta}\,[\hat{g}]\,(\mathbf{p}_f, \mathbf{r})$$

Trajectories $\mathbf{v}(\mathbf{p}_f)$
in Real Space

Momentum Space

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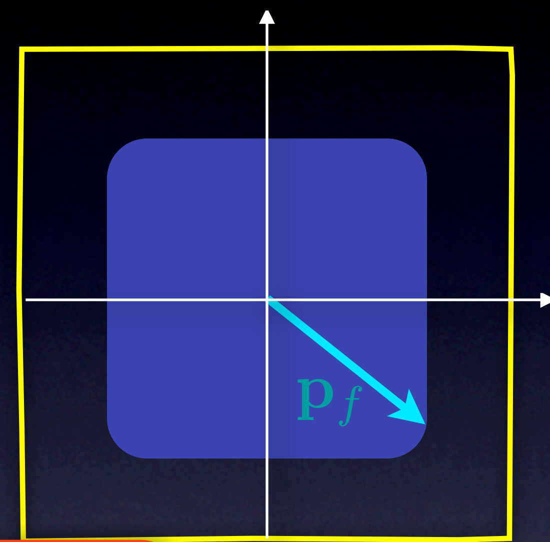



$$\Delta(\mathbf{p}_f, \mathbf{r}) = \sum_{\Gamma} \Delta_\Gamma(\mathbf{r})\,\eta_\Gamma(\mathbf{p}_f)$$

$$p_x + i p_y \sim e^{i\vartheta_{\mathbf{p}_f}}$$

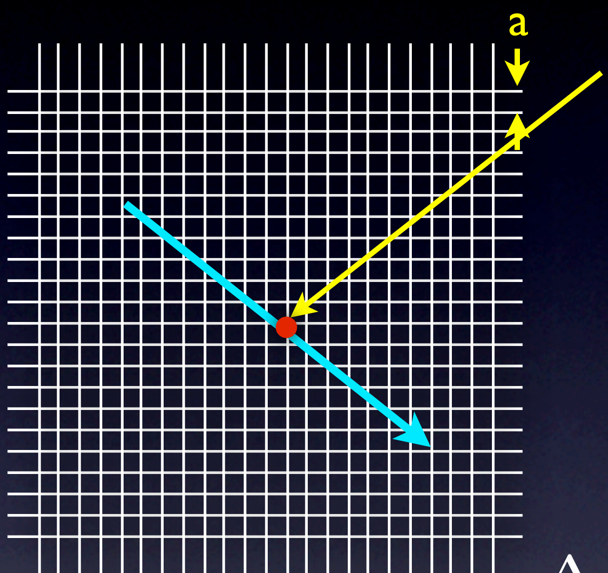
$$\left[ i\epsilon_n \widehat{\tau}_3 - \widehat{\Delta} - \widehat{\Sigma}\,,\, \widehat{g} \right] + i\mathbf{v}_f \cdot \boldsymbol{\nabla}_{\mathbf{r}} \widehat{g} = 0$$

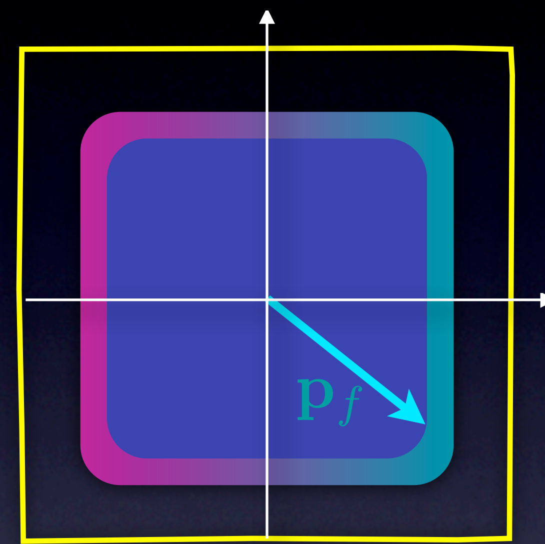
$$\widehat{\Sigma}\,[\widehat{g}]\,(\mathbf{p}_f, \mathbf{r}, \epsilon_n) \qquad \widehat{\Delta}\,[\widehat{g}]\,(\mathbf{p}_f, \mathbf{r})$$

Trajectories in Real Space $\mathbf{v}(\mathbf{p}_f)$

Momentum Space

$$\mathbf{r} = a\,(m\,\hat{\mathbf{x}} + n\,\hat{\mathbf{y}})$$




$$\Delta(\mathbf{p}_f, \mathbf{r}) = \sum_{\Gamma} \Delta_{\Gamma}(\mathbf{r})\,\eta_{\Gamma}(\mathbf{p}_f)$$

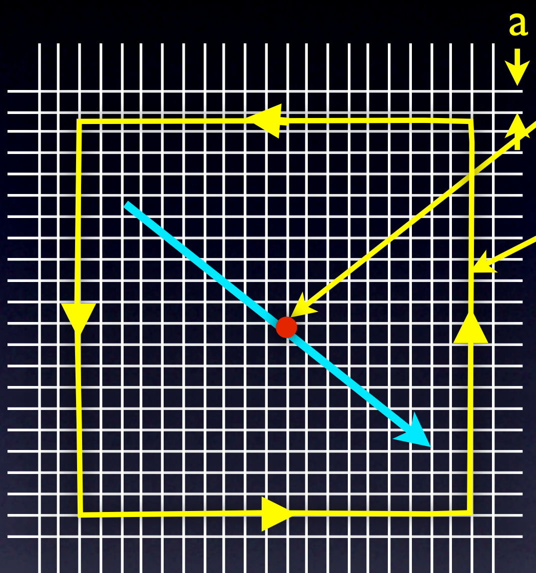
$$p_x + i p_y \sim e^{i\vartheta_{\mathbf{p}_f}}$$

$$\left[ i\epsilon_n \widehat{\tau}_3 - \widehat{\Delta} - \widehat{\Sigma}\,,\, \widehat{g} \right] + i\mathbf{v}_f \cdot \boldsymbol{\nabla}_{\mathbf{r}} \widehat{g} = 0$$

$$\widehat{\Sigma}\,[\widehat{g}]\,(\mathbf{p}_f, \mathbf{r}, \epsilon_n) \qquad \widehat{\Delta}\,[\widehat{g}]\,(\mathbf{p}_f, \mathbf{r})$$

# Trajectories $\mathbf{v}(\mathbf{p}_f)$ in Real Space

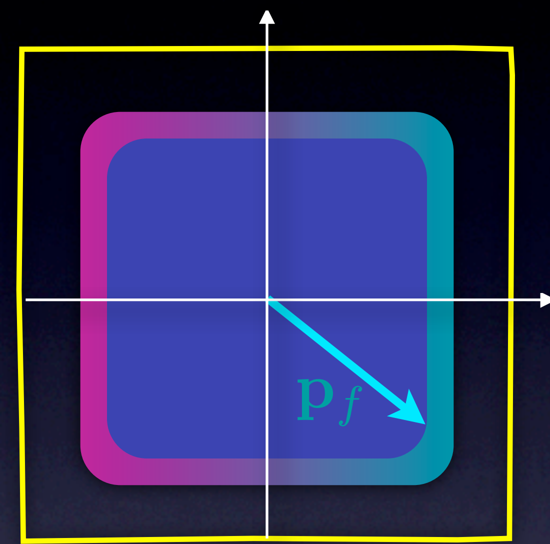
# Momentum Space




$$\mathbf{r} = a\,(m\,\hat{\mathbf{x}} + n\,\hat{\mathbf{y}})$$

$$N_{\mathrm{C}} = \frac{1}{2\pi}\oint_{\mathrm{C}} d\boldsymbol{\ell}\cdot\boldsymbol{\nabla}\vartheta$$

$$\Delta(\mathbf{r}) = |\Delta(\mathbf{r})|e^{i\vartheta(\mathbf{r})}$$




$$\Delta(\mathbf{p}_f,\mathbf{r}) = \sum_{\Gamma}\Delta_{\Gamma}(\mathbf{r})\,\eta_{\Gamma}(\mathbf{p}_f)$$

$$p_x + ip_y \sim e^{i\vartheta_{\mathbf{p}_f}}$$

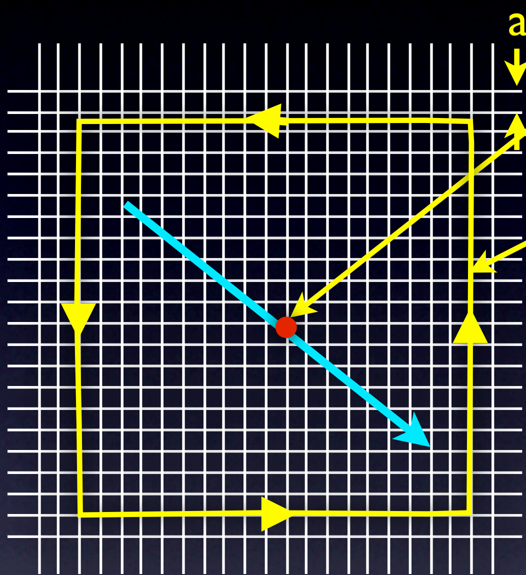
$$\left[i\epsilon_n\widehat{\tau}_3 - \widehat{\Delta} - \widehat{\Sigma}\,,\,\widehat{g}\right] + i\mathbf{v}_f\cdot\boldsymbol{\nabla}_{\mathbf{r}}\widehat{g} = 0$$

# Vortex Structures

$$\widehat{\Sigma}\,[\widehat{g}]\,(\mathbf{p}_f,\mathbf{r},\epsilon_n) \qquad \widehat{\Delta}\,[\widehat{g}]\,(\mathbf{p}_f,\mathbf{r})$$

## Trajectories $\mathbf{v}(\mathbf{p}_f)$ in Real Space

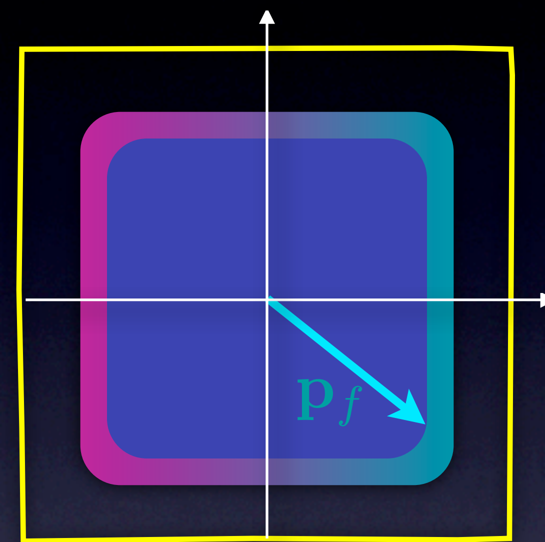
## Momentum Space




$$\mathbf{r} = a\,(m\,\hat{\mathbf{x}} + n\,\hat{\mathbf{y}})$$

$$N_{\text{C}} = \frac{1}{2\pi}\oint_{\text{C}} d\boldsymbol{\ell}\cdot\boldsymbol{\nabla}\vartheta$$

$$\Delta(\mathbf{r}) = |\Delta(\mathbf{r})|e^{i\vartheta(\mathbf{r})}$$




$$\Delta(\mathbf{p}_f,\mathbf{r}) = \sum_{\Gamma}\Delta_{\Gamma}(\mathbf{r})\,\eta_{\Gamma}(\mathbf{p}_f)$$

$$p_x + ip_y \sim e^{i\vartheta_{\mathbf{p}_f}}$$

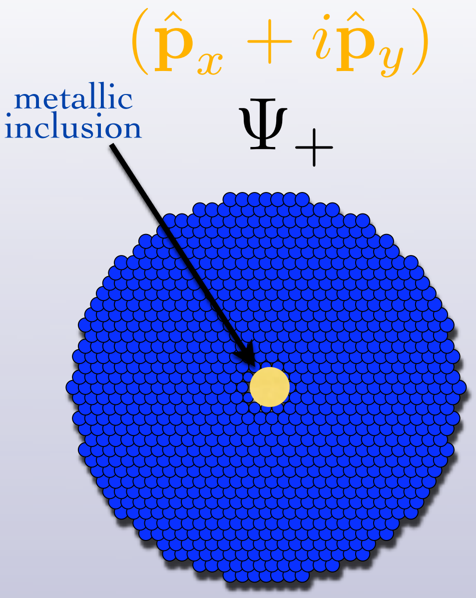
$$\left[i\epsilon_n\widehat{\tau}_3 - \widehat{\Delta} - \widehat{\Sigma}\,,\,\widehat{g}\right] + i\mathbf{v}_f\cdot\boldsymbol{\nabla}_{\mathbf{r}}\widehat{g} = 0$$

## Vortex Structures

$$\widehat{\Sigma}\,[\widehat{g}]\,(\mathbf{p}_f,\mathbf{r},\epsilon_n) \qquad \widehat{\Delta}\,[\widehat{g}]\,(\mathbf{p}_f,\mathbf{r})$$

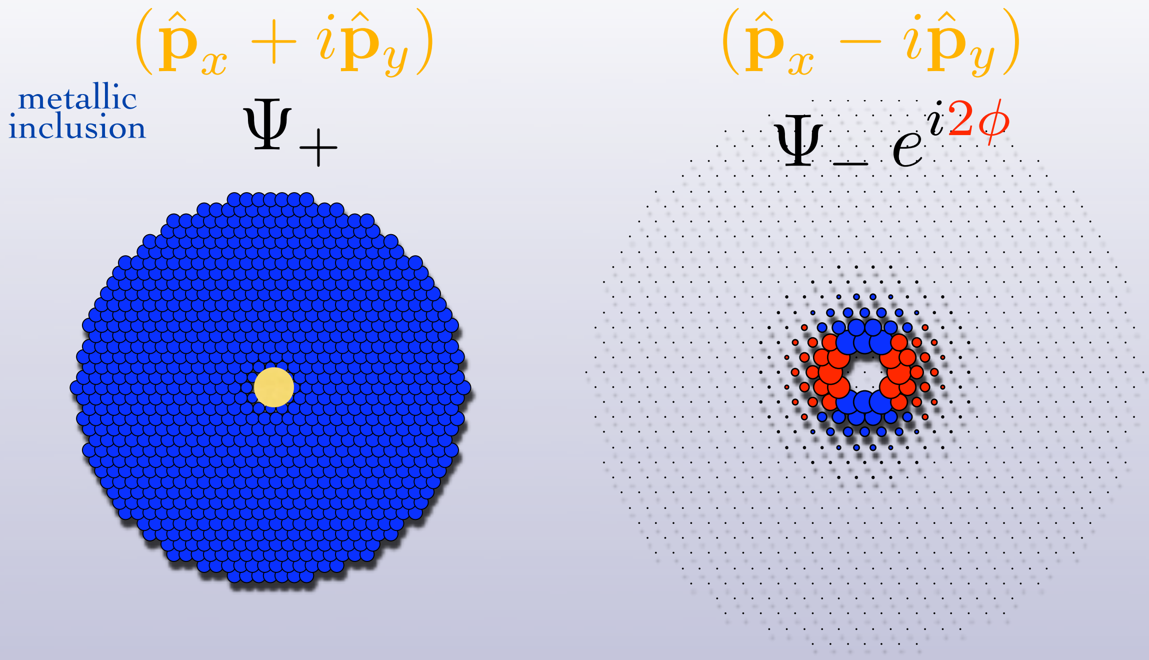
$$\frac{\partial\Delta}{\partial t} = -\frac{1}{N_f\,\tau_{\Delta}}\,\frac{\delta F}{\delta\Delta^*}$$

# Structure of a Defect in the Meissner Phase of a Chiral Superconductor




JAS & M. Eschrig, New J. Phys. 11, 075008 (2009)

# Structure of a Defect in the Meissner Phase of a Chiral Superconductor



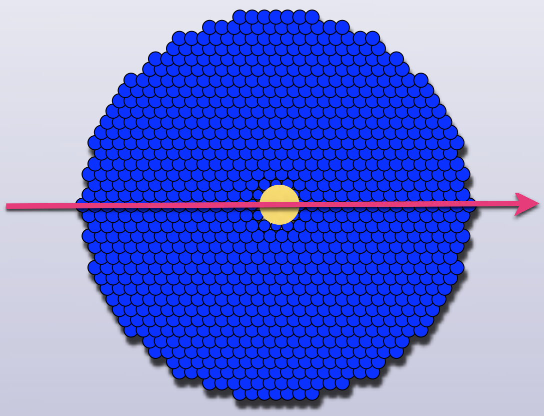

JAS & M. Eschrig, New J. Phys. 11, 075008 (2009)

# Structure of a Defect in the Meissner Phase of a Chiral Superconductor

$(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$

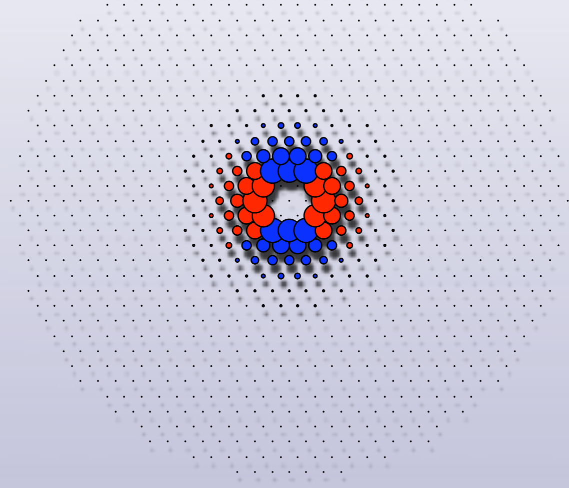
metallic inclusion

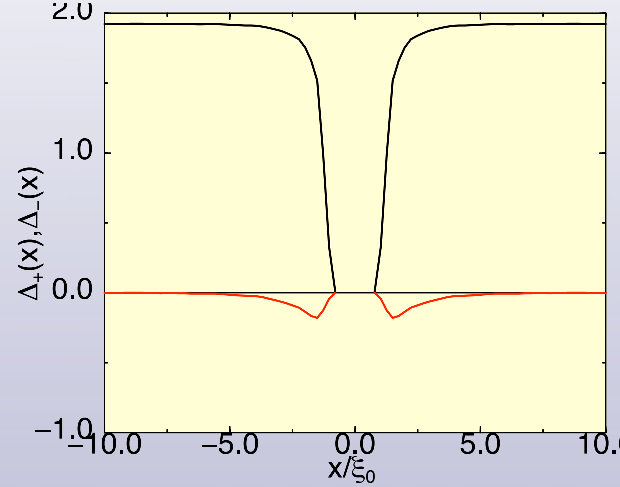
$\Psi_+$



$(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$

$\Psi_- \, e^{i2\phi}$






JAS & M. Eschrig, New J. Phys. 11, 075008 (2009)

# Structure of a Defect in the Meissner Phase of a Chiral Superconductor

$(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$

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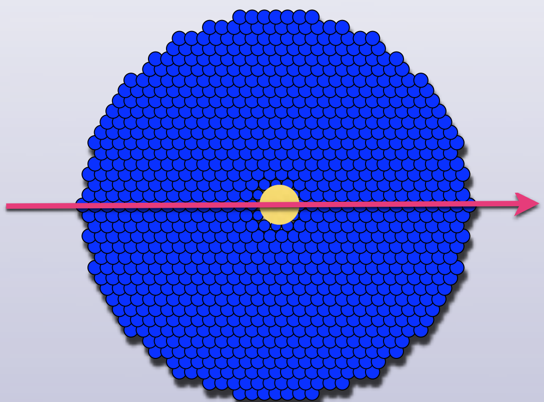
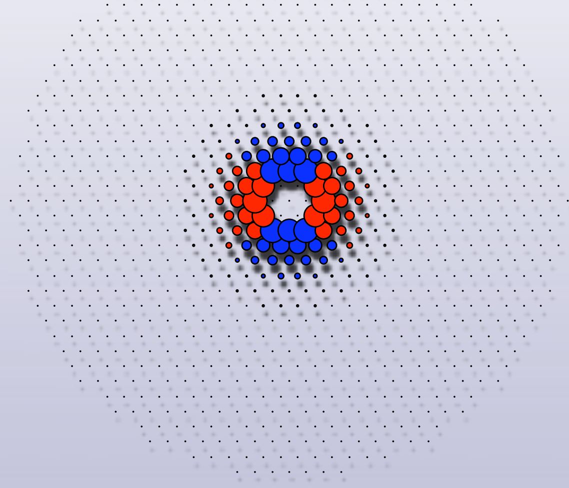
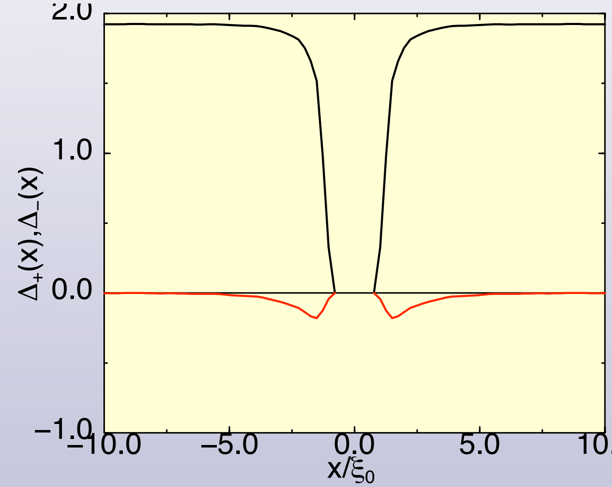


$(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$

$\Psi_- \, e^{i2\phi}$



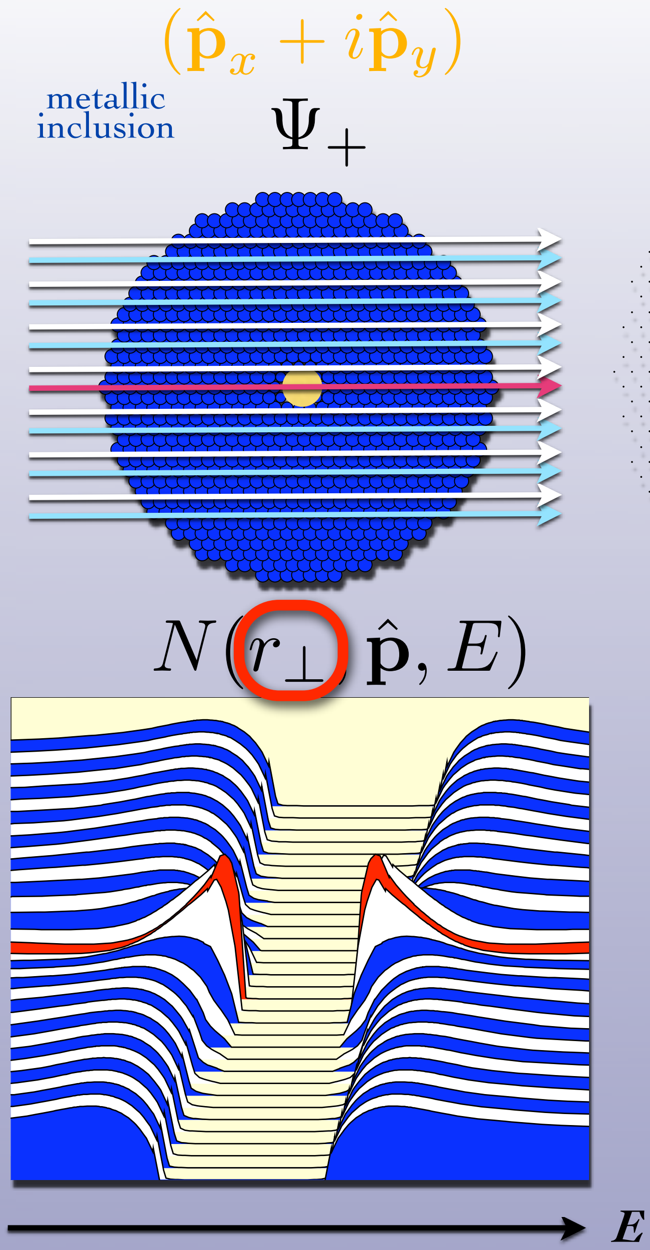


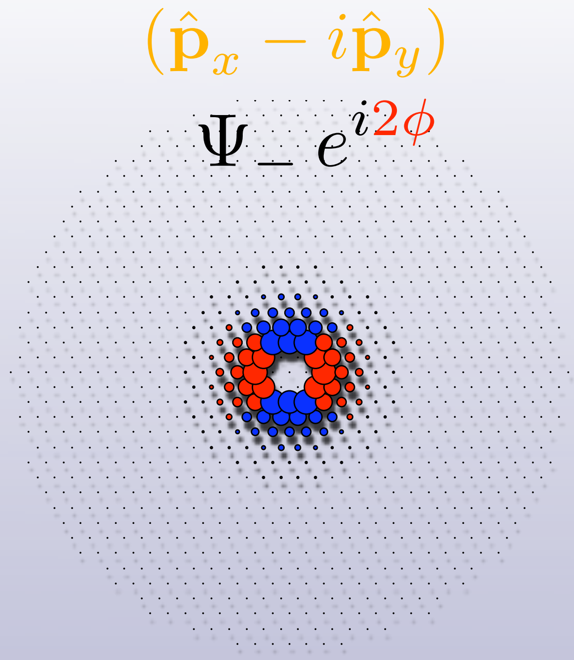

$N(r_\perp, \hat{\mathbf{p}}, E)$

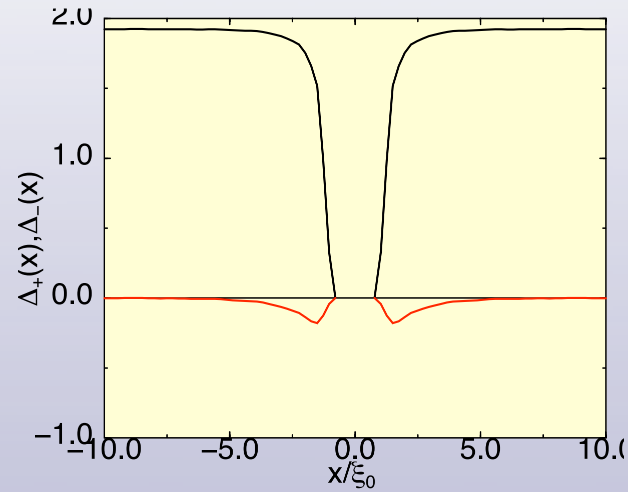
$E$

JAS & M. Eschrig, New J. Phys. 11, 075008 (2009)

# Structure of a Defect in the Meissner Phase of a Chiral Superconductor



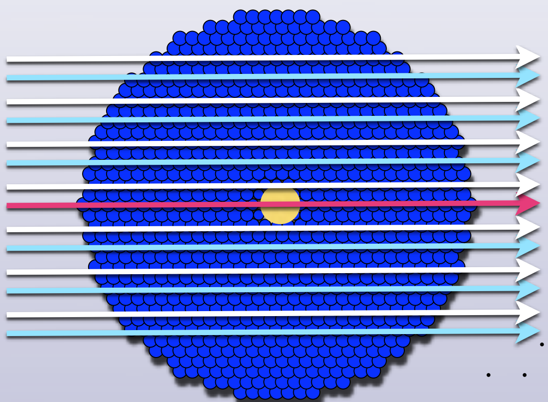

JAS & M. Eschrig, New J. Phys. 11, 075008 (2009)

# Structure of a Defect in the Meissner Phase of a Chiral Superconductor

$(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$

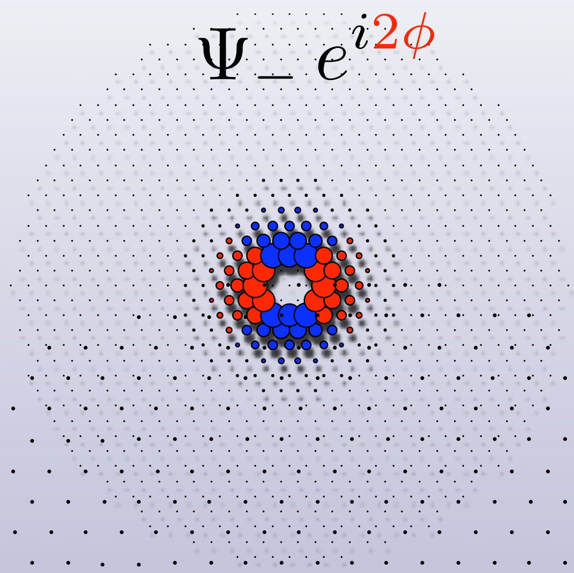
metallic inclusion

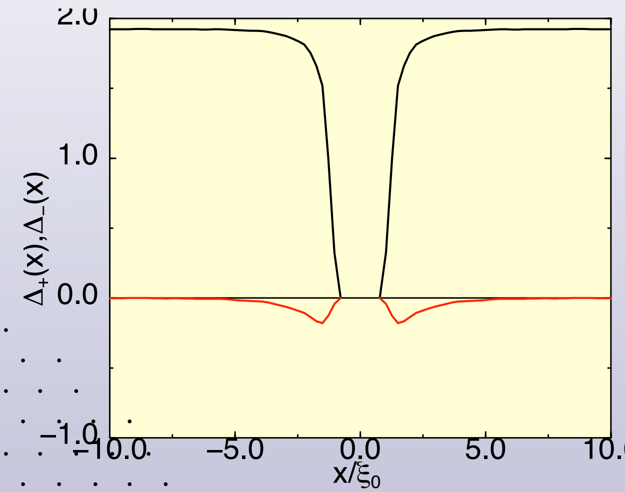
$\Psi_+$



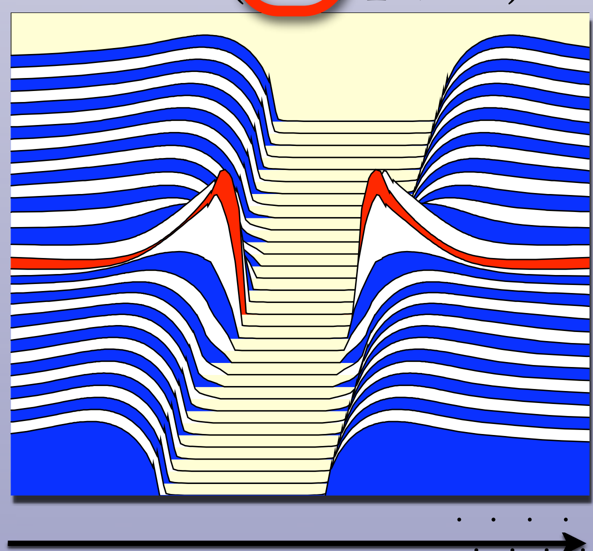
$(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$

$\Psi_- \, e^{i2\phi}$

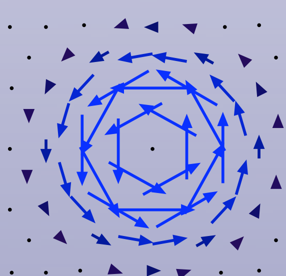





$N(r_\perp, \hat{\mathbf{p}}, E)$



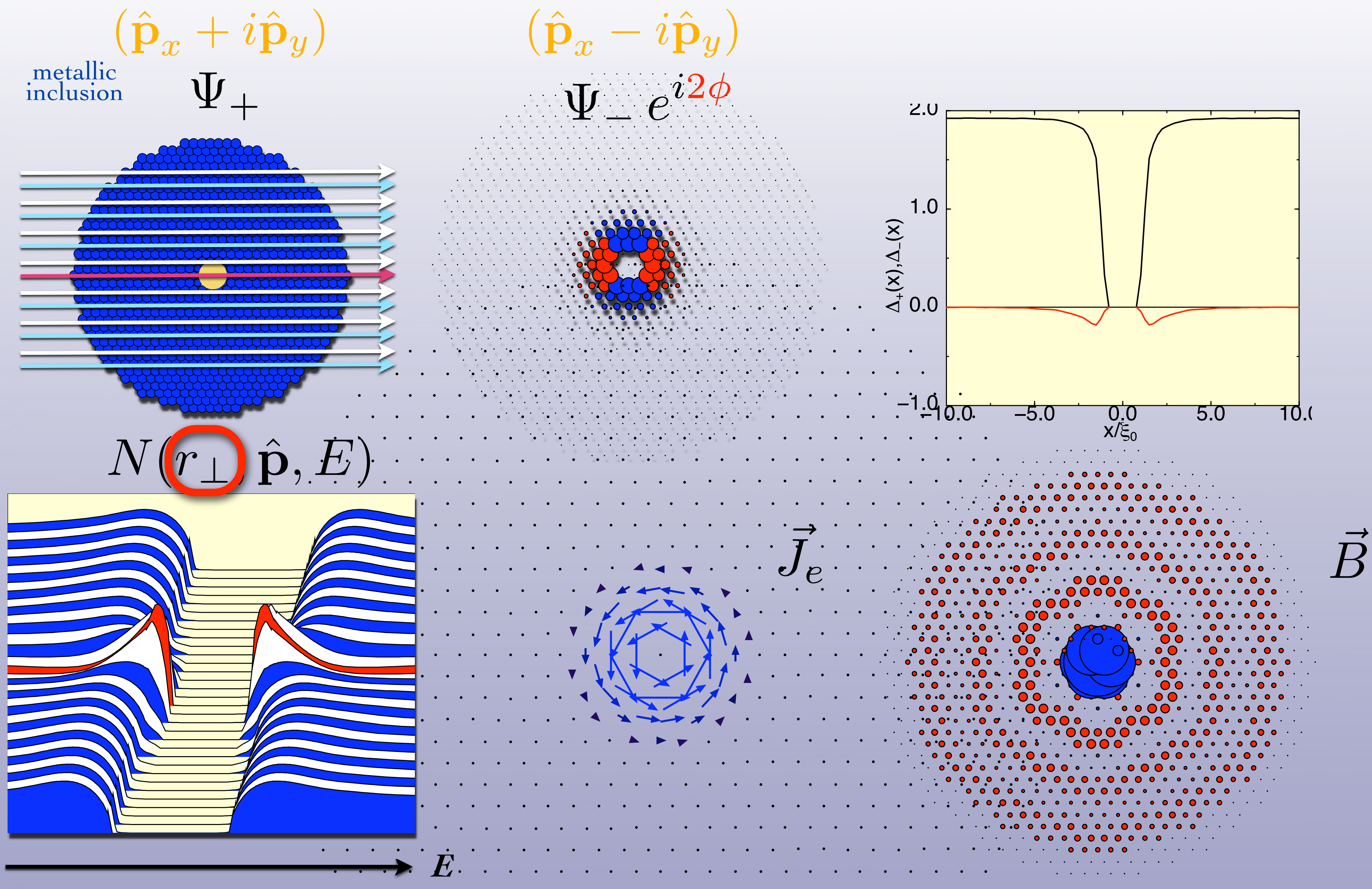

$\vec{J}_e$



Spontaneous Current

JAS & M. Eschrig, New J. Phys. 11, 075008 (2009)

# Structure of a Defect in the Meissner Phase of a Chiral Superconductor




Spontaneous Current ... and Field

Would provide key signature of chirality

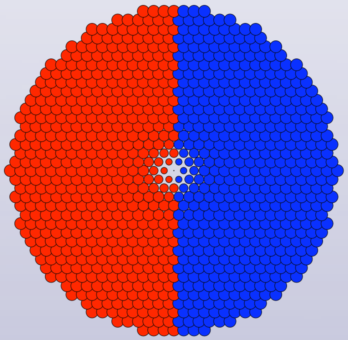
JAS & M. Eschrig, New J. Phys. 11, 075008 (2009)

D. Rainer & M. Vurio (1977) - 3He-A

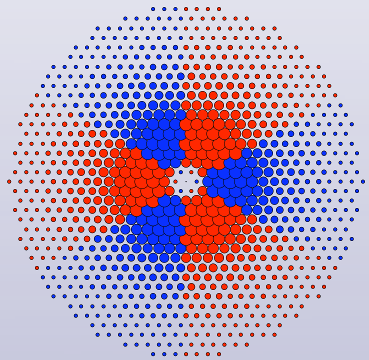
C. Choi & P. Muzikar (1989) - chiral SC

# Chiral Superconductors - 2 Inequivalent Singly Quantized Vortices

$(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$ $\Psi_+ \, e^{i\phi}$



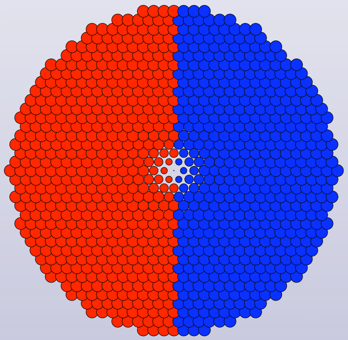
$(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$ $\Psi_- \, e^{i3\phi}$

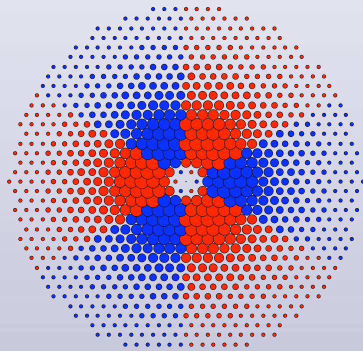


# Chiral Superconductors - 2 Inequivalent Singly Quantized Vortices

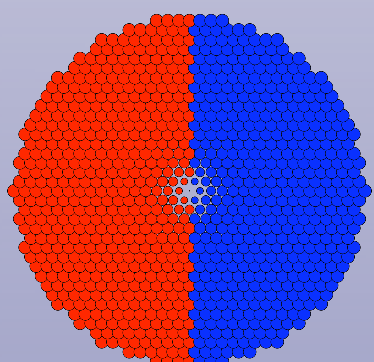
$(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$ $(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$

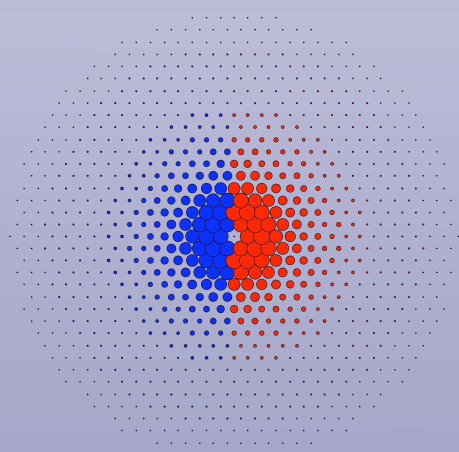
$\Psi_+ \, e^{i\phi}$ $\Psi_- \, e^{i3\phi}$





$\Psi_+ \, e^{-i\phi}$ $\Psi_- \, e^{+i\phi}$

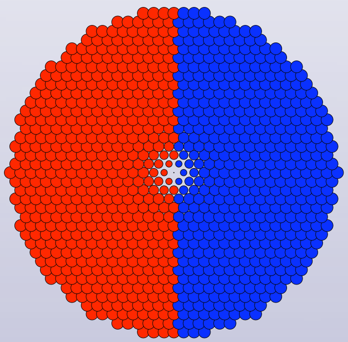


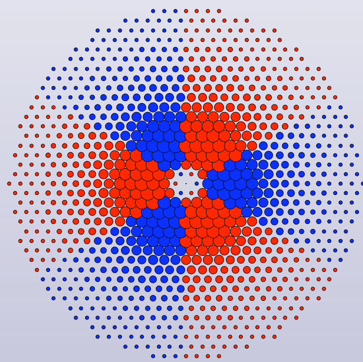


# Chiral Superconductors - 2 Inequivalent Singly Quantized Vortices

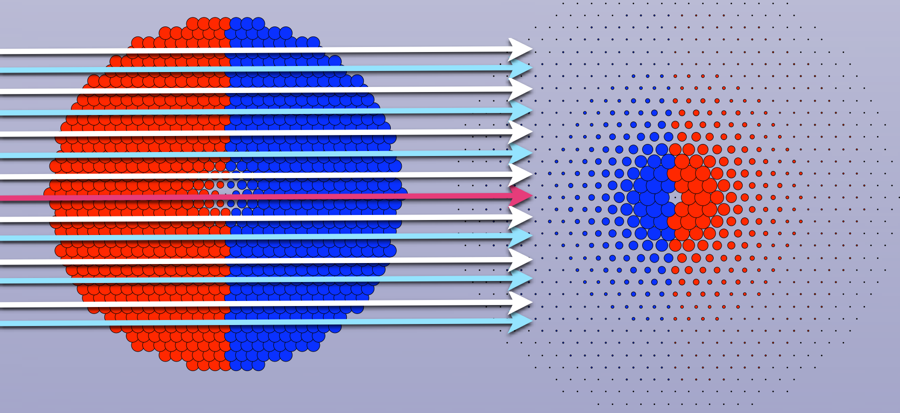
$(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$ $(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$

$\Psi_+ \, e^{i\phi}$ $\Psi_- \, e^{i3\phi}$

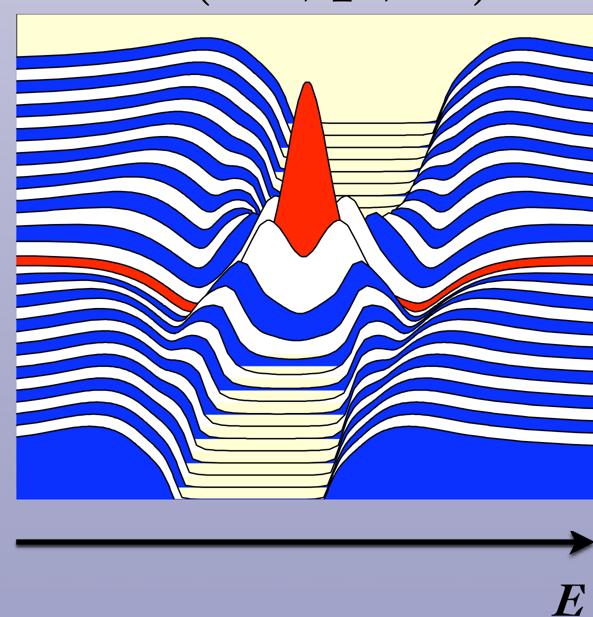




$\Psi_+ \, e^{-i\phi}$ $\Psi_- \, e^{+i\phi}$



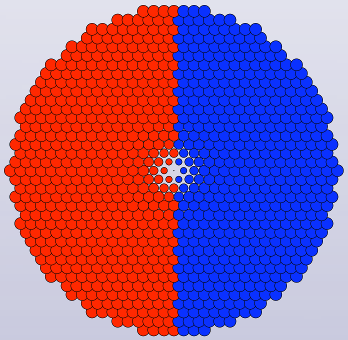
$N(r_\perp, \hat{\mathbf{p}}, E)$



# Chiral Superconductors - 2 Inequivalent Singly Quantized Vortices

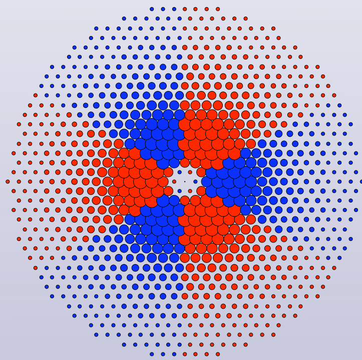
$(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$

$\Psi_+ \, e^{i\phi}$

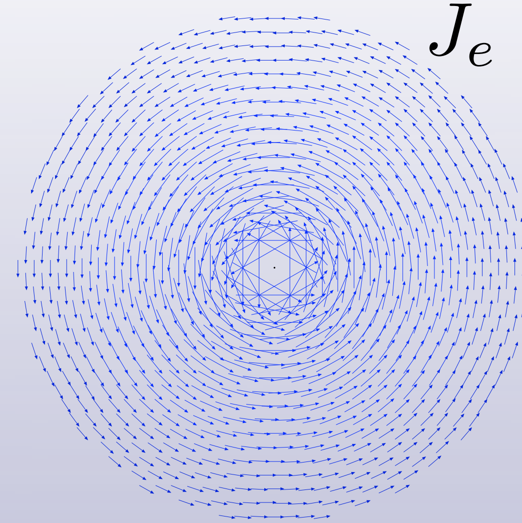


$(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$

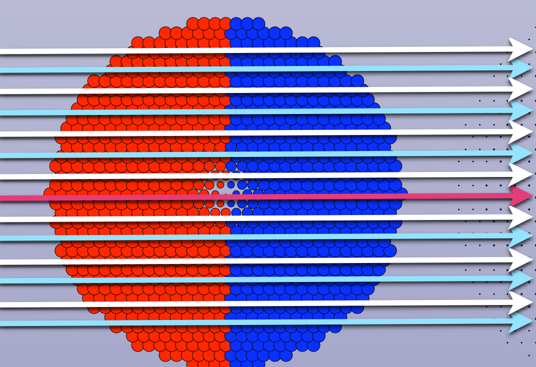
$\Psi_- \, e^{i3\phi}$



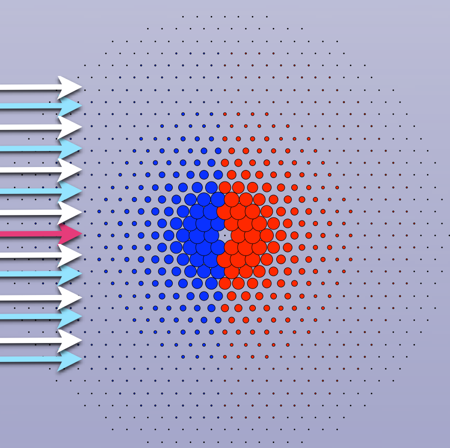
$\vec{J}_e$



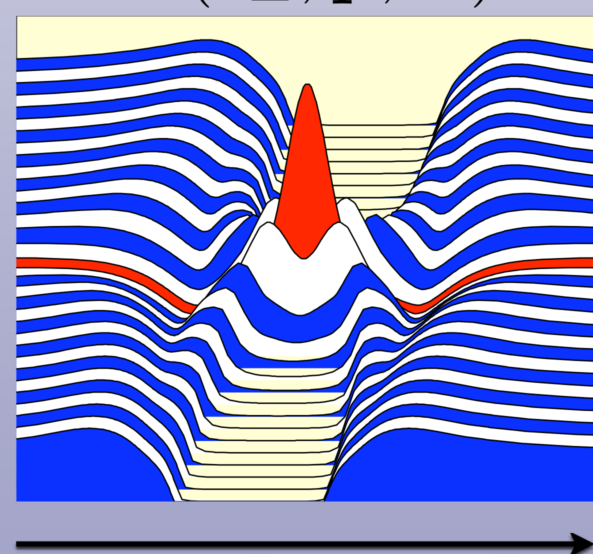
$\Psi_+ \, e^{-i\phi}$



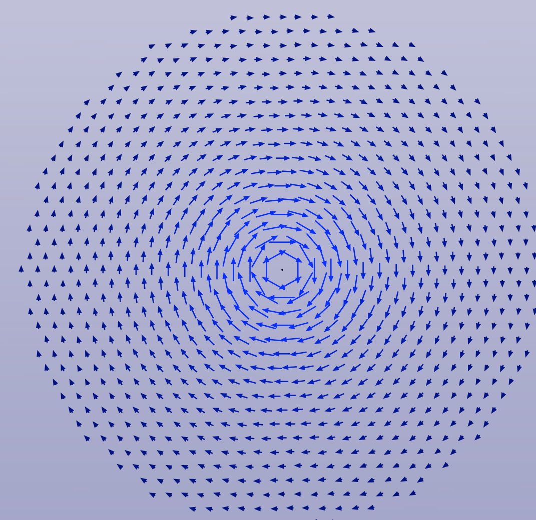
$\Psi_- \, e^{+i\phi}$



$N(r_\perp, \hat{\mathbf{p}}, E)$



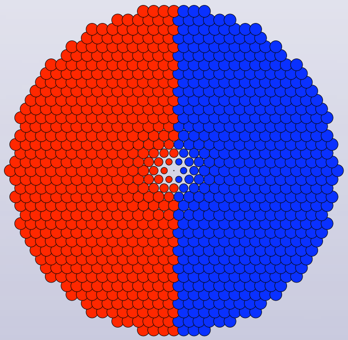
$\boldsymbol{E}$



# Chiral Superconductors - 2 Inequivalent Singly Quantized Vortices

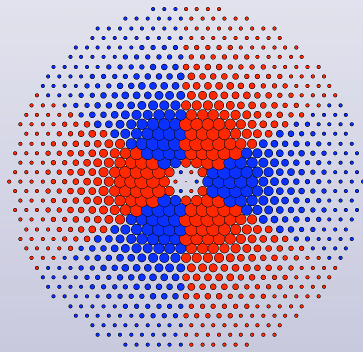
$(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$

$\Psi_+ \, e^{i\phi}$



$(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$

$\Psi_- \, e^{i3\phi}$

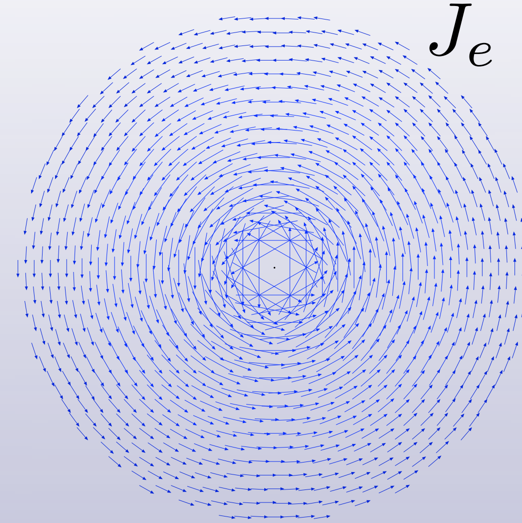


Signature of Broken T-symmetry

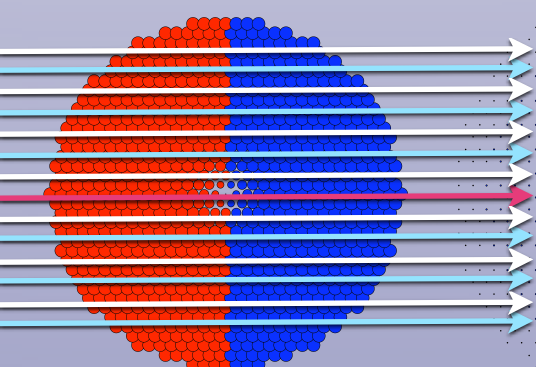
$$H_{c_1+} \neq H_{c_1-}$$

Tokuyasu, et al. PRB (1989)

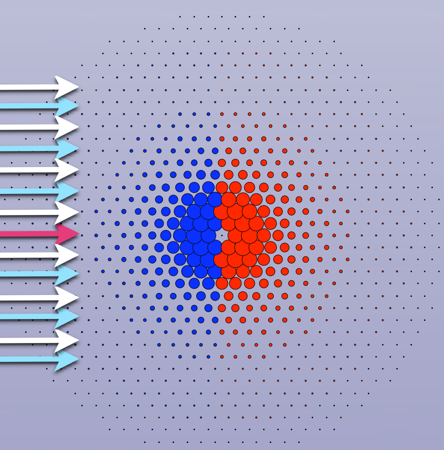
$\vec{J}_e$



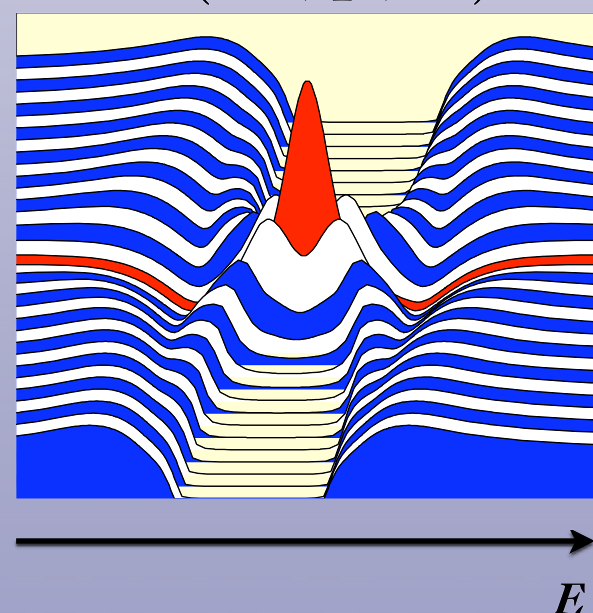
$\Psi_+ \, e^{-i\phi}$

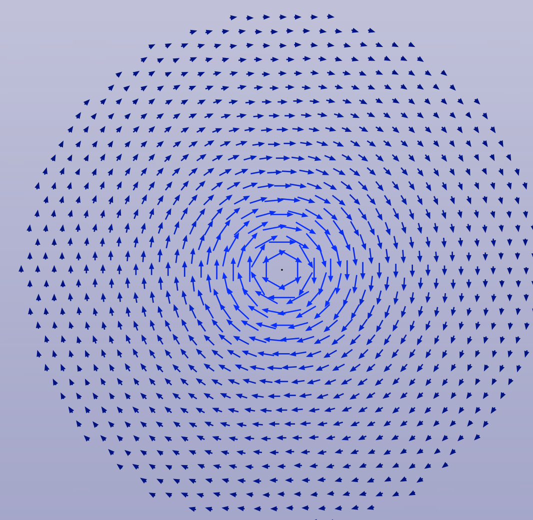


$\Psi_- \, e^{+i\phi}$



$N(r_\perp, \hat{\mathbf{p}}, E)$

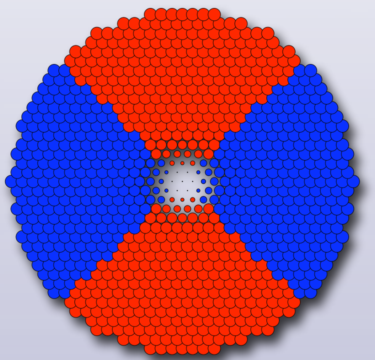


# Double Quantum Vortices

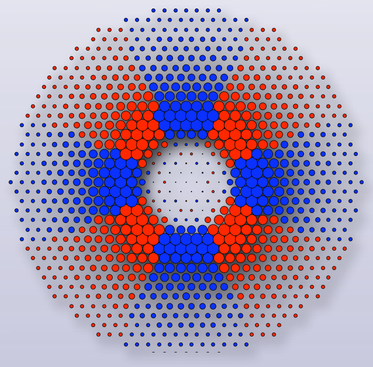
$(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$

$\Psi_+ \, e^{i2\phi}$



$(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$

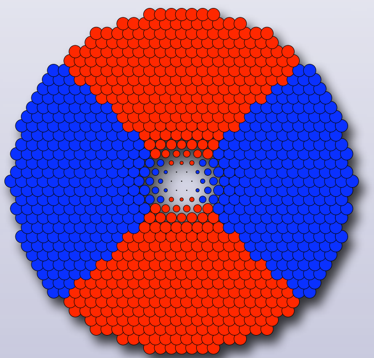
$\Psi_- \, e^{i4\phi}$



# Double Quantum Vortices

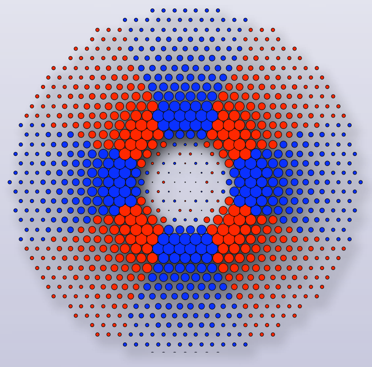
$(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$

$\Psi_+ \, e^{i2\phi}$

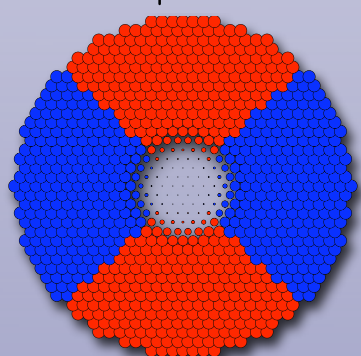


$(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$

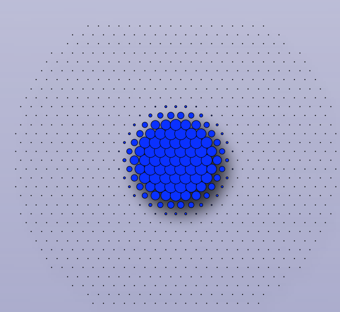
$\Psi_- \, e^{i4\phi}$



$\Psi_+ \, e^{-i2\phi}$



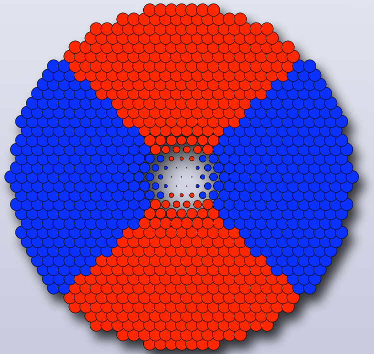
$\Psi_-$



# Double Quantum Vortices

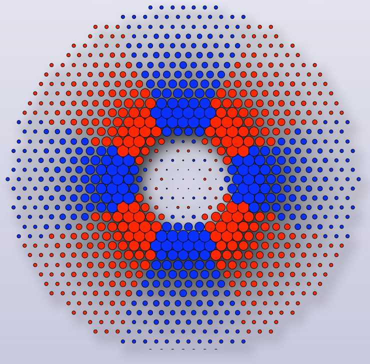
$(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$

$\Psi_+ \, e^{i2\phi}$

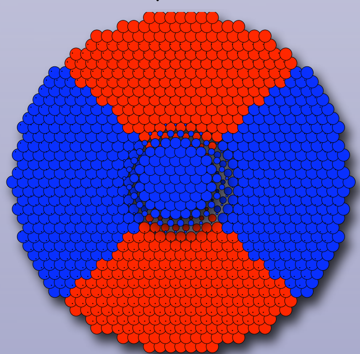


$(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$

$\Psi_- \, e^{i4\phi}$



$\Psi_+ \, e^{-i2\phi}$



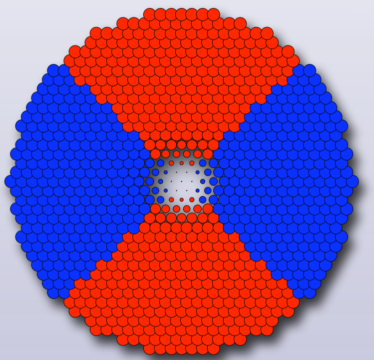
$\Psi_-$

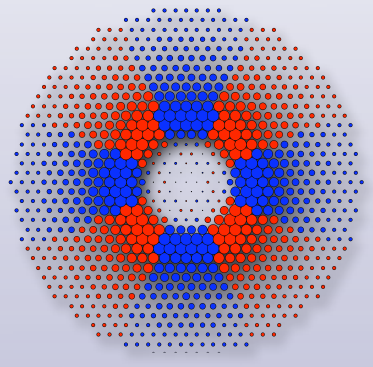
"coreless"

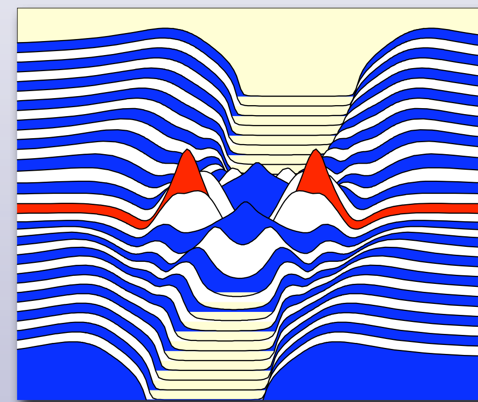
# Double Quantum Vortices

$(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$ $(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$

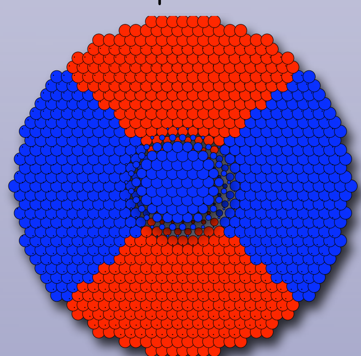
$\Psi_+ \, e^{i2\phi}$ $\Psi_- \, e^{i4\phi}$ $N(r_\perp, \hat{\mathbf{p}}, E)$

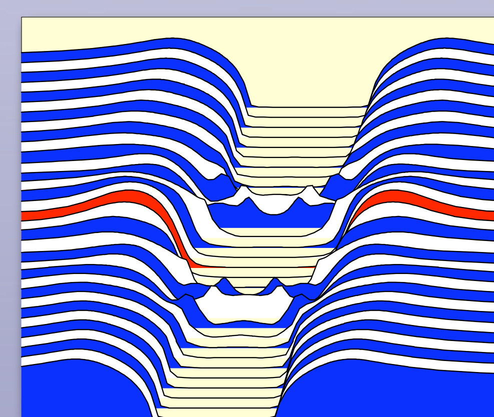






$\Psi_+ \, e^{-i2\phi}$ $\Psi_-$

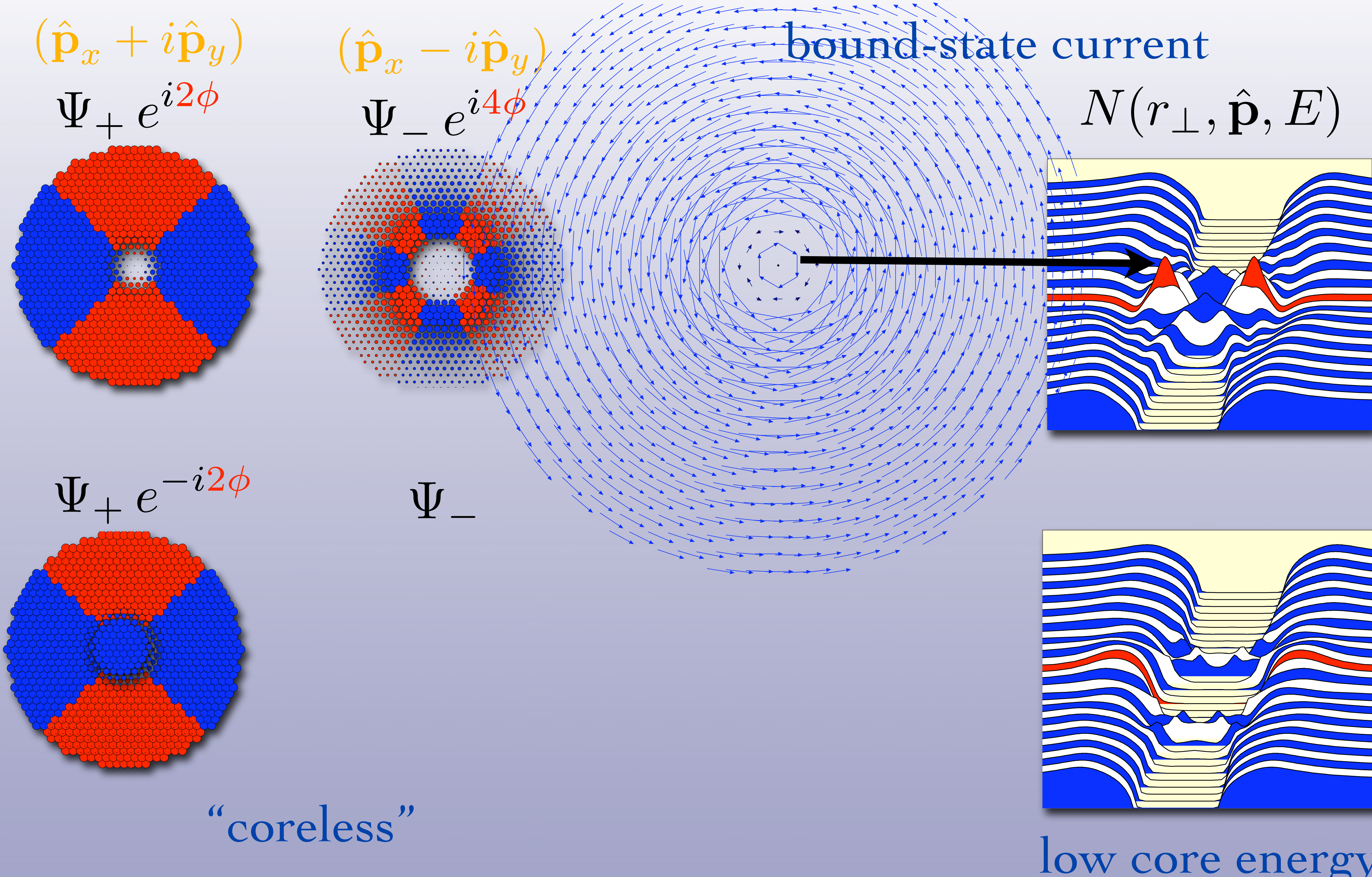




"coreless"

low core energy

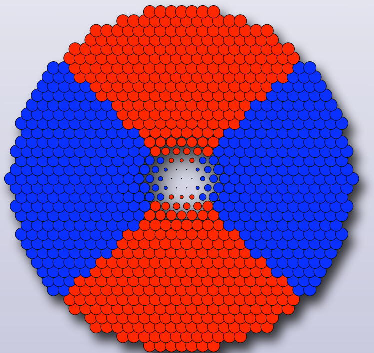
# Double Quantum Vortices



# Double Quantum Vortices

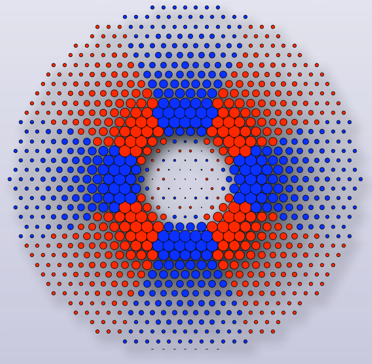
$(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$

$\Psi_+ \, e^{i2\phi}$

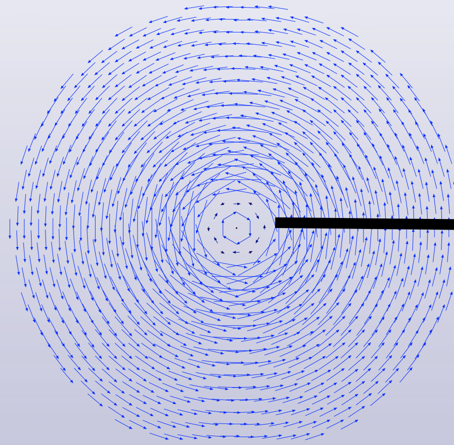


$(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$

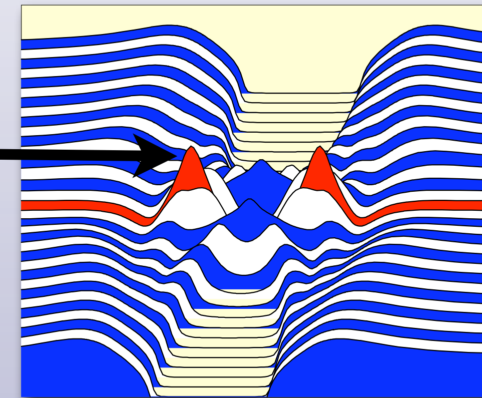
$\Psi_- \, e^{i4\phi}$



bound-state current

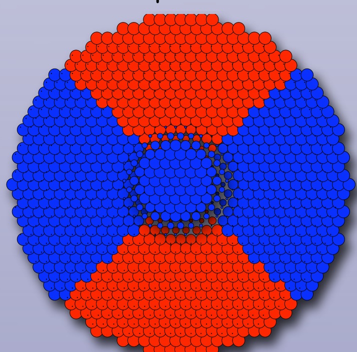


$N(r_\perp, \hat{\mathbf{p}}, E)$



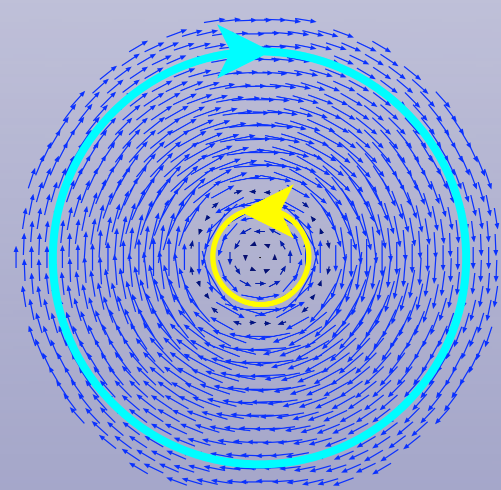
$\Psi_+ \, e^{-i2\phi}$

$\Psi_-$

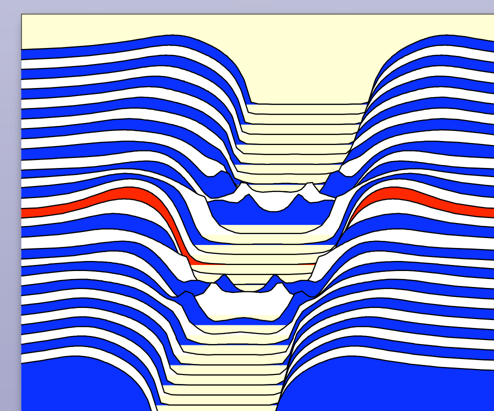


"coreless"

Current Reversal



domain wall



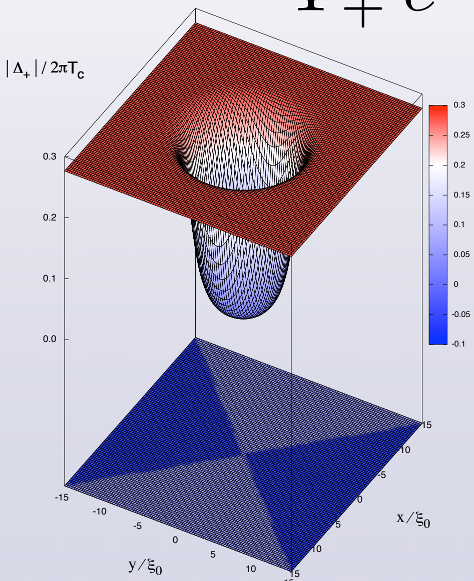
low core energy

Energetically Stable - $H_{c1} < H < H_{c2}$

# Double Quantum Vortices

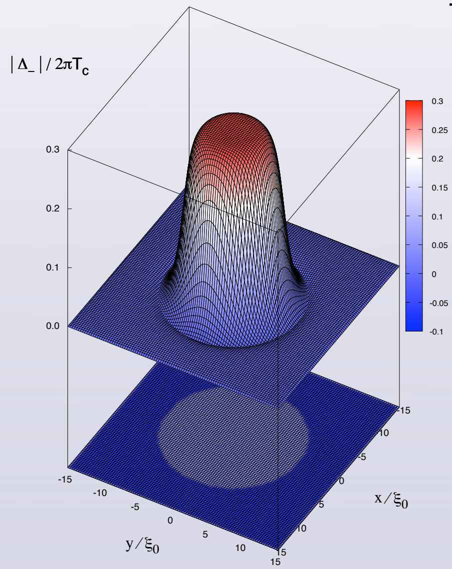
$(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$

$\Psi_+ \, e^{-i2\phi}$




$(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$

$\Psi_-$



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$(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$

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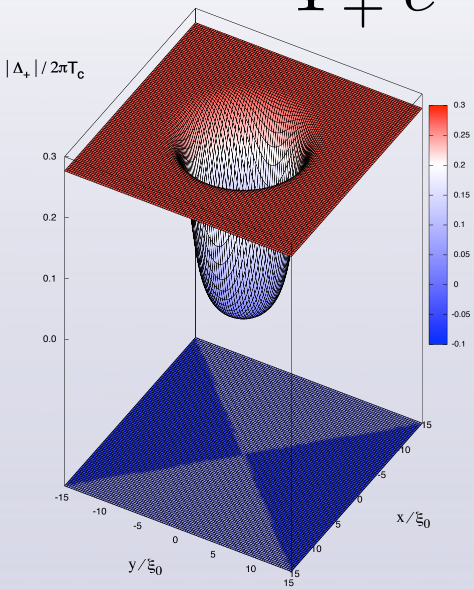
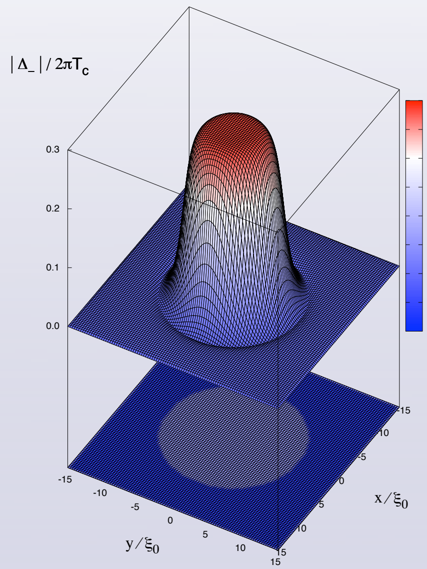
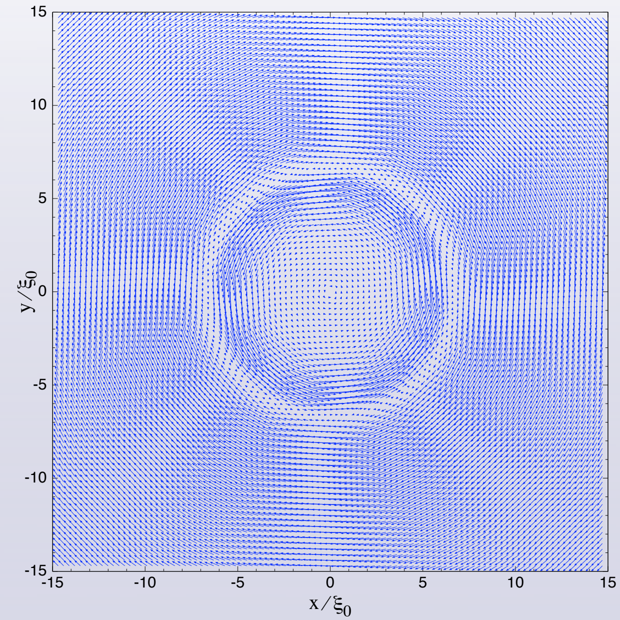



$(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$

$\Psi_-$



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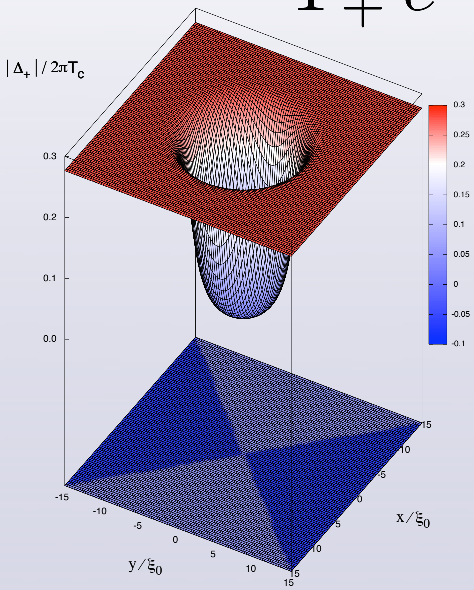
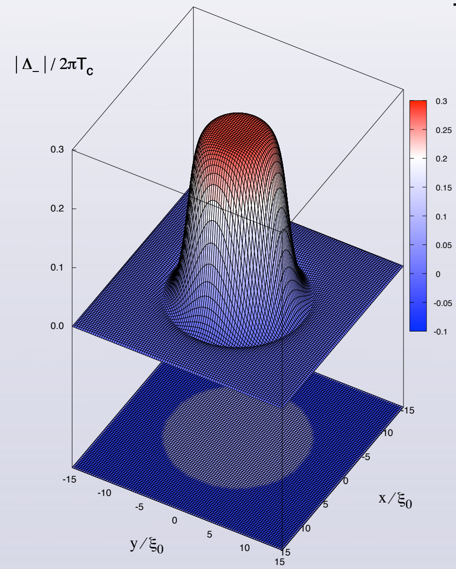
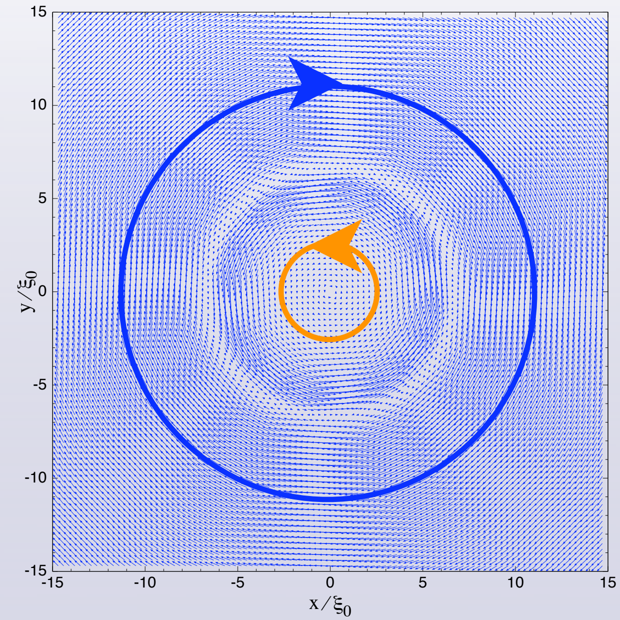
$\Psi_+ \, e^{-i2\phi}$




$(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$

$\Psi_-$



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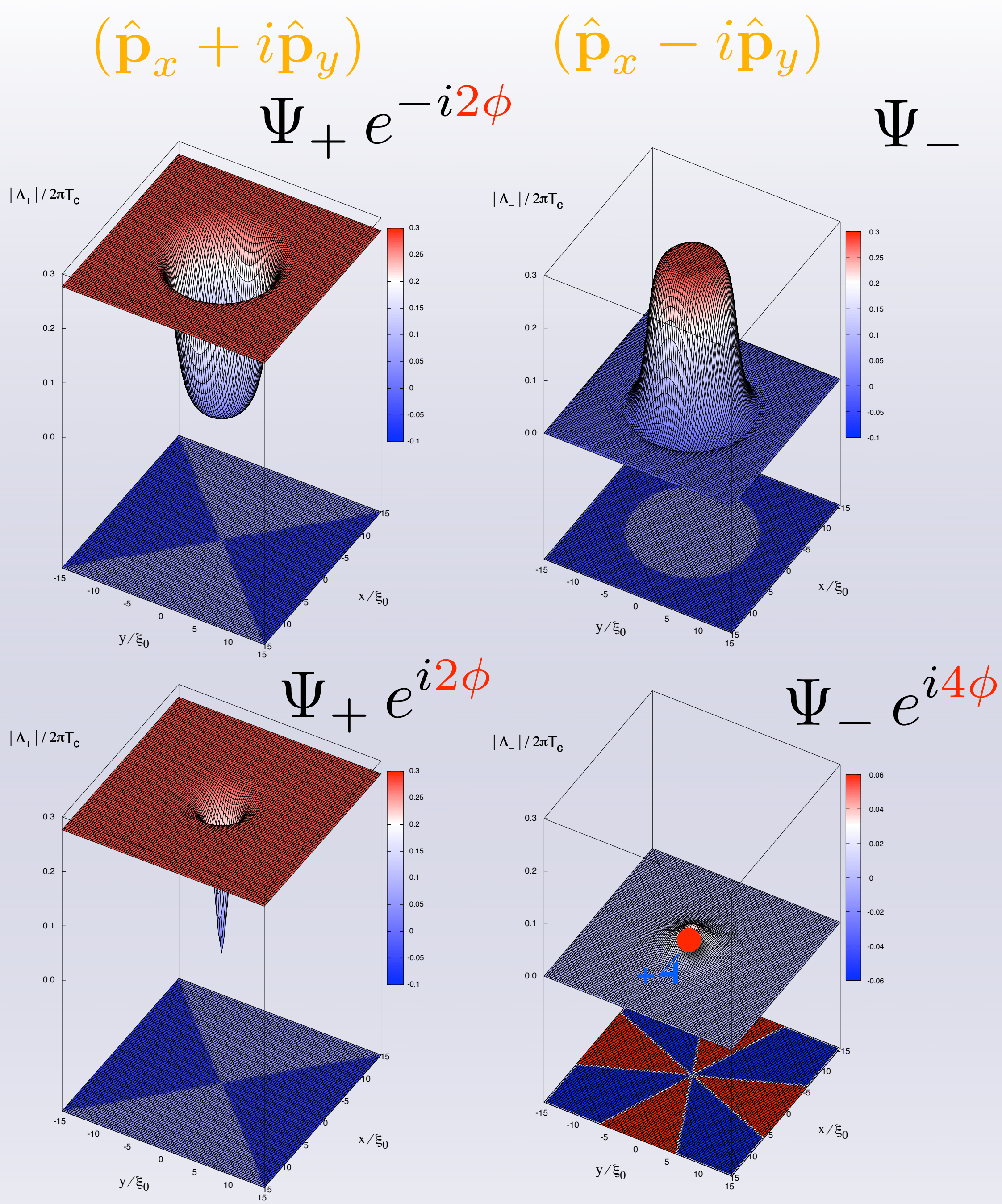
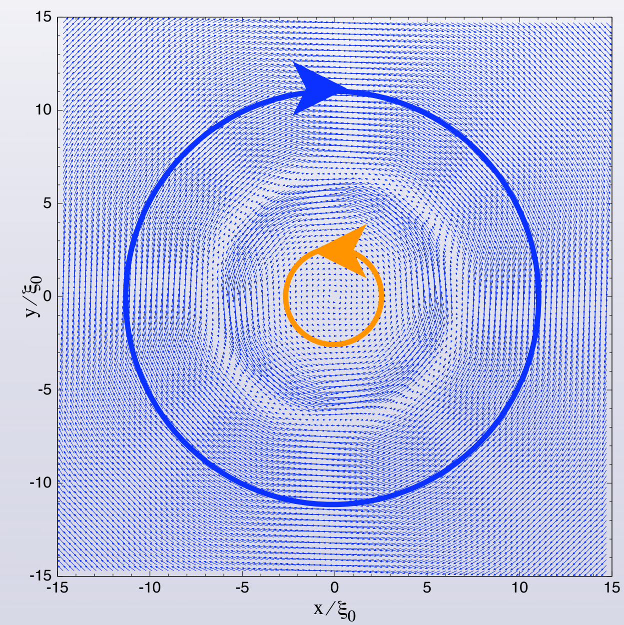
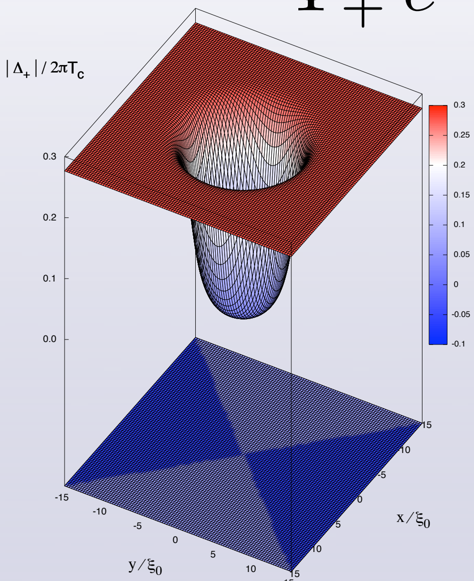


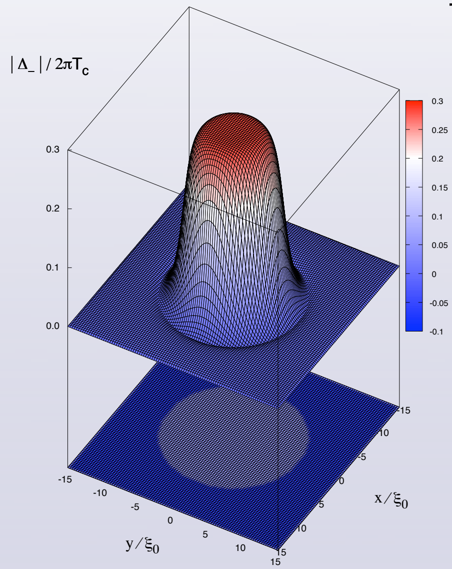
$(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y)$ $(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y)$

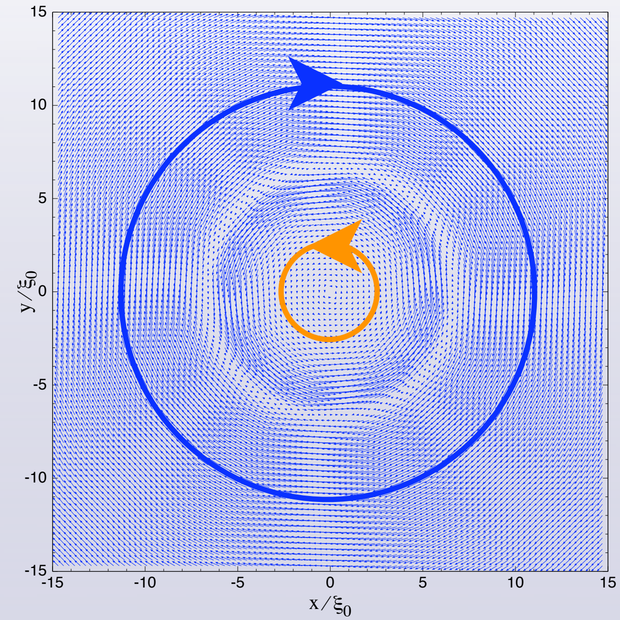
# Double Quantum Vortices

$\Psi_+ \, e^{-i2\phi}$

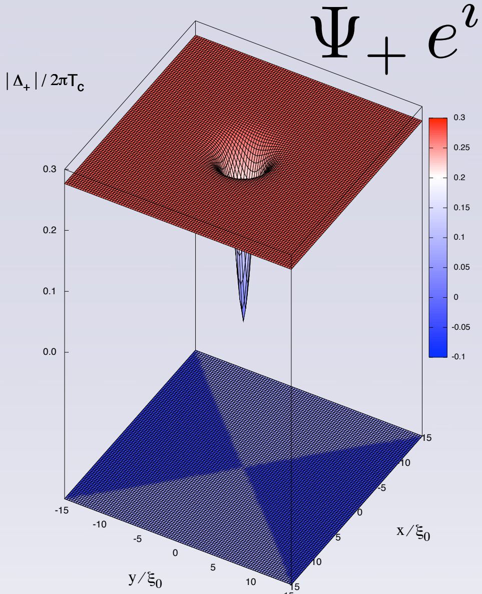



$\Psi_-$

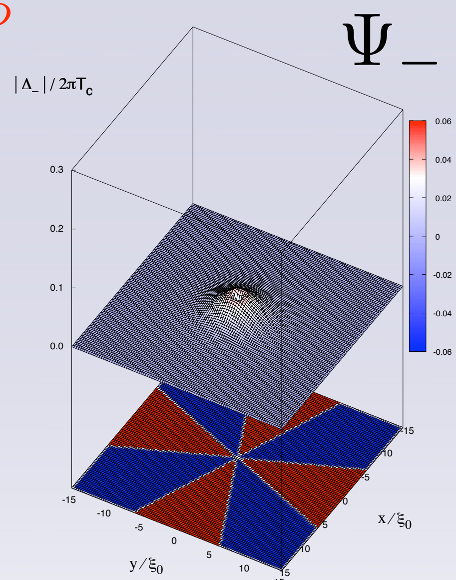


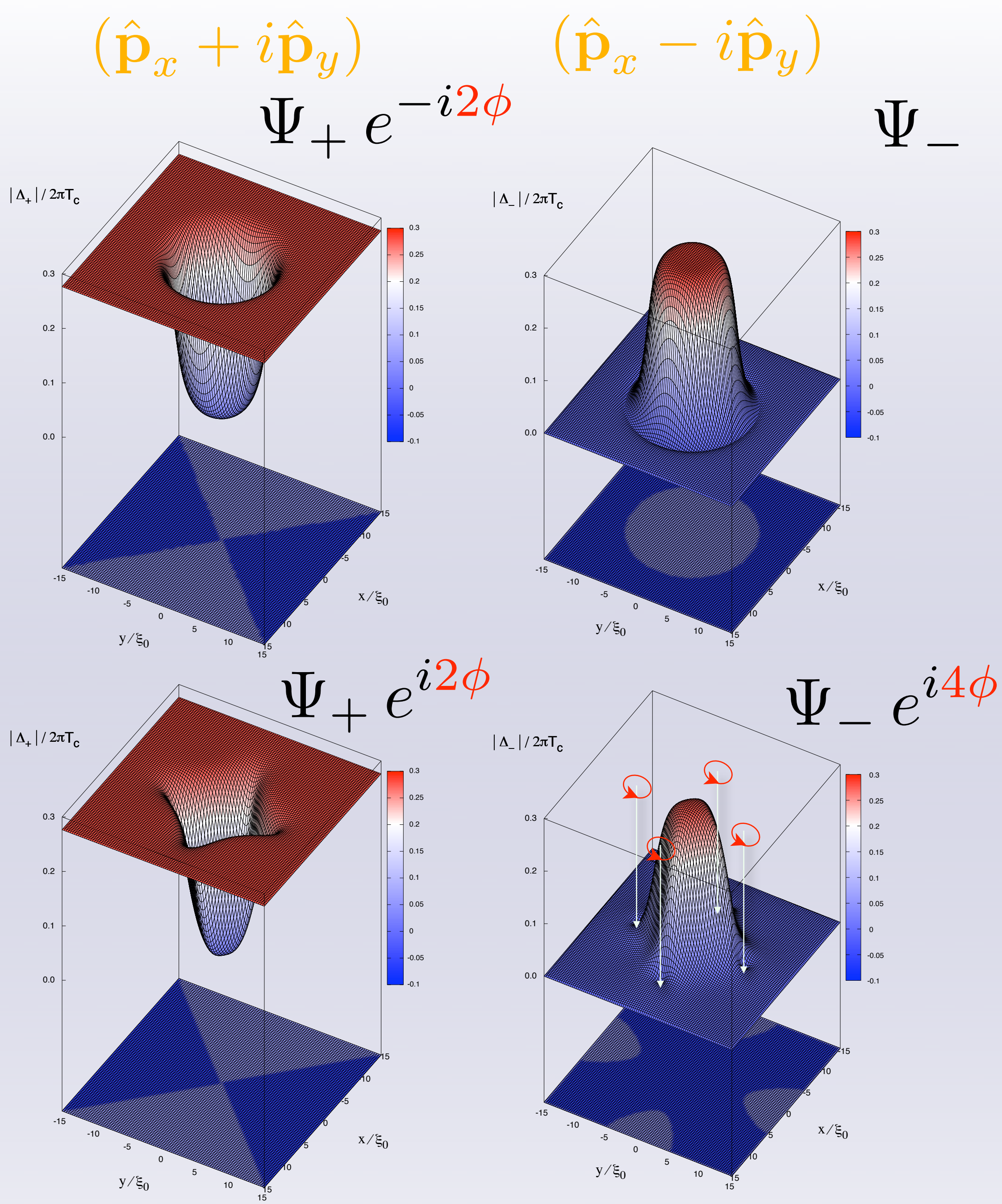

$\Psi_+ \, e^{i2\phi}$

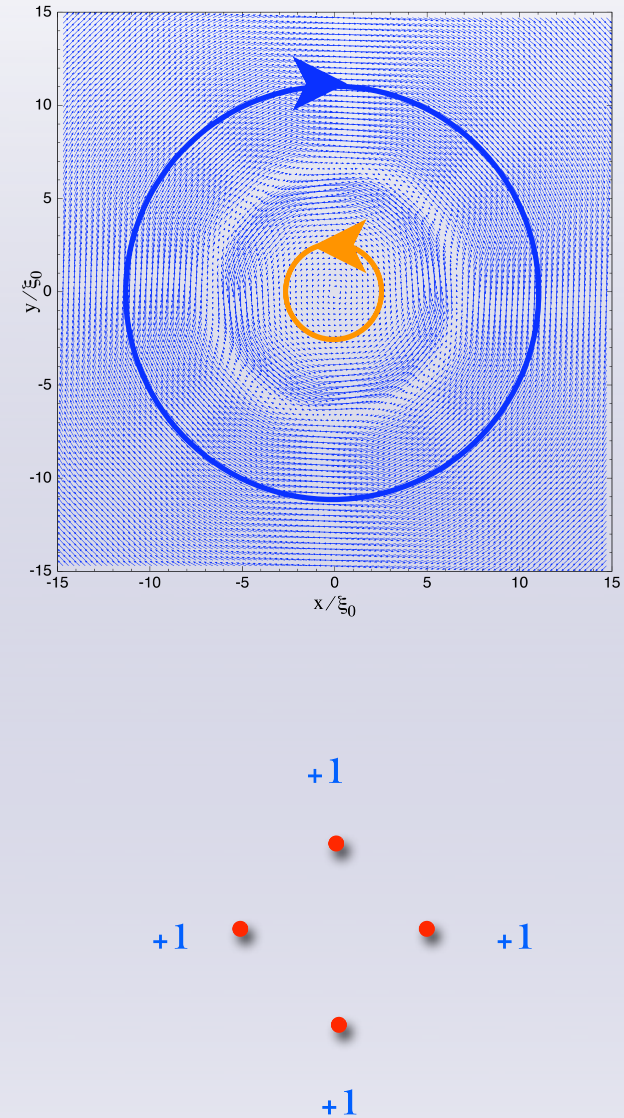



$\Psi_- \, e^{i4\phi}$

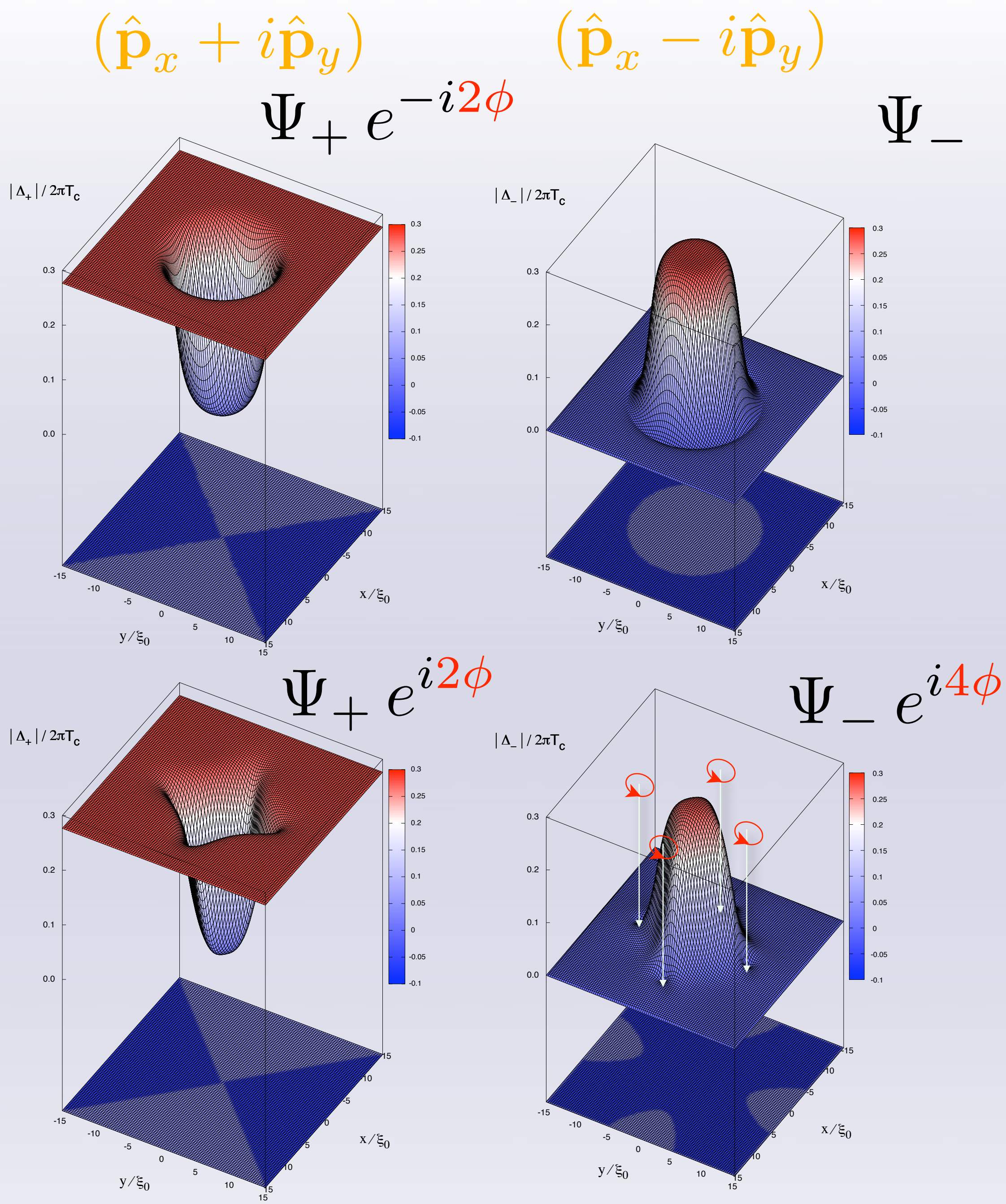



# Double Quantum Vortices

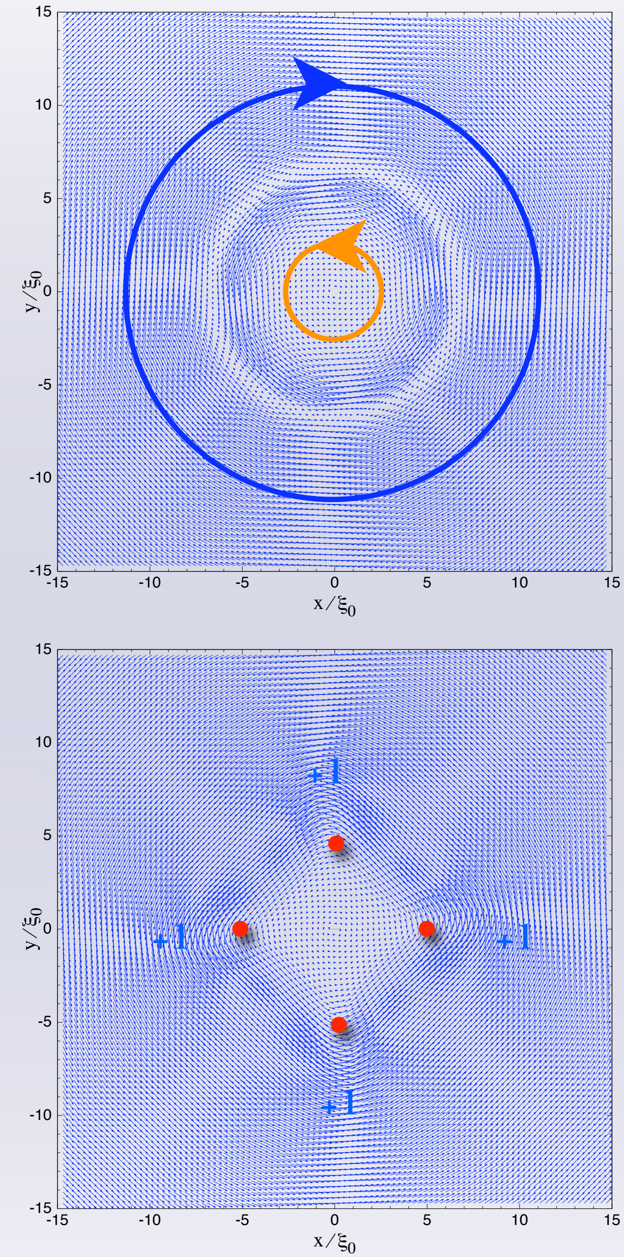



Spontaneously Broken Axial Symmetry




# Double Quantum Vortices




Hexagonal vs. Tetragonal Vortex Lattice

Spontaneously Broken Axial Symmetry

# What inhibits dissociation of 2-quantum vortices in chiral p-wave SCs be stable?

Core Energies & *2* topological quantum numbers

$$\vec{\Psi}(\hat{\mathbf{p}},\mathbf{r}) = \vec{\mathbf{d}} \left[ |\Psi_+|e^{ip\phi}(\hat{\mathbf{p}}_x + i\hat{\mathbf{p}}_y) + |\Psi_-|e^{im\phi}(\hat{\mathbf{p}}_x - i\hat{\mathbf{p}}_y) \right]$$

| p | m |
|---|---|
| +1 | +3 |
| -1 | +1 |
| +2 | +4 |
| -2 | 0 |
| 0 | +2 |

Dissociation

(2,4) ⟶ (1,3) + (1,3)

(-2,0) ⟶ (-1,1) + (-1,1)

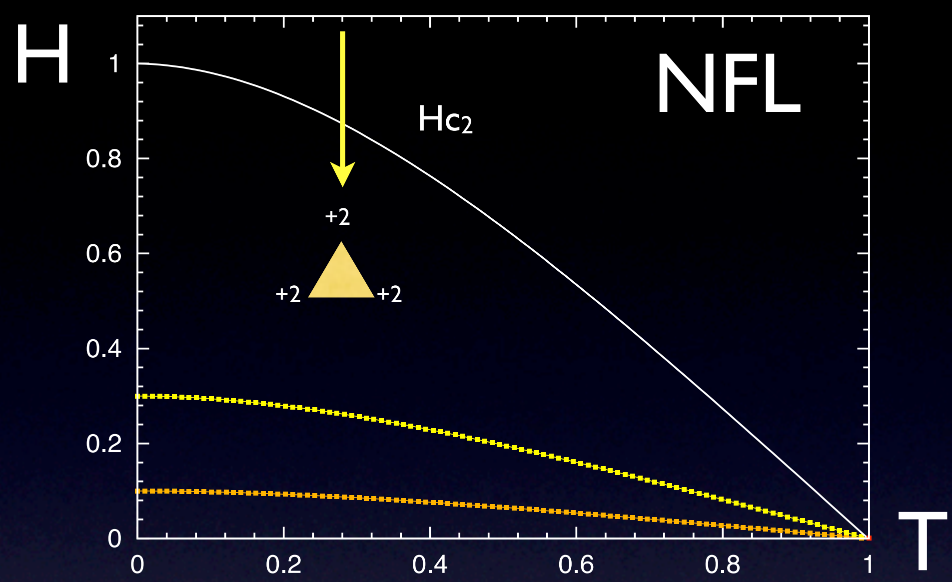
violates $p + 1 = m - 1 \;\; +4n$

➡ Axial Constraint can be violated in Finite geometries, e.g. *Vortex Lattice:*

➡ Vortex creation & annihilation at cell boundaries

# Chiral Ground State Effects of the H-T Phase Diagram

*Cylindrical Fermi Surface*



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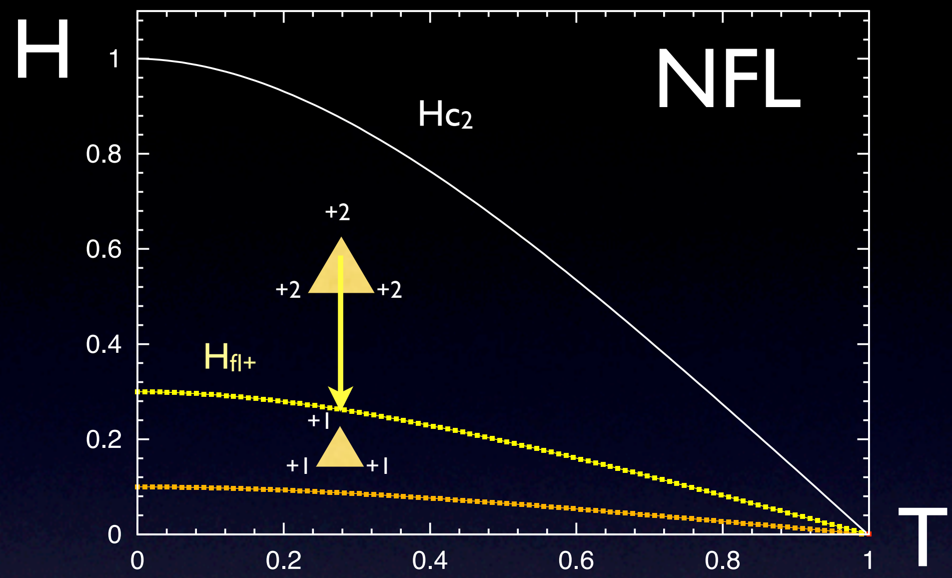


*Cylindrical Fermi Surface*

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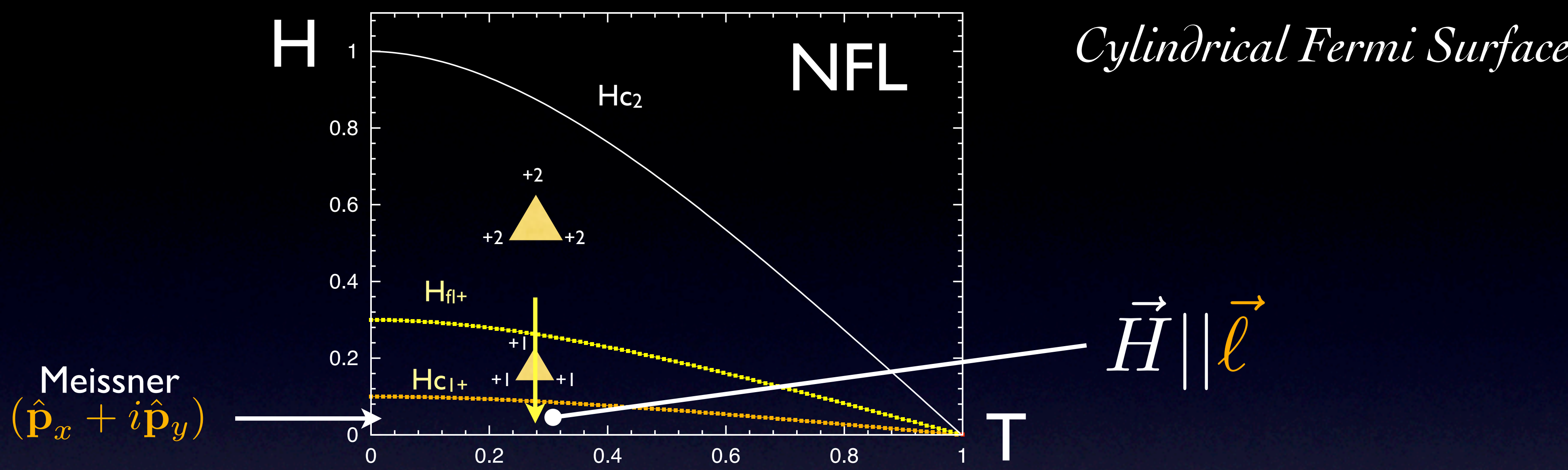

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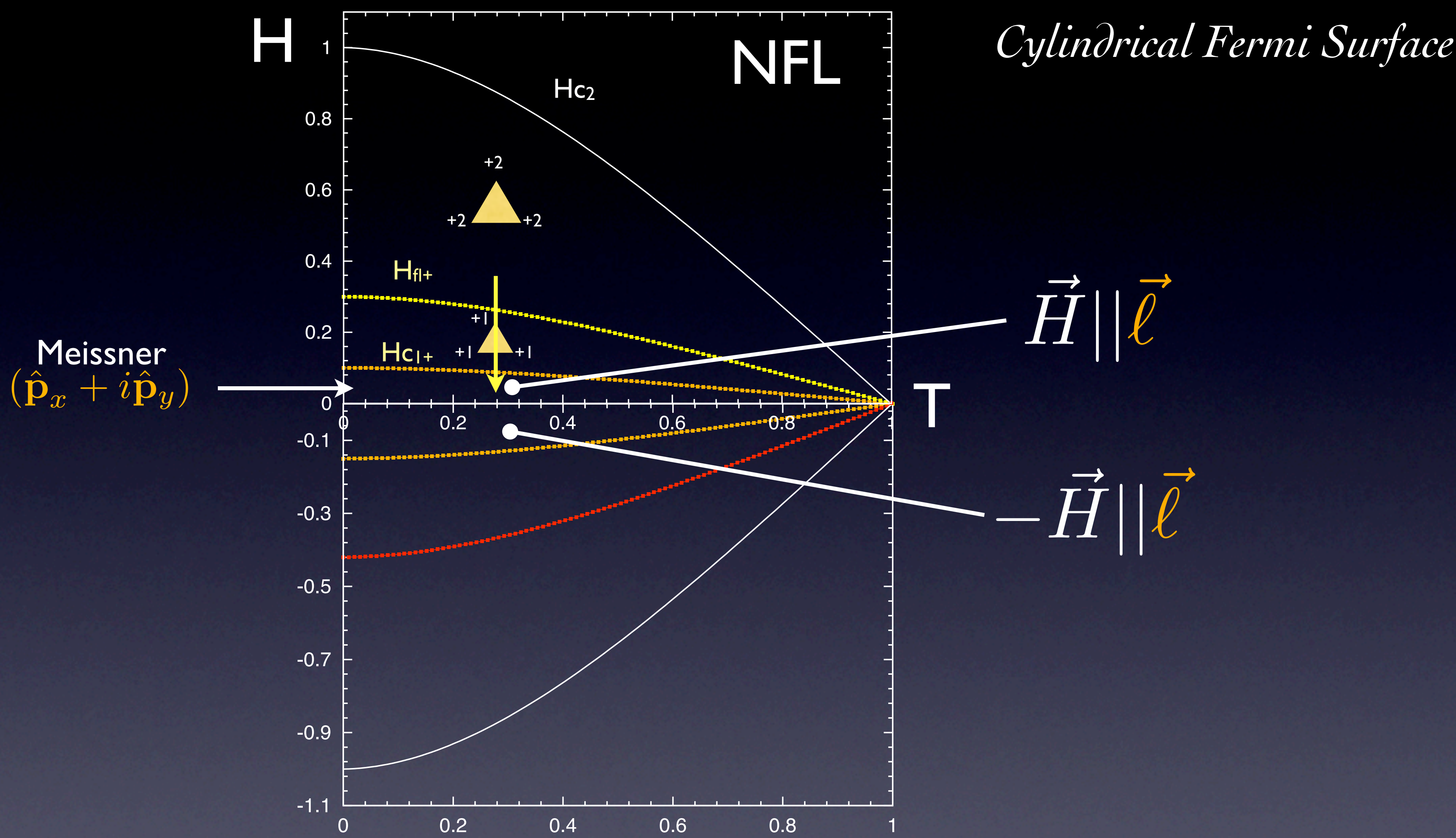

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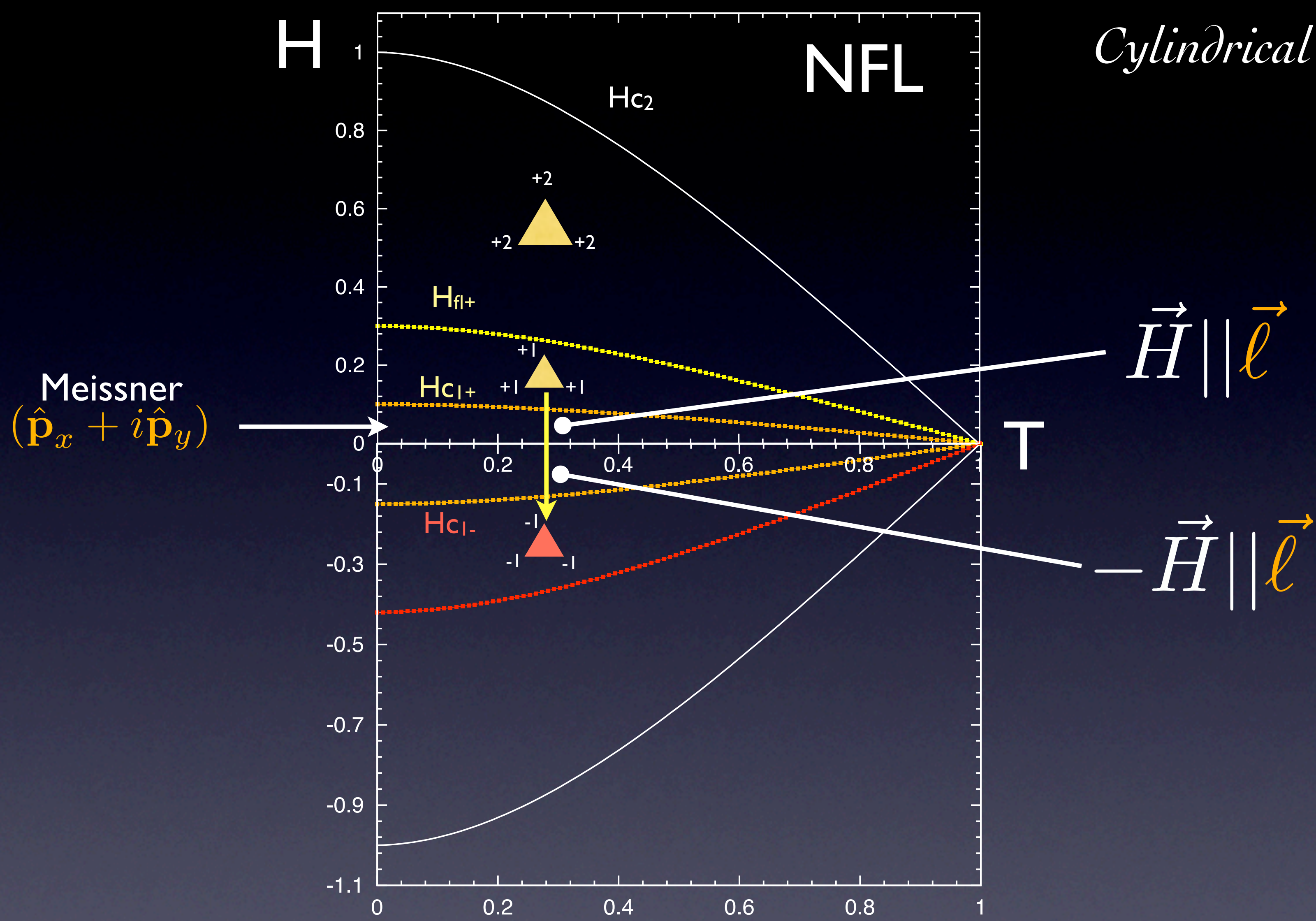

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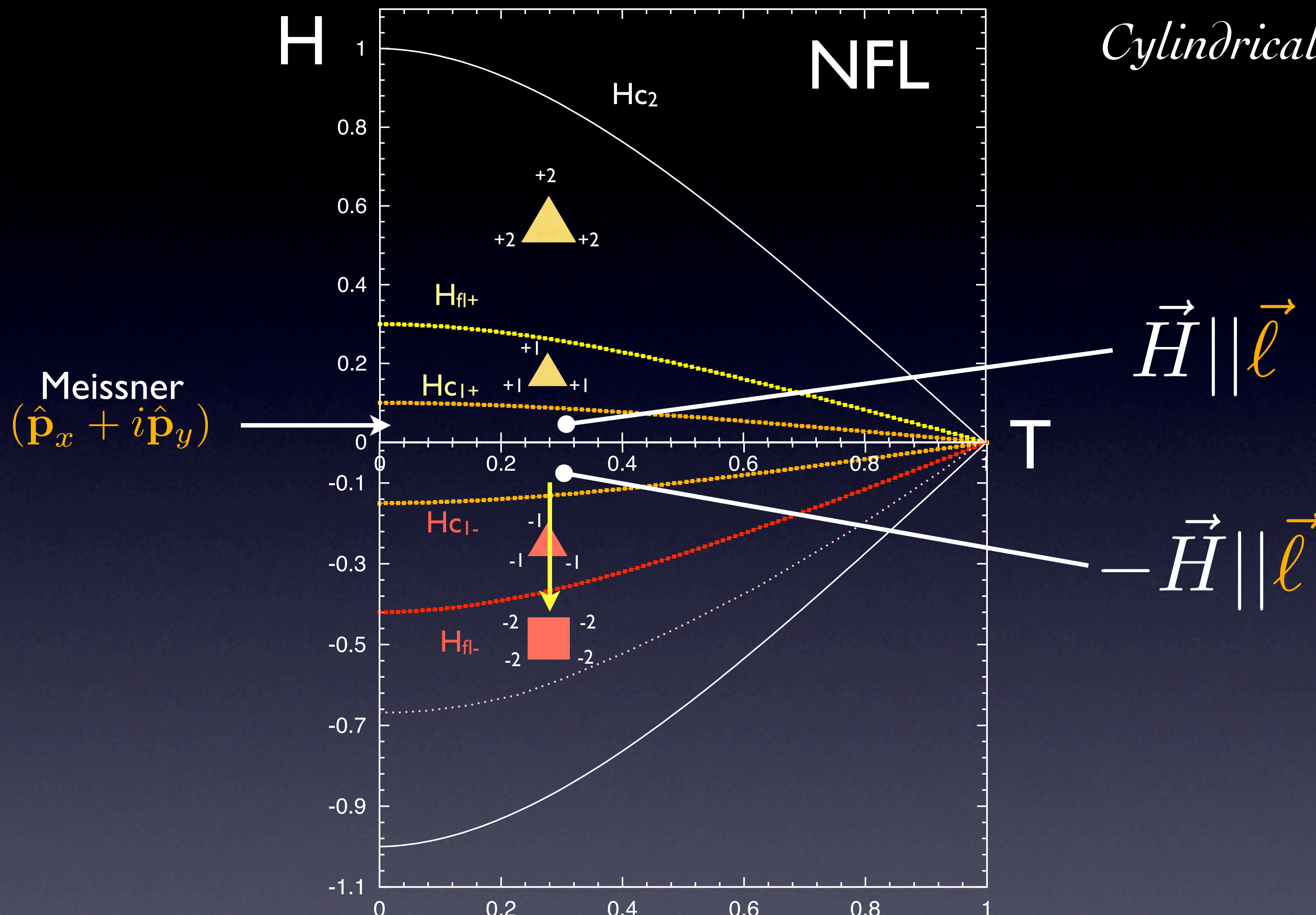

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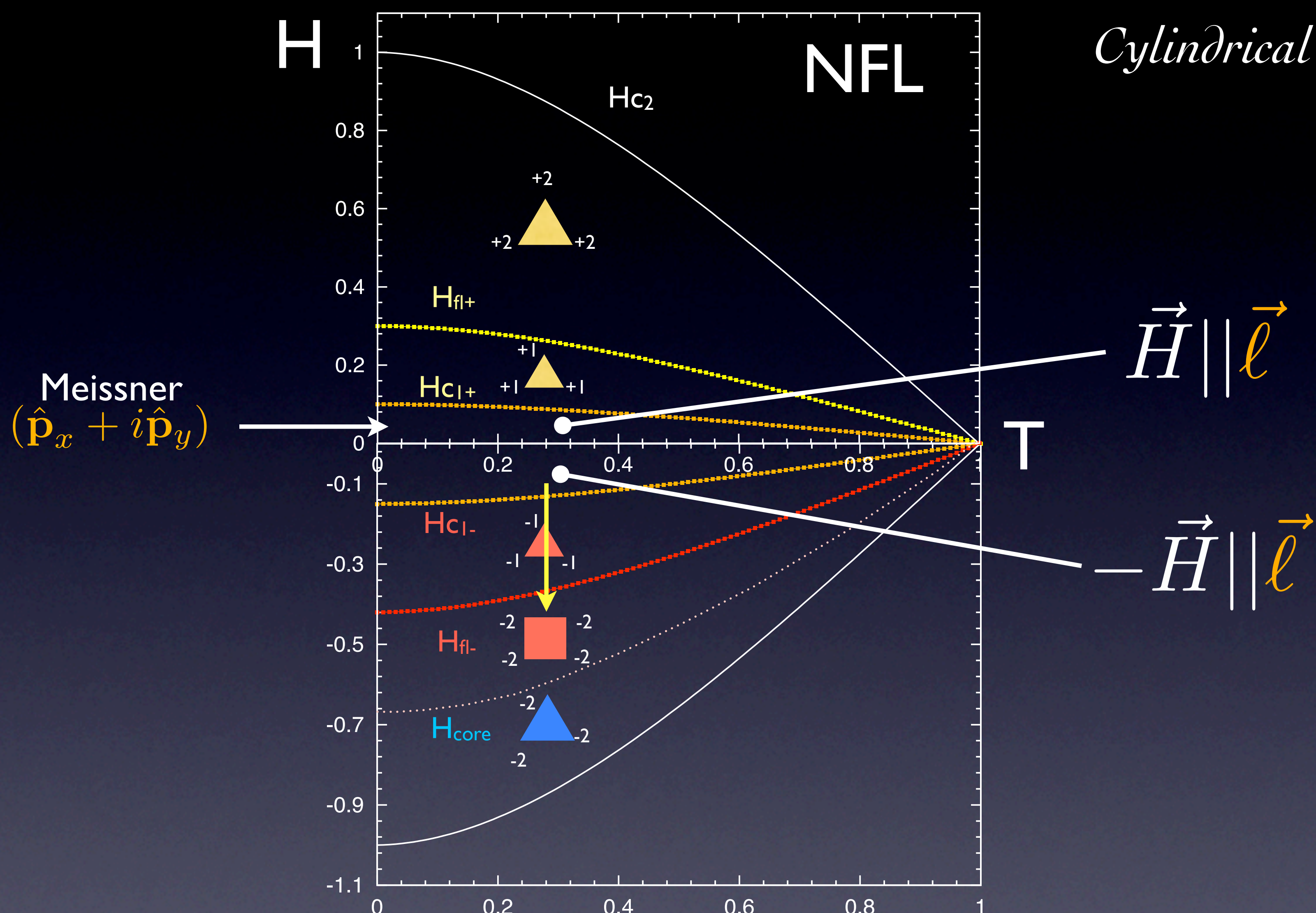

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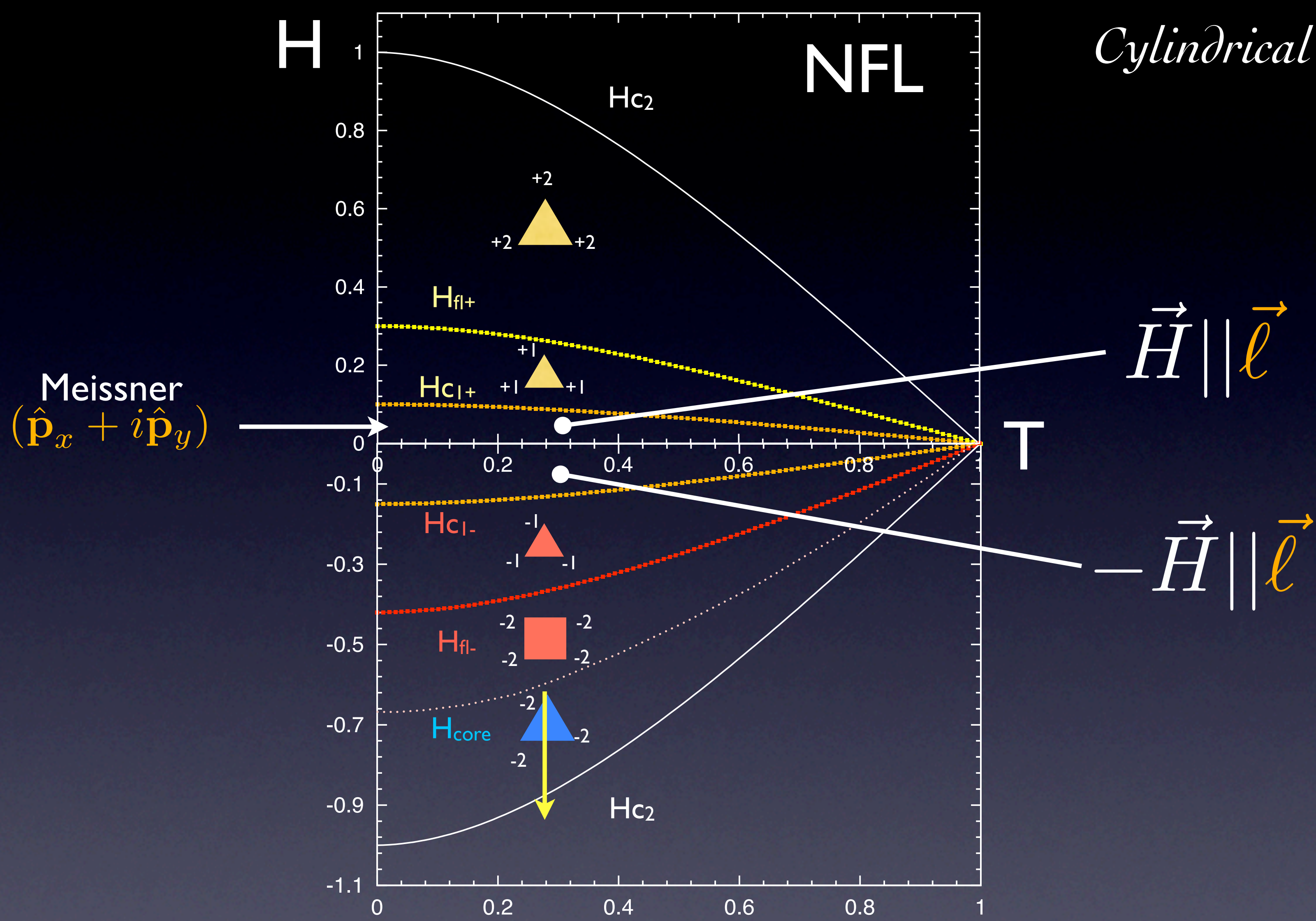

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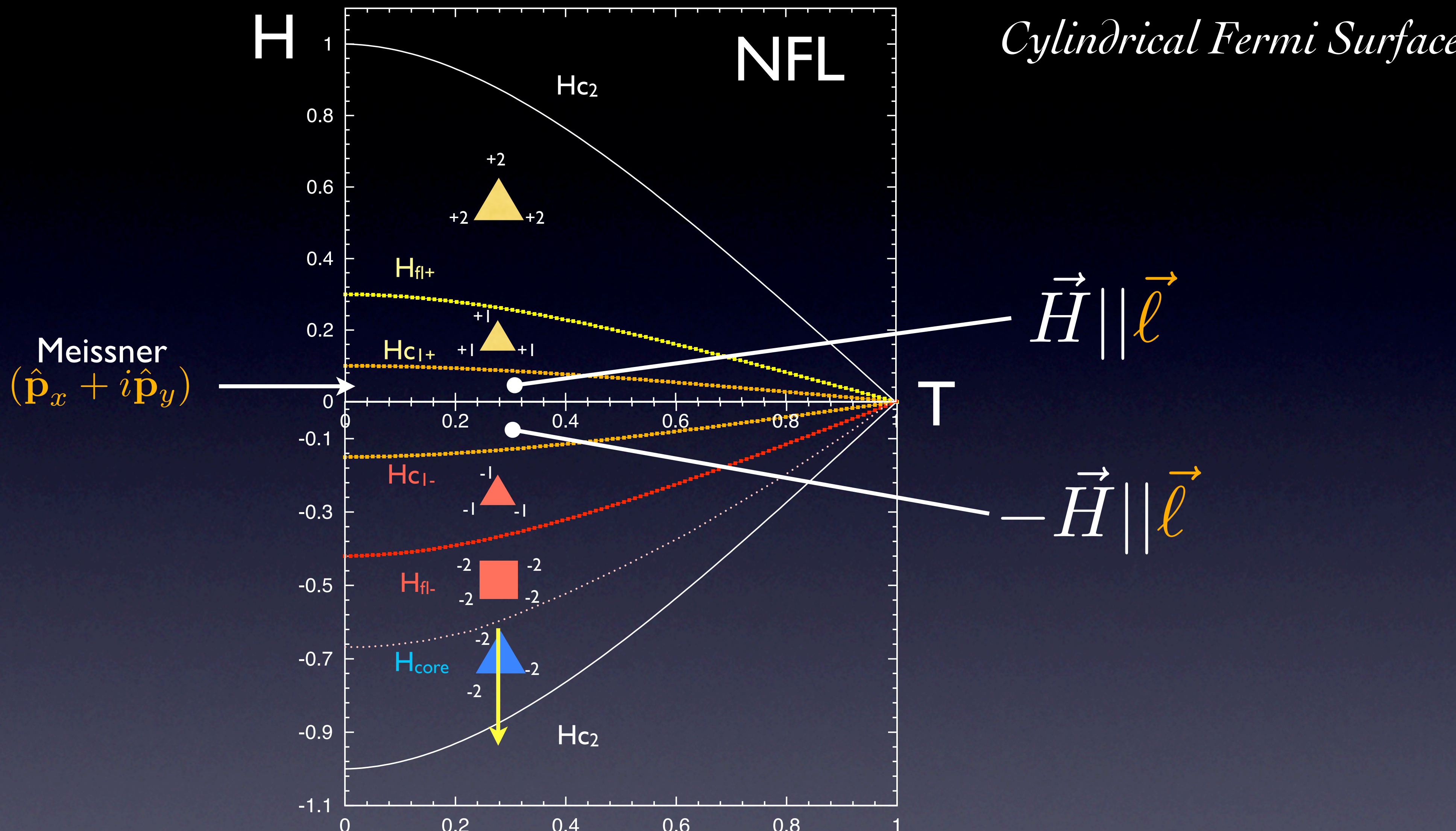



- ✓ $H_{c1+} = H_{c1-}$ due to inequivalent Vortex Core Energies
- ✓ Lattices of 2-quantum vortices stable for intermediate to high fields
- ✓ History-dependent Phase diagram with additional vortex lattice phase

M. Ichioka, Okayama & Northwestern 2010

# Vortex structure in p- wave superconductors

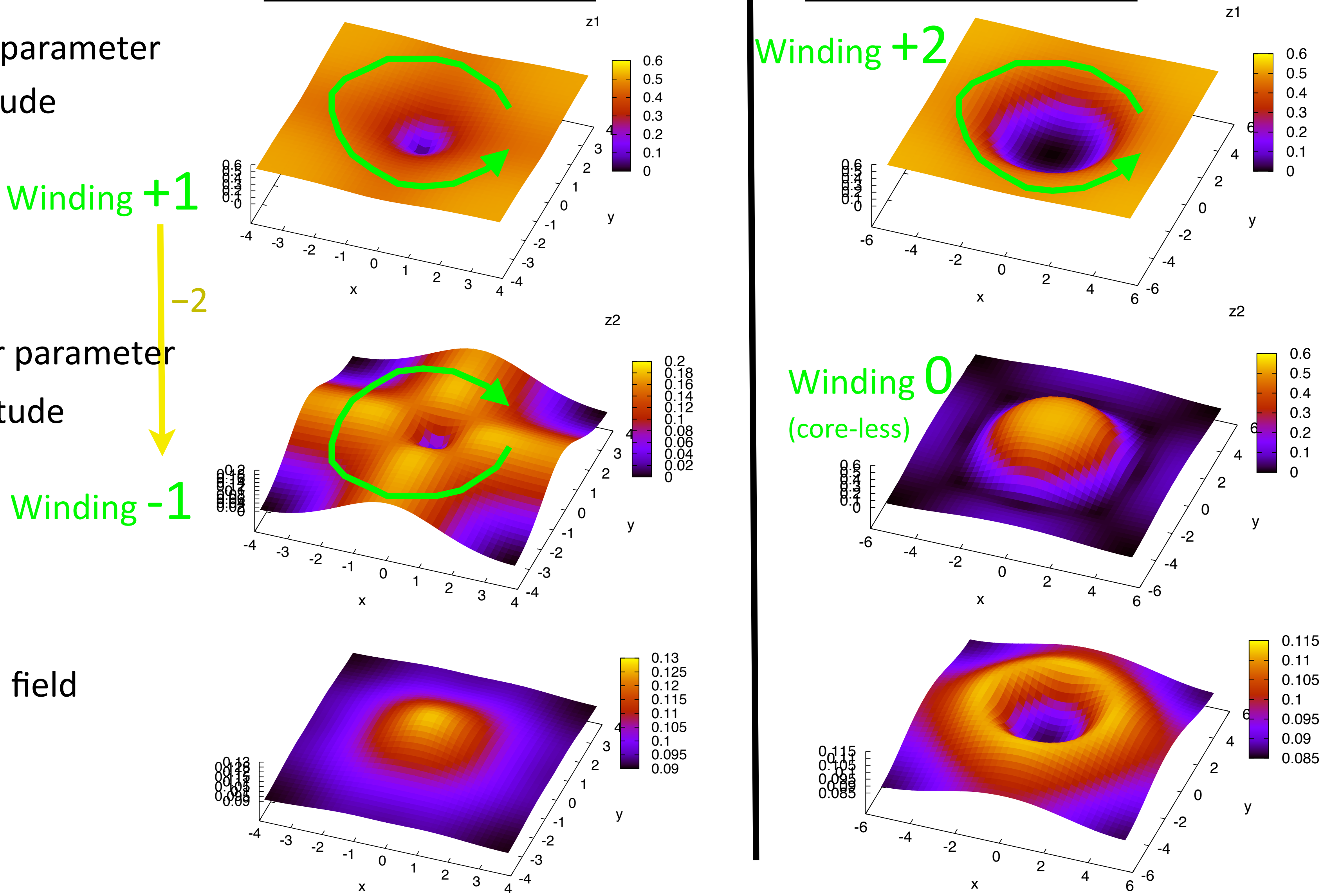
B=0.1 (low field), T=0.5, k=2.7(GL parameter), Square vortex lattice, 2D Fermi surface

Vortex winding +1 | Vortex winding +2

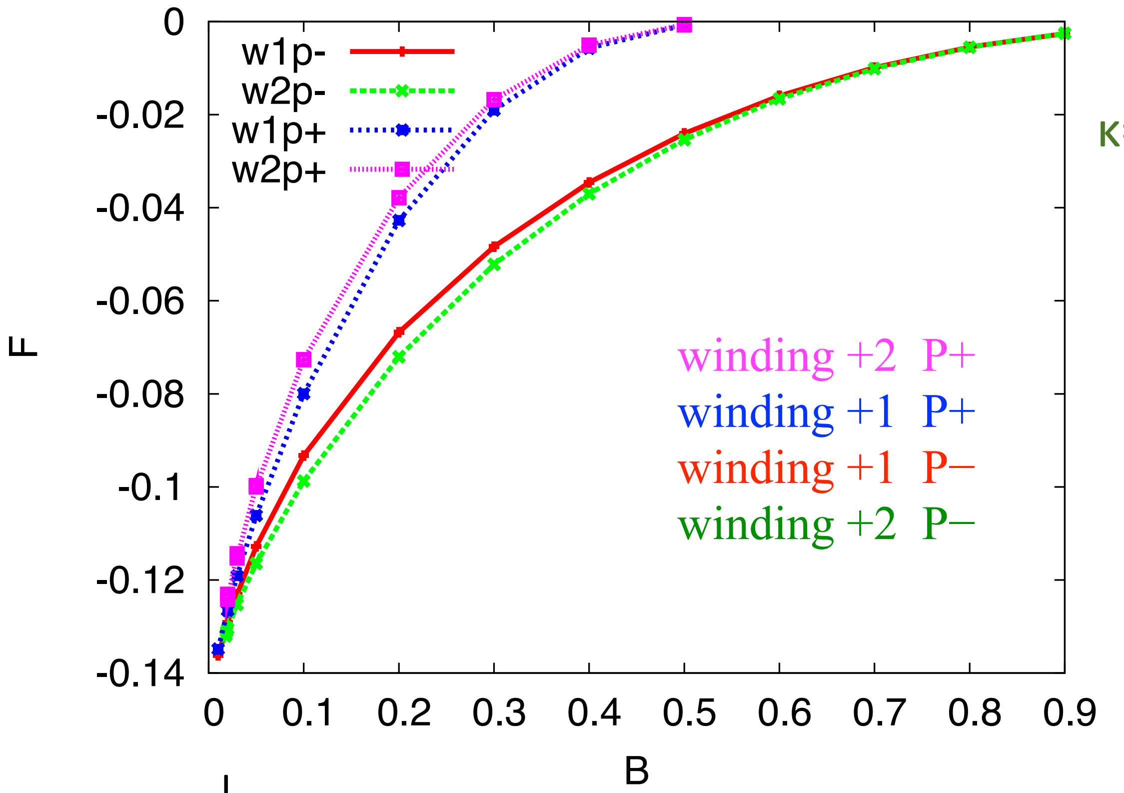
P- order parameter Amplitude

P+ order parameter Amplitude

Internal field



# B – dependence of free energy F  T=0.2Tc

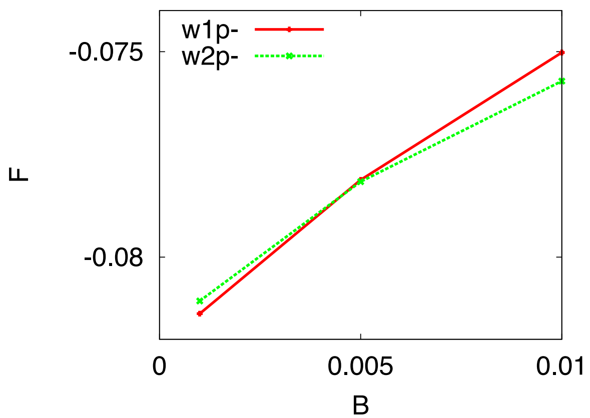



κ= 2.7

P+
H // z in p+ wave SC
H//-z in p- wave SC

↓

Low fields  κ= 30




[Stable vortex states in isotropic
p- wave superconductors]  T=0.2Tc
0 < B < 0.005 : winding +1 vortex
0.005 < B < 0.9 : winding +2 vortex
0.9 < B < 1(Hc2) : winding +1 vortex

# SANS experiment on $Sr_2RuO_4$

Small Angle Neutron Scattering

Riseman, *et al.*, Nature **396** (1998) 242
& **404** (2000) 629 (E).

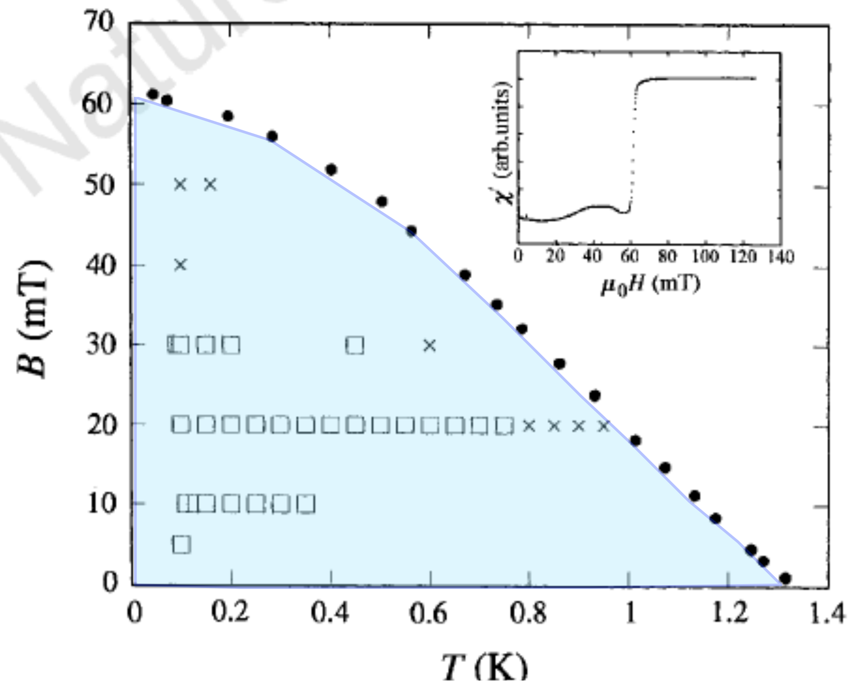



**Figure 2** Observations in the $B$–T plane of a square FLL. By neutron scattering, a square FLL was observed at points marked with a square; at those marked with a cross, there was insufficient intensity to detect a FLL. The temperature dependence of $B_{c2}$ for our sample with field parallel to **c** is also shown (filled circles). The transition was determined by measurement of the in-phase response of the a.c. susceptibility, $\chi'$; a typical trace (at $T = 70$ mK) is shown as the inset.

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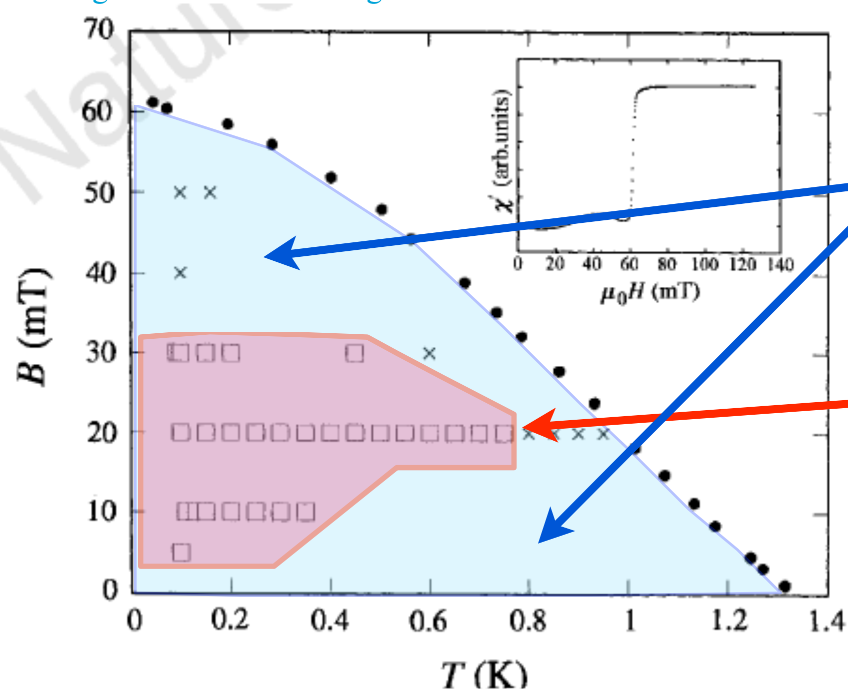


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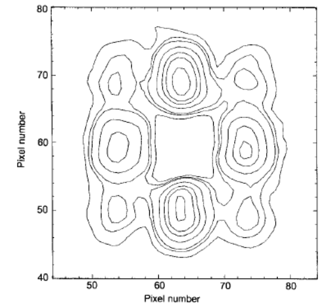
No VL
Detected




(110)

In wide (T,B) region
winding +1 vortex
square ◇ vortex lattice

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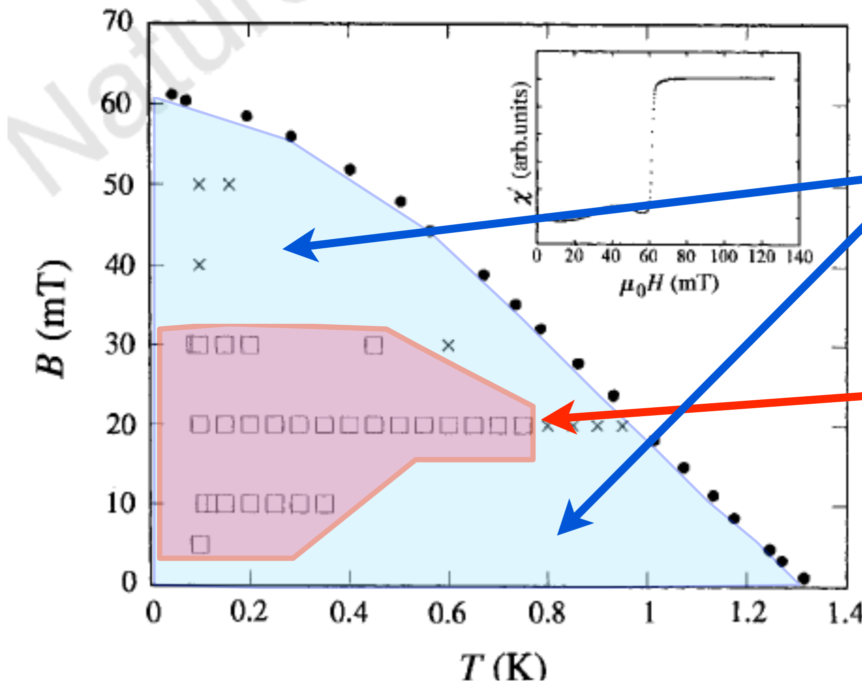


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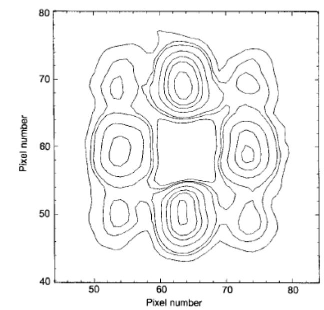


→ (110)

In wide (T,B) region
winding +1 vortex
square ◇ vortex lattice

GL theory (chiral p-wave SC & Fermi surface)

⇒ Square □ vortex lattice

Agterberg *et al.*

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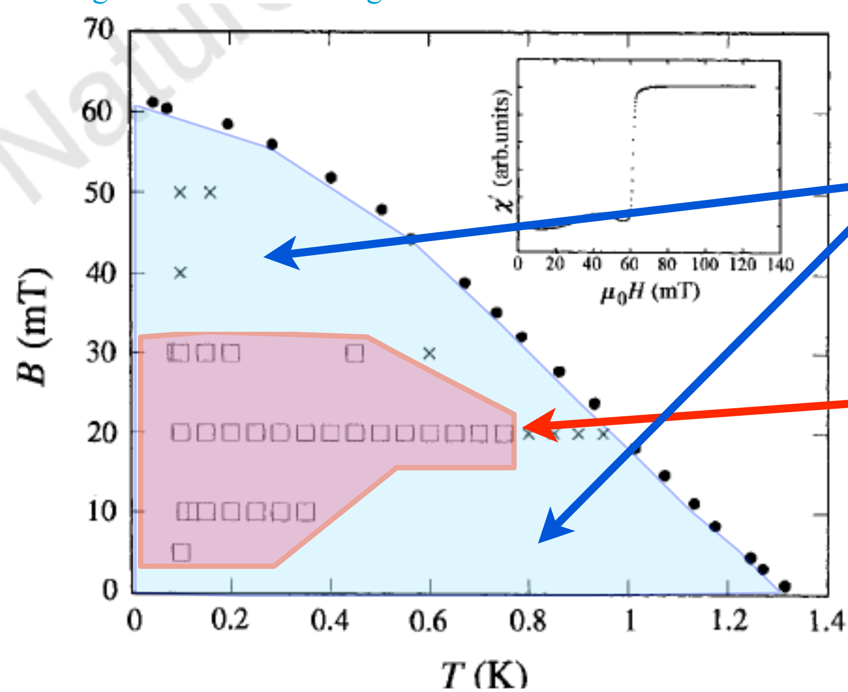


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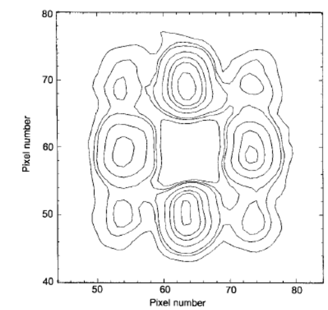
No VL Detected




→ (110)

In wide (T,B) region
winding +1 vortex
square ◇ vortex lattice

GL theory (chiral p-wave SC & Fermi surface)
⟹ Square □ vortex lattice

Agterberg *et al.*

Winding +2 vortex is not observed.
↓
$Sr_2RuO_4$ is not
an isotropic chiral p-wave

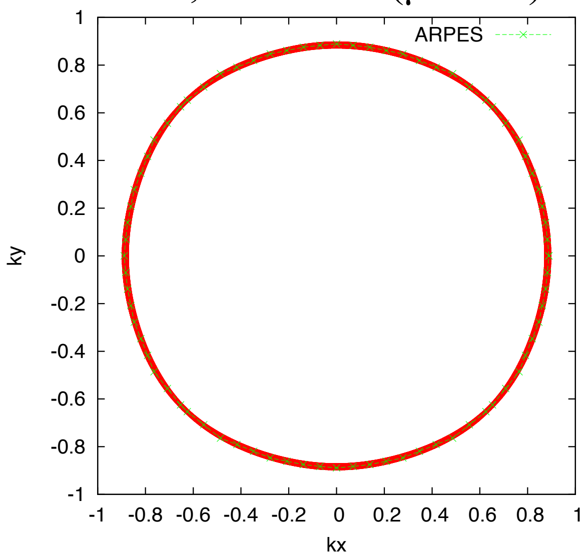
➡ Include:
➡ Realistic Fermi Surface
➡ Gap Anisotropy

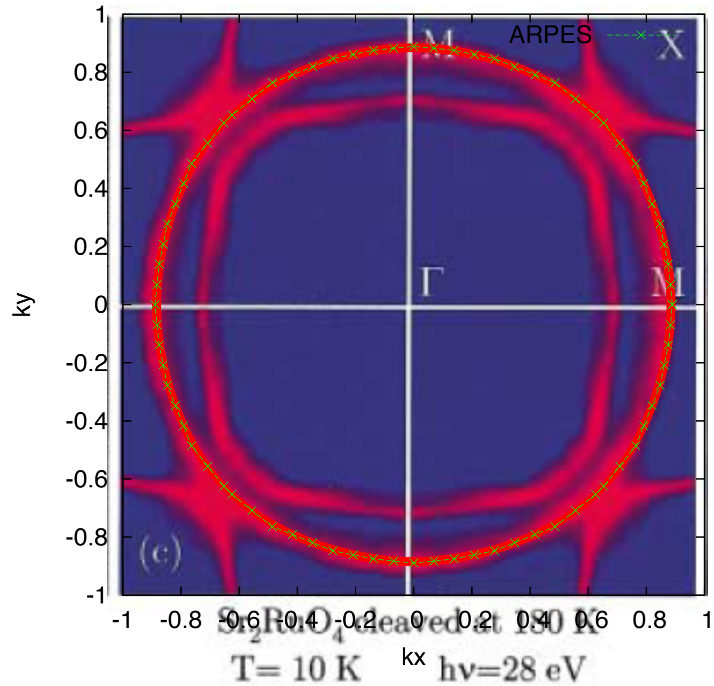
# $Sr_2RuO_4$ γ-Fermi surface

Fitting by tight-binding model

$$E(\mathbf{k}) = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y = \mu$$

t'/t=0.41, n=1.3845 (μ=1.25)




ARPES : Damascelli et al. PRL85(2000)5194

## Gap function of chiral p-wave

$p_x \pm ip_y \rightarrow \phi_\pm = \sin p_x \pm i \sin p_y$

$p_{x+y} \pm ip_{-x+y} \rightarrow \phi_\pm = \sin(p_x+p_y) \pm i \sin(-p_x+p_y)$

Arita, Onari, Kuroki, Aoki, PRL 92 (2004) 247006

Hubbard model U+V (γ-band) → Pairing symmetry $p_{x+y} \pm ip_{-x+y}$

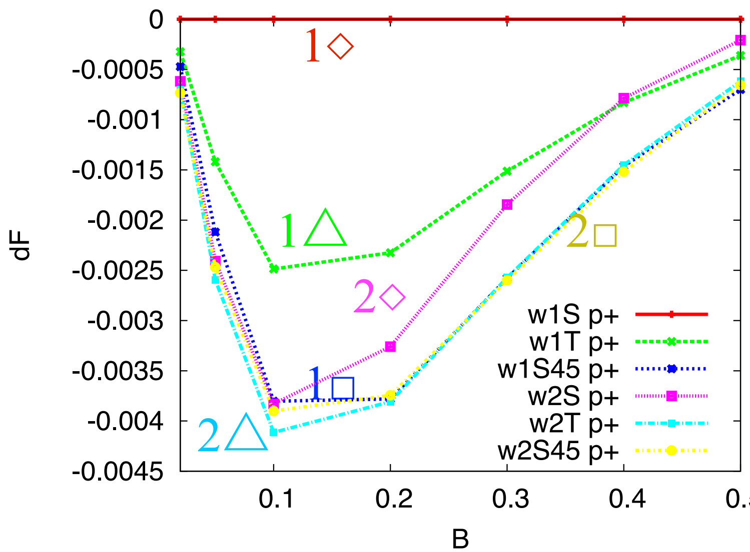
M. Ichioka, Okayama & Northwestern 2010 (Preliminary Results)

# Free-energy difference between some vortex states : B-dependence

Tight binding model : t'/t=0.41, n=1.3845 (μ=1.25)     T=0.5Tc, κ=2.7

## vortex winding +1 & +2

### $p_x \pm ip_y$



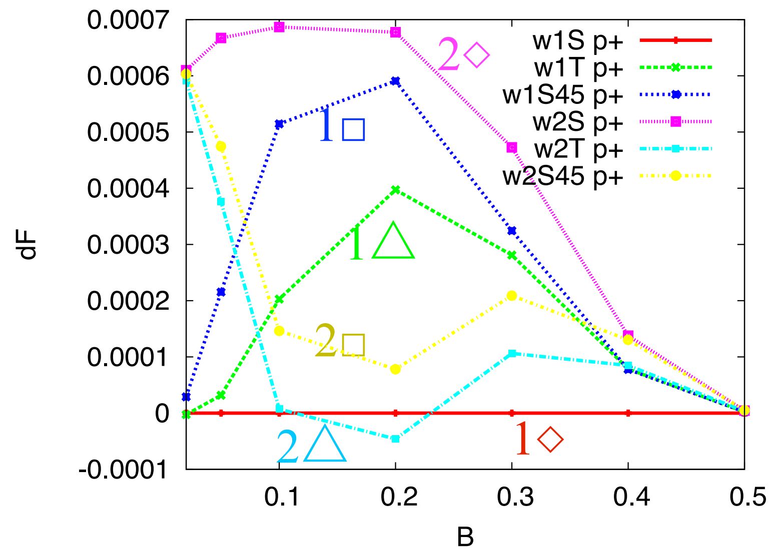

F□ < F△ < F◇ (within winding +1)

Stable

Consistent to GL theory
(Agterberg)

Wide region of winding +2 vortex
(B < 0.5 )

### $p_{x+y} \pm ip_{-x+y}$




w1: winding-1
w2: winding-2
p+: p+ main
p- : p- main
S: Square ◇ vortex lattice
S45: Square □ vortex lattice
T: Triangular vortex lattice

F◇ < F△ < F□ (within winding +1)

Stable

Consistent to experiment on $Sr_2RuO_4$
(Square ◇ lattice in wide region)

Stable region of winding +2 vortex becomes smaller. (only B ~ 0.2 )

# a.c. EM Response of Pancake Vortices

Dirty Type II SC $\ell << \xi_0$

``normal core'' $\sigma_{\mathrm{core}} = \sigma_{\mathrm{N}}$

Bardeen-Stephen Model

$$\rho_{\mathrm{ff}} = (B/\Phi_0) \; \rho_{\mathrm{N}}$$

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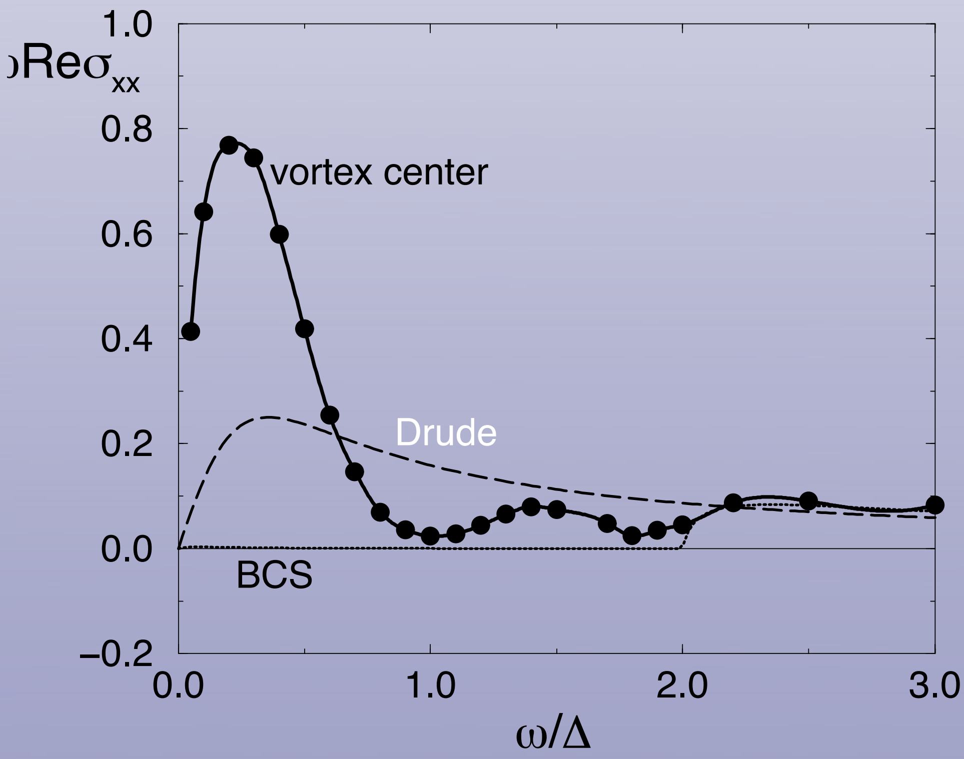
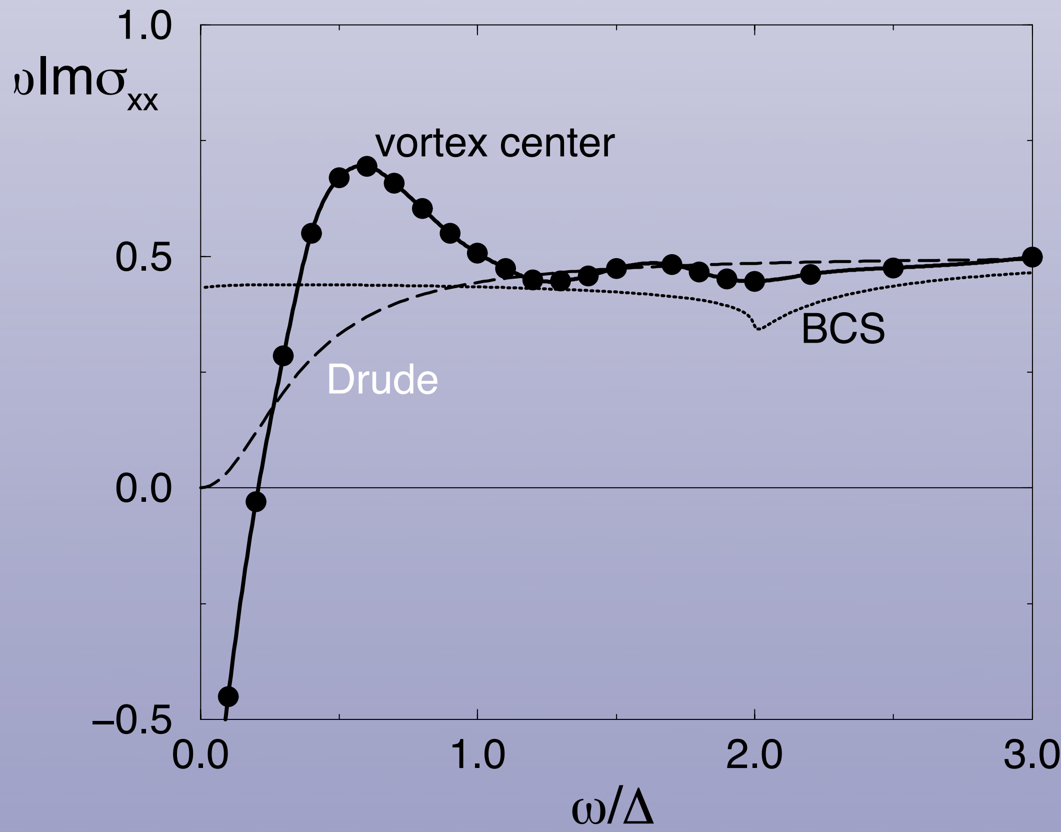
L. Gorkov & N. Kopnin, A. Larkin & Ovchinnikov

## Clean Type II SC: dynamics of Andreev Levels & collective mode




. Eschrig, D. Rainer & JAS, PRB 60 10447 (1999)

Pinned Single Quantum Vortex

# Vortex Charge and Current Response

| superclean case | moderately clean | dirty limit |
| --- | --- | --- |
| $\ell \gg \xi_0 \dfrac{E_f}{\Delta}$ | $\xi_0 \dfrac{E_f}{\Delta} \gg \ell \gg \xi_0$ | $\xi_0 \gg \ell \gg \xi_0 \dfrac{\Delta}{E_f}$ |
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M. Eschrig and J.A. Sauls, New J. Phys. **11**, 075009 (2009).

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# Dynamical Charge Response of Vortices in Chiral Superconductor

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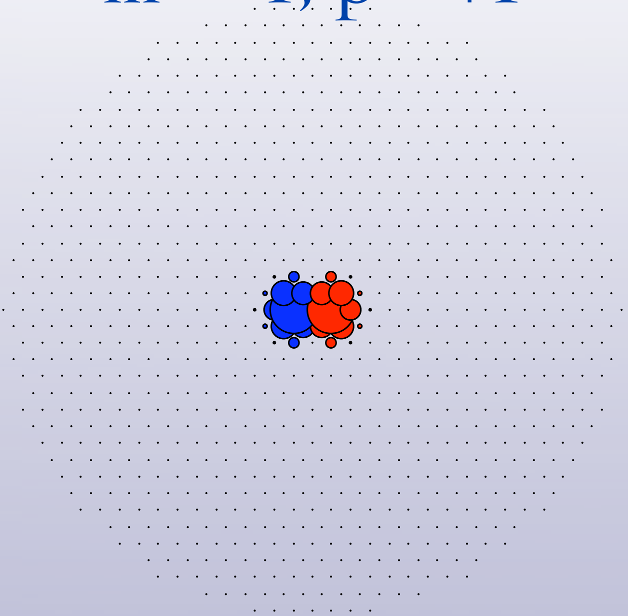
m = -1, p = +1



$\delta\rho_\omega$

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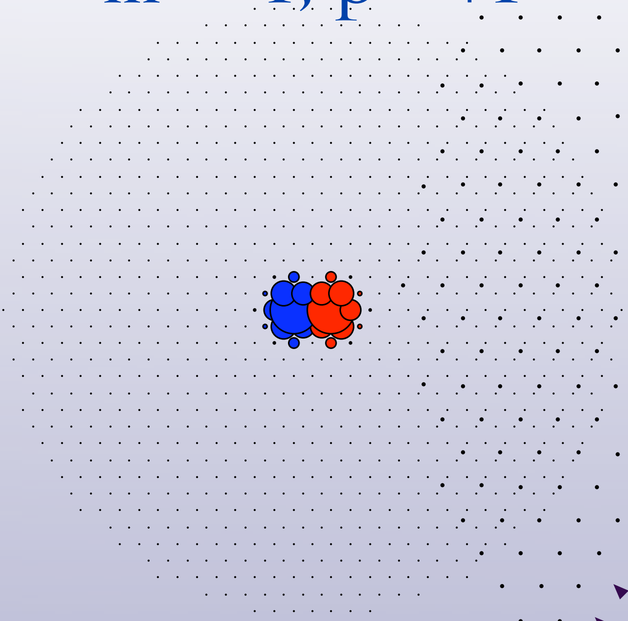

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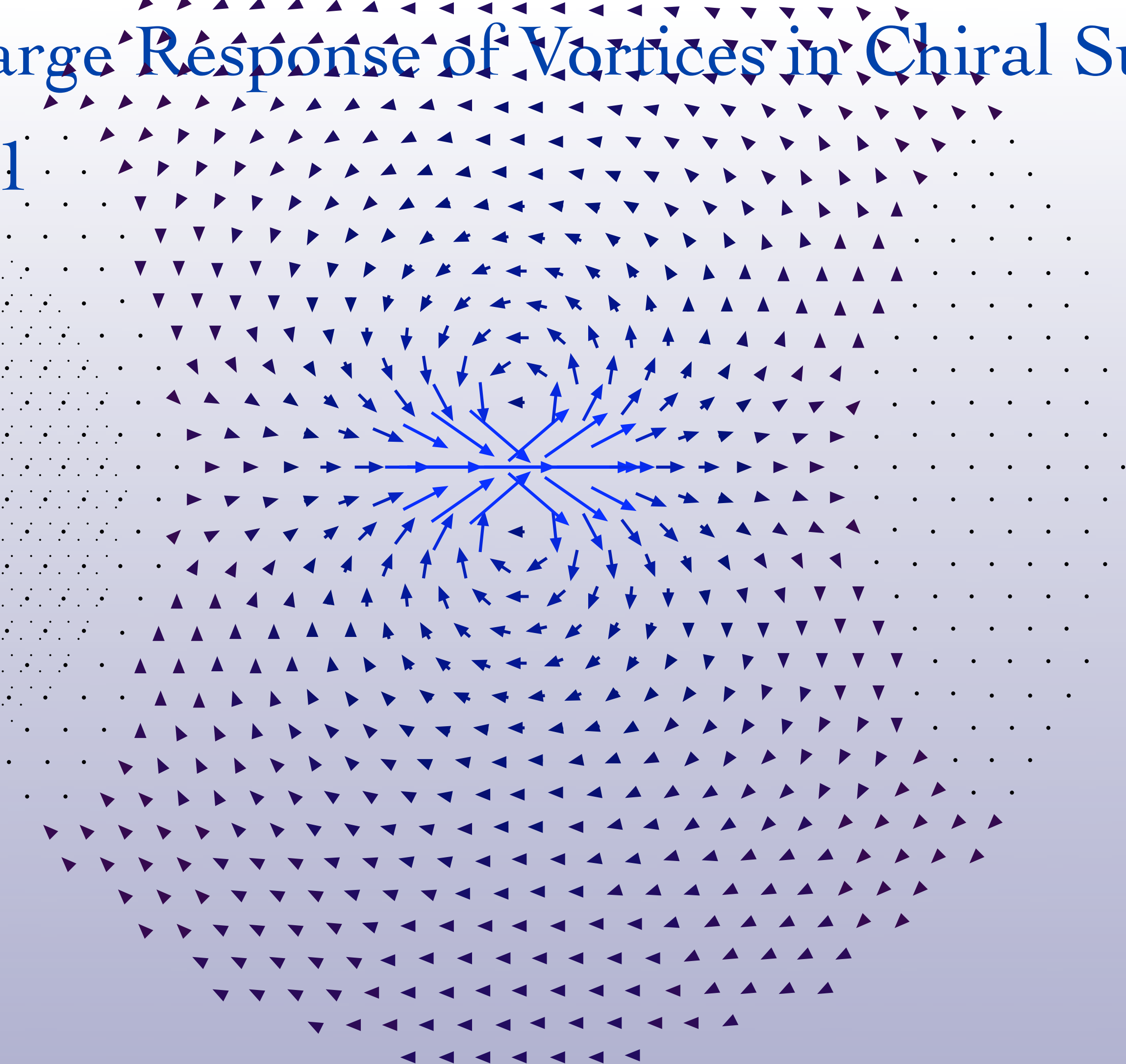

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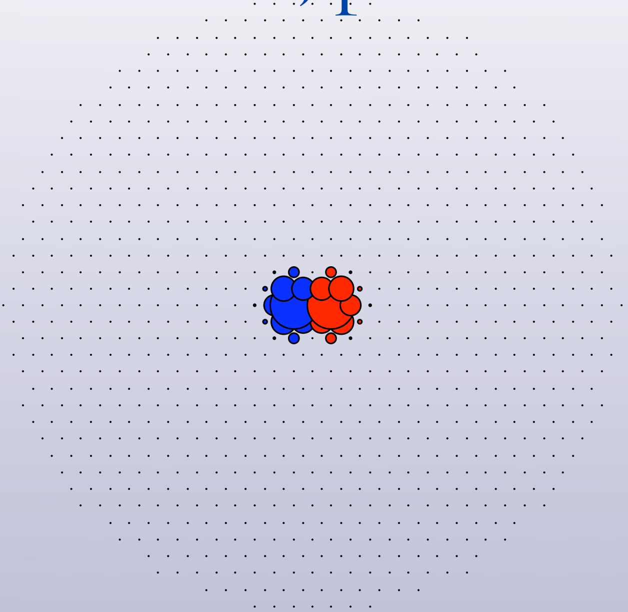

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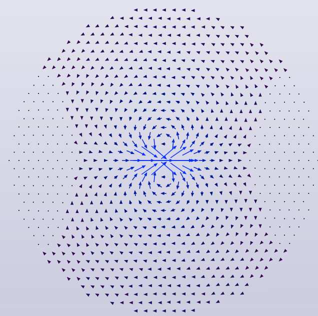

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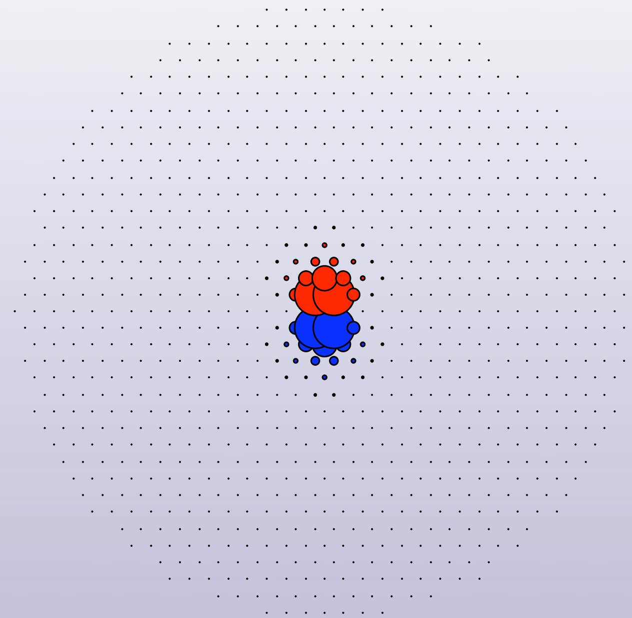

$\delta\vec{\mathbf{B}}_\omega = \boldsymbol{\nabla}\times\vec{\mathbf{A}}_\omega$

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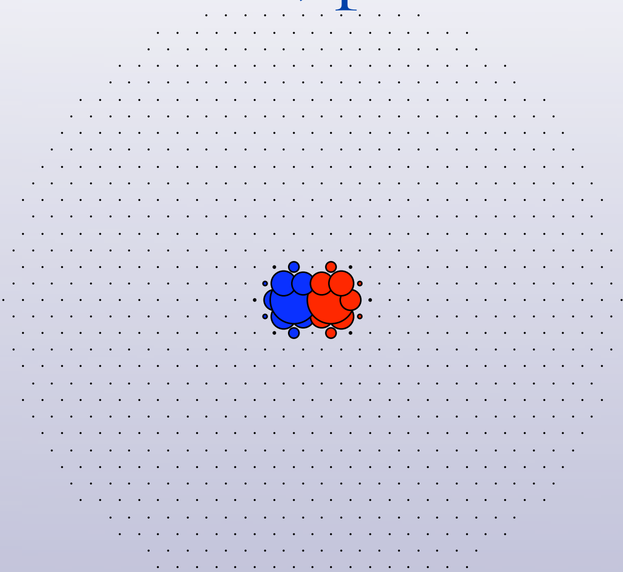

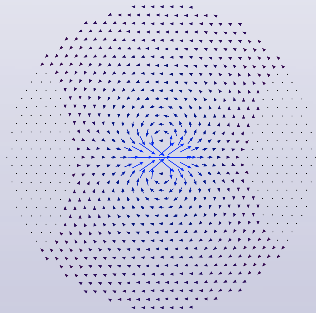

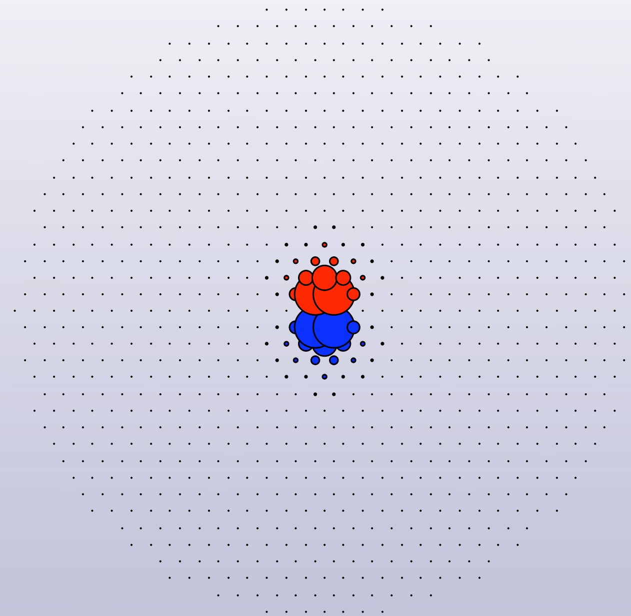

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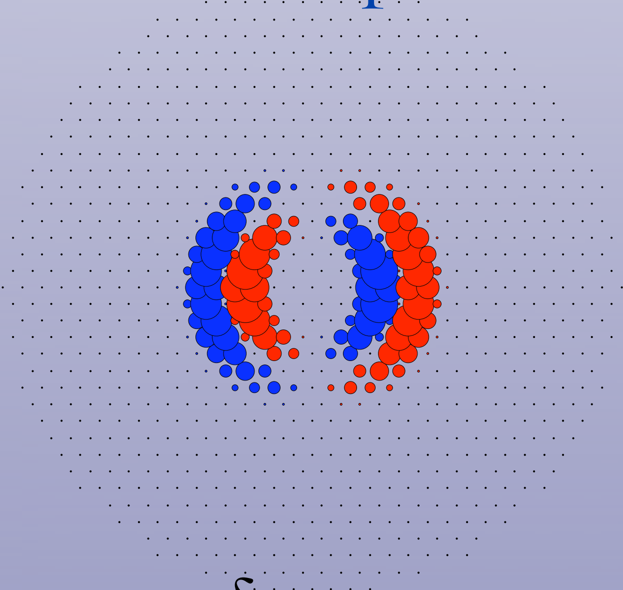

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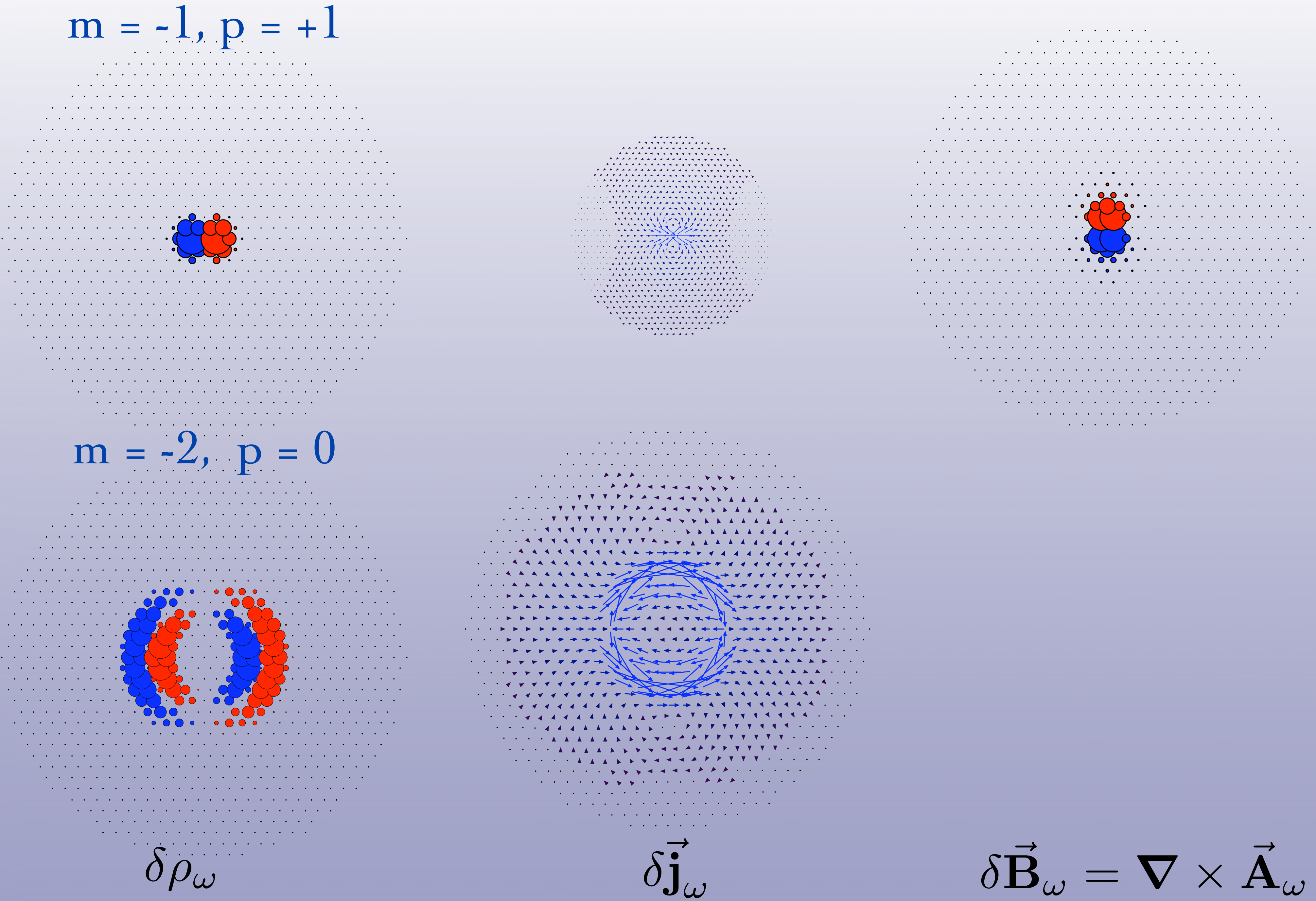

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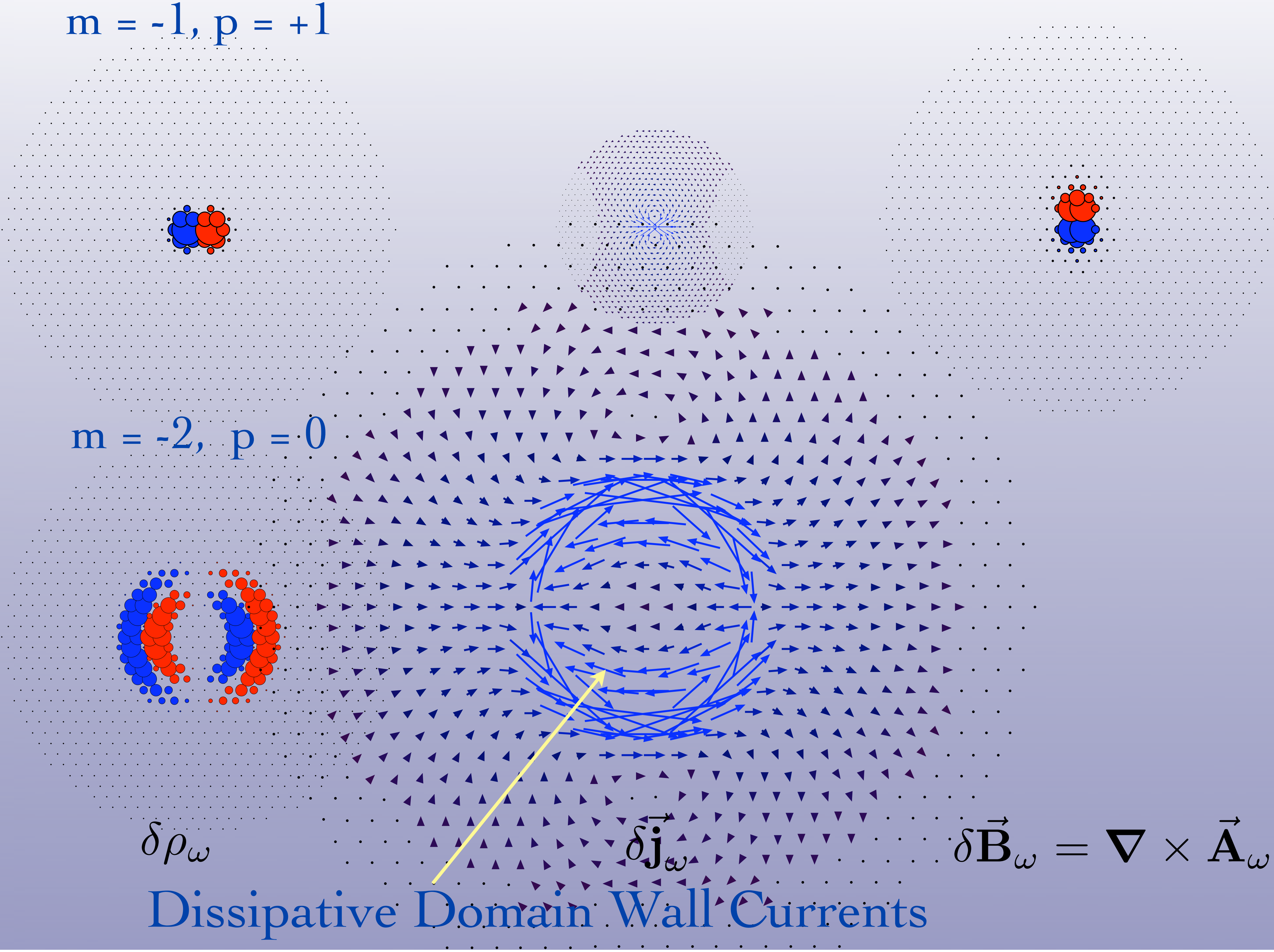

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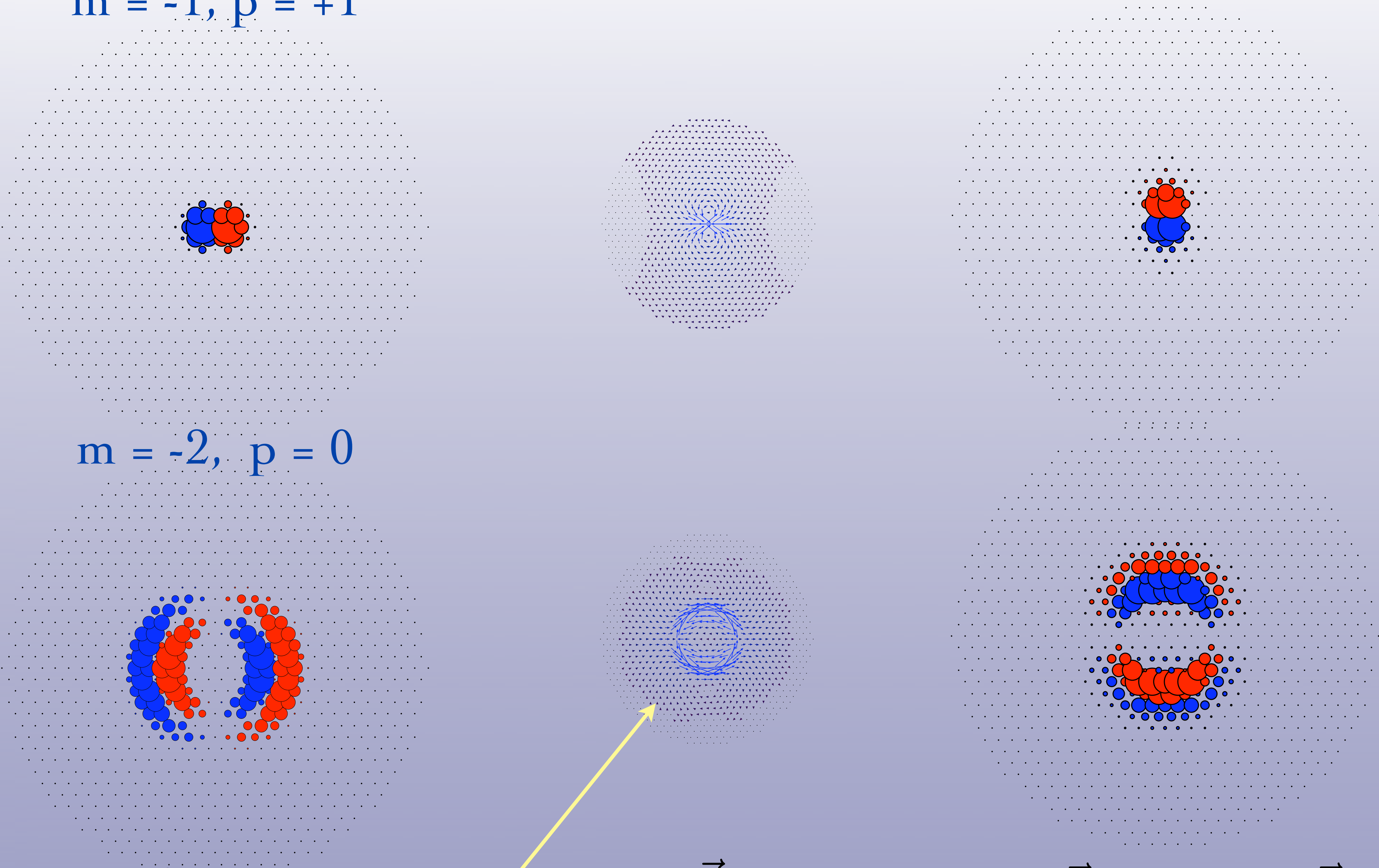

$\delta\rho_\omega$ $\delta\vec{\mathbf{j}}_\omega$ $\delta\vec{\mathbf{B}}_\omega = \boldsymbol{\nabla} \times \vec{\mathbf{A}}_\omega$

Dissipative Domain Wall Currents

# Summary

Topological Defects - signatures of chiral ground state

Structure of the core - asymmetry in $H_{c1}$

Doubly Quantized vortices with low core energy

Vortices with broken axial symmetry

Multiple Flux Lattice structures at Low T/High B

Vortex core electrodynamics dominated by ABS dynamics

Large dynamical charge response

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✓ Novel Topological Defects and Phases waiting to be discovered in novel superfluid and superconducting materials