

# *Signatures of Broken Symmetries in $^3\text{He}$ and Chiral Superconductors*

J. A. Sauls

Northwestern University

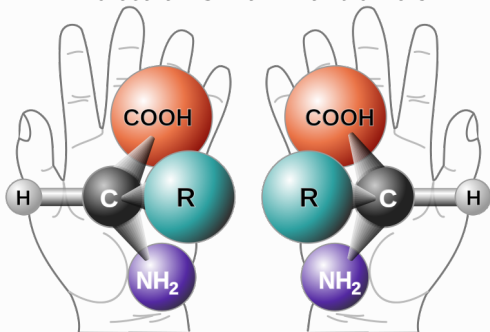
• Wave Ngampruetikorn • Oleksii Shevtsov • Joshua Wiman

- ▶ Broken P & T Symmetry -  $^3\text{He-A}$
- ▶ Anomalous Hall Effect in  $^3\text{He-A}$
- ▶ Edge Fermions & Left-Handed Electrons
- ▶ An Unsolved Problem ... or two

▶ Supported by National Science Foundation Grant DMR-1508730

# Chirality in Nature

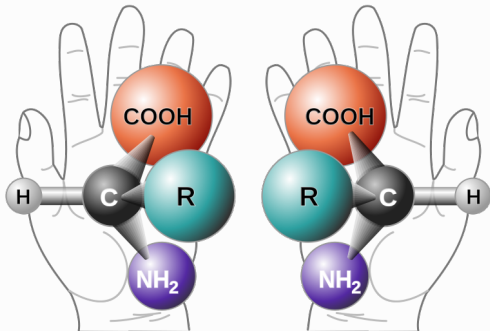
Molecular Chiral Enantiomers



Handedness: Broken Mirror Symmetry

# Chirality in Nature

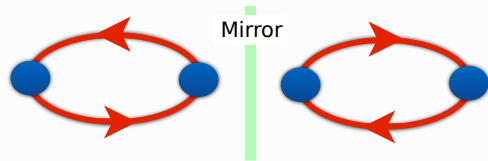
## Molecular Chiral Enantiomers



Handedness: Broken Mirror Symmetry

## Chiral Diatomic Molecules

$$\Psi(\mathbf{r}) = f(r) (x + iy)$$



## Broken Mirror Symmetries

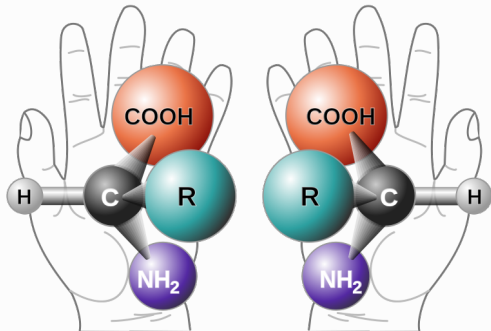
$$\Pi_{zx} \Psi(\mathbf{r}) = f(r) (x - iy)$$

## Broken Time-Reversal Symmetry

$$\mathcal{T} \Psi(\mathbf{r}) = f(r) (x - iy)$$

# Chirality in Nature

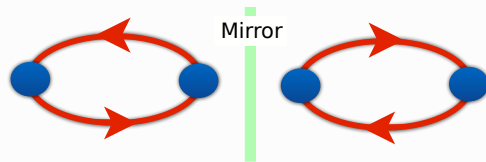
Molecular Chiral Enantiomers



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Broken Mirror Symmetries

$$\Pi_{zx} \Psi(\mathbf{r}) = f(r) (x - iy)$$

Broken Time-Reversal Symmetry

$$\mathcal{T} \Psi(\mathbf{r}) = f(r) (x - iy)$$

Realized in Superfluid <sup>3</sup>He-A & possibly the ground states in unconventional superconductors

# Chiral Superconductors

## Ground states exhibiting:

- ▶ Emergent Topology of a Broken-Symmetry Vacuum of Cooper Pairs
- ▶ Weyl-Majorana excitations of the Vacuum
- ▶ Ground-State Edge Currents and Angular Momentum
- ▶ Broken P and T  $\rightsquigarrow$  **Anomalous Hall Transport**

# Chiral Superconductors

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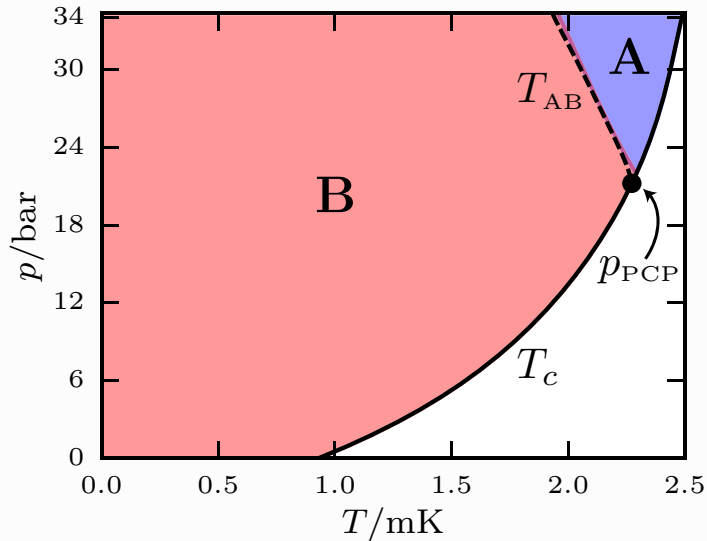
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## Where are They?

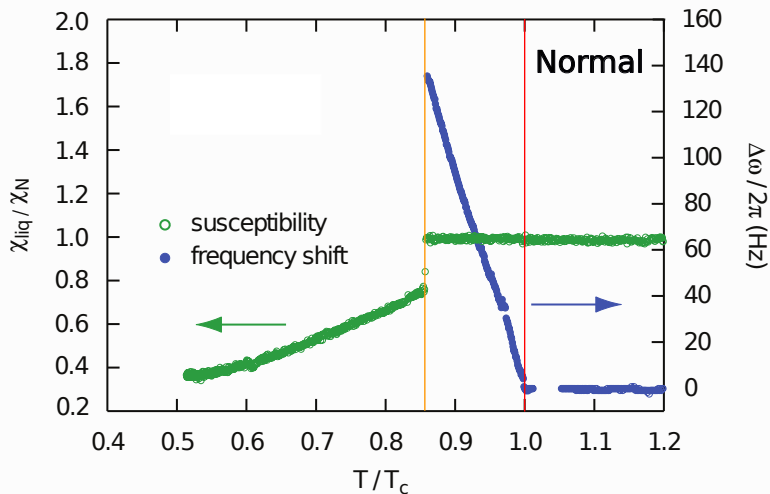
- ▶  $^3\text{He-A}$ : definitive chiral p-wave condensate; quantitative theory-experimental confirmation
- ▶  $\text{Sr}_2\text{RuO}_4$ : proposed as the electronic analog of  $^3\text{He-A}$ ; evidence of chirality
- ▶  $\text{UPt}_3$ : electronic analog to  $^3\text{He}$ : Multiple Superconducting Phases; evidence of chirality
- ▶ Other candidates:  $\text{URu}_2\text{Si}_2$ ;  $\text{SrPtAs}$ , doped graphene ...

## The Pressure-Temperature Phase Diagram for Liquid $^3\text{He}$

Maximal Symmetry:  $G = \text{SO}(3)_S \times \text{SO}(3)_L \times \text{U}(1)_N \times \text{P} \times \text{T} \rightarrow$  Superfluid Phases of  $^3\text{He}$



## NMR frequency shift and Magnetic Susceptibility





Interpretation of Recent Results on  $\text{He}^3$  below 3 mK: A New Liquid Phase?

A. J. Leggett

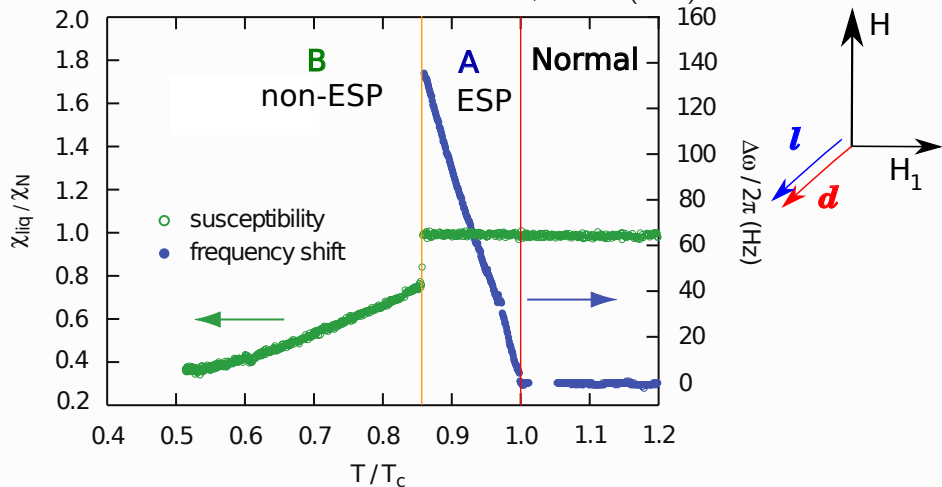
*School of Mathematical and Physical Sciences, University of Sussex, England*

(Received 5 September 1972)

It is demonstrated that recent NMR results in  $^3\text{He}$  indicate that at 2.65 mK, the liquid makes a second-order transition to a phase in which the "spin-orbit" symmetry is spontaneously broken. The hypothesis that this phase is a BCS-type phase in which pairs form with  $l$  odd,  $S=1$ ,  $S_z=\pm 1$  leads to reasonable agreement with both NMR and thermodynamic data, but involves some difficulties as to stability.

$$\omega^2 = (\gamma H)^2 + \Omega^2(T)$$
$$\Omega^2 = -\frac{2\gamma^2}{\chi} \langle \mathcal{H}_D \rangle \quad \Omega \neq 0 \implies \text{Broken Spin-Orbit Symmetry}$$
$$\omega - \gamma H \simeq \frac{\Omega^2(T)}{2\gamma H} \propto (1 - T/T_c)$$

J. Pollanen et al. PRL 107, 195301 (2011)



►  $\mathbf{d}_B(\mathbf{p}) = \Delta_B(T) \left( \hat{p}_x \hat{\mathbf{d}}_x + \hat{p}_y \hat{\mathbf{d}}_y + \hat{p}_z \hat{\mathbf{d}}_z \right)$   
 $\rightsquigarrow G \rightarrow SO(3)_J \times T$

►  $\mathbf{d}_A(\mathbf{p}) = \Delta_A(T) \hat{\mathbf{d}} (\hat{p}_x + i\hat{p}_y)$   
 $\rightsquigarrow G \rightarrow SO(2)_S \times U(1)_{L_z-N} \times Z_2$

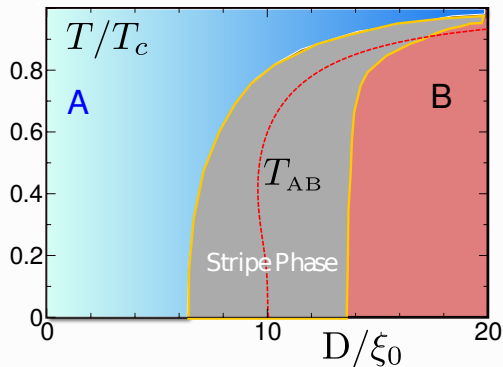
# Realization of Broken Time-Reversal and Mirror Symmetry by the Vacuum State of $^3\text{He}$ Films

## ► Length Scale for Strong Confinement:

$$\xi_0 = \hbar v_f / 2\pi k_B T_c \approx 20 - 80 \text{ nm}$$

► L. Levitov et al., Science 340, 6134 (2013)

► A. Vorontsov & J. A. Sauls, PRL 98, 045301 (2007)

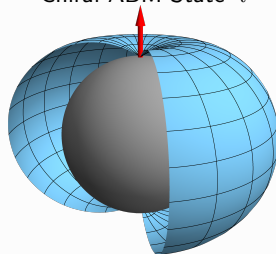


$$\text{SO}(3)_S \times \text{SO}(3)_L \times \text{U}(1)_N \times \text{T} \times \text{P}$$

↓

$$\text{SO}(2)_S \times \text{U}(1)_{N-L_z} \times \text{Z}_2$$

Chiral ABM State  $\vec{l} = \hat{z}$



$$L_z = 1, S_z = 0$$

Ground-State Angular Momentum

$$\langle \hat{L}_z \rangle = \frac{N}{2} \hbar ?$$

Open Question

$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix}_{ABM} = \begin{pmatrix} p_x + ip_y \sim e^{+i\phi} & 0 \\ 0 & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

## Signatures of Broken T and P Symmetry in $^3\text{He-A}$

What is the Evidence for Chirality of Superfluid  $^3\text{He-A}$ ?

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Broken T and P  $\rightsquigarrow$  Anomalous Hall Effect for Electrons in  $^3\text{He-A}$

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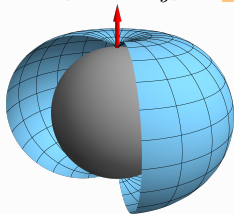
Broken T and P  $\rightsquigarrow$  Anomalous Hall Effect for Electrons in  $^3\text{He-A}$

Broken Symmetries  $\rightsquigarrow$  Topology of  $^3\text{He-A}$

Chirality + Topology  $\rightsquigarrow$  Chiral Edge States

Topology in Momentum Space

$$\Psi(\mathbf{p}) = \Delta(p_x \pm ip_y) \sim e^{\pm i\varphi_{\mathbf{p}}}$$



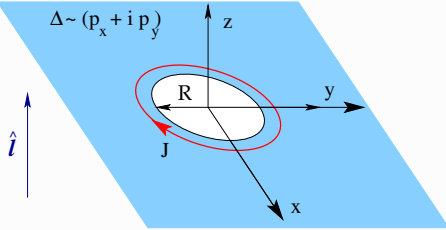
Winding Number of the Phase:

$$L_z = \pm 1$$

$$N_{2D} = \frac{1}{2\pi} \oint d\mathbf{p} \cdot \frac{1}{|\Psi(\mathbf{p})|} \text{Im}[\nabla_{\mathbf{p}} \Psi(\mathbf{p})] = L_z$$

- ▶ Massless Chiral Fermions
- ▶ Nodal Fermions in 3D
- ▶ Edge Fermions in 2D

## Chiral Edge Current Circulating a Hole or Defect in a Chiral Superfluid



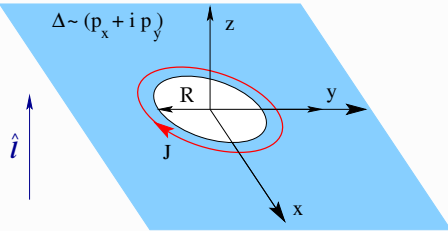
►  $R \gg \xi_0 \approx 100 \text{ nm}$

► Sheet Current :

$$J \equiv \int dx J_\varphi(x)$$



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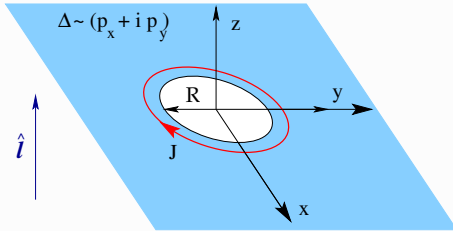
► Sheet Current :

$$J \equiv \int dx J_\varphi(x)$$

► Quantized Sheet Current:  $\frac{1}{4} n \hbar$  ( $n = N/V = {}^3\text{He}$  density)

► Edge Current *Counter-Circulates*:  $J = -\frac{1}{4} n \hbar$  w.r.t. Chirality:  $\hat{l} = +\mathbf{z}$

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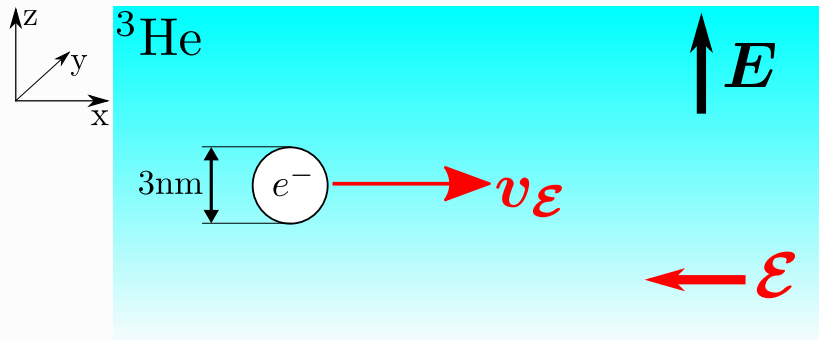
► Edge Current *Counter-Circulates*:  $J = -\frac{1}{4} n \hbar$  w.r.t. Chirality:  $\hat{l} = +\mathbf{z}$

► Angular Momentum:  $L_z = 2\pi \hbar R^2 \times (-\frac{1}{4} n \hbar) = -(N_{\text{hole}}/2) \hbar$

$N_{\text{hole}}/2 = \text{Number of } {}^3\text{He} \text{ Cooper Pairs excluded from the Hole}$

∴ An object in  ${}^3\text{He-A}$  *inherits* angular momentum from the Condensate of Chiral Pairs!

## Electron bubbles in the Normal Fermi liquid phase of $^3\text{He}$

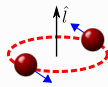


- ▶ Bubble with  $R \simeq 1.5 \text{ nm}$ ,  
 $0.1 \text{ nm} \simeq \lambda_f \ll R \ll \xi_0 \simeq 80 \text{ nm}$
- ▶ Effective mass  $M \simeq 100m_3$   
( $m_3$  – atomic mass of  $^3\text{He}$ )

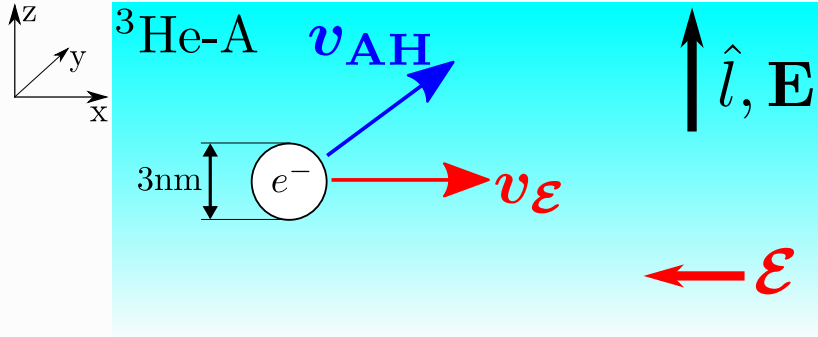
- ▶ QPs mean free path  $l \gg R$
- ▶ Mobility of  $^3\text{He}$  is *independent of  $T$*  for  
 $T_c < T < 50 \text{ mK}$

B. Josephson and J. Leckner, PRL 23, 111 (1969)

Electron bubbles in chiral superfluid  $^3\text{He-A}$



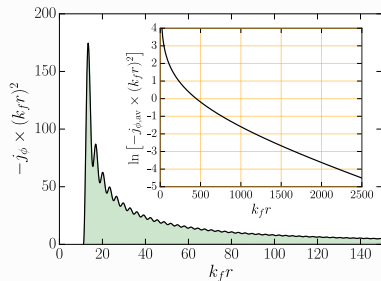
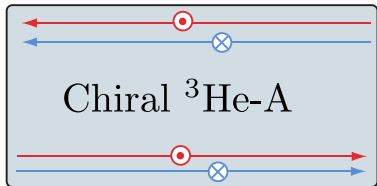
$$\Delta(\hat{k}) = \Delta(\hat{k}_x + i\hat{k}_y) = \Delta e^{i\phi_{\mathbf{k}}}$$



- Current:  $\mathbf{v} = \underbrace{\mu_{\perp} \boldsymbol{\mathcal{E}}}_{\mathbf{v}_{\mathcal{E}}} + \underbrace{\mu_{\text{AH}} \boldsymbol{\mathcal{E}} \times \hat{\mathbf{l}}}_{\mathbf{v}_{\text{AH}}}$  R. Salmelin, M. Salomaa & V. Mineev, PRL **63**, 868 (1989)

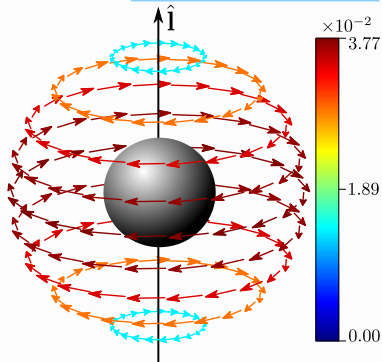
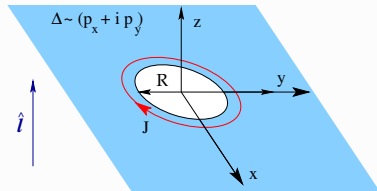
- Hall ratio:  $\tan \alpha = v_{\text{AH}}/v_{\mathcal{E}} = |\mu_{\text{AH}}/\mu_{\perp}|$

# Current bound to an electron bubble ( $k_f R = 11.17$ )



$$\mathbf{j}(\mathbf{r})/v_f N_f k_B T_c = j_\phi(\mathbf{r}) \hat{\mathbf{e}}_\phi$$

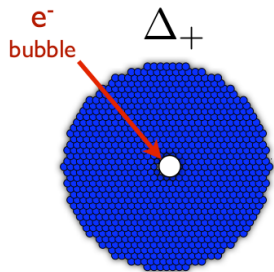
► O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)



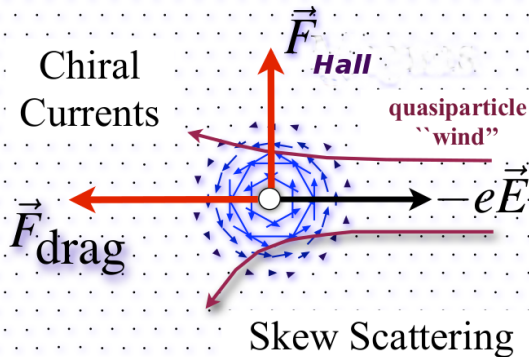
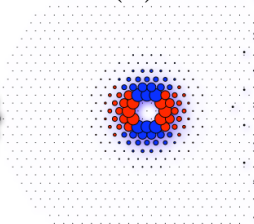
$$\mathbf{L}(T \rightarrow 0) \approx -\hbar N_{\text{bubble}}/2 \hat{\mathbf{i}} \approx -100 \hbar \hat{\mathbf{i}}$$

$\vec{\ell} = +\hat{z}$       Structure of an Ion embedded in  $^3\text{He-A}$

$(p_x + ip_y)$        $(p_x - ip_y)$

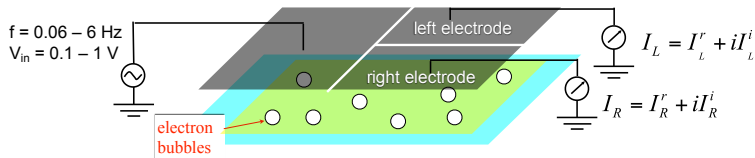


$\Delta_-(r) e^{+i2\phi}$

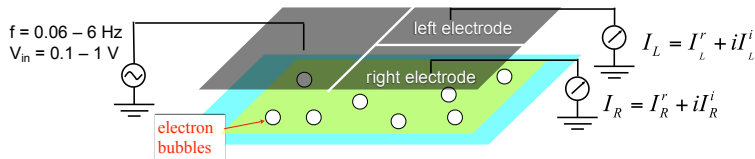


$\hbar/p_f \ll R \lesssim \xi_0$

## Measurement of the Transverse $e^-$ mobility in Superfluid $^3\text{He}$ Films



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Transverse Force from **Skew Scattering**

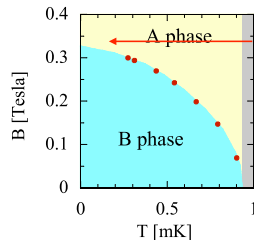
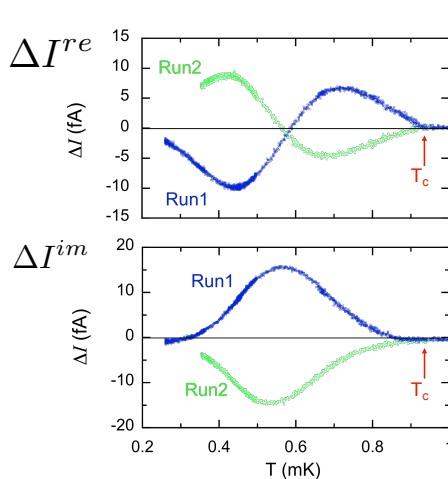
$$\rightsquigarrow \Delta I = I_R - I_L \neq 0$$

$$\vec{v} = \left[ \mu_{\perp} \vec{E} + \mu_{xy} \hat{\ell} \times \vec{E} \right]$$

The diagram shows a green parallelogram representing the film. Two red arrows originate from white circles (electron bubbles) and curve in opposite directions. To the right, two blue arrows represent the unit vector  $\hat{\ell}$ : an upward arrow labeled  $\vec{\ell} = +\hat{z}$  and a downward arrow labeled  $\vec{\ell} = -\hat{z}$ .



# Transverse $e^-$ bubble current in $^3\text{He-A}$    $\Delta I = I_R - I_L$



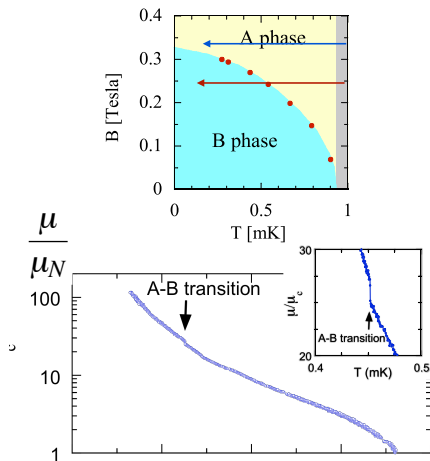
Single Domains:

Run 1     $\vec{\ell} = +\hat{z}$

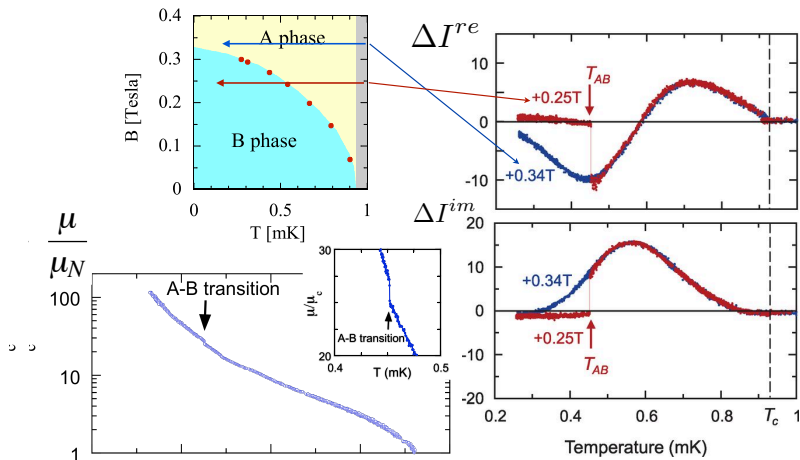
Run 2     $\vec{\ell} = -\hat{z}$

$$\frac{|I_R - I_L|}{I_R + I_L} \approx 6\%$$

**Zero Transverse  $e^-$  current in  $^3\text{He-B}$  ( $T$ -symmetric phase)**



**Zero Transverse  $\mathbf{e}^-$  current in  $^3\text{He-B}$  ( $T$ -symmetric phase)**



## Forces on the Electron bubble in $^3\text{He-A}$ :

- ▶  $M \frac{d\mathbf{v}}{dt} = e\mathcal{E} + \mathbf{F}_{QP}, \quad \mathbf{F}_{QP} - \text{force from quasiparticle collisions}$

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- ▶  $\mathbf{F}_{QP} = -\overleftrightarrow{\eta} \cdot \mathbf{v}$ ,  $\overleftrightarrow{\eta}$  – generalized Stokes tensor
- ▶  $\overleftrightarrow{\eta} = \begin{pmatrix} \eta_{\perp} & \eta_{AH} & 0 \\ -\eta_{AH} & \eta_{\perp} & 0 \\ 0 & 0 & \eta_{\parallel} \end{pmatrix}$  for broken PT symmetry with  $\hat{\mathbf{l}} \parallel \mathbf{e}_z$

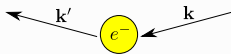
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- ▶  $M \frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} - \eta_{\perp} \mathbf{v} + \frac{e}{c} \mathbf{v} \times \mathbf{B}_{\text{eff}}$ , for  $\boldsymbol{\mathcal{E}} \perp \hat{\mathbf{l}}$
- ▶  $\mathbf{B}_{\text{eff}} = -\frac{c}{e} \eta_{\text{AH}} \hat{\mathbf{l}} \quad B_{\text{eff}} \simeq 10^3 - 10^4 \text{ T} \quad !!!$

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- ▶  $\mathbf{B}_{\text{eff}} = -\frac{c}{e} \eta_{\text{AH}} \hat{\mathbf{l}}$   $B_{\text{eff}} \simeq 10^3 - 10^4 \text{ T}$  !!!
- ▶ Mobility:  $\frac{d\mathbf{v}}{dt} = 0 \rightsquigarrow \mathbf{v} = \overleftrightarrow{\mu} \boldsymbol{\mathcal{E}}$ , where  $\overleftrightarrow{\mu} = e \overleftrightarrow{\eta}^{-1}$

## T-matrix description of Quasiparticle-Ion scattering



► Lippmann-Schwinger equation for the  $T$ -matrix ( $\varepsilon = E + i\eta$ ;  $\eta \rightarrow 0^+$ ):

$$\hat{T}_S^R(\mathbf{k}', \mathbf{k}, E) = \hat{T}_N^R(\mathbf{k}', \mathbf{k}) + \int \frac{d^3 k''}{(2\pi)^3} \hat{T}_N^R(\mathbf{k}', \mathbf{k}'') \left[ \hat{G}_S^R(\mathbf{k}'', E) - \hat{G}_N^R(\mathbf{k}'', E) \right] \hat{T}_S^R(\mathbf{k}'', \mathbf{k}, E)$$

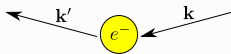
$$\hat{G}_S^R(\mathbf{k}, E) = \frac{1}{\varepsilon^2 - E_{\mathbf{k}}^2} \begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad E_{\mathbf{k}} = \sqrt{\xi_k^2 + |\Delta(\hat{\mathbf{k}})|^2}, \quad \xi_k = \frac{\hbar^2 k^2}{2m^*} - \mu$$

► Normal-state  $T$ -matrix:

$$\hat{T}_N^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = \begin{pmatrix} t_N^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}) & 0 \\ 0 & -[t_N^R(-\hat{\mathbf{k}}', -\hat{\mathbf{k}})]^\dagger \end{pmatrix} \quad \text{in p-h (Nambu) space}$$



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$$\hat{G}_S^R(\mathbf{k}, E) = \frac{1}{\varepsilon^2 - E_{\mathbf{k}}^2} \begin{pmatrix} \varepsilon + \xi_{\mathbf{k}} & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_{\mathbf{k}} \end{pmatrix}, \quad E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + |\Delta(\hat{\mathbf{k}})|^2}, \quad \xi_{\mathbf{k}} = \frac{\hbar^2 k^2}{2m^*} - \mu$$

► Normal-state  $T$ -matrix:

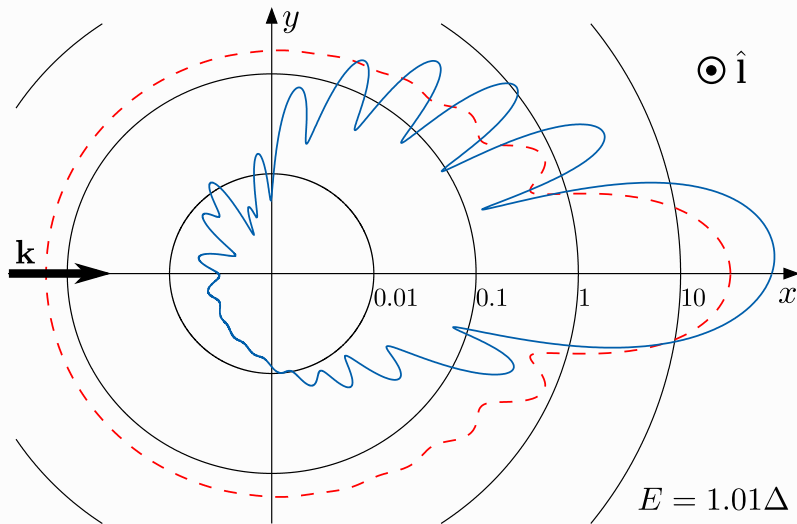
$$\hat{T}_N^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = \begin{pmatrix} t_N^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}) & 0 \\ 0 & -[t_N^R(-\hat{\mathbf{k}}', -\hat{\mathbf{k}})]^\dagger \end{pmatrix} \quad \text{in p-h (Nambu) space, where}$$

$$t_N^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = -\frac{1}{\pi N_f} \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin \delta_l P_l(\hat{\mathbf{k}}' \cdot \hat{\mathbf{k}}), \quad P_l(x) - \text{Legendre function}$$

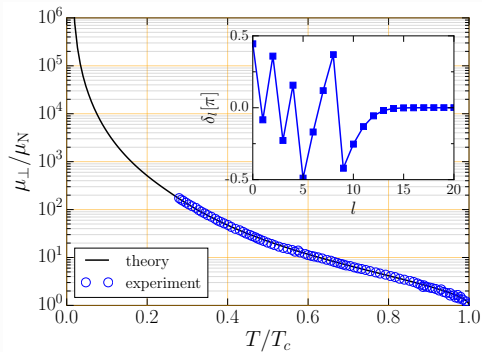
► Hard-sphere potential  $\rightsquigarrow \tan \delta_l = j_l(k_f R)/n_l(k_f R)$  – spherical Bessel functions

►  $k_f R$  – determined by the Normal-State Mobility  $\rightsquigarrow k_f R = 11.17$  ( $R = 1.42 \text{ nm}$ )

Differential cross section for Bogoliubov QP-Ion Scattering  $k_f R = 11.17$

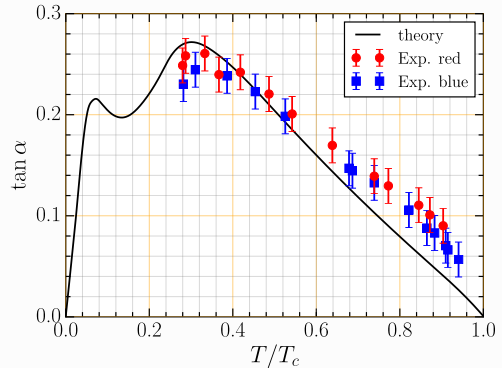


## Comparison between Theory and Experiment for the Drag and Transverse Forces



$$\begin{aligned} \blacktriangleright \mu_{\perp} &= e \frac{\eta_{\perp}}{\eta_{\perp}^2 + \eta_{\text{AH}}^2} \\ \blacktriangleright \mu_{\text{AH}} &= -e \frac{\eta_{\text{AH}}}{\eta_{\perp}^2 + \eta_{\text{AH}}^2} \end{aligned}$$

► O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)



$$\begin{aligned} \blacktriangleright \tan \alpha &= \left| \frac{\mu_{\text{AH}}}{\mu_{\perp}} \right| = \frac{\eta_{\text{AH}}}{\eta_{\perp}} \\ \blacktriangleright \text{Hard-Sphere Model:} \\ k_f R &= 11.17 \end{aligned}$$

► O. Shevtsov and JAS, JLTP 187, 340353 (2017)

## Summary

- ▶ Electrons in  $^3\text{He-A}$  are “dressed” by a spectrum of Chiral Fermions
- ▶ Electrons in  $^3\text{He-A}$  are “Left handed” in a Right-handed Chiral Vacuum  
 $\rightsquigarrow L_z \approx -(N_{\text{bubble}}/2)\hbar \approx -100 \hbar$
- ▶ Experiment: RIKEN mobility experiments  $\rightsquigarrow$  Observation an AHE in  $^3\text{He-A}$
- ▶ Scattering of Bogoliubov QPs by the dressed Ion  
 $\rightsquigarrow$  Drag Force  $(-\eta_{\perp} \mathbf{v})$  and Transverse Force  $(\frac{e}{c} \mathbf{v} \times \mathbf{B}_{\text{eff}})$  on the Ion
- ▶ *Anomalous Hall Field*:  $\mathbf{B}_{\text{eff}} \approx \frac{\Phi_0}{3\pi^2} k_f^2 (k_f R)^2 \left( \frac{\eta_{\text{AH}}}{\eta_{\text{N}}} \right) \mathbf{1} \simeq 10^3 - 10^4 \text{ T}$
- ▶ Mechanism: Skew/Andreev Scattering of Bogoliubov QPs by the dressed Ion
- ▶ Origin: Broken Mirror & Time-Reversal Symmetry  $\rightsquigarrow W(\mathbf{k}, \mathbf{k}') \neq W(\mathbf{k}', \mathbf{k})$
- ▶ Theory:  $\rightsquigarrow$  Quantitative account of RIKEN mobility experiments

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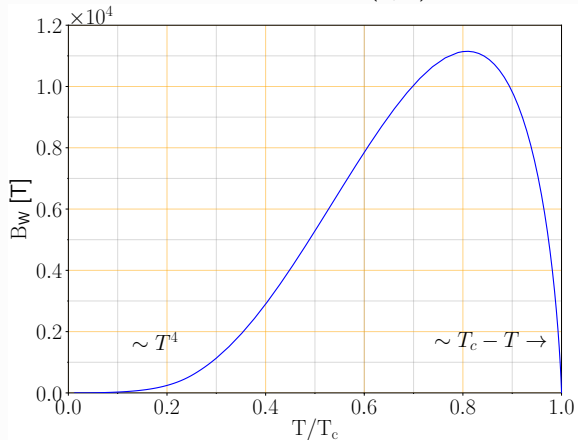
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- ▶ Open Problem: Bulk Signature of BTRS in  $\text{UPt}_3, \text{Sr}_2\text{RuO}_4 \rightsquigarrow$  Thermal Hall Effects?

## Vanishing of the Effective Magnetic Field for $T \rightarrow 0$

## Breakdown of Laminar Flow

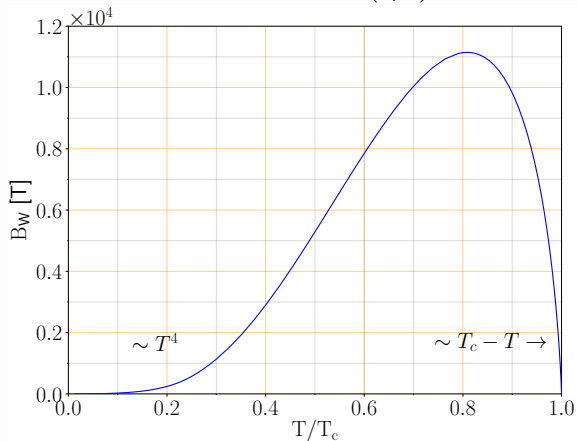
$$B_W = 5.9 \times 10^5 \text{ T} \left( \frac{\eta_{xy}}{\eta_N} \right)$$



$$\eta_{xy}/\eta_N|_{T=0.8 T_c} \approx \frac{\hbar}{p_f R}$$

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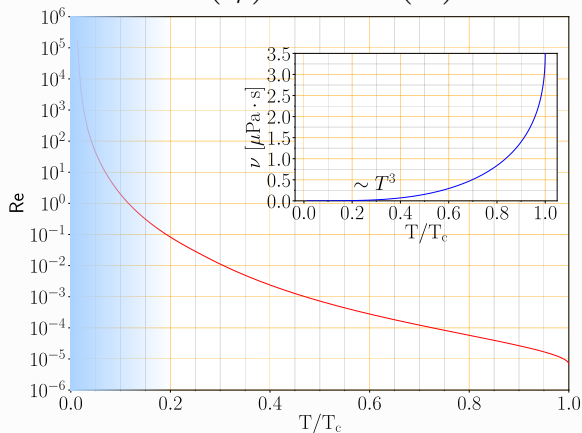
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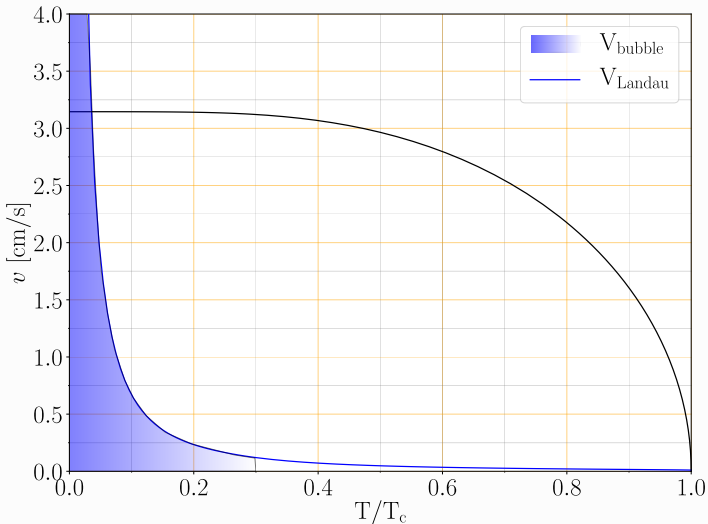
$$Re = Re_N \left( \frac{\eta_N}{\eta} \right)^{3/2} \xrightarrow{T \rightarrow 0} \sim \left( \frac{T_c}{T} \right)^{9/2}$$



$$Re_N = 6.7 \times 10^{-6}$$



## Breakdown of Scattering Theory for $T \rightarrow 0$



### Electron Bubble Velocity

▶  $V_N = \mu_N E_N = 1.01 \times 10^{-4} \text{ m/s}$

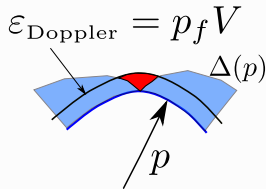
▶  $V = \mu_N E_N \sqrt{\frac{\eta_N}{\eta}}$

### Maximum Landau critical velocity

▶  $V_c^{\text{max}} \approx 155 \times 10^{-4} \text{ m/s} \frac{\Delta_A(T)}{k_b T_c}$

### Nodal Superfluids:

▶  $V_c = \Delta(p)/p_f \rightarrow 0$  for  $p \rightarrow p_{\text{node}}$

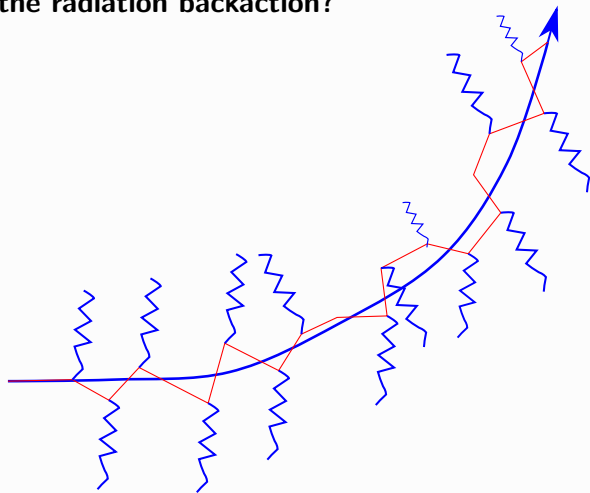


▶ Radiation Dominated Damping for  $T \lesssim 0.1 T_c$

**Is there a transverse component of the radiation backaction?**

Stochastic Radiative Dynamics

Fluctuations of the Chiral Vacuum



► Mesoscopic Ion coupled and driven through a Chiral “Bath”

Happy Birthday Tony!

Thanks for the beautiful physics you created and stimulated !