*$28^{th}$ International Conference on Low Temperature Physics, Gothenburg, Sweden, August 11, 2017*

# Signatures of Majorana-Weyl Fermions in Superfluid $^3$He

J. A. Sauls

Northwestern University

- Oleksii Shevtsov

- Parity violation
- Superfluid $^3$He
- Edge States & Currents
- Electron Bubbles in $^3$He
- Anomalous Hall Effect
- Electron Transport in $^3$He

- NSF Grant DMR-1508730

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# The Left Hand of the Electron in Superfluid $^3$He

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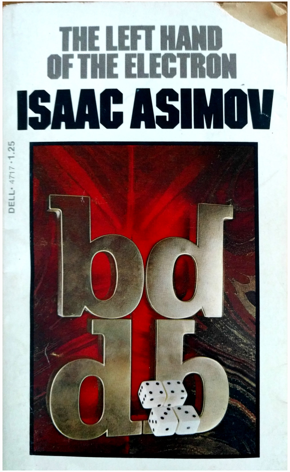
# The Left Hand of the Electron, Issac Asimov, circa 1971

- An Essay on the Discovery of Parity Violation by the Weak Interaction




- ... And Reflections on Mirror Symmetry in Nature

# Parity Violation in Beta Decay of $^{60}$Co - Physical Review 105, 1413 (1957)

**Experimental Test of Parity Conservation in Beta Decay***

C. S. Wu, *Columbia University, New York, New York*

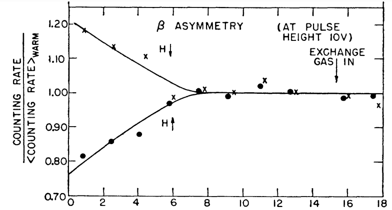
AND

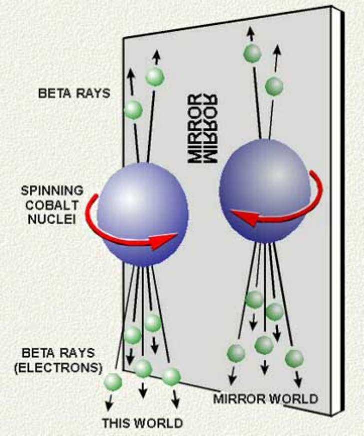
E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson,
*National Bureau of Standards, Washington, D. C.*

(Received January 15, 1957)

- T. D. Lee and C. N. Yang, Phys Rev 104, 204 (1956)

$$^{60}Co \to {}^{60}Ni + e^- + \bar{\nu}$$




- Current of Beta electrons is (anti) correlated with the Spin of the $^{60}$Co nucleus.

$$\langle \vec{S} \cdot \vec{p} \rangle \neq 0 \rightsquigarrow \text{Parity violation}$$

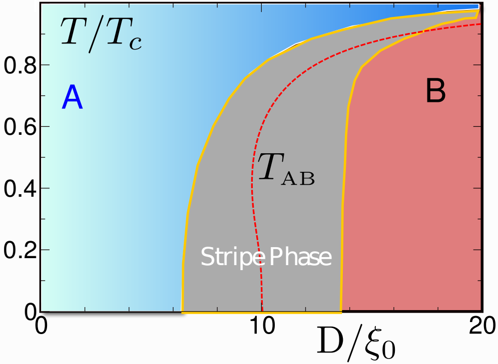
# Realization of Broken Time-Reversal and Mirror Symmetry by the Vacuum State of $^3$He Films

- Length Scale for Strong Confinement:

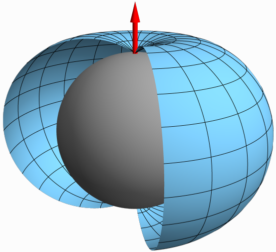
$$\xi_0 = \hbar v_f / 2\pi k_B T_c \approx 20 - 80\,\text{nm}$$

- L. Levitov et al., Science 340, 6134 (2013)
- A. Vorontsov & J. A. Sauls, PRL 98, 045301 (2007)

$$\mathrm{SO}(3)_{\mathrm{S}} \times \mathrm{SO}(3)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{N}} \times \mathrm{T} \times \mathrm{P}$$
$$\Downarrow$$
$$\mathrm{SO}(2)_{\mathrm{S}} \times \mathrm{U}(1)_{\mathrm{N-L}_z} \times \mathrm{Z}_2$$




Chiral AM State $\vec{l} = \hat{\mathbf{z}}$



$L_z = 1,\ S_z = 0$

Ground-State Angular Momentum

$$\langle \widehat{L}_z \rangle = \frac{N}{2}\hbar$$

- M. McClure and S. Takagi PRL 43, 596 (1979)

$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix}_{AM} = \begin{pmatrix} p_x + ip_y \sim e^{+i\phi} & 0 \\ 0 & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

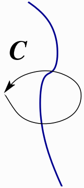
# Signatures of Broken T and P Symmetry in $^3$He-A

- Spontaneous Symmetry Breaking $\rightsquigarrow$ Emergent Topology of the $^3$He-A Ground State
- Chirality + Topology $\rightsquigarrow$ Weyl-Majorana Edge States $\rightsquigarrow$ Chiral Edge Currents
- Broken T and P $\rightsquigarrow$ Anomalous Hall Effects in Chiral Superfluids, e.g. $^3$He-A
- Confinement $\rightsquigarrow$ Edge State Hybridization and *New Broken Symmetry Phases of $^3$He*

## Real-Space vs. Momentum-Space Topology

Topology in Real Space

$\Psi(\mathbf{r}) = |\Psi(r)|\, e^{i\vartheta(\mathbf{r})}$



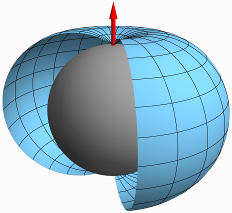

Phase Winding

$$N_C = \frac{1}{2\pi}\oint_C d\vec{l}\cdot \frac{1}{|\Psi|}\text{Im}[\boldsymbol{\nabla}\Psi] \in \{0, \pm 1, \pm 2, \ldots\}$$

- Massless Fermions confined in the Vortex Core

Chiral Symmetry $\rightsquigarrow$
Topology in Momentum Space

$\Psi(\mathbf{p}) = \Delta(p_x \pm i p_y) \sim e^{\pm i\varphi_{\mathbf{p}}}$



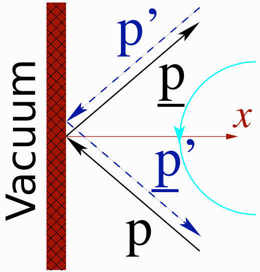
Topological Quantum Number: $L_z = \pm 1$

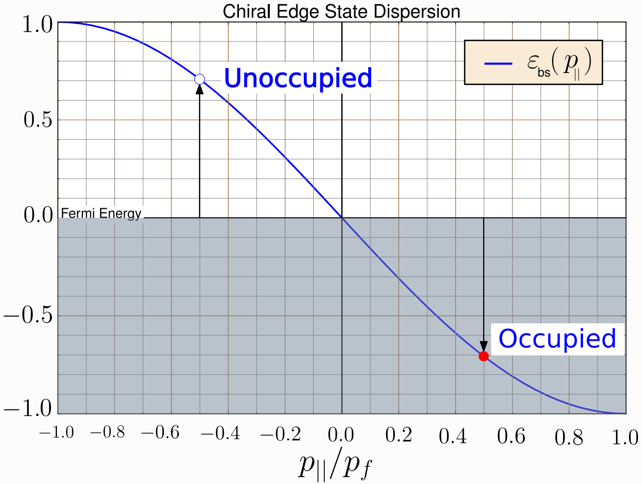
$$N_{2D} = \frac{1}{2\pi}\oint d\mathbf{p}\cdot \frac{1}{|\Psi(\mathbf{p})|}\text{Im}[\boldsymbol{\nabla}_{\mathbf{p}}\Psi(\mathbf{p})] = L_z$$

- Massless Chiral Fermions
  - Nodal Fermions in 3D
  - Edge Fermions in 2D

Edge Fermions: $G^{\text{R}}_{\text{edge}}(\mathbf{p},\varepsilon;x) = \dfrac{\pi\Delta|\mathbf{p}_x|}{\varepsilon + i\gamma - \varepsilon_{\text{bs}}(\mathbf{p}_{||})}\, e^{-x/\xi_\Delta}$ $\quad \xi_\Delta = \hbar v_f/2\Delta \approx 10^2\,\mathring{A} \gg \hbar/p_f$

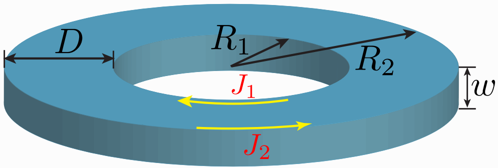
- $\varepsilon_{\text{bs}} = -c\,p_{||}$ with $c = \Delta/p_f \ll v_f$
- Broken P & T $\rightsquigarrow$ Edge Current




- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

## $^3$He-A confined in a toroidal cavity




- $R_1, R_2, R_1 - R_2 \gg \xi_0$

- Sheet Current: $J = \frac{1}{4}\, n\, \hbar$ ($n = N/V =$ $^3$He density)
- Counter-propagating Edge Currents: $J_1 = -J_2 = \frac{1}{4}\, n\, \hbar$
- Angular Momentum:

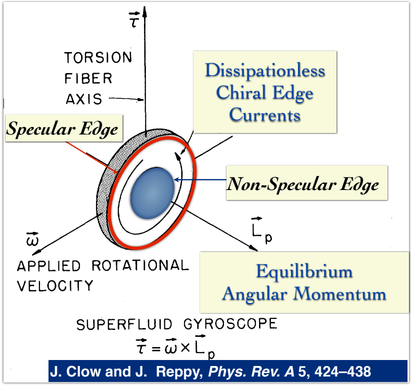
$$L_z = 2\pi\, h\, (R_1^2 - R_2^2) \times \frac{1}{4}\, n\, \hbar = (N/2)\, \hbar$$

McClure-Takagi's Global Symmetry Result PRL 43, 596 (1979)

# Long-Standing Challenge: Detect the Ground-State Angular Momentum of $^3$He-A

Possible Gyroscopic Experiment to Measure of $L_z(T)$

- Hyoungsoon Choi (KAIST) [micro-mechanical gyroscope @ 200 $\mu$K]




**Thermal Signature of Massless Chiral Fermions**

- Power Law for $T \lesssim 0.5 T_c$

$$L_z = (N/2)\hbar \, (1 - c\,(T/\Delta)^2)$$

Toroidal Geometry with Engineered Surfaces

- Incomplete Screening

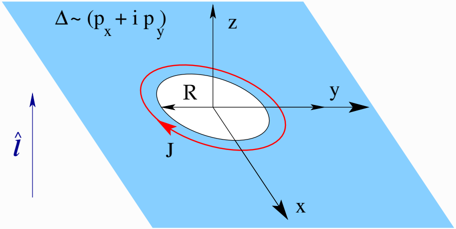
$$L_z > (N/2)\hbar$$

**Direct Signature of Edge Currents**

- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)
- Y. Tsutsumi, K. Machida, JPSJ 81, 074607 (2012)

# Anomalous Hall Effect for Electrons in Chiral Superfluid $^3$He

# Chiral Edge Current Circulating a Hole or Defect in a Chiral Superfluid




- $R \gg \xi_0 \approx 100\,\text{nm}$
- Sheet Current :
  $J \equiv \int dx\, J_\varphi(x)$

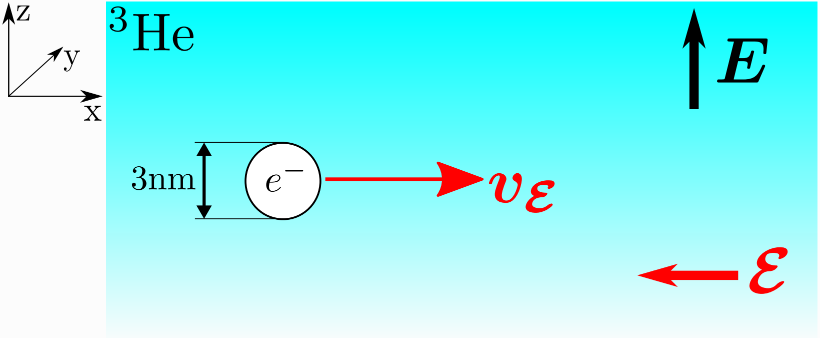
- Quantized Sheet Current: $\frac{1}{4}\, n\, \hbar$ $(n = N/V = {}^3\text{He density})$
- Edge Current *Counter*-Circulates: $J = -\frac{1}{4}\, n\, \hbar$ w.r.t. Chirality: $\hat{\mathbf{l}} = +\mathbf{z}$
- Angular Momentum: $L_z = 2\pi\, h\, R^2 \times (-\frac{1}{4}\, n\, \hbar) = -(N_{\text{hole}}/2)\, \hbar$

$N_{\text{hole}}$ = Number of ${}^3$He atoms excluded from the Hole

∴ An object in ${}^3$He-A *inherits* angular momentum from the Condensate of Chiral Pairs!

- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

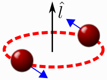
# Electron bubbles in the Normal Fermi liquid phase of $^3$He



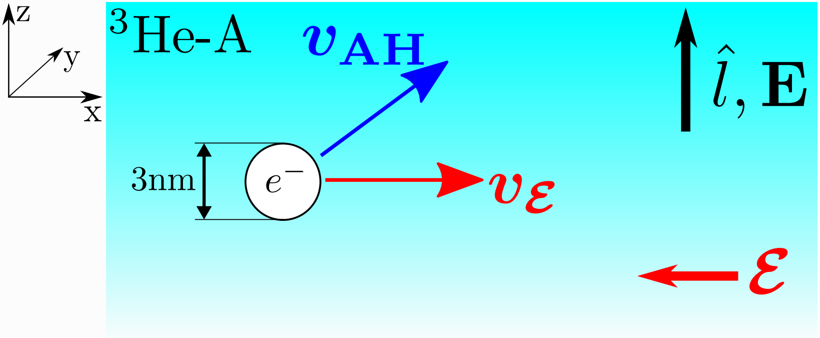

- Bubble with $R \simeq 1.5$ nm, $0.1\,\text{nm} \simeq \lambda_f \ll R \ll \xi_0 \simeq 80\,\text{nm}$
- Effective mass $M \simeq 100 m_3$ ($m_3$ – atomic mass of $^3$He)
- QPs mean free path $l \gg R$
- Mobility of $^3$He is *independent of* $T$ for $T_c < T < 50$ mK

B. Josephson and J. Leckner, PRL 23, 111 (1969)

Electron bubbles in chiral superfluid $^3$He-A

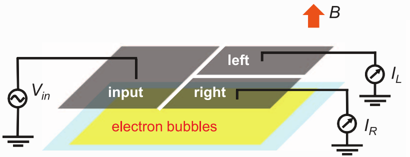


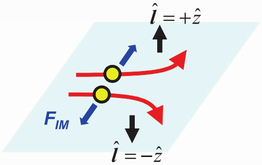

$\Delta(\hat{k}) = \Delta(\hat{k}_x + i\hat{k}_y) = \Delta\, e^{i\phi_{\mathbf{k}}}$

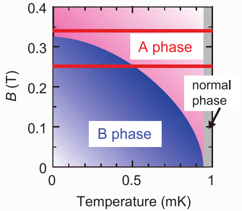



- Current: $\mathbf{v} = \overbrace{\mu_\perp \boldsymbol{\mathcal{E}}}^{\mathbf{v}_{\mathcal{E}}} + \overbrace{\mu_{\text{AH}} \boldsymbol{\mathcal{E}} \times \hat{\mathbf{l}}}^{\mathbf{v}_{\text{AH}}}$ R. Salmelin, M. Salomaa & V. Mineev, PRL **63**, 868 (1989)
- Hall ratio: $\tan\alpha = v_{\text{AH}}/v_{\mathcal{E}} = |\mu_{\text{AH}}/\mu_\perp|$

# Mobility of Electron Bubbles in $^3$He-A

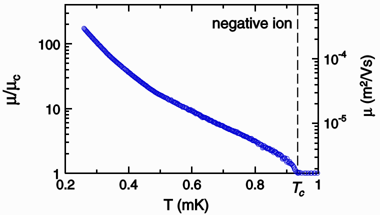


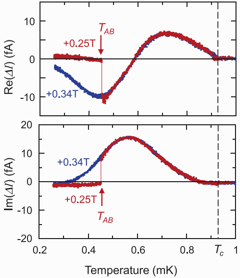



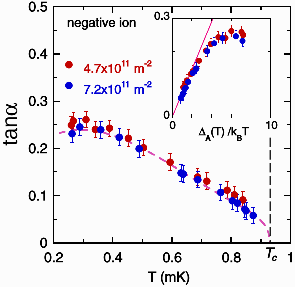




Electric current: $\mathbf{v} = \overbrace{\mu_\perp \boldsymbol{\mathcal{E}}}^{\mathbf{v}_{\mathcal{E}}} + \overbrace{\mu_{AH} \boldsymbol{\mathcal{E}} \times \hat{\mathbf{l}}}^{\mathbf{v}_{AH}}$

▶ Hall ratio: $\tan\alpha = v_{AH}/v_{\mathcal{E}} = |\mu_{AH}/\mu_\perp|$



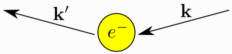

▶ H. Ikegami et al., Science **341**, 59 (2013); JPSJ **82**, 124607 (2013); JPSJ **84**, 044602 (2015)

# Forces on the Electron bubble in $^3$He-A:

- $M\frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} + \mathbf{F}_{\text{QP}}$, $\mathbf{F}_{QP}$ – force from quasiparticle collisions
- $\mathbf{F}_{QP} = -\overleftrightarrow{\eta}\cdot\mathbf{v}$, $\overleftrightarrow{\eta}$ – generalized Stokes tensor
- $\overleftrightarrow{\eta} = \begin{pmatrix} \eta_\perp & \eta_{\text{AH}} & 0 \\ -\eta_{\text{AH}} & \eta_\perp & 0 \\ 0 & 0 & \eta_\parallel \end{pmatrix}$ for chiral symmetry with $\hat{\mathbf{l}} \parallel \mathbf{e}_z$
- $M\frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} - \eta_\perp\mathbf{v} + \frac{e}{c}\mathbf{v}\times\mathbf{B}_{\text{eff}}$, for $\boldsymbol{\mathcal{E}} \perp \hat{\mathbf{l}}$
- $\mathbf{B}_{\text{eff}} = -\frac{c}{e}\eta_{\text{AH}}\hat{\mathbf{l}}$ $\quad B_{\text{eff}} \simeq 10^3 - 10^4$ T !!!
- Mobility: $\frac{d\mathbf{v}}{dt} = 0 \rightsquigarrow \mathbf{v} = \overleftrightarrow{\mu}\boldsymbol{\mathcal{E}}$, where $\overleftrightarrow{\mu} = e\overleftrightarrow{\eta}^{-1}$

▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

## T-matrix description of Quasiparticle-Ion scattering




- Lippmann-Schwinger equation for the $T$-matrix ($\varepsilon = E + i\eta\,;\ \eta \to 0^+$):

$$\hat{T}^R_S(\mathbf{k}',\mathbf{k},E) = \hat{T}^R_N(\mathbf{k}',\mathbf{k}) + \int \frac{d^3k''}{(2\pi)^3}\hat{T}^R_N(\mathbf{k}',\mathbf{k}'')\Big[\hat{G}^R_S(\mathbf{k}'',E) - \hat{G}^R_N(\mathbf{k}'',E)\Big]\hat{T}^R_S(\mathbf{k}'',\mathbf{k},E)$$

$$\hat{G}^R_S(\mathbf{k},E) = \frac{1}{\varepsilon^2 - E^2_{\mathbf{k}}}\begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad E_{\mathbf{k}} = \sqrt{\xi_k^2 + |\Delta(\hat{\mathbf{k}})|^2}, \ \ \xi_k = \frac{\hbar^2 k^2}{2m^*} - \mu$$

- Normal-state $T$-matrix:

$$\hat{T}^R_N(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = \begin{pmatrix} t^R_N(\hat{\mathbf{k}}',\hat{\mathbf{k}}) & 0 \\ 0 & -[t^R_N(-\hat{\mathbf{k}}',-\hat{\mathbf{k}})]^\dagger \end{pmatrix}$$ in p-h (Nambu) space, where

$$t^R_N(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = -\frac{1}{\pi N_f}\sum_{l=0}^{\infty}(2l+1)e^{i\delta_l}\sin\delta_l P_l(\hat{\mathbf{k}}'\cdot\hat{\mathbf{k}}),$$ $P_l(x)$ – Legendre function

- Hard-sphere potential $\rightsquigarrow \tan\delta_l = j_l(k_f R)/n_l(k_f R)$ – spherical Bessel functions

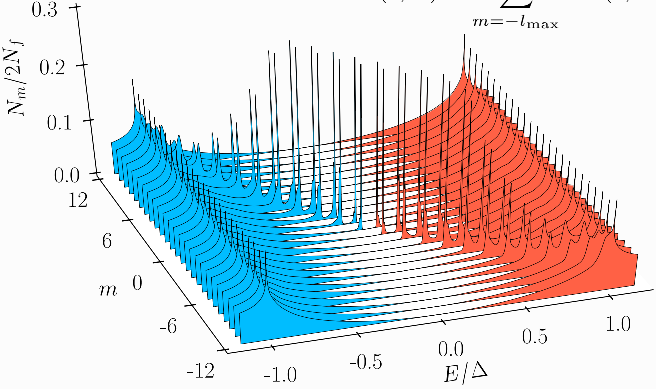
  - $k_f R$ – determined by the Normal-State Mobility

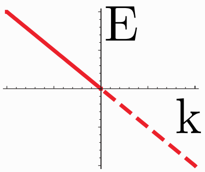
# Weyl Fermion Spectrum bound to the Electron Bubble

$$\mu_{\mathsf{N}} = \frac{e}{n_3 p_f \sigma_{\mathsf{N}}^{\mathsf{tr}}} \quad \Leftarrow \quad \mu_{\mathsf{N}}^{\mathsf{exp}} = 1.7 \times 10^{-6}\, \frac{m^2}{V\,s}$$

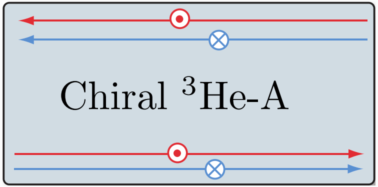
$$\tan\delta_l = j_l(k_f R)/n_l(k_f R) \;\Rightarrow\; \sigma_{\mathsf{N}}^{\mathsf{tr}} = \frac{4\pi}{k_f^2}\sum_{l=0}^{\infty}(l+1)\sin^2(\delta_{l+1}-\delta_l) \;\rightsquigarrow\; k_f R = 11.17$$

$$N(\mathbf{r},E) = \sum_{m=-l_{\max}}^{l_{\max}} N_m(\mathbf{r},E), \quad l_{\max} \simeq k_f R$$

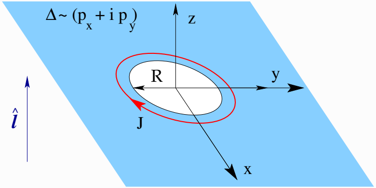


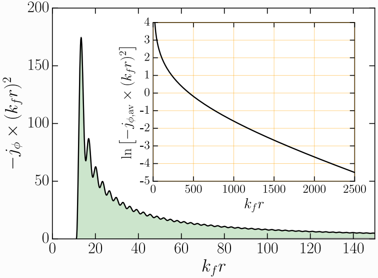



# Current bound to an electron bubble ($k_f R = 11.17$)

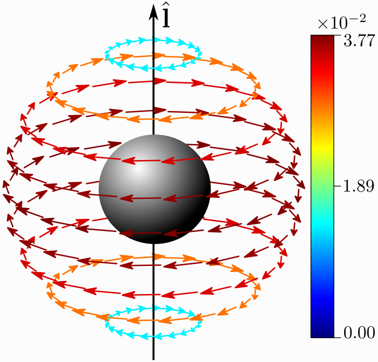



$\Longrightarrow$




$\Longrightarrow$




$$\mathbf{j}(\mathbf{r})/v_f N_f k_B T_c = j_\phi(\mathbf{r})\hat{\mathbf{e}}_\phi$$

▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

$$\mathbf{L}(T \to 0) \approx -\hbar N_{\mathrm{bubble}}/2\,\hat{\mathbf{l}} \approx -100\,\hbar\,\hat{\mathbf{l}}$$

▶ JAS PRB **84**, 214509 (2011)

## Determination of the Stokes Tensor from the QP-Ion T-matrix

(i) Fermi's golden rule and the QP scattering rate:

$$\Gamma(\mathbf{k}',\mathbf{k}) = \frac{2\pi}{\hbar} W(\hat{\mathbf{k}}',\hat{\mathbf{k}})\delta(E_{\mathbf{k}'} - E_{\mathbf{k}}), \;\; W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = \frac{1}{2}\sum_{\tau'\sigma';\tau\sigma} |\overbrace{\langle\,\mathbf{k}',\sigma',\tau'\,|}^{\text{outgoing}}\hat{T}_S\overbrace{|\,\mathbf{k},\sigma,\tau\,\rangle}^{\text{incoming}}|^2$$

(ii) Drag force from QP-ion collisions (linear in $\mathbf{v}$): ▶ Baym et al. PRL **22**, 20 (1969)

$$\mathbf{F}_{\text{QP}} = -\sum_{\mathbf{k},\mathbf{k}'} \hbar(\mathbf{k}'-\mathbf{k})\left[\hbar\mathbf{k}'\mathbf{v} f_{\mathbf{k}}\left(-\frac{\partial f_{\mathbf{k}'}}{\partial E}\right) - \hbar\mathbf{k}\mathbf{v}(1-f_{\mathbf{k}'})\left(-\frac{\partial f_{\mathbf{k}}}{\partial E}\right)\right]\Gamma(\mathbf{k}',\mathbf{k})$$

(iii) Microscopic reversibility condition: $W(\hat{\mathbf{k}}',\hat{\mathbf{k}}:+\mathbf{l}) = W(\hat{\mathbf{k}},\hat{\mathbf{k}}':-\mathbf{l})$

Broken T and mirror symmetries in $^3$He-A $\Rightarrow$ fixed $\hat{\mathbf{l}}$ $\rightsquigarrow$ $W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) \neq W(\hat{\mathbf{k}},\hat{\mathbf{k}}')$

(iv) Generalized Stokes tensor:

$$\mathbf{F}_{\text{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v} \;\;\rightsquigarrow\;\; \eta_{ij} = n_3 p_f \int_0^\infty dE\left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E), \quad \overleftrightarrow{\eta} = \begin{pmatrix} \eta_\perp & \eta_{\text{AH}} & 0 \\ -\eta_{\text{AH}} & \eta_\perp & 0 \\ 0 & 0 & \eta_\parallel \end{pmatrix}$$

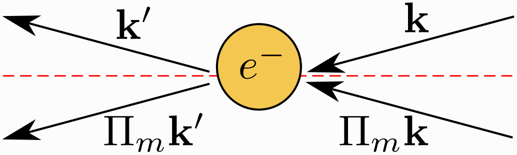
$n_3 = \dfrac{k_f^3}{3\pi^2}$ – $^3$He particle density, $\sigma_{ij}(E)$ – transport scattering cross section,

$f(E) = [\exp(E/k_B T) + 1]^{-1}$ – Fermi Distribution

# Mirror-symmetric scattering $\Rightarrow$ longitudinal drag force

$$\mathbf{F}_{\text{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^{\infty} dE \left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E)$$

Subdivide by mirror symmetry:




$$W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}),$$

$$\sigma_{ij}(E) = \sigma_{ij}^{(+)}(E) + \sigma_{ij}^{(-)}(E),$$

$$\sigma_{ij}^{(+)}(E) = \frac{3}{4}\int_{E\geq|\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'} \int_{E\geq|\Delta(\hat{\mathbf{k}})|} \frac{d\Omega_{\mathbf{k}}}{4\pi}\left[(\hat{\mathbf{k}}'_i-\hat{\mathbf{k}}_i)(\hat{\mathbf{k}}'_j-\hat{\mathbf{k}}_j)\right]\frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E)$$

Mirror-symmetric cross section: $W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W(\hat{\mathbf{k}},\hat{\mathbf{k}}')]/2$

$$\frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2 \frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}}')|^2}} W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) \frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}})|^2}}$$

$\rightsquigarrow$ Stokes Drag $\eta_{xx}^{(+)} = \eta_{yy}^{(+)} \equiv \eta_{\perp},\ \eta_{zz}^{(+)} \equiv \eta_{\parallel}$ , No transverse force $\left[\eta_{ij}^{(+)}\right]_{i\neq j} = 0$

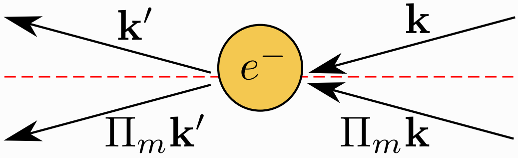
# Mirror-antisymmetric scattering ⇒ transverse force

$$\mathbf{F}_{\text{QP}} = -\overset{\leftrightarrow}{\eta} \cdot \mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^\infty dE \left(-2\frac{\partial f}{\partial E}\right) \sigma_{ij}(E)$$

Subdivide by mirror symmetry:

$$W(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = W^{(+)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}) + W^{(-)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}),$$

$$\sigma_{ij}(E) = \sigma_{ij}^{(+)}(E) + \sigma_{ij}^{(-)}(E),$$




$$\sigma_{ij}^{(-)}(E) = \frac{3}{4}\int_{E \geq |\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'} \int_{E \geq |\Delta(\hat{\mathbf{k}})|} \frac{d\Omega_{\mathbf{k}}}{4\pi} \left[\epsilon_{ijk}(\hat{\mathbf{k}}' \times \hat{\mathbf{k}})_k\right] \frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E)\left[f(E) - \frac{1}{2}\right]$$

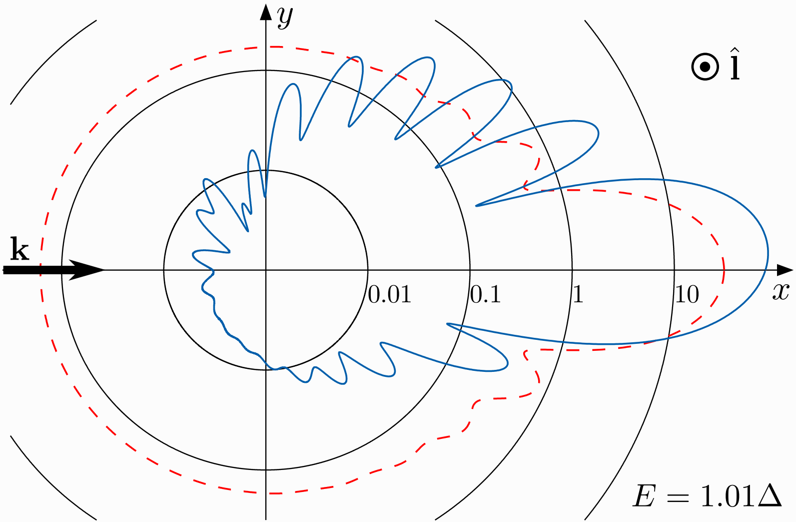
Mirror-antisymmetric cross section: $W^{(-)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}', \hat{\mathbf{k}}) - W(\hat{\mathbf{k}}, \hat{\mathbf{k}}')]/2$

$$\frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2 \frac{E}{\sqrt{E^2 - |\Delta(\hat{\mathbf{k}}')|^2}} W^{(-)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}) \frac{E}{\sqrt{E^2 - |\Delta(\hat{\mathbf{k}})|^2}}$$

Transverse force $\eta_{xy}^{(-)} = -\eta_{yx}^{(-)} \equiv \eta_{\text{AH}}$ ⇒ anomalous Hall effect

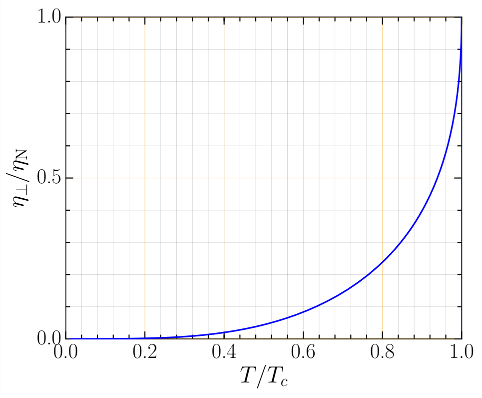
▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

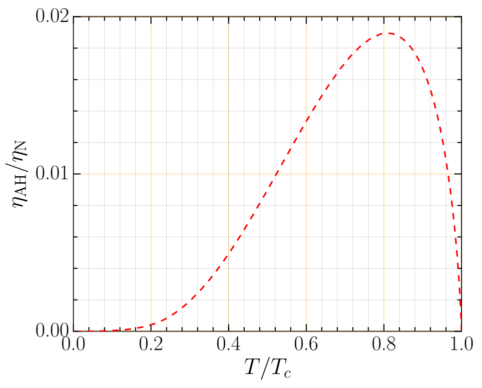
# Differential cross section for Bogoliubov QP-Ion Scattering $k_f R = 11.17$




▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

# Theoretical Results for the Drag and Transverse Forces




- $\Delta p_x \approx p_f \qquad \sigma^{\rm tr}_{xx} \approx \sigma^{\rm tr}_{\rm N} \approx \pi R^2$
- $F_x \approx n\, v_x\, \Delta p_x\, \sigma^{\rm tr}_{xx}$
  $\approx n\, v_x\, p_f\, \sigma^{\rm tr}_{\rm N}$

$$|F_y/F_x| \approx \frac{\hbar}{p_f R}\,(\Delta(T)/k_B T_c)^2$$

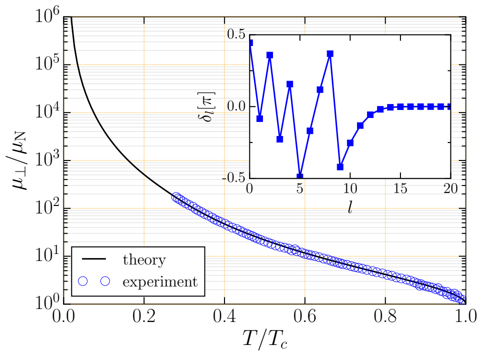
- $\Delta p_y \approx \hbar/R\; \sigma^{\rm tr}_{xy} \approx (\Delta(T)/k_{\rm B}T_c)^2 \sigma^{\rm tr}_{\rm N}$
- $F_y \approx n\, v_x\, \Delta p_y\, \sigma^{\rm tr}_{xy}$
  $\approx n\, v_x\, (\hbar/R)\, \sigma^{\rm tr}_{\rm N} (\Delta(T)/k_{\rm B}T_c)^2$

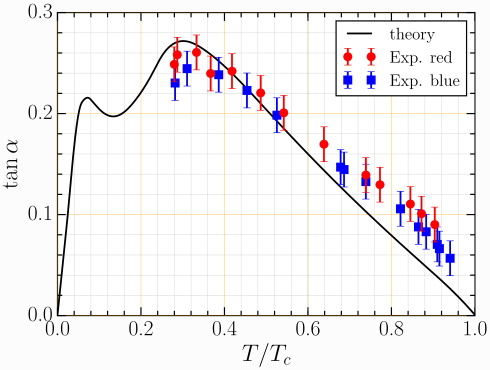
$k_f R = 11.17$

Branch Conversion Scattering

- O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

# Comparison between Theory and Experiment for the Drag and Transverse Forces




- $\mu_\perp = e\,\frac{\eta_\perp}{\eta_\perp^2 + \eta_{AH}^2}$
- $\mu_{AH} = -e\,\frac{\eta_{AH}}{\eta_\perp^2 + \eta_{AH}^2}$

- $\tan\alpha = \left|\frac{\mu_{AH}}{\mu_\perp}\right| = \frac{\eta_{AH}}{\eta_\perp}$
- Hard-Sphere Model: $k_f R = 11.17$

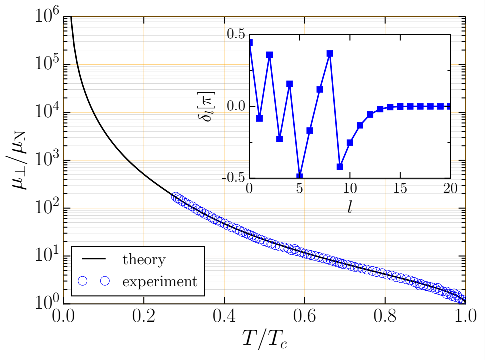
- O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)
- O. Shevtsov and JAS, JLTP 187, 340353 (2017)

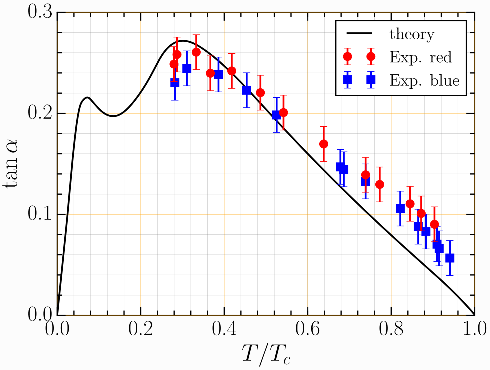
# Summary

- Electrons in $^3$He-A are "dressed" by a spectrum of Weyl Fermions
- Electrons in $^3$He-A are "Left handed" in a Right-handed Chiral Vacuum
  $\rightsquigarrow L_z \approx -(N_{bubble}/2)\hbar \approx -100\,\hbar$
- Experiment: RIKEN mobility experiments $\rightsquigarrow$ Observation an AHE in $^3$He-A
- Scattering of Bogoliubov QPs by the dressed Ion
  $\rightsquigarrow$ Drag Force $(-\eta_\perp \mathbf{v})$ and Transverse Force $(\frac{e}{c}\mathbf{v} \times \mathbf{B}_{eff})$ on the Ion
- *Anomalous Hall Field:* $\mathbf{B}_{\text{eff}} \approx \frac{\Phi_0}{3\pi^2}\, k_f^2\,(k_f R)^2 \left(\frac{\eta_{\text{AH}}}{\eta_{\text{N}}}\right) \mathbf{l} \simeq 10^3 - 10^4\,\text{T}\,\mathbf{l}$
- Mechanism: Skew/Andreev Scattering of Bogoliubov QPs by the dressed Ion
- Origin: Broken Mirror & Time-Reversal Symmetry $\rightsquigarrow W(\mathbf{k},\mathbf{k}') \neq W(\mathbf{k}',\mathbf{k})$
- Theory: $\rightsquigarrow$ Quantitative account of RIKEN mobility experiments
- New directions for Transport in $^3$He-A & Chiral Superconductors

Anomalous Hall and Thermal Hall Effects in Chiral Superconductors: $UPt_3$ & $Sr_2RuO_4$

# Comparison between Theory and Experiment for the Drag and Transverse Forces




- $\mu_\perp = e\,\dfrac{\eta_\perp}{\eta_\perp^2 + \eta_{\text{AH}}^2}$
- $\mu_{\text{AH}} = -e\,\dfrac{\eta_{\text{AH}}}{\eta_\perp^2 + \eta_{\text{AH}}^2}$

- $\tan\alpha = \left|\dfrac{\mu_{\text{AH}}}{\mu_\perp}\right| = \dfrac{\eta_{\text{AH}}}{\eta_\perp}$
- Hard-Sphere Model: $k_f R = 11.17$


- O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)
- O. Shevtsov and JAS, JLTP 187, 340353 (2017)

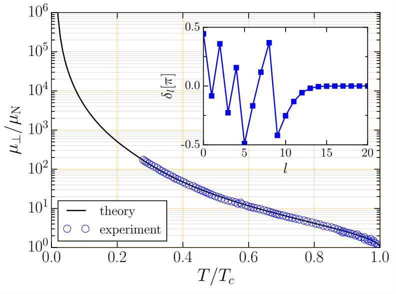
# Theoretical Models for the QP-ion potential

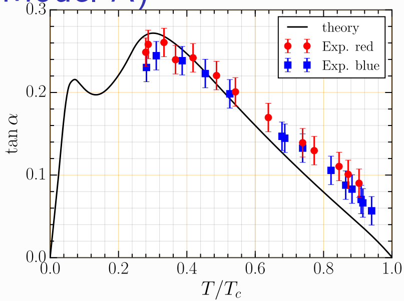
- $$U(r) = \begin{cases} U_0, & r < R, \\ -U_1, & R < r < R', \\ 0, & r > R'. \end{cases}$$
- $\rightsquigarrow$ Hard-Sphere Potential: $U_1 = 0$, $R' = R$, $U_0 \to \infty$
- $U(x) = U_0\,[1 - \tanh[(x-b)/c]]$, $\quad x = k_f r$
- $U(x) = U_0/\cosh^2[\alpha x^n]$, $\quad x = k_f r$ (Pöschl-Teller-like potential)
- Random phase shifts: $\{\delta_l |\, l = 1 \ldots l_{\max}\}$ are generated with $\delta_0$ is an adjustable parameter
- Parameters for all models are chosen to fit the experimental value of the normal-state mobility, $\mu_N^{\exp} = 1.7 \times 10^{-6}\, m^2/V \cdot s$

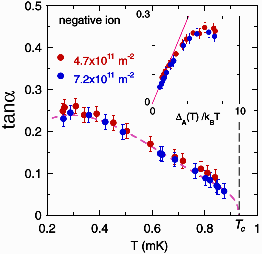
# Theoretical Models for the QP-ion potential

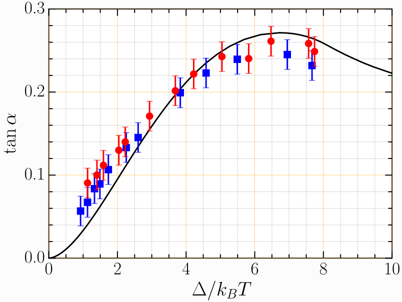
| Label | Potential | Parameters |
|---|---|---|
| Model A | hard sphere | $k_f R = 11.17$ |
| Model B | repulsive core & attractive well | $U_0 = 100E_f, U_1 = 10E_f, k_f R' = 11, R/R' = 0.36$ |
| Model C | random phase shifts model 1 | $l_{\text{max}} = 11$ |
| Model D | random phase shifts model 2 | $l_{\text{max}} = 11$ |
| Model E | Pöschl-Teller-like | $U_0 = 1.01E_f, k_f R = 22.15, \alpha = 3 \times 10^{-5}, n = 4$ |
| Model F | Pöschl-Teller-like | $U_0 = 2E_f, k_f R = 19.28, \alpha = 6 \times 10^{-5}, n = 4$ |
| Model G | hyperbolic tangent | $U_0 = 1.01E_f, k_f R = 14.93, b = 12.47, c = 0.246$ |
| Model H | hyperbolic tangent | $U_0 = 2E_f, k_f R = 14.18, b = 11.92, c = 0.226$ |
| Model I | soft sphere 1 | $U_0 = 1.01E_f, k_f R = 12.48$ |
| Model J | soft sphere 2 | $U_0 = 2E_f, k_f R = 11.95$ |

# Hard-sphere model with $k_f R = 11.17$ (Model A)



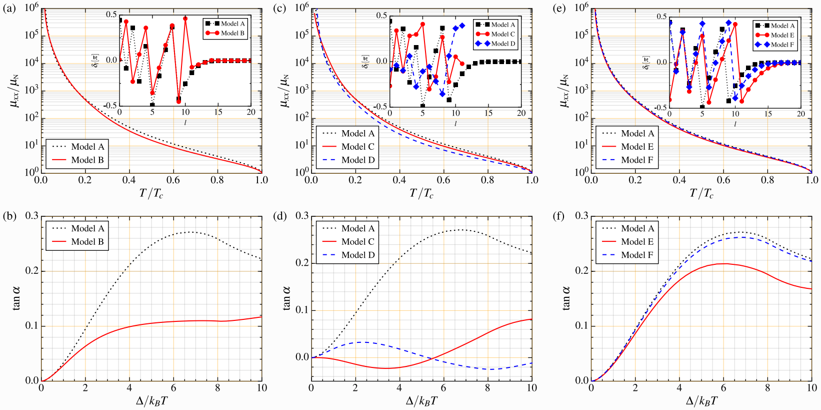



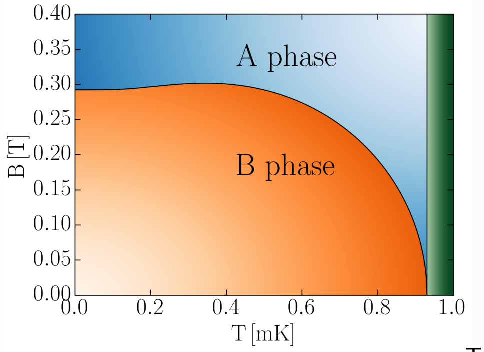



# Comparison with Experiment for Models for the QP-ion potential

| Label | Potential | Parameters |
|---|---|---|
| Model A | hard sphere | $k_f R = 11.17$ |
| Model B | attractive well with a repulsive core | $U_0 = 100E_f, U_1 = 10E_f, k_f R' = 11, R/R' = 0.36$ |
| Model C | random phase shifts model 1 | $l_{\max} = 11$ |
| Model D | random phase shifts model 2 | $l_{\max} = 11$ |
| Model E | Pöschl-Teller-like | $U_0 = 1.01E_f, k_f R = 22.15, \alpha = 3 \times 10^{-5}, n = 4$ |
| Model F | Pöschl-Teller-like | $U_0 = 2E_f, k_f R = 19.28, \alpha = 6 \times 10^{-5}, n = 4$ |



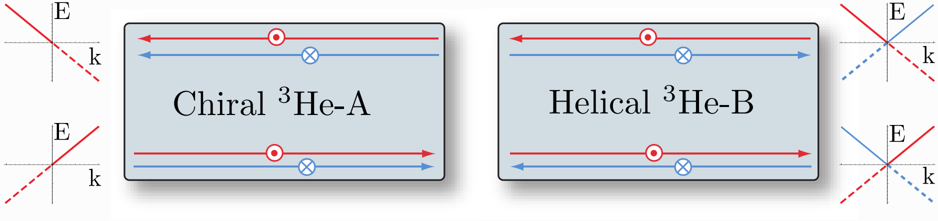
# Stabilizing the A-phase at Low Temperatures




Magnetic field $\mathbf{B}$:

- suppresses $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ Cooper pairs:
  $\rightsquigarrow$ disfavors the B-phase
- favors the chiral, $p_x + ip_y$, A-phase with:
  $((1+\eta B)|\uparrow\uparrow\rangle + (1-\eta B)|\downarrow\downarrow\rangle)$
- critical field: $B_c(0) \approx 0.3\,\mathrm{T}$

Topological Edge states:



## Calculation of LDOS and Current Density

$$\hat{\mathcal{G}}_S^R(\mathbf{r}',\mathbf{r},E) = \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} e^{i\mathbf{k}'\mathbf{r}'} e^{-i\mathbf{k}\mathbf{r}} \hat{\mathcal{G}}_S^R(\mathbf{k}',\mathbf{k},E)$$

$$\hat{\mathcal{G}}_S^R(\mathbf{k}',\mathbf{k},E) = (2\pi)^3 \hat{G}_S^R(\mathbf{k},E)\delta_{\mathbf{k}',\mathbf{k}} + \hat{G}_S^R(\mathbf{k}',E)\hat{T}_S(\mathbf{k}',\mathbf{k},E)\hat{G}_S^R(\mathbf{k},E)$$

$$\hat{G}_S^R(\mathbf{k},E) = \frac{1}{\varepsilon^2 - E_{\mathbf{k}}^2} \begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad \varepsilon = E + i\eta, \quad \eta \to 0^+$$

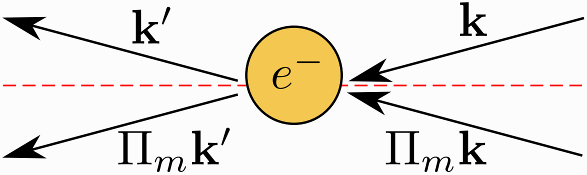
$$N(\mathbf{r},E) = -\frac{1}{2\pi} \mathrm{Im}\left\{ \mathrm{Tr}\left[ \hat{\mathcal{G}}_S^R(\mathbf{r},\mathbf{r},E) \right] \right\}$$

$$\mathbf{j}(\mathbf{r}) = \frac{\hbar}{4mi} k_B T \sum_{n=-\infty}^{\infty} \lim_{\mathbf{r}\to\mathbf{r}'} \mathrm{Tr}\left[ (\boldsymbol{\nabla}_{\mathbf{r}'} - \boldsymbol{\nabla}_{\mathbf{r}}) \hat{\mathcal{G}}^M(\mathbf{r}',\mathbf{r},\epsilon_n) \right]$$

$$\hat{\mathcal{G}}_S^R(\mathbf{r}',\mathbf{r},E) = \hat{\mathcal{G}}_S^M(\mathbf{r}',\mathbf{r},\epsilon_n)\Big|_{i\epsilon_n \to \varepsilon}, \text{ for } n \geq 0$$

$$\hat{\mathcal{G}}_S^M(\mathbf{k},\mathbf{k}',-\epsilon_n) = \left[ \hat{\mathcal{G}}_S^M(\mathbf{k}',\mathbf{k},\epsilon_n) \right]^\dagger$$

# Broken Time-Reversal ($\mathtt{T}$) & mirror ($\Pi_m$) symmetries in Chiral Superfluids




- Broken TRS: $\mathtt{T}\cdot(\hat{p}_x + i\hat{p}_y) = (\hat{p}_x - i\hat{p}_y)$
- Broken mirror symmetry: $\Pi_m \cdot (\hat{p}_x + i\hat{p}_y) = (\hat{p}_x - i\hat{p}_y)$
- Chiral symmetry: $\mathtt{C} = \mathtt{T}\times\Pi_m \quad \rightsquigarrow \quad \mathtt{C}\cdot(\hat{p}_x + i\hat{p}_y) = (\hat{p}_x + i\hat{p}_y)$
- Microscopic reversibility for chiral superfluids: $W(\hat{\mathbf{k}}', \hat{\mathbf{k}}; +\hat{\mathbf{l}}) = W(\hat{\mathbf{k}}, \hat{\mathbf{k}}'; -\hat{\mathbf{l}})$
- $\therefore$ For BTRS: the chiral axis $\hat{\mathbf{l}}$ is fixed $\rightsquigarrow$ $W(\hat{\mathbf{k}}', \hat{\mathbf{k}}; \hat{\mathbf{l}}) \neq W(\hat{\mathbf{k}}, \hat{\mathbf{k}}'; \hat{\mathbf{l}})$

# Determination of the Electron Bubble Radius

(i) Energy required to create a bubble:

$$E(R,P) = E_0(U_0,R) + 4\pi R^2\gamma + \frac{4\pi}{3}R^3 P, \quad P \text{ – pressure}$$

(ii) For $U_0 \to \infty$: $E_0 = -U_0 + \pi^2\hbar^2/2m_e R^2$ – ground state energy

(iii) Surface Energy: hydrostatic surface tension $\rightsquigarrow \gamma = 0.15\,\mathrm{erg/cm^2}$

(iv) Minimizing E w.r.t. $R \rightsquigarrow P = \pi\hbar^2/4m_e R^5 - 2\gamma/R$

(v) For zero pressure, $P = 0$:

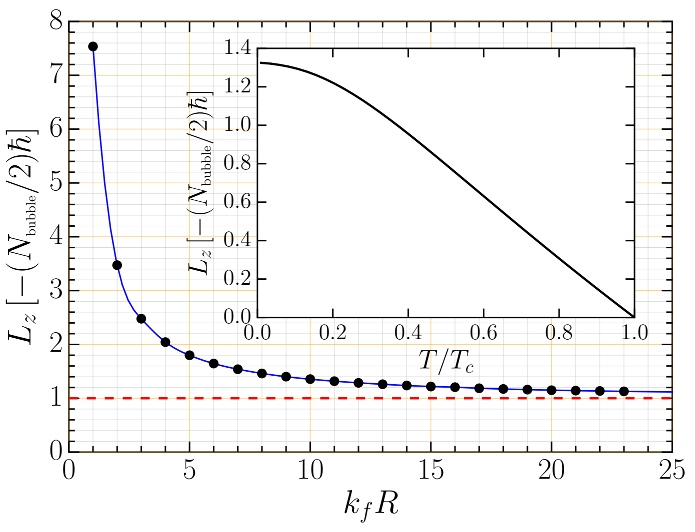
$$R = \left(\frac{\pi\hbar^2}{8m_e\gamma}\right)^{1/4} \approx 2.38\,\mathrm{nm} \quad \rightsquigarrow \quad k_f R = 18.67$$

Transport $\rightsquigarrow k_f R = 11.17$

- A. Ahonen et al., J. Low Temp. Phys., 30(1):205228, 1978

Angular momentum of an electron bubble in $^3$He-A ($k_f R = 11.17$)

$$\mathbf{L}(T \to 0) \approx -\hbar N_{\text{bubble}} \hat{\mathbf{l}}/2\,; \qquad N_{\text{bubble}} = n_3\,\frac{4\pi}{3} R^3 \approx 200\ ^3\text{He atoms}$$



# Mobility of an electron bubble in the Normal Fermi Liquid

(i) $$t_{\mathsf{N}}^{R}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E) = \sum_{l=0}^{\infty} (2l+1) t_l^R(E) P_l(\hat{\mathbf{k}}' \cdot \hat{\mathbf{k}})$$

(ii) $$t_l^R(E) = -\frac{1}{\pi N_f} e^{i\delta_l} \sin \delta_l$$

(iii) $$\frac{d\sigma}{d\Omega_{\mathbf{k}'}} = \left( \frac{m^*}{2\pi\hbar^2} \right)^2 |t_{\mathsf{N}}^{R}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E)|^2$$

(iv) $$\sigma_{\mathsf{N}}^{\mathsf{tr}} = \int \frac{d\Omega_{\mathbf{k}'}}{4\pi} (1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}') \frac{d\sigma}{d\Omega_{\mathbf{k}'}} = \frac{4\pi}{k_f^2} \sum_{l=0}^{\infty} (l+1) \sin^2(\delta_{l+1} - \delta_l)$$

(v) $$\mu_{\mathsf{N}} = \frac{e}{n_3 p_f \sigma_{\mathsf{N}}^{\mathsf{tr}}}, \quad p_f = \hbar k_f, \quad n_3 = \frac{k_f^3}{3\pi^2}$$

## Calculation of LDOS and Current Density

$$\hat{\mathcal{G}}_S^R(\mathbf{r}', \mathbf{r}, E) = \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} e^{i\mathbf{k}'\mathbf{r}'} e^{-i\mathbf{k}\mathbf{r}} \hat{\mathcal{G}}_S^R(\mathbf{k}', \mathbf{k}, E)$$

$$\hat{\mathcal{G}}_S^R(\mathbf{k}', \mathbf{k}, E) = (2\pi)^3 \hat{G}_S^R(\mathbf{k}, E)\delta_{\mathbf{k}',\mathbf{k}} + \hat{G}_S^R(\mathbf{k}', E)\hat{T}_S(\mathbf{k}', \mathbf{k}, E)\hat{G}_S^R(\mathbf{k}, E)$$

$$\hat{G}_S^R(\mathbf{k}, E) = \frac{1}{\varepsilon^2 - E_{\mathbf{k}}^2} \begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad \varepsilon = E + i\eta, \quad \eta \to 0^+$$

$$N(\mathbf{r}, E) = -\frac{1}{2\pi} \mathrm{Im} \left\{ \mathrm{Tr} \left[ \hat{\mathcal{G}}_S^R(\mathbf{r}, \mathbf{r}, E) \right] \right\}$$

$$\mathbf{j}(\mathbf{r}) = \frac{\hbar}{4mi} k_B T \sum_{n=-\infty}^{\infty} \lim_{\mathbf{r}\to\mathbf{r}'} \mathrm{Tr} \left[ (\boldsymbol{\nabla}_{\mathbf{r}'} - \boldsymbol{\nabla}_{\mathbf{r}}) \hat{\mathcal{G}}^M(\mathbf{r}', \mathbf{r}, \epsilon_n) \right]$$

$$\hat{\mathcal{G}}_S^R(\mathbf{r}', \mathbf{r}, E) = \hat{\mathcal{G}}_S^M(\mathbf{r}', \mathbf{r}, \epsilon_n) \Big|_{i\epsilon_n \to \varepsilon}, \ \text{for } n \geq 0$$

$$\hat{\mathcal{G}}_S^M(\mathbf{k}, \mathbf{k}', -\epsilon_n) = \left[ \hat{\mathcal{G}}_S^M(\mathbf{k}', \mathbf{k}, \epsilon_n) \right]^\dagger$$

# Temperature scaling of the Stokes tensor components

- For $1 - \frac{T}{T_c} \to 0^+$:

$$\frac{\eta_\perp}{\eta_{\mathsf{N}}} - 1 \propto -\Delta(T) \propto \sqrt{1 - \frac{T}{T_c}}$$

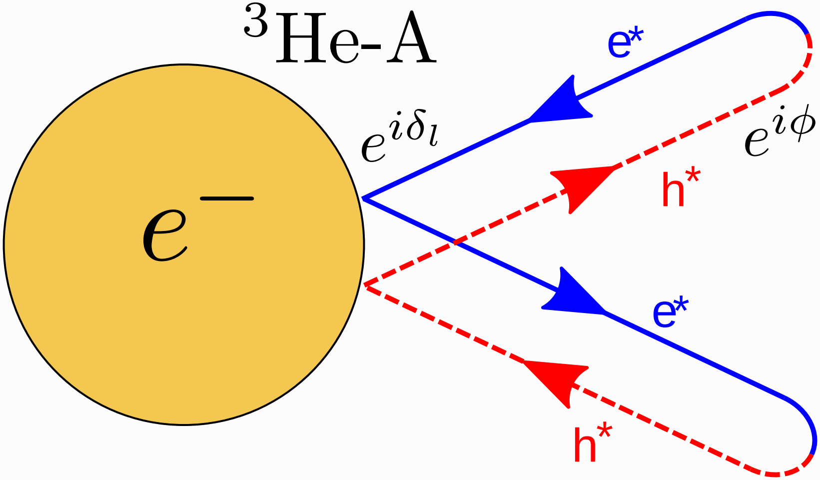
$$\frac{\eta_{\mathsf{AH}}}{\eta_{\mathsf{N}}} \propto \Delta^2(T) \propto 1 - \frac{T}{T_c}$$

- For $\frac{T}{T_c} \to 0^+$:

$$\frac{\eta_\perp}{\eta_{\mathsf{N}}} \propto \left(\frac{T}{T_c}\right)^2$$

$$\frac{\eta_{\mathsf{AH}}}{\eta_{\mathsf{N}}} \propto \left(\frac{T}{T_c}\right)^3$$

## Multiple Andreev Scattering ⇝ Formation of Weyl fermions on $e$-bubbles




$$\Delta(\hat{\mathbf{k}}) = \Delta \sin\theta e^{i\phi}$$