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Superfluid Phases of Liquid ^{3}He in Random Media & Confined Space

James Avery Sauls

Department of Physics & Astronomy
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August 15, 2017

- Research supported by US National Science Foundation Grant DMR-1508730

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Kathy Burgess

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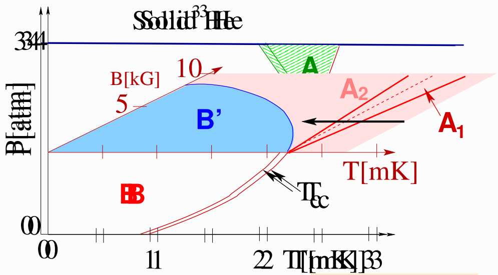
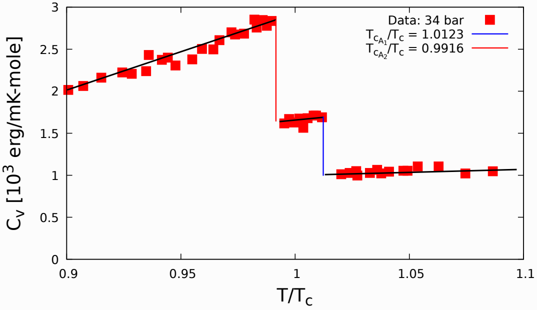
The Phases of Pure Superfluid ^{3}He- Spin-Triplet Pairing





- Suppression of ↑↓ Cooper pairs: $T_{\mathrm{AB}} = T_c - g\,B^2$
- Linear splitting of T_c for ESP Cooper pairs:
  - ↑↑ Cooper pairs condense at $T_c^{A_1} = T_c + \lambda^{\mathrm{A}_1} B$
  - ↓↓ Cooper pairs condense at $T_c^{A_2} = T_c - \lambda^{\mathrm{A}_2} B$

$$\Delta T_c \equiv (T_c^{A_1} - T_c^{A_2}) = (\lambda^{\mathrm{A}_1} + \lambda^{\mathrm{A}_2})\,B$$

$$\lambda^{\mathrm{A}_1} + \lambda^{\mathrm{A}_2} \simeq 6.1\,\mu\mathrm{K/kG}$$

- Theory: $\lambda^{\mathrm{A}_1} \approx \left|\frac{\gamma\hbar}{2}\right| \left(\frac{k_{\mathrm{B}}T_c}{E_f}\right)$

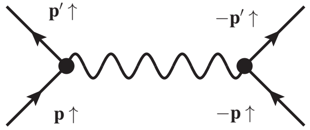
- D. Osheroff, R. Richardson, D. Lee, Evidence for a New Phase of Solid ^{3}He, PRL 28, 885 (1972)
- W. Halperin, C. Archie, F. Rasmussen, T. Alvesalo, and R. Richardson, Specific heat of superfluid ^{3}He on the melting curve, PRB 13, 2124 (1976)
- V. Ambegaokar and N. D. Mermin, Thermal Anomalies of ^{3}He: Pairing in a Magnetic Field, PRL 30, 81 (1973)

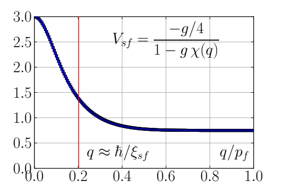
Superfluid ^{3}He

Spin-Fluctuation Mediated Pairing

Paramagnon Exchange: Ferromagnetic Spin Fluctuations $\rightsquigarrow$ Odd-Parity, Spin-Triplet Pairing for ^{3}He

- A. Layzer and D. Fay, Int. J. Magn. 1, 135 (1971)

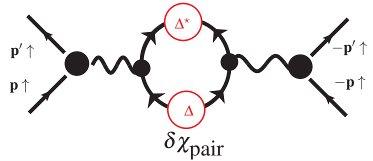
$V_{\text{sf}}(\mathbf{q}) =$  $= \dfrac{g}{1 - g\,\chi(\mathbf{q})}$



$$-g_l = (2l+1)\int \frac{d\Omega_{\hat{p}}}{4\pi}\int \frac{d\Omega_{\hat{p}'}}{4\pi}\, V_{\text{sf}}(\mathbf{p},\mathbf{p}')\, P_l(\hat{p}\cdot\hat{p}')$$

- $-g_l$ is a function of $g \approx 0.75$ & $\xi_{\text{sf}} \approx 5\hbar/p_f$
- $l = 1$ (p-wave) is dominant pairing channel
  - $\hat{p}_x + i\hat{p}_y \sim \sin\theta_{\hat{p}}\; e^{+i\phi_{\hat{p}}} \rightsquigarrow l_z = +1$
  - $\hat{p}_z \sim \cos\theta_{\hat{p}} \rightsquigarrow l_z = 0$
  - $\hat{p}_x - i\hat{p}_y \sim \sin\theta_{\hat{p}}\; e^{-i\phi_{\hat{p}}} \rightsquigarrow l_z = -1$
- $S = 1$, $S_z = 0$, ± 1 pairing fluctuations

- Feedback on $V_{sf} \rightsquigarrow$ Multiple Stable Superfluid Phases



$\chi_A \approx \chi_N > \chi_B \rightsquigarrow \frac{1}{3}\chi_N \rightsquigarrow$ Superfluid A-phase

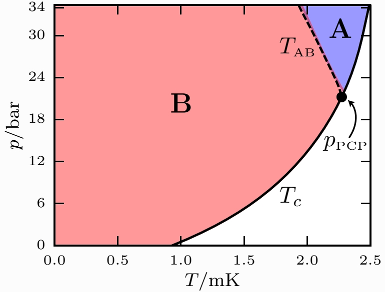
- W. Brinkman, J. Serene, and P. Anderson, PRA 10, 2386 (1974)

- Not the Whole Story: Liquid ^{3}He is near a Mott transition & Solid is AFM Ordered
- Normal ^{3}He: an almost localized Fermi liquid, D. Vollhardt, RMP 56, 99 (1984) See Poster P.966, Joshua Wiman

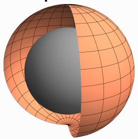
Maximal Symmetry of ^{3}He: $\mathtt{G} = \mathtt{SO(3)}_{\mathsf{S}} \times \mathtt{SO(3)}_{\mathsf{L}} \times \mathtt{U(1)}_{\mathsf{N}} \times \mathtt{P} \times \mathtt{T} \rightarrow$

Superfluid Phases of ^{3}He

J. Wiman & J. A. Sauls, PRB 92, 144515 (2015)



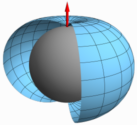
"Isotropic" BW State



$J = 0,\ J_z = 0$

$H = \mathtt{SO(3)}_{\mathsf{J}} \times \mathtt{T}$

Chiral AM State $\vec{l} = \hat{\mathbf{z}}$



$L_z = 1,\ S_{z'} = 0$

$H = \mathtt{U(1)}_{\mathsf{S}} \times \mathtt{U(1)}_{\mathsf{L_z\text{-}N}} \times \mathtt{Z_2}$

Spin-Triplet BCS Condensate Amplitude: $\widehat{\Psi} = \begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix} \leftarrow \Psi_{\alpha\beta}(p) = \langle \psi_\alpha(p)\psi_\beta(-p)\rangle$

$$\widehat{\Psi}_{BW} = \begin{pmatrix} p_x - ip_y \sim e^{-i\phi} & p_z \\ p_z & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

$$\widehat{\Psi}_{AM} = \begin{pmatrix} p_x + ip_y \sim e^{+i\phi} & 0 \\ 0 & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

Fully Gapped: $\widehat{\Psi}^{\dagger}_{BW}\widehat{\Psi}_{BW} = |\Delta|^2$

Nodal Points: $\widehat{\Psi}^{\dagger}_{AM}\widehat{\Psi}_{AM} = |\Delta|^2 \sin^2\theta$

Ginzburg-Landau Functional for Superfluid ^{3}He

- Maximal Symmetry of ^{3}He: $\mathtt{G} = \mathtt{SO(3)_L} \times \mathtt{SO(3)_S} \times \mathtt{U(1)_N} \times \mathtt{P} \times \mathtt{T} \times \mathtt{C}$

- Order Parameter for P-wave $(L=1)$, Spin-Triplet $(S=1)$ Pairing

$$\widehat{\Psi}(\hat{p}) = \overbrace{\begin{pmatrix} S_{\mathbf{x}} & S_{\mathbf{y}} & S_{\mathbf{z}} \end{pmatrix}}^{\text{Spin Basis}} \times \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix} \times \overbrace{\begin{pmatrix} \hat{p}_x \\ \hat{p}_y \\ \hat{p}_z \end{pmatrix}}^{\text{Orbital Basis}}$$

- GL Functional: $A_{\alpha i}$ $\rightsquigarrow$ vector under both $\mathtt{SO(3)_S}$ $[\alpha]$ and $\mathtt{SO(3)_L}$ $[i]$

$$\begin{aligned} \mathscr{U}[A] &= \int d^3r \Big[\alpha(T)\,\mathrm{Tr}\left\{AA^\dagger\right\} + \beta_1 \left|\mathrm{Tr}\left\{AA^{\mathrm{tr}}\right\}\right|^2 + \beta_2 \left(\mathrm{Tr}\left\{AA^\dagger\right\}\right)^2 \\ &+ \beta_3\,\mathrm{Tr}\left\{AA^{\mathrm{tr}}(AA^{\mathrm{tr}})^*\right\} + \beta_4\,\mathrm{Tr}\left\{(AA^\dagger)^2\right\} + \beta_5\,\mathrm{Tr}\left\{AA^\dagger(AA^\dagger)^*\right\} \\ &+ \kappa_1 \partial_i A_{\alpha j} \partial_i A^*_{\alpha j} + \kappa_2 \partial_i A_{\alpha i} \partial_j A^*_{\alpha j} + \kappa_3 \partial_i A_{\alpha j} \partial_j A^*_{\alpha i} \Big] \end{aligned}$$

- Mermin, Ambegaokar, Brinkman, Anderson, circa 1974

Dynamical Consequences of Spontaneous Symmetry Breaking

Acoustic Cavity Modes

Lagrangian Field Theory for Bosonic Excitations of Superfluid ^{3}He-B

^{3}He-B: $B_{\alpha i} = \frac{1}{\sqrt{3}}\Delta\,\delta_{\alpha i}$ $L=1$, $S=1$ $\rightsquigarrow$ $J=0$ $C=+1$

- Symmetry of ^{3}He-B: $\mathtt{H} = \mathtt{SO(3)}_\mathtt{J} \times \mathtt{T}$
- Fluctuations: $\mathscr{D}_{\alpha i}(\mathbf{r},t) = A_{\alpha i}(\mathbf{r},t) - B_{\alpha i} = \sum_{J,m} D_{J,m}(\mathbf{r},t)\, t^{(J,m)}_{\alpha i}$
- Lagrangian:

$$\mathscr{L} = \int d^3r \left\{ \tau\,\mathrm{Tr}\left\{\dot{\mathscr{D}}\dot{\mathscr{D}}^\dagger\right\} - \alpha\,\mathrm{Tr}\left\{\mathscr{D}\mathscr{D}^\dagger\right\} - \sum_{p=1}^{5}\beta_p\,u_p(\mathscr{D}) - \sum_{l=1}^{3} K_l\,v_l(\partial\mathscr{D}) \right\}$$

$$\partial_t^2 D^{(\mathtt{C})}_{J,m} + E^{(\mathtt{C})}_{J,m}(\mathbf{q})^2\, D^{(\mathtt{C})}_{J,m} = \frac{1}{\tau}\eta^{(\mathtt{C})}_{J,m}$$

with $J = \{0,1,2\}\,, m = -J \ldots +J\,, \mathtt{C} = \pm 1$

- *Time-Dependent GL Theory for Bosonic Excitations of Superfluid ^{3}He-B*: JAS & T. Mizushima, PRB 95, 094515 (2017)

Spectrum of Bosonic Modes of Superfluid ^{3}He-B : Condensate is $J^{\mathtt{C}} = 0^+$

▶ 4 Nambu-Goldstone Modes & 14 Higgs modes

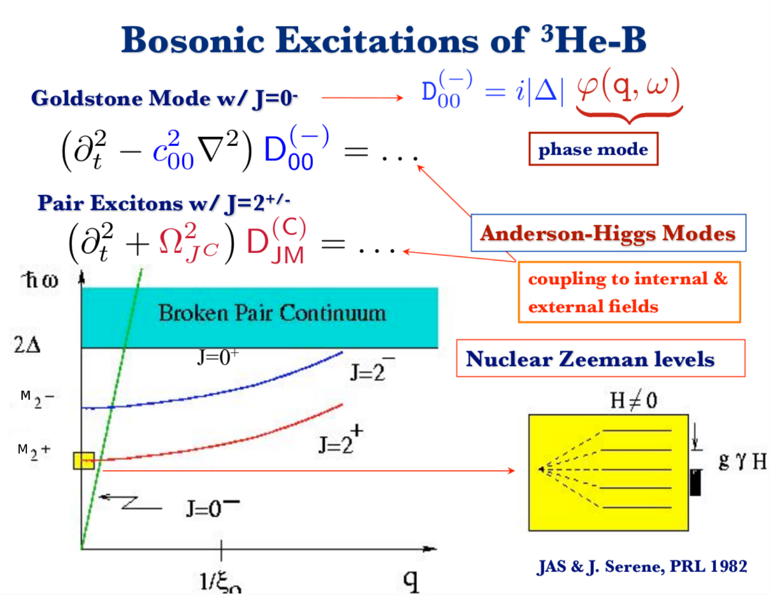
$$E^{(\mathtt{C})}_{J,m}(\mathbf{q}) = \sqrt{M^2_{J,\mathtt{C}} + \left(c^{(\mathtt{C})}_{J,|m|}|\mathbf{q}|\right)^2}$$

Mode	Symmetry	Mass	Name
$D^{(+)}_{0,m}$	$J=0,\ \mathtt{C}=+1$	2Δ	Amplitude Higgs
$D^{(-)}_{0,m}$	$J=0,\ \mathtt{C}=-1$	0	NG Phase Mode
$D^{(+)}_{1,m}$	$J=1,\ \mathtt{C}=+1$	0	NG Spin-Orbit Modes
$D^{(-)}_{1,m}$	$J=1,\ \mathtt{C}=-1$	2Δ	AH Spin-Orbit Modes
$D^{(+)}_{2,m}$	$J=2,\ \mathtt{C}=+1$	$\sqrt{\frac{8}{5}}\Delta$	2^+ AH Modes
$D^{(-)}_{2,m}$	$J=2,\ \mathtt{C}=-1$	$\sqrt{\frac{12}{5}}\Delta$	2^- AH Modes

▶ Vdovin, Maki, Wölfle, Serene, Nagai, Volovik, Schopohl, McKenzie, JAS ...

▶ JAS and J. W. Serene, Coupling of Order-Parameter Modes with L>1 to Zero Sound in ^{3}He-B, Phys. Rev. B 23, 4798 (1982)

Collective Mode Spectrum for ^{3}He-B



Dynamical Consequences of Spontaneous Symmetry Breaking

First Observations of Higgs Bosons in a BCS Condensate - Superfluid ^{3}He-B

Observation of a New Sound-Attenuation Peak in Superfluid ^{3}He-*B*

R. W. Giannetta,[a] A. Ahonen,[b] E. Polturak, J. Saunders,
E. K. Zeise, R. C. Richardson, and D. M. Lee
Laboratory of Atomic and Solid State Physics and Materials Science Center, Cornell University, Ithaca, New York 14853
(Received 25 March 1980)

Results of zero-sound attenuation measurements in ^{3}He-*B*, at frequencies up to 60 MHz and pressures between 0 and 20 bars, are reported. At frequencies of 30 MHz and above, a new attenuation feature is observed which bears the signature of a collective mode of the superfluid.

VOLUME 45, NUMBER 4 PHYSICAL REVIEW LETTERS 28 JULY 1980

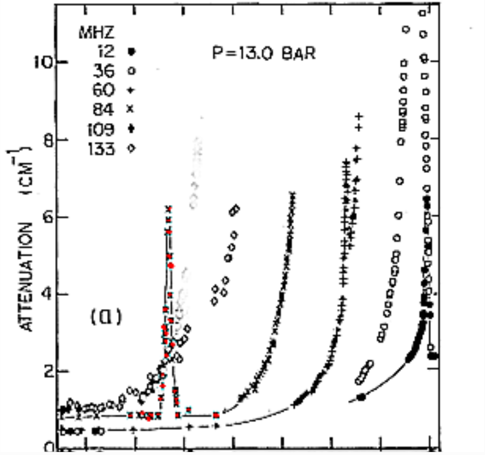
Measurements of High-Frequency Sound Propagation in ^{3}He-*B*

D. B. Mast, Bimal K. Sarma, J. R. Owers-Bradley, I. D. Calder,
J. B. Ketterson, and W. P. Halperin
Department of Physics and Astronomy and Materials Research Center, Northwestern University, Evanston, Illinois 60201
(Received 10 April 1980)

Measurements of the attenuation and velocity of pulsed high-frequency sound have been performed up to 133 MHz in superfluid ^{3}He-*B*. A new collective mode of the order parameter was discovered at a frequency extrapolated to T_c of $\omega = (1.165 \pm 0.05)\Delta_{\rm BCS}(T_c)$, where $\Delta_{\rm BCS}(T)$ is the energy gap in the weak-coupling BCS theory. The group velocity has been observed to decrease by as much as $\frac{2}{3}$ of the zero-sound velocity.

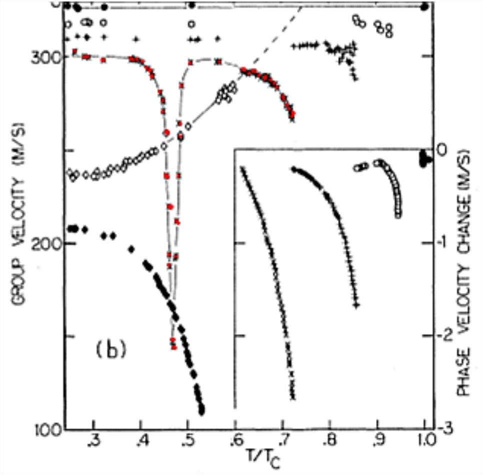
Dynamical Consequences of Spontaneous Symmetry Breaking

Higgs Mode with mass: $M = 500$ neV and spin $J^{\mathrm{C}} = 2^{+}$ at ULT-Northwestern



T/T_c

Group Velocity

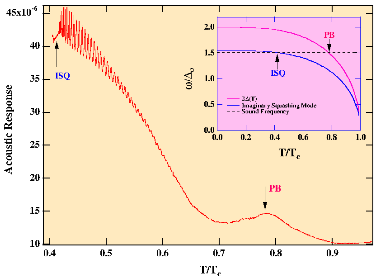


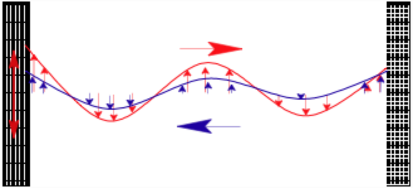
$J = 2^-$, $m = \pm 1$ Higgs Modes Transport Mass and Spin

- "Transverse Waves in Superfluid ^{3}He-B", G. Moores and JAS, JLTP 91, 13 (1993)
- "Electromagnetic Absorption in Anisotropic Superconductors", P. Hirschfeld et al., PRB 40, 6695 (1989)

$$C_t(\omega) = \sqrt{\frac{F_1^s}{15}} v_f \left[\rho_n(\omega) + \frac{2}{5}\rho_s(\omega) \underbrace{\left\{ \frac{\omega^2}{(\omega + i\Gamma)^2 - \frac{12}{5}\Delta^2 - \frac{2}{5}(q^2 v_f^2)} \right\}}_{D^{(-)}_{2,\pm 1}} \right]^{\frac{1}{2}}$$

Transverse Zero Sound Propagation in Superfluid ^{3}He-B: *Cavity Oscillations of TZS*





- Y. Lee et al. Nature 400 (1999)

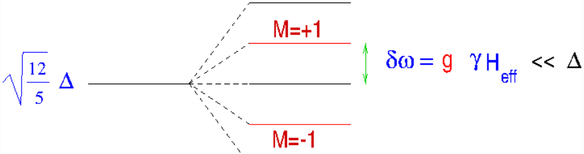
B ⟶

Faraday Rotation: Magneto-Acoustic Birefringence of Transverse Currents

- "Magneto-Acoustic Rotation of Transverse Waves in ^{3}He-B", J. A. Sauls et al., Physica B, 284,267 (2000)

$$C_{\substack{\text{RCP}\\\text{LCP}}}(\omega) = v_f \left[\frac{F_1^s}{15}\rho_n(\omega) + \frac{2F_1^s}{75}\rho_s(\omega) \underbrace{\left\{ \frac{\omega^2}{(\omega + i\Gamma)^2 - \Omega_{2,\pm}^{(-)}(\mathbf{q})} \right\}}_{D_{2,\pm 1}^{(-)}} \right]^{\frac{1}{2}}$$

$$\Omega_{2,\pm}^{(-)}(\mathbf{q}) = \sqrt{\frac{12}{5}}\Delta \pm g_{2^-}\,\gamma H_{\text{eff}}$$



- Circular Birefringence $\Longrightarrow$ $C_{\text{RCP}} \neq C_{\text{LCP}}$ $\Longrightarrow$ Faraday Rotation

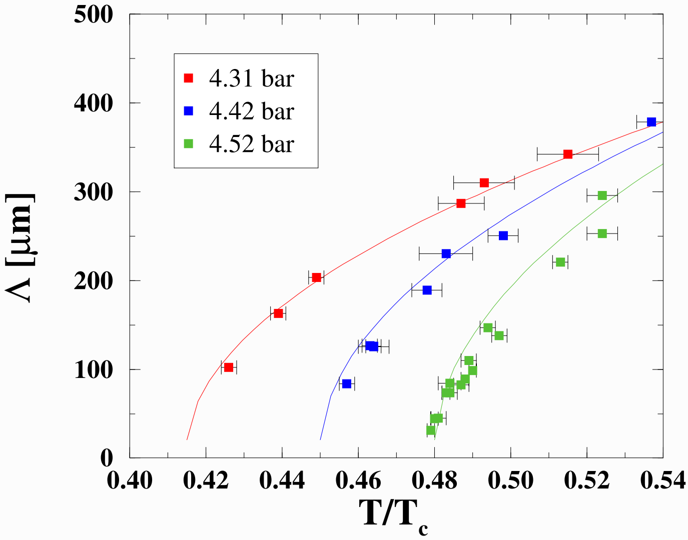
$$\left(\frac{C_{\text{RCP}} - C_{\text{LCP}}}{C_t}\right) \simeq g_{2^-} \left(\frac{\gamma H_{\text{eff}}}{\omega}\right)$$

- Faraday Rotation Period ($\gamma H_{\text{eff}} \ll (\omega - \Omega_2^{(-)})$):

$$\Lambda \simeq \frac{4\pi C_t}{g_{2^-}\gamma H} \simeq 500\,\mu m\,, \quad H = 200\,G$$

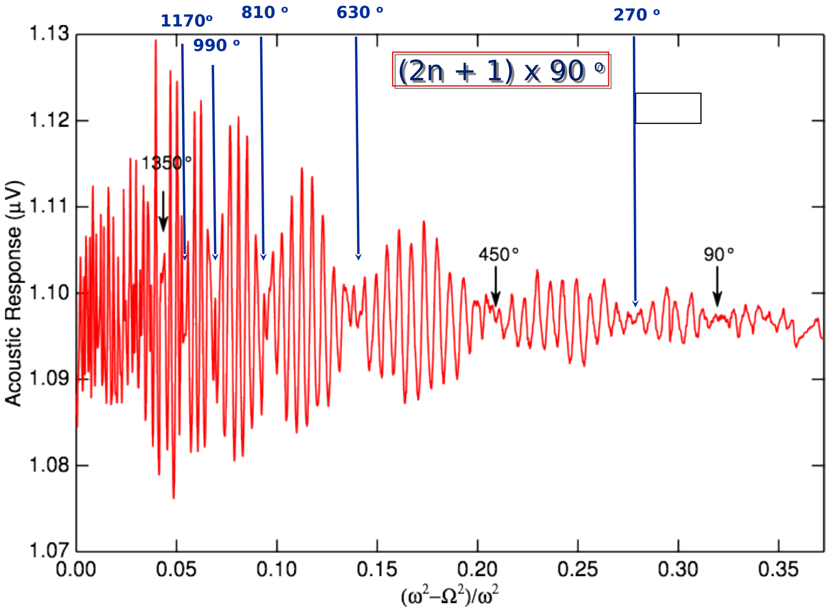
- *Discovery of the acoustic Faraday effect in superfluid ^{3}He-B*, Y. Lee, et al. Nature 400, 431 (1999)

Faraday Rotation: Magneto-Acoustic Birefringence of Transverse Currents



- "Broken Symmetry & Non-Equilibrium Superfluid ^{3}He", J. A. Sauls, *Lecture Notes - Les Houches 1999*, Eds. H. Godfrin & Y. Bunkov, Elsievier (2000)

Large Faraday Rotations vs. ``Blue Tuning'' B = 1097 G



Liquid ^{3}He in Random Media

Superfluidity in the Presence of Disorder

Liquid ^{3}He is the purest known form of condensed matter

- ^{3}He is Chemically Inert
- Large Zero-point Motion
- Even ^{4}He is expelled at $T \ll 1\,\mathrm{K}$

The discovery of superfluidity of ^{3}He infused into Silica Aerogel opened up new and wide-ranging investigations into the effects of disorder, confinement and random fields on matter with well-understood broken symmetry breaking phases.

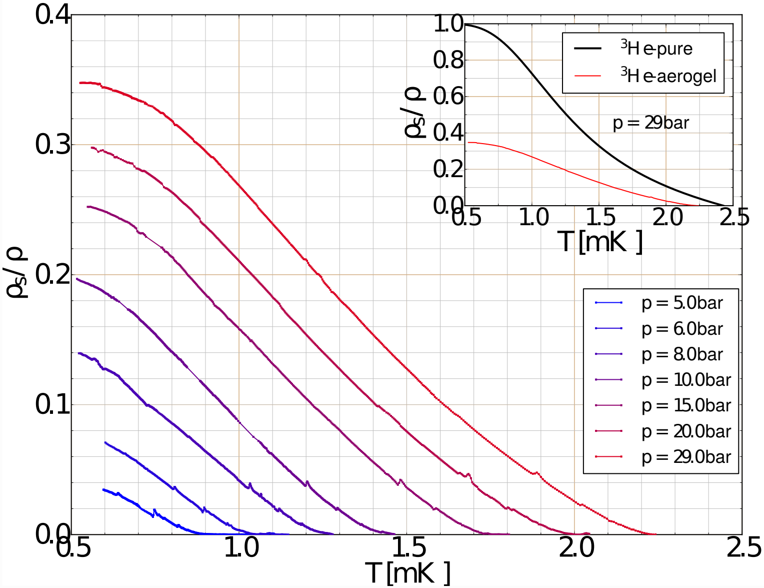
If you had asked me in 1994 if Superfluidity would be able to exist if ^{3}He were infused into Silica Aerogel, I likely would have bet against.

- Size of Pure ^{3}He Cooper Pairs: $\xi_0 \approx 100\,\mathrm{nm}$
- For 98% Porosity Silica Aerogel: $\rho_a = 0.02/a^3$ with $a \approx 1\,\mathrm{nm}$
- ^{3}He Quasiparticle-Impurity Cross-Section: $\sigma = a^2$
- Mean-Free Path: $l = 1/\rho_a \sigma = 50\,\mathrm{nm} < \xi_0$

$\rightsquigarrow$ Collisions would destroy the Orbital Correlations of P-wave Cooper Pairs

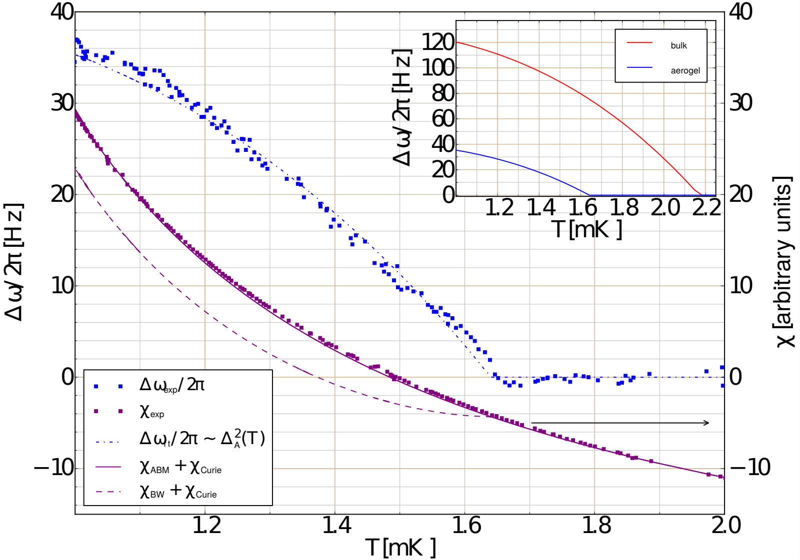
However, mean values can be misleading

Discovery of Superfluidity of ^{3}He in Silica Aerogel - Torsional Oscillator



- Porto and J. Parpia, Superfluid ^{3}He in Aerogel, Phys. Rev. Lett. 74, 4667 (1995)

Discovery of Superfluidity of ^{3}He in Silica Aerogel - NMR



- $\Delta\omega > 0 \rightsquigarrow$ "A-like" Phase
- Solid ^{3}He $\rightsquigarrow \chi_{\text{solid}} = C/T$
- $\chi_{\text{liquid}} = \chi_N \rightsquigarrow$ ESP State
- **ESP @ $P = 12\,$bar**

- Strong Suppression of T_c
- Stronger Suppression of $\Delta\omega$

- D. Sprague, T. Haard, J. Kycia, M. Rand, Y. Lee, P. Hamot, and W. Halperin, Homogeneous Equal-Spin Pairing Superfluid State of ^{3}He in Aerogel, Phys. Rev. Lett. 75, 661 (1995)

Research on Unconventional Superconductivity circa 1995

PHYSICAL REVIEW B VOLUME 51, NUMBER 22 1 JUNE 1995-II

Nonlinear Meissner effect in unconventional superconductors

D. Xu, S. K. Yip, and J. A. Sauls
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(Received 16 August 1994)

PHYSICAL REVIEW B VOLUME 52, NUMBER 14 1 OCTOBER 1995-II

Infrared conductivity in layered *d*-wave superconductors

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PHYSICAL REVIEW B VOLUME 53, NUMBER 22 1 JUNE 1996-II

Electronic thermal conductivity and the Wiedemann-Franz law for unconventional superconductors

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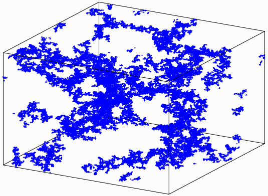
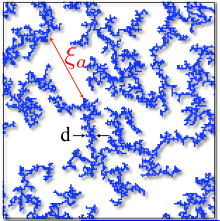
Liquid ^{3}He in $(1-\rho) \approx 98\,\%$ Silica Aerogel

Northwestern Aerogel Lab



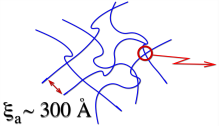
DLCA Simulations - S. Ali & JAS

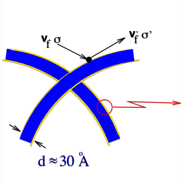


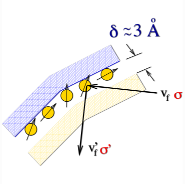
Local Anisotropy

Characteristic Length Scales in Silica Aerogel

Strand/Cluster Model







2D Solid ^{3}He

Quasiparticle Mean-free Path in ^{3}He-aerogel

$$\ell_{\text{geom}} = 3\pi/8\,(d/\rho) \approx 2000 \text{Å}$$

Cooper Pair Radius in Superfluid ^{3}He

$$\xi_0 = \frac{\hbar v_f}{2\pi k_B T_c^{\text{bulk}}} \approx 200 - 800 \text{Å}$$

- E. Thuneberg, S.K. Yip and JAS, 1996

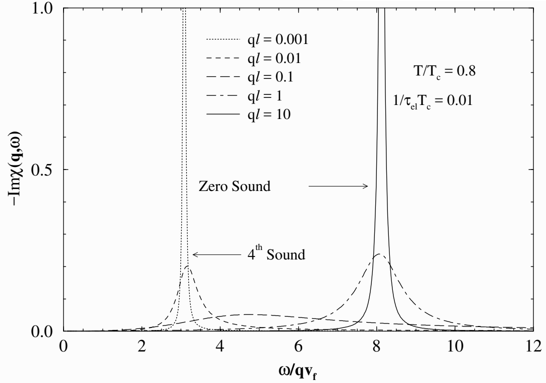
Sound Propagation and Transport Properties of Liquid ^{3}He in Aerogel

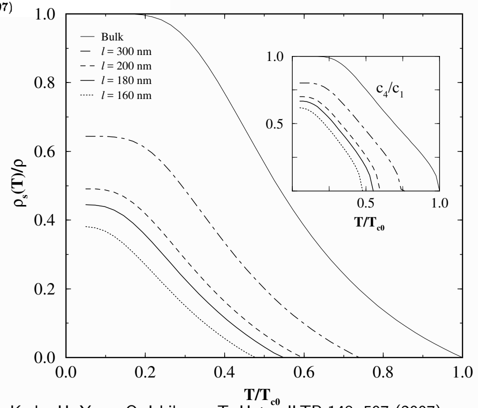
D. Rainer[1] and J. A. Sauls[2]

[1] *Physikalisches Institut, Universität Bayreuth, Bayreuth, Germany*

[2] *Department of Physics & Astronomy, Northwestern University, Evanston, Illinois*

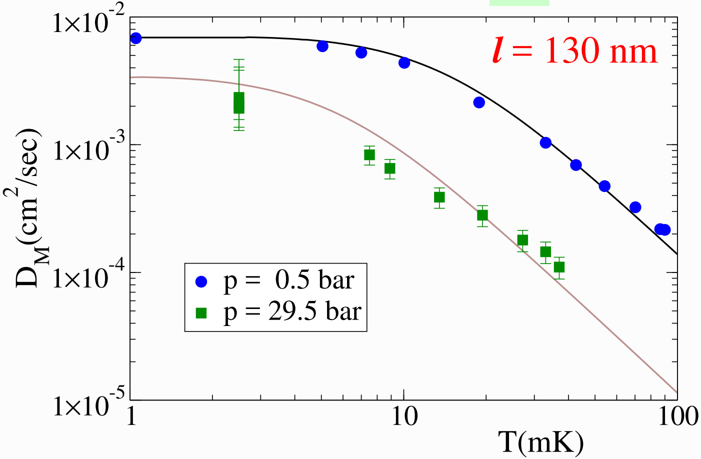
(Received August 31, 1997)





- "Fourth Sound Measurement of Superfluid ^{3}He in Aerogel", Y. Nago, K. Obara, R. Kado, H. Yano, O. Ishikawa, T. Hata, JLTP 148, 597 (2007)
- "High-Frequency Acoustics of ^{3}He in Aerogel", R. Nomura, G. Gervais, T. Haard, Y. Lee, N. Mulders, and W. P. Halperin, PRL 85, 4325 (2000)
- "Ultrasound Attenuation of Superfluid ^{3}He in Aerogel", H. C. Choi, N. Masuhara, B. H. Moon, P. Bhupathi, M. W. Meisel, Y. Lee, N. Mulders, S. Higashitani, M. Miura, and K. Nagai, PRL 98, 225301 (2007)
- "Transverse Sound in Liquid-^{3}HeAerogel System", S. Higashitani, M. Miura, T. Ichikawa, M. Yamamoto, and K. Nagai PRL 89, 215301 (2002)
- "Collision Drag Eect on Propagation of Sound in Liquid ^{3}He in Aerogel", T. Ichikawa , M. Yamamoto, S. Higashitani and K. Nagai, JPSJ 70, 3483 (2001)

Spin Diffusion in Normal ^{3}He-Aerogel: $\vec{J}_M = -D_M \vec{\nabla} M$

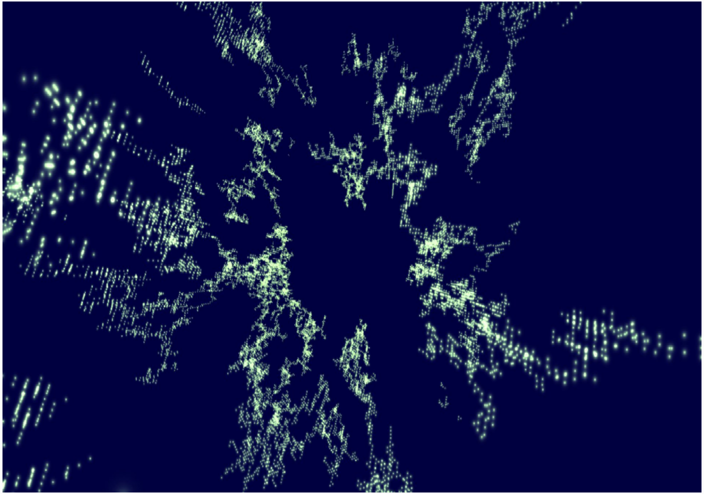


Exact Solutions to the Landau-Boltzmann Transport Equation: $D_M = \frac{1}{3}(1+F_0^a)\, v_f^2\, \tau_D(T,\ell)$

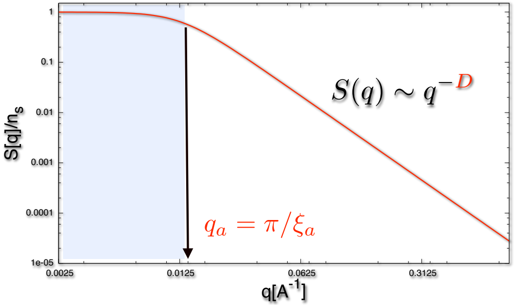
$$\tau_D(T,\ell) \to \begin{cases} \tau_D^{pure} \approx \quad \tau_{inel}(T) \propto T^{-2}, & T \gg T^* \\ \tau_D^{dirty} \approx \quad \tau_{elastic} = \ell/v_f, & T \ll T^* \approx 10-30\,\mathrm{mK} \end{cases}$$

- Transport properties of normal liquid ^{3}He in aerogel, P. Venkataramani (Sharma), J.A. Sauls, Physica B 284-288, 297 (2000)
- Magnetization & spin diffusion of liquid ^{3}He in Aerogel, J. A. Sauls, Yu. M. Bunkov, E. Collin, H. Godfrin and P. Sharma, PRB 72, 024507 (2005)

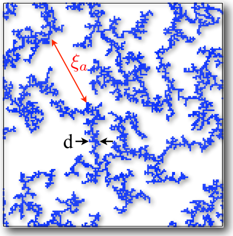
DLCA Numerical Simulation of 98% Aerogel, Sarosh Ali and JAS



Fractal Structure of Silica Aerogel



DLCA Simulations - S. Ali & JAS



Radial mass distribution

Static Structure Factor S. Sinha (ANL 1999)

$$S(q) = \int d^3r\, e^{iq\cdot r}\, g(r)$$

$$g(r) = n_{\rm clusters}(r)/r^3$$

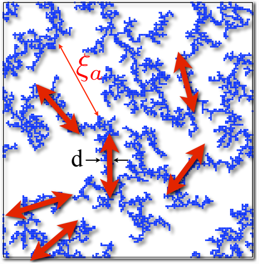
$$n_{\rm clusters}(r) \sim r^D$$

There is another distribution characterizing the random fractal corresponding free flights in the open structure.

$D \simeq 1.7$

T. Witten and Sander, PRL (1981)
P. Meakin, PRL (1983)

Effects of Aerogel Anisotropy on Superfluidity of ^{3}He



$\langle \hat{\mathbf{s}}(\mathbf{r}) \cdot \hat{\mathbf{s}}(0) \rangle \sim e^{-r^2/2\,\xi_s^2}$

Short-range Orientational Order

Local Anisotropy

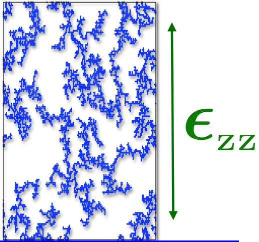
Local "Strand" Orientation

$\Longleftarrow$ $\hat{\mathbf{s}}(\mathbf{r})$

Short-range correlations:

$\xi_s \approx \xi_a$

Global Anisotropy



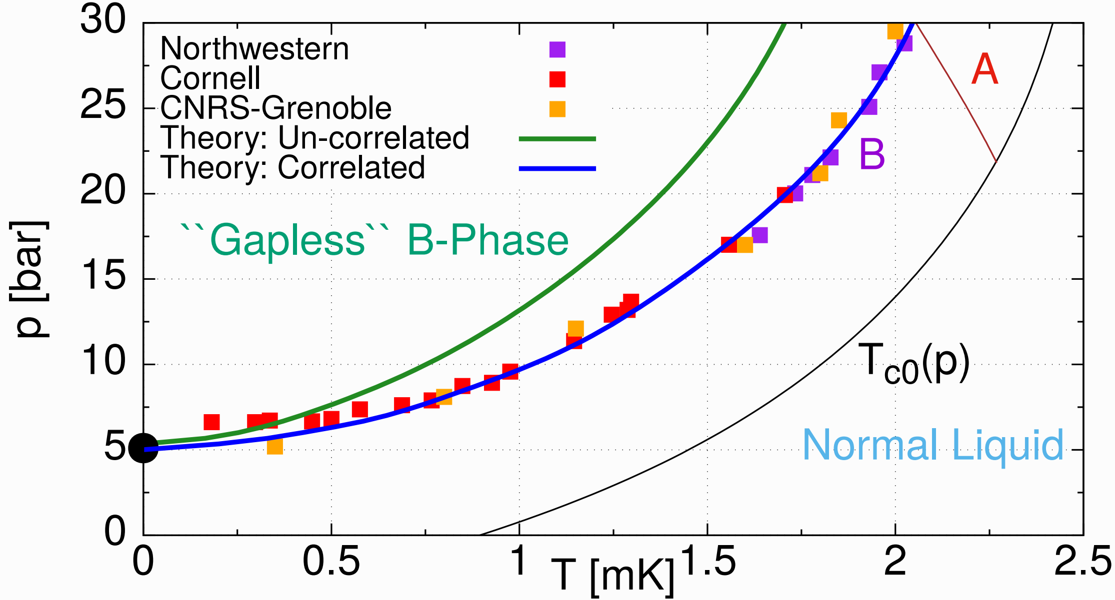
Long-range Orientational Order

$\langle \hat{\mathbf{s}}(\mathbf{r}) \cdot \hat{\mathbf{s}}(0) \rangle \rightarrow \mathbf{s}^2 \,, \; 0 < \mathbf{s} \leq 1$

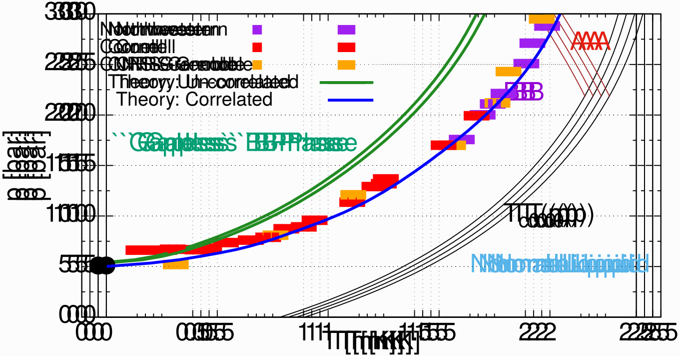
- **Local Anisotropy:** $\xi_s \approx \xi_a < \xi = \hbar v_f / 2\pi k_B T_c \rightsquigarrow$ ^{3}He in "Isotropic" Aerogels $\rightsquigarrow$ "Gapless" B-phase
- **Random Anisotropy:** fluctuations in the nemaic axis: $\hat{\mathbf{s}}_\perp(\mathbf{r}) \equiv \hat{\mathbf{s}}(\mathbf{r}) - s\hat{\mathbf{z}} \rightsquigarrow$ Orbital "Glass Phases"
- **Global Anisotropy:** (e.g. "Stretched" Aerogels) $\rightsquigarrow$ Chiral-ESP, Polar-ESP and Bi-Axial-ESP Phases

- Models for Superfluid ^{3}He in Aerogel, E. Thuneberg, S.K. Yip, M. Fogelström, J.A. Sauls, Arxiv:cond-mat/9601148 (1996) & PRL 80, 2861 (1998)
- Glass state of superfluid ^{3}He-A in an aerogel, G. Volovik, JETP Lett 63, 301 (1996)
- Pairing States of ^{3}He in a Uniaxially Anisotropic Aerogel, K. Aoyama and R. Ikeda, Phys. Rev. B 73, 060504 (2006).
- Calculation of Orientational Effect of Deformed Aerogel on the Order Parameter of Superfluid ^{3}He, E. V. Surovtsev & I. A. Fomin, JLTP 159, 487 (2008)
- On Larkin-Imry-Ma State of ^{3}He-A in Aerogel, G. Volovik, JLTP 150, 453 (2008)
- Chiral Phases of Superfluid ^{3}He in an Anisotropic Medium, J. A. Sauls, PRB 88, 214503 (2013)

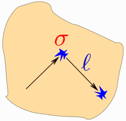
Phase Diagram for Superfluid ^{3}He in "Isotropic" 98 % Silica Aerogel



Phase Diagram for Superfluid ^{3}He in "Isotropic" 98 % Silica Aerogel

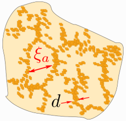


- Un-correlated Impurities (mean field)



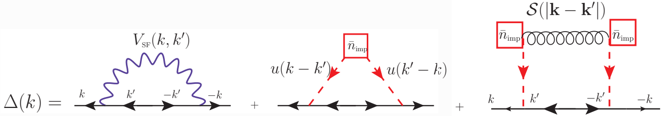
  - $\ell = \dfrac{1}{n_{imp}\,\sigma} \rightsquigarrow \ell_{geom}$
  - $\mathscr{L}_{hsm}(p) = \dfrac{\xi(p)}{\ell}$

- Correlated Disorder $\rightsquigarrow$ Random Voids



  - $\xi_a \gtrsim \sqrt{\xi(p)\,\ell}$
  - $\mathscr{L}_{voids}(p) = \left(\dfrac{\xi(p)}{\xi_a}\right)^2$

- 98% Aerogel: $\xi_a \approx 420\,\text{Å}$, $\ell \approx 1450\,\text{Å}$
- ^{3}He-aerogel: $\xi(p) \equiv \hbar v_f / 2\pi k_B T_c(p)$

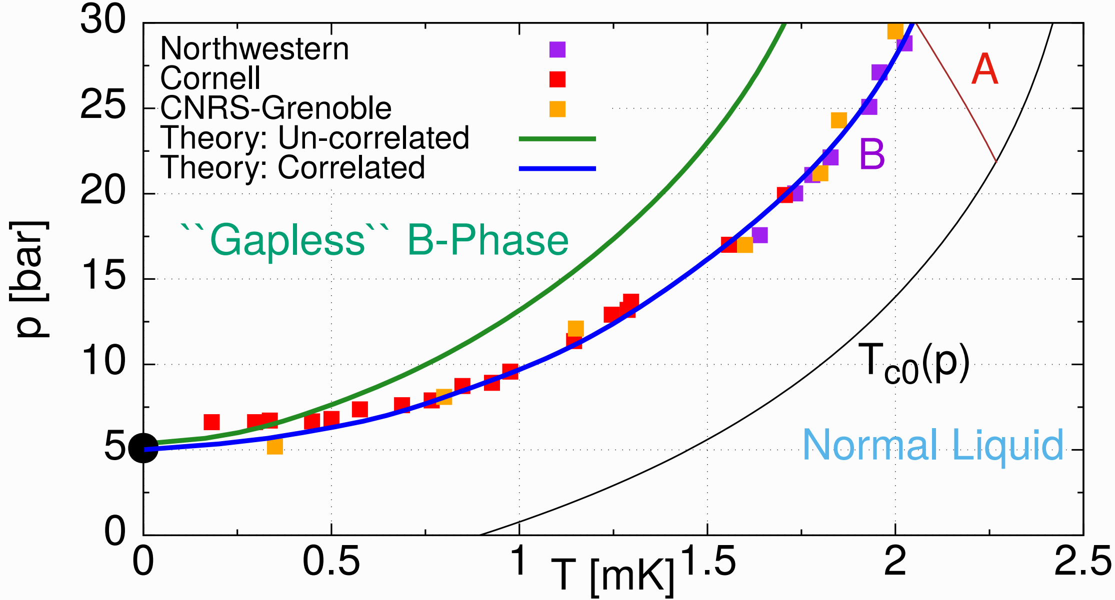


$$1/\mathscr{L} = 1/\mathscr{L}_{hsm} + 1/\mathscr{L}_{voids}$$

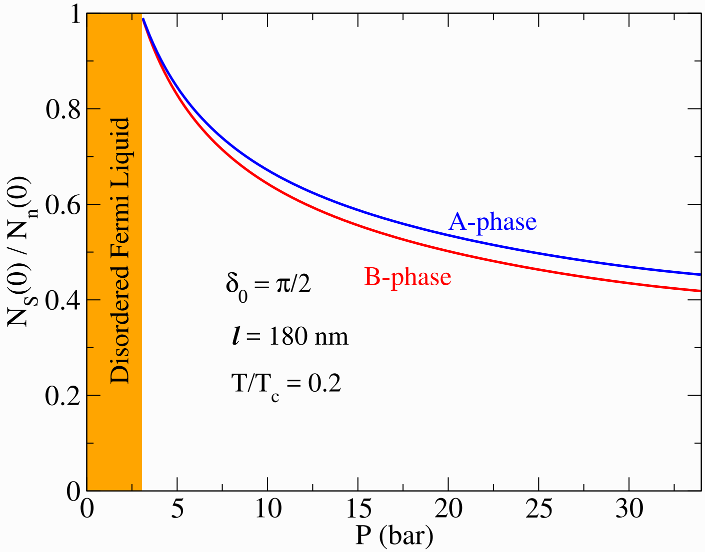
- $\xi_0(p_c) = 0.28\,\ell$
- $p_c \approx 5\,\text{bar}$

- Models for Superfluid ^{3}He in Aerogel, E. Thuneberg, S.K. Yip, M. Fogelström, J.A. Sauls, Arxiv:cond-mat/9601148 (1996) & PRL 80, 2861 (1998)
- Impurity effects on the A_1-A_2 splitting of superfluid ^{3}He in Aerogel, J. A. Sauls & P. Sharma, PRB 68, 224502 (2003)
- Quantum Phase Transition of ^{3}He in Aerogel, K. Matsumoto, J. V. Porto, L. Pollack, E. N. Smith, T.L. Ho, and J. M. Parpia, PRL 79, 253 (1997)
- Phase diagram of the superfluid phases of 3He in 98% aerogel, G. Gervais, K.Yawata, N. Mulders, and W.P. Halperin, PRB 66, 054528 (2002)
- Magnetization & spin diffusion of liquid ^{3}He in Aerogel, J. A. Sauls, Yu. M. Bunkov, E. Collin, H. Godfrin and P. Sharma, PRB 72, 024507 (2005)

Phase Diagram for Superfluid ^{3}He in “Isotropic” 98 % Silica Aerogel



Gapless B-phase in "Isotropic" 98 % Silica Aerogel



- Magnetic Susceptibility of the Balian-Werthamer Phase of ^{3}He in Aerogel, P. Sharma and J. A. Sauls, JLTP 125, 115-142 (2001)

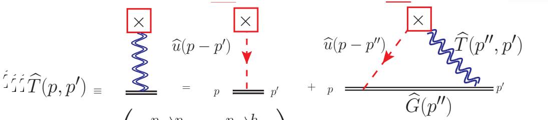
Gapless B-phase in "Isotropic" 98 % Silica Aerogel

▶ Quasiparticle Density of States

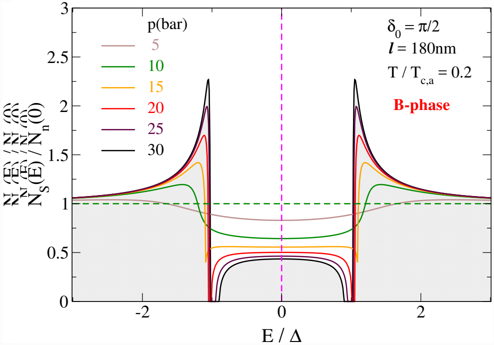
$$N_S(\mathbf{p},E) = -\frac{N_f}{4\pi}\,\mathrm{Im}\,\mathrm{Tr}\left\{\hat{\tau}_3 \widehat{G}^{R}(\mathbf{p},E)\right\}$$

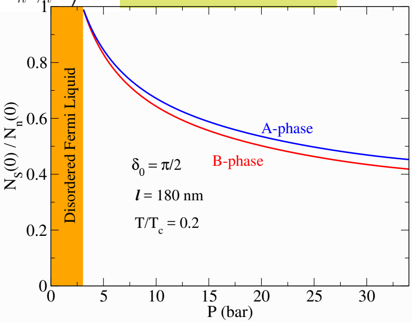
$$N_S(E) = \int \frac{d\Omega_{\hat{p}}}{4\pi} N_S(\mathbf{p},E)$$

▶ Pair-breaking ⟵ Quasiparticle-Aerogel Scattering



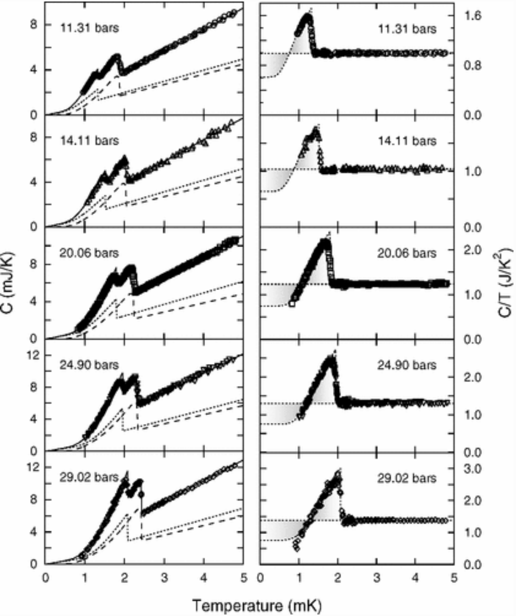
$$= \begin{pmatrix} \overbrace{T(p,p')}^{p\to p} & \overbrace{\mathbf{A}(\mathbf{p},\mathbf{p}')}^{p\to h} \\ \underbrace{\bar{\mathbf{A}}(\mathbf{p},\mathbf{p}')}_{h\to p} & \underbrace{\bar{T}(p,p')}_{h\to h} \end{pmatrix} \rightsquigarrow \widehat{\Sigma}_{imp}(p) = \begin{pmatrix} \Sigma_{imp}(p) & \Delta_{imp}(p) \\ \bar{\Delta}_{imp}(p) & \bar{\Sigma}_{imp}(p) \end{pmatrix}$$



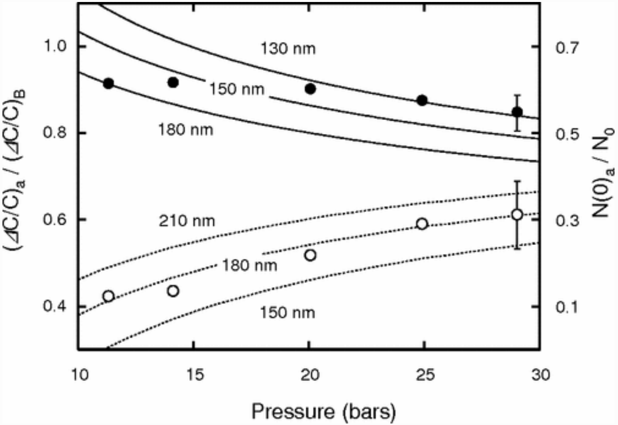


▶ Magnetic Susceptibility of the Balian-Werthamer Phase of ^{3}He in Aerogel, P. Sharma and J. A. Sauls, JLTP 125, 115-142 (2001)

Specific Heat of ^{3}He in 98% Silica Aerogel



- Entropy balance: $\gamma_{\text{n}} T_c = \int_0^{T_c} \frac{C_{\text{S}}(T)}{T}\, dT \rightsquigarrow N_{\text{S}}(0) \neq 0$



- Specific Heat of Disordered Superfluid ^{3}He, H. Choi, K. Yawata, T. Haard, J. Davis, G. Gervais, N. Mulders, P. Sharma, J. A. Sauls, and W. P. Halperin, PRL 93, 145301 (2004)
- Models for Superfluid ^{3}He in Aerogel, E. Thuneberg, S.K. Yip, M. Fogelström, J.A. Sauls, Arxiv:cond-mat/9601148 (1996) & PRL 80, 2861 (1998)

Extending the Ginzburg-Landau Theory for ^{3}He-aerogel

- Quasiclassical Reduction of the Luttinger-Ward Functional - Serene, Rainer, Thuneberg, Kurkijärvi, JAS:

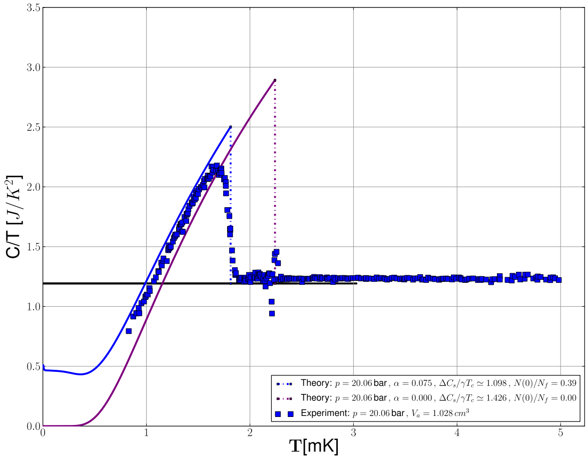
$$\Delta\Omega\left[\widehat{G},\widehat{\Sigma}\right]=-N_f\int d^3R\int\frac{d\Omega_{\hat{\mathbf{p}}}}{4\pi}T\sum_{\varepsilon_n}\mathrm{Tr}_4\left\{\widehat{\Sigma}\widehat{G}+\int d\xi_{\mathbf{p}}\left[\ln\left(-i\varepsilon_n\widehat{\tau}_3+\xi_{\mathbf{p}}+\widehat{\Sigma}+\widehat{V}_{\mathrm{imp}}[\{\mathbf{R}_i\}]\right)-\ln\left(-i\varepsilon_n\widehat{\tau}_3+\xi_{\mathbf{p}}\right)\right]\right\}+\Phi[\widehat{G}]$$

- Ensemble Averaged Propagator: $\widehat{\mathfrak{G}}(\hat{p},\mathbf{R};\varepsilon_n)=\int d\xi_{\mathbf{p}}\prod_{i=1}^{N_s}\int d^3R_i\,P(\{\mathbf{R}_i\})\left(i\varepsilon_n\widehat{\tau}_3-\xi_{\mathbf{p}}-\widehat{\Delta}-\widehat{V}_{\mathrm{imp}}[\{\mathbf{R}_i\}]\right)^{-1}$

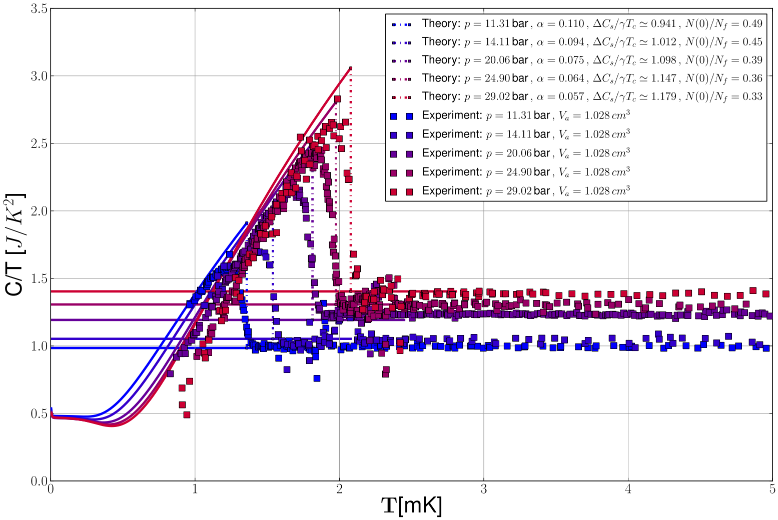
- Thermodynamic Potential for Superfluid ^{3}He in Aerogel, S. Ali, L. Zhang and J. A. Sauls, JLTP 162, 233242 (2010)

Specific Heat of Superfluid ^{3}He in Aerogel - Experiment vs. Theory

$P = 20.06\,\text{bar} \quad \alpha = 0.075 \quad \Delta C_S/\gamma_S T_c = 1.098 \quad N_S/N_n = 0.39$

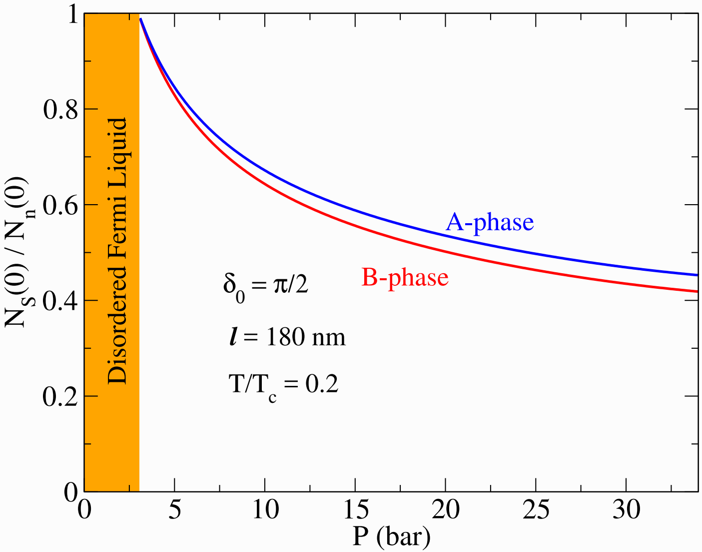


- Specific Heat of Disordered Superfluid ^{3}He, H. Choi, K. Yawata, T. Haard, J. Davis, G. Gervais, N. Mulders, P. Sharma, J. A. Sauls, and W. P. Halperin, PRL 93, 145301 (2004)



- $\lim_{T\to 0} C/T \approx 0.5\text{J}/\text{K}^2$ *independent of Pressure* $\rightsquigarrow N_S(0) \propto$ Surface Area of SiO_2
- $\lim_{T\to 0} C/T \propto$ Number of Zero-Energy Surface Andreev Bound-State Fermions

Gapless B-phase in "Isotropic" 98 % Silica Aerogel



- Magnetic Susceptibility of the Balian-Werthamer Phase of ^{3}He in Aerogel, P. Sharma and J. A. Sauls, JLTP 125, 115-142 (2001)

Nuclear Magnetization of the Gapless Balian-Werthamer Phase of ^{3}He

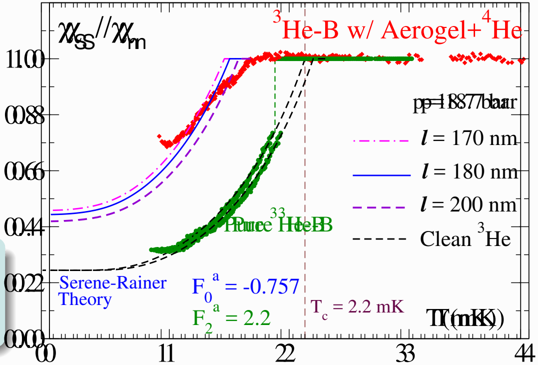
- Nuclear Magnetization of ^{3}He Fermi Liquid:

$$\chi_n = \mu^2 \frac{2\,N_f}{1+F_0^a} \quad \text{with } \mu = \frac{\gamma\hbar}{2}, \quad F_0^a \approx -0.75$$

DOS: $N_n(E) \approx N_n(0) \equiv N_f = \dfrac{m^* p_f}{2\pi^2\hbar^3} \quad m^*/m_3 \approx 6$

- Superfluid ^{3}He: $\mathbf{M} = \chi_N \left(\mathbf{H} + \dfrac{\mathbf{m}}{2\mu}\right)$

$$\frac{\chi_S}{\chi_n} = \frac{(1+F_0^a)\left[\frac{2}{3}+\tilde{Y}\left(\frac{1}{3}+\frac{1}{5}F_2^a\right)\right]}{1+F_0^a\left(\frac{2}{3}+\frac{1}{3}\tilde{Y}\right)+\frac{1}{5}F_2^a\left(\frac{1}{3}+\frac{2}{3}\tilde{Y}\right)+\frac{1}{5}F_2^aF_0^a\,\tilde{Y}}$$



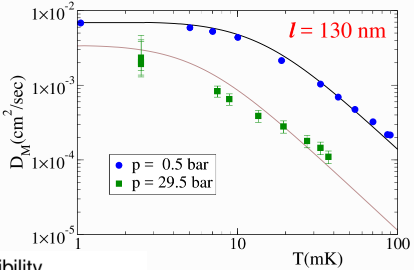
- $\tilde{Y}(T,\Gamma) = 1 - \pi T \sum_n \dfrac{\Delta^2}{[\tilde{\varepsilon}_n^2+\Delta^2]^{3/2}} \left\{ \dfrac{1}{1+\frac{1}{3}\frac{\Delta^2}{\tilde{\varepsilon}_n^2}\frac{\Gamma}{\sqrt{\tilde{\varepsilon}_n^2+\Delta^2}}} \right\}, \quad \tilde{\varepsilon}_n = \varepsilon_n + \Gamma \dfrac{\sqrt{\tilde{\varepsilon}_n^2+\Delta^2}}{\tilde{\varepsilon}_n}, \quad \Gamma = \hbar v_f/2\ell$

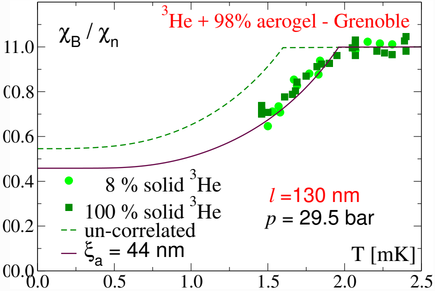
- Effect of Magnetic Scattering on Superfluid ^{3}He-aerogel, D. Sprague, T. Haard, J. Kycia, M. Rand, Y. Lee, P. Hamot & W. Halperin, PRL 77, 4568 (1996)
- Magnetic Susceptibility of the Balian-Werthamer Phase of ^{3}He in Aerogel, P. Sharma and J. A. Sauls, JLTP 125, 115-142 (2001)
- Spin susceptibility of the superfluid ^{3}He-B in aerogel, V. P. Mineev and P. L. Krotkov, PRB, 65, 024501 (2001)
- Models for Superfluid ^{3}He in Aerogel, E. Thuneberg, S.K. Yip, M. Fogelström, J.A. Sauls, Arxiv:cond-mat/9601148 (1996) & PRL 80, 2861 (1998)
- The Quasiclassical Approach to ^{3}He, J. W. Serene and D. Rainer, Phys. Rep. 101, 221 (1983)

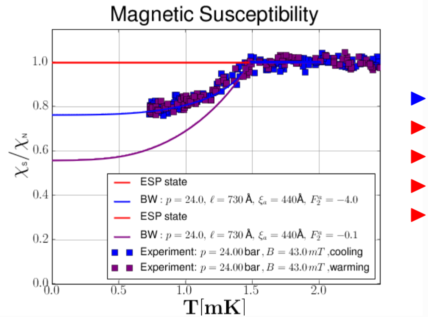
Spin Diffusion & Magnetization ⇝ Aerogel Correlation Length

- Spin Diffusion of ^{3}He Fermi Liquid: $\vec{J}_{\mathsf{M}} = -\, D_{\mathsf{M}}\, \vec{\nabla} M$

$$D_{\mathsf{M}} = \frac{1}{3}\,(1+F_0^a)\; v_f^2\; \tau_{\mathsf{D}}(T,\ell)$$



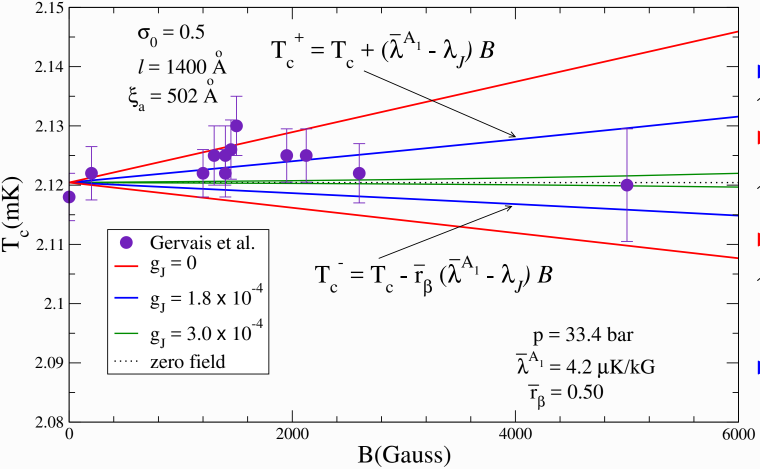




- Y. Sasaki, Kyoto (LT26)
- 97.5% Aerogel
- $P = 24\,$bar
- $\ell = 73\,$nm (D_{M})
- $\xi_a = 44\,$nm

- Magnetization & spin diffusion of liquid ^{3}He in Aerogel, J. A. Sauls, Yu. M. Bunkov, E. Collin, H. Godfrin and P. Sharma, PRB 72, 024507 (2005)
- Transport properties of normal liquid ^{3}He in aerogel, P. Venkataramani (Sharma), J.A. Sauls, Physica B 284-288, 297 (2000)

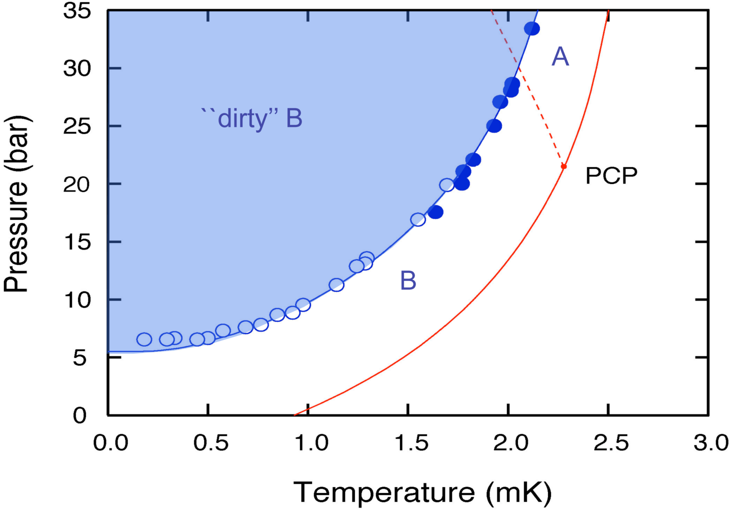
Suppression of the A_1 - A_2 Splitting in Superfluid ^{3}He-Aerogel



- Anti-Ferromagnetic Exchange ($J > 0$)
  $\rightsquigarrow$ suppressed A_1-A_2 Splitting ($B \lesssim 1$ Tesla)
- For $g_J = 1.8 \times 10^{-4}$
  $\rightsquigarrow$ $J \simeq 0.1\,\text{mK/liquid-}^3\text{He spin}$
- High B/T: $\mu B/k_B T \rightarrow \mathscr{P}(B/B_S, T/\Theta_C)$
  $\rightsquigarrow$ Solid ^{3}He AFM Order: $\Theta_C = 0.4\,\text{mK}$
- Non-linear field evolution of
  $T_c^{A_1}$ - $T_c^{A_2}$ for $B \approx B_C \approx 1\,\text{T}$

- Phase diagram of the superfluid phases of 3He in 98% aerogel, G. Gervais, K.Yawata, N. Mulders, and W.P. Halperin, PRB 66, 054528 (2002)
- Impurity effects on the A_1-A_2 splitting of superfluid ^{3}He in Aerogel, J. A. Sauls & P. Sharma, PRB 68, 224502 (2003)
- Two-dimensional nuclear magnets, H. Godfrin and R.E. Rapp, Adv. Phys. 44, 113 1995

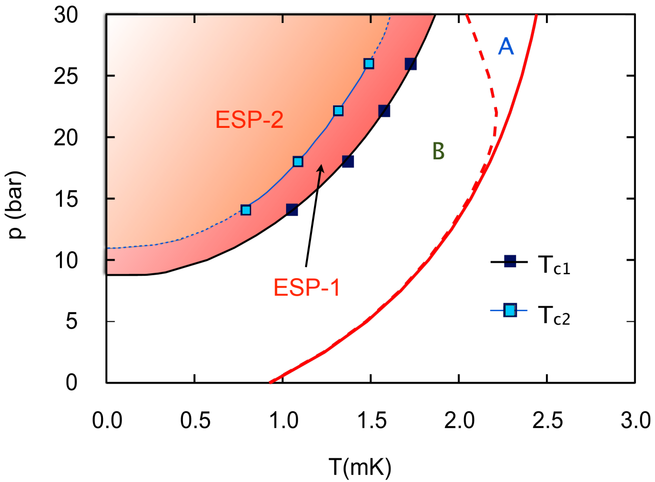
Superfluid ^{3}He in Globally Isotropic Aerogel

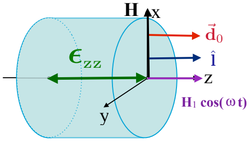


- <u>No</u> A-phase
- Suppressed Transition: $T_{c_a} < T_c$
- QCP at $P \approx 5\,\text{bar}$
- Aerogel Structure $\rightsquigarrow \Delta T_c \lll T_c$
- Order survives the Random Field
- ^{3}He-B is a “Robust phase”

Torsional Oscillator: Matsumoto et al. PRL 79, 253 (1997) Acoustic Attenuation: Gervais et al., PRL 87 035701 (2002)

Superfluid ^{3}He in Globally Ansotropic Aerogels





- Two Equal-Spin-Pairing Phases
- No Polar Phase for $T > T_{c_1}$

- ESP-1 $\rightsquigarrow$ Strain-aligned A phase: $\hat{\mathbf{l}} \parallel \hat{\mathbf{z}}$
- ESP-2 $\rightsquigarrow$ Bi-axial/Chiral: $\hat{\mathbf{l}}(\mathbf{r}) \nparallel \hat{\mathbf{z}}$
- Random Anisotropy $\rightsquigarrow$ ESP-2: variant of the Larkin-Imry-Ma Phase

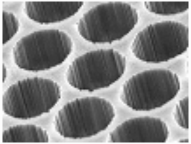
"Engineering" New Phases of ^{3}He

- V. Dmitriev (Kapitza): Nematic Aerogels $\rightsquigarrow$ Polar Phase
- A. Zimmerman (Northwestern): Compressed Aerogels
- J. Wiman, JAS (Northwestern): S. Laine, E. Thuneberg (Oulu) GL Theory Nematic Aerogels

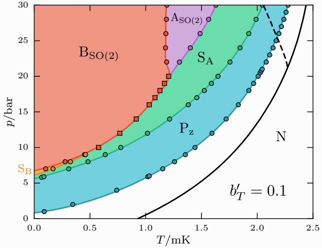
- New Chiral phases of superfluid ^{3}He in anisotropic aerogel, J. Pollanen, J. Li, C. Collett, W. Gannon, W. Halperin & J. Sauls, Nat Phys. 8, 317 (2012)
- The Angular Momentum in ^{3}He-A in a Stretched Aerogel, J. Li, A. Zimmerman, J. Pollanen, C. Collett, W. Gannon, W. Halperin, JLTP 175, 31 (2014)
- Chiral phases of superfluid He in an anisotropic medium, J. A. Sauls, Phys. Rev. B 88, 214503 (2013)
- Pairing states of superfluid ^{3}He in uniaxially anisotropic aerogel, K. Aoyama and R. Ikeda, PRB 73, 060504(R) (2006)
- Anisotropic strong-coupling effects on superfluid ^{3}He in aerogels: Conventional spin-fluctuation approach, R. Ikeda, PRB B 91, 174515 (2015)

New Phases of Superfluid ^{3}He Under Strong Confinement

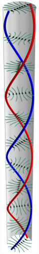
100nm Pores



- J.J. Wiman & JAS, PRB 92, 144515 (2015)



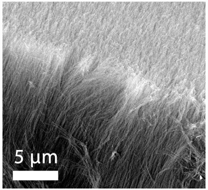
S_A - Spiral Phase



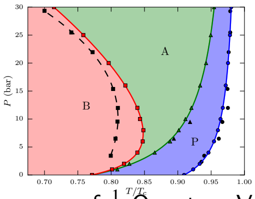
Chiral Anomaly

- 5 phases of ^{3}He
- Polar P_z Phase
- Spiral Phase

- V. Dmitriev et al.,PRL (2015)



- J.J. Wiman, S. Laine, E. Thuneberg & JAS, LT28.

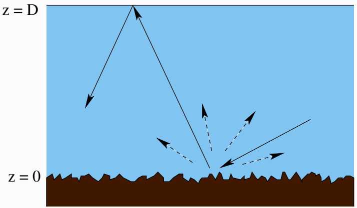


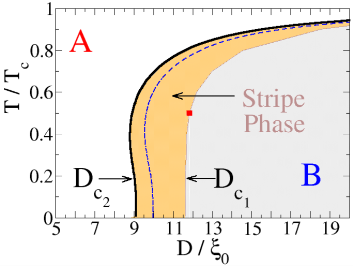
- ^{3}He Confined in Nematic Aerogel
- Discovery of $\frac{1}{2}$ Quantum Vortices - S. Autti et al., PRL (2016)

Superfluid ^{3}He Under Strong Confinement

New Phases with Spontaneously Broken *Translational & Time-Reversal* Symmetries

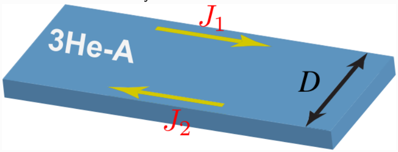
- A. Vorontsov & JAS, PRL (2007)
- J. J. Wiman & JAS JLTP (2016)





- *Dynamical instability of the $J = 2^+, m_J = 0$ Higgs field in confined superfluid* ^{3}*He*, T. Mizushima & JAS, LT28

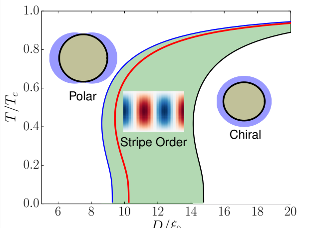
Laterally confined films of ^{3}He



Chiral Edge Currents in ^{3}He-A Films

⇝ Anomalous Hall Effects

- Hao Wu, PhD Thesis (2016) Hybridization of Chiral Edge States ⟶



Superfluid ^{3}He as Topological Quantum Matter

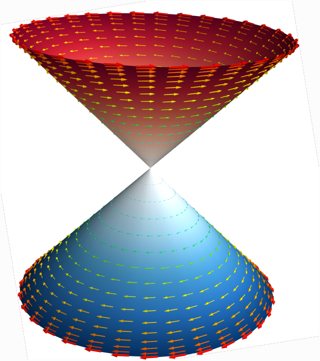
Confinement, Excitations & New Phases

Spontaneously Broken Relative Spin-Orbit Symmetry in ^{3}He-B

$\Downarrow$

Symmetry Protected Topology in Momentum Space

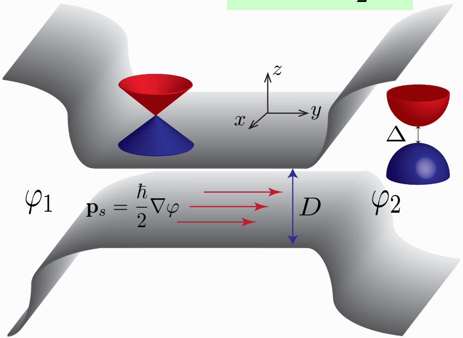
- Massless Helical Majorana Fermions
- Spin-Momentum Locked Vacuum Spin-Currents
- Ising Magnetic Response
- Ground-State Helical Spin-Currents
- Thermal Signatures of Helical Majorana Fermions
- Topological Phase Transitions



- G. Volovik, The Universe in a Helium Droplet, Oxford Press (2003)
- T. Mizushima, Y. Tsutsumi, M. Sato & K. Machida, Symmetry protected topological ^{3}He-B, J. Phys. Cond. Mat. 27 113203 (2015)
- Y. Nagato, S. Higashitani & K. Nagai, Strong Anisotropy in Spin Suceptibility of ^{3}He-B Films, JPSJ 78, 123603 (2009)
- Hao Wu and J. Sauls, Majorana excitations, spin & mass currents on the surface of topological superfluid ^{3}He-B, Phys. Rev. B 88, 184506 (2013)

Condensate Flow and Backflow from Majorana Excitations

Condensate Flow: $\mathbf{p}_s \equiv m\mathbf{v}_s = \frac{\hbar}{2}\nabla\varphi$

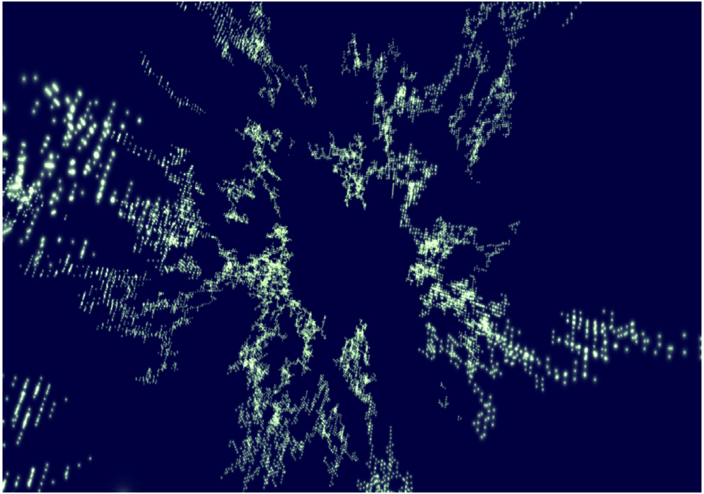


- Flow Field Breaks T-symmetry, but not Topological Protection
- Doppler Shifted Majorana Spectrum: $\varepsilon \to \varepsilon = c|\mathbf{p}_{||}| + \mathbf{p}_{||}\cdot\mathbf{v}_s$
- Thermal Signature: $\vec{J} = n\,\mathbf{p}_s \times \left(1 - \frac{27\pi\zeta(3)}{2}\,\frac{\xi_\Delta}{D}\,\frac{\Delta_\perp}{\Delta_{||}}\,\frac{m^*}{m_3}\left(\frac{T}{\Delta_{||}}\right)^3\right)$

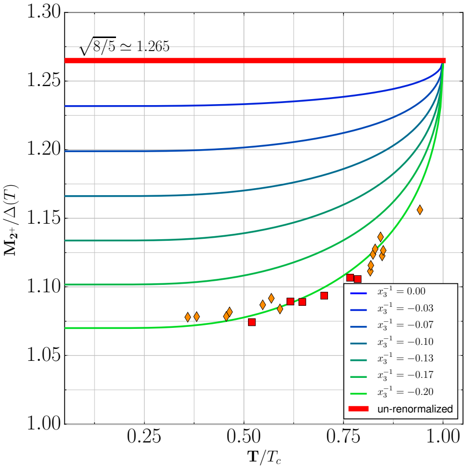
- T. Mizushima et al., PRL 109, 165301, (2012)
- Hao Wu, JAS, PRB 88, 18 184506 (2013)

The End

DLCA Numerical Simulation of 98% Aerogel, Sarosh Ali and JAS



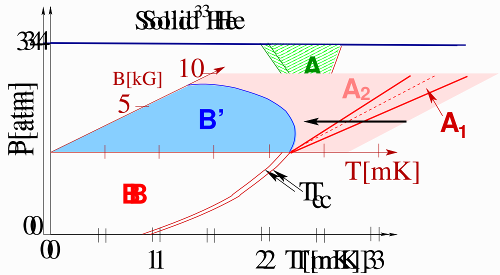
Vacuum Polarization $\rightsquigarrow$ Mass shift of the $J^{\mathrm{C}} = 2^{+}$ Higgs Mode in ^{3}He-B

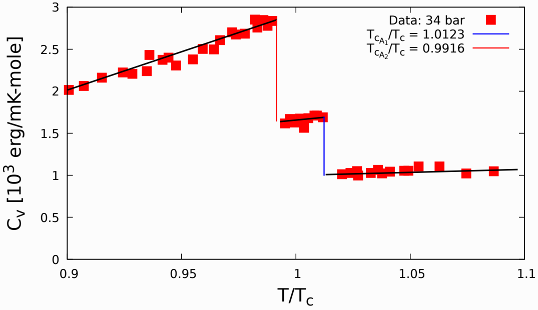


- Measurements: D. Mast et al. PRL 45, 266 (1980)
- exchange p-h channel: $F_2^a = -0.88$ (from Magnetic susceptibility of ^{3}He-B)
  - *attractive* f-wave interaction $\rightsquigarrow$ Higgs Modes with $J = 4^{\pm}$ with $M \lesssim 2\Delta$!

- JAS and J. W. Serene, Coupling of Order-Parameter Modes with L>1 to Zero Sound in ^{3}He-B, Phys. Rev. B 23, 4798 (1982)
- JAS and T. Mizushima, On Nambu's Boson-Fermion Mass Relations, Phys. Rev. B 95, 094515 (2017)

The A_1 - A_2 Transitions of Superfluid ^{3}He





$$\Delta T_c \equiv (T_c^{A_1} - T_c^{A_2}) = (\lambda^{A_1} + \lambda^{A_2})\, B$$

$$\lambda^{A_1} + \lambda^{A_2} \simeq 6.1 \mu\text{K/kG}$$

Linear splitting of T_c for ESP Cooper pairs:

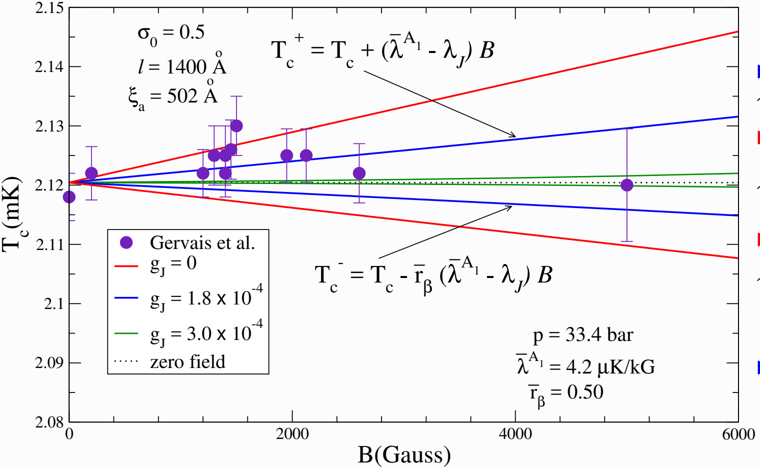
- $\uparrow\uparrow$ Cooper pairs condense at $T_c^{A_1} = T_c + \lambda^{A_1} B$
- $\downarrow\downarrow$ Cooper pairs condense at $T_c^{A_2} = T_c - \lambda^{A_2} B$

- Theory: $\lambda^{A_1} \approx \left|\frac{\gamma\hbar}{2}\right| \left(\frac{k_B T_c}{E_f}\right)$

Requires: Broken Time-Reversal & Particle-Hole Symmetries

- W. Halperin, C. Archie, F. Rasmussen, T. Alvesalo, and R. Richardson, Specific heat of superfluid ^{3}He on the melting curve, PRB 13, 2124 (1976)
- U. Israelsson, B. Crooker, H. Bozler, and C. Gould, Phase Diagram of Superfluid ^{3}He-A, PRL 53, 1943 (1984)
- V. Ambegaokar and N. D. Mermin, Thermal Anomalies of ^{3}He: Pairing in a Magnetic Field, PRL 30, 81 (1973)

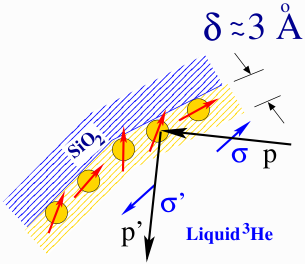
Suppression of the A_1 - A_2 Splitting in Superfluid ^{3}He-Aerogel



- Anti-Ferromagnetic Exchange ($J > 0$)
- $\rightsquigarrow$ suppressed A_1-A_2 Splitting ($B \lesssim 1$ Tesla)
- For $g_J = 1.8 \times 10^{-4}$
- $\rightsquigarrow$ $J \simeq 0.1\,$mK/liquid-^{3}He spin
- High B/T: $\mu B/k_B T \rightarrow \mathscr{P}(B/B_S, T/\Theta_C)$
- $\rightsquigarrow$ Solid ^{3}He AFM Order: $\Theta_C = 0.4\,$mK
- Non-linear field evolution of $T_c^{A_1}$ - $T_c^{A_2}$ for $B \approx B_C \approx 1\,$T

- Phase diagram of the superfluid phases of 3He in 98% aerogel, G. Gervais, K.Yawata, N. Mulders, and W.P. Halperin, PRB 66, 054528 (2002)
- Impurity effects on the A_1-A_2 splitting of superfluid ^{3}He in Aerogel, J. A. Sauls & P. Sharma, PRB 68, 224502 (2003)
- Two-dimensional nuclear magnets, H. Godfrin and R.E. Rapp, Adv. Phys. 44, 113 1995

Liquid-Solid Exchange Coupling in ^{3}He-aerogel

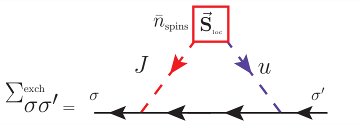


Liquid-Solid Exchange

$$U = u_{\text{pot}} + J\,\boldsymbol{\sigma}\cdot\mathbf{S}_{\text{loc}}$$

- $\boldsymbol{\sigma}$ = itinerant ^{3}He spin
- $\mathbf{S}_{\text{loc}}$ = localized ^{3}He spin
- J = exchange interaction
- Polarized Solid: $\langle\vec{\mathbf{S}}_{\text{loc}}\rangle = \mathbf{s}\mathscr{P}(B)\,\hat{\mathbf{B}}$

$$\mathscr{P}(B) \approx \left(\frac{\mu B}{k_{\text{B}}T}\right) \Longrightarrow$$



- Asymmetry in Scattering Cross-Sections for ↑ (+) versus ↓ (−) Quasiparticles:

$$\frac{1}{\tau_\pm} = \frac{1}{\bar{\tau}} \pm \frac{1}{\tau_{\text{S}}} \quad \text{with} \quad \frac{1}{\tau_{\text{S}}} = \frac{1}{\bar{\tau}}\left(\frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}\right)$$

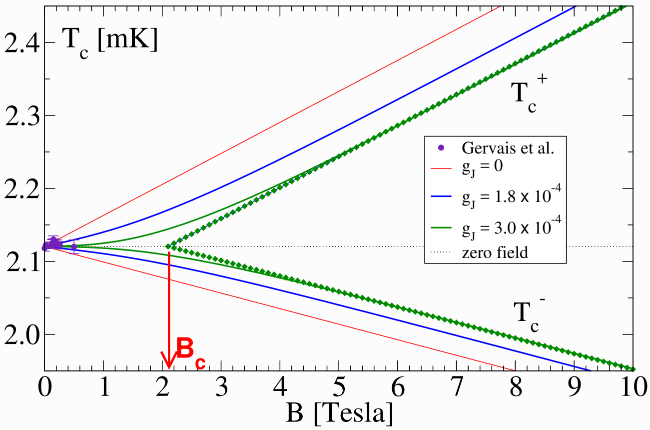
Asymmetry in QP scattering: $$\frac{\hbar}{2\tau_{\text{S}}} = n_s\,\mathbf{s}J\;\mathscr{P}(T,B)\;\sqrt{\sigma_0}(1-\sigma_0)^{3/2}$$

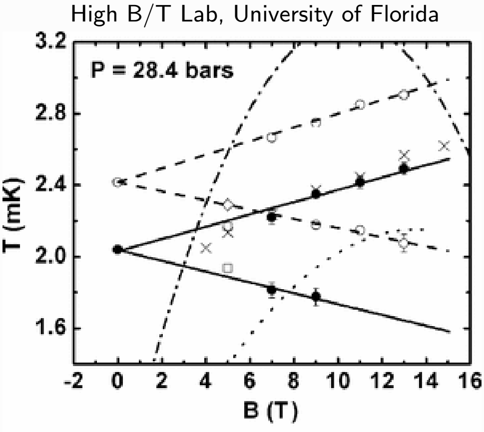
Asymmetric pair-breaking of ↑↑ (+) and ↓↓ (−) Cooper pairs: $T_c^{\pm} = T_c \pm \left(\bar{\lambda}^{A_1} - \lambda_J\right) B$ for $\mu B \ll k_{\text{b}}T$

$$\lambda_{\text{J}} = g_{\text{J}}\left(\frac{|\mu|}{k_B}\right)\left(\frac{(1-\sigma_0)^{3/2}}{\sqrt{\sigma_0}}\right) R(\xi_0/\ell, \xi_a/\xi_0) \quad \text{where} \quad g_{\text{J}} = 2\pi N_f\,J\,\mathbf{s}$$

- Impurity effects on the A_1-A_2 splitting of superfluid ^{3}He in Aerogel, J. A. Sauls & P. Sharma, PRB 68, 224502 (2003)
- Cooper pairing in ^{3}He in the presence of spin-polarized scattering centers, G. Baramidze and G. Kharadze, Physica B 284, 305 (2000)

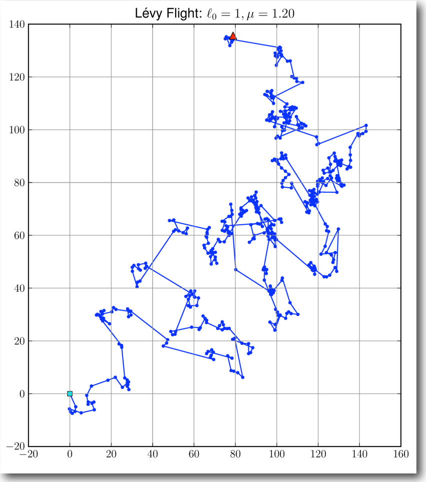
The A_1 - A_2 Splitting at High Magnetic Fields in Superfluid ^{3}He-Aerogel





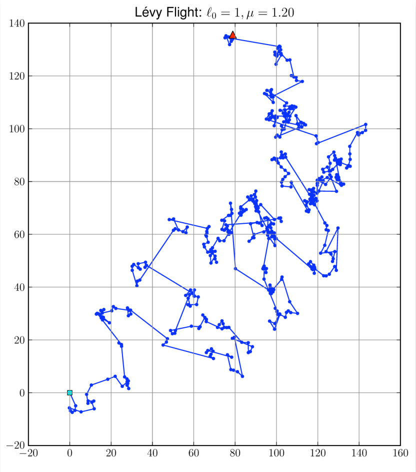
- A_1 & A_2 Transitions in 98% ^{3}He Aerogel, H.C. Choi, A. Gray, C. Vicente, J. Xia, G. Gervais, W. Halperin, N. Mulders, and Y. Lee, PRL 93, 145302 (2004)
- Phase diagram of the superfluid phases of 3He in 98% aerogel, G. Gervais, K.Yawata, N. Mulders, and W.P. Halperin, PRB 66, 054528 (2002)
- Impurity effects on the A_1-A_2 splitting of superfluid ^{3}He in Aerogel, J. A. Sauls & P. Sharma, PRB 68, 224502 (2003)

Ballistic Flights in a Random Fractal

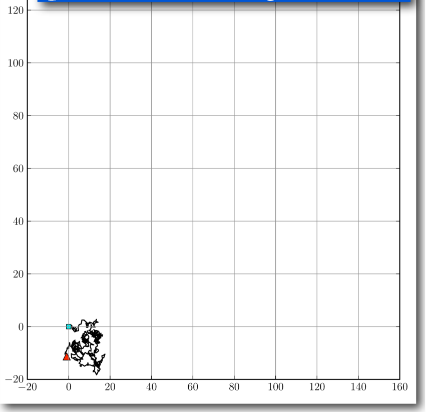


- Ballistic flights interrupted by elastic scattering from aerogel clusters
- Most probable ballistic flight = $\ell_0 \approx \xi_a$

Ballistic Flights in a Random Fractal

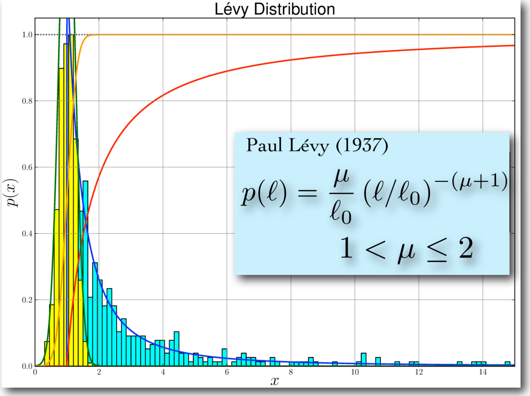


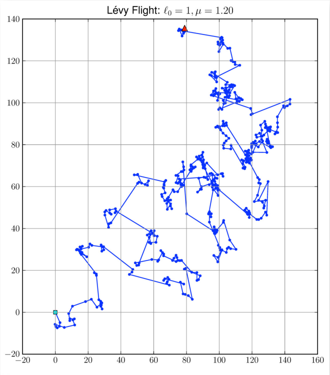
Gaussian Random Walk with probable ballistic flight = $\ell_0 \approx \xi_a$



- ❖Ballistic flights interrupted by elastic scattering from aerogel clusters
- ❖Most probable ballistic flight = $\ell_0 \approx \xi_a$

Ballistic Flights in a Random Fractal





$$\langle \ell \rangle \equiv \int d\ell \, p(\ell) \, \ell = \frac{\mu}{\mu - 1} \ell_0$$

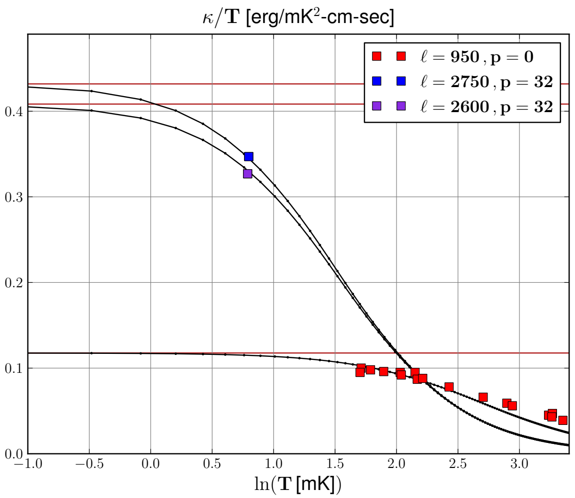
$$\langle \ell^2 \rangle \equiv \int d\ell \, p(\ell) \, \ell^2 = \infty$$

- ✓ Diffusion in turbulent fluids, Richardson (1926); Kolmogorov (1941)
- ✓ Rotating Flow in 2D, Solomon et al. PRL (1993)
- ✓ Foraging patterns of albatrosses, ... (c.f. Edwards et al. Nature 2007)
- ✓ Human mobility patterns, spread of infectious diseases, ... (c.f. D. Brockmann et al. 2006 & M. Shleshinger (Nature Phys. 2006)

Anomalous Diffusion

$$\langle \ell(t)^2 \rangle = \mathcal{D} \, t^{\gamma} \qquad 0 < \gamma < 2$$

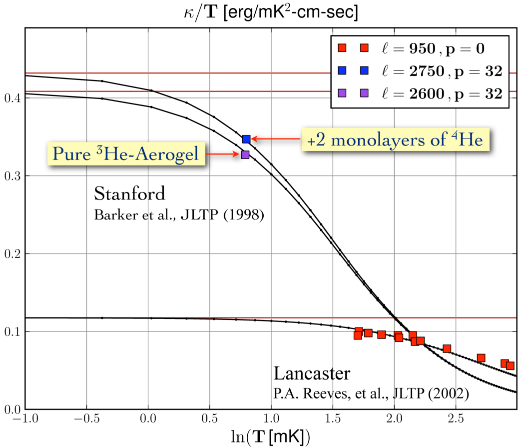
Thermal Conductivity - κ/T - in Normal ^{3}He-Aerogel

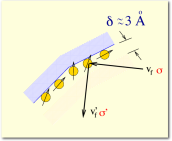


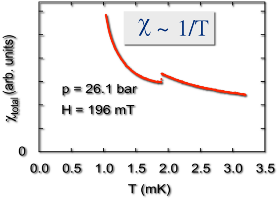
- Thermal Conductivity of Normal Liquid ^{3}He in Aerogel P. Reeves, G. Tvalashvili, S. Fisher, A. Gunault, and G. Pickett, J. Low Temp. Phys. 129, 185 (2002)
- Spin Dynamics of 3He in Aerogel, B. Barker, L. Polukhina, J. Poco, L. Hrubesh, D. Osheroff, JLTP 113, 635 (1998)
- Theory of heat transport of normal liquid ^{3}He in aerogel, J. A. Sauls & P. Sharma, NJP 12, 083056 (2010)

Existing κ vs. T in ^{3}He-Aerogel

Evidence for Spin-Flip Scattering between *Solid* ^{3}He and Liquid ^{3}He







$$u = -\frac{J_{\text{ind}}}{n}\sum_{i=1}^{N_s}\mathbf{S}_i\cdot\boldsymbol{\sigma}\,\delta(\mathbf{r}-\mathbf{R}_i)$$

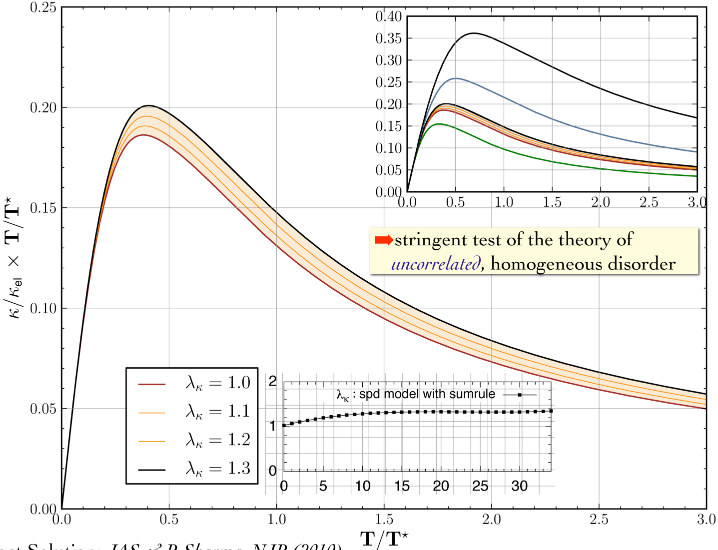
$$\hbar/\tau_{\text{spin}} = \frac{4\pi n_s}{N_f}\left(J_{\text{ind}}/n\right)^2 S(S+1)$$

$$\Delta\left[\frac{1}{\tau}\right] = v_f\left(\frac{1}{\ell_{^3\text{He}}} - \frac{1}{\ell_{^3\text{He}+^4\text{He}}}\right) \rightsquigarrow \frac{1}{\tau_{\text{spin}}} \rightsquigarrow J_{\text{ind}} \simeq 0.5\,\text{mK/spin}$$

Predicted Scaling Behavior for ^{3}He-Aerogel

$$\frac{\kappa}{\kappa_{\rm el}} \times (T/T^\star) = x\,F(x, \lambda_\kappa)$$

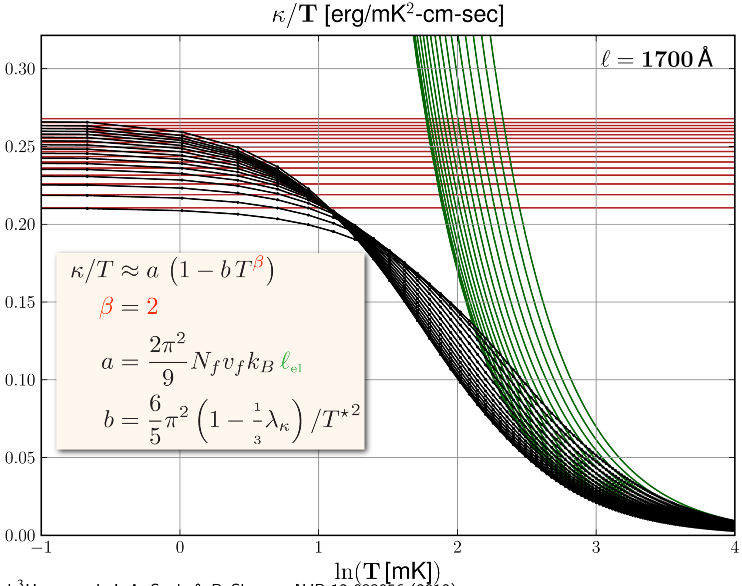
✓ All pressures
✓ All M.F.P.
✓ All temperatures



➡ stringent test of the theory of *uncorrelated*, homogeneous disorder

Exact Solution: *JAS & P. Sharma, NJP (2010)*

Cross-Over in $\varkappa$ vs. T for ^{3}He-Aerogel - Pressure Dependence



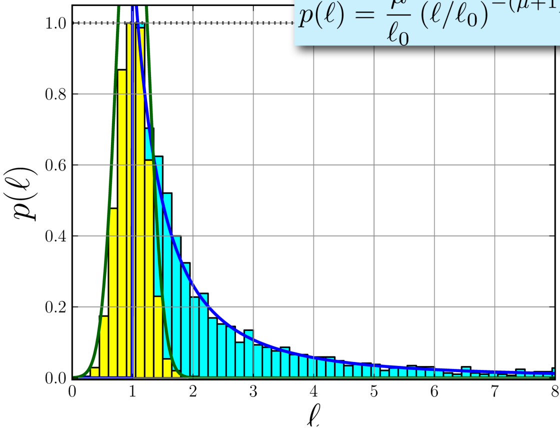
- $\lim_{T\to 0} \kappa/T \propto \ell_{\mathrm{el}}$

Gaussian Disorder

$\beta = 2$

- Theory of Transport in Normal ^{3}He-aerogel, J. A. Sauls & P. Sharma, NJP 12 083056 (2010)

Possible signature of Lévy Flights in ^{3}He-Aerogel



$$p(\ell) = \frac{\mu}{\ell_0} \left(\ell/\ell_0\right)^{-(\mu+1)}$$

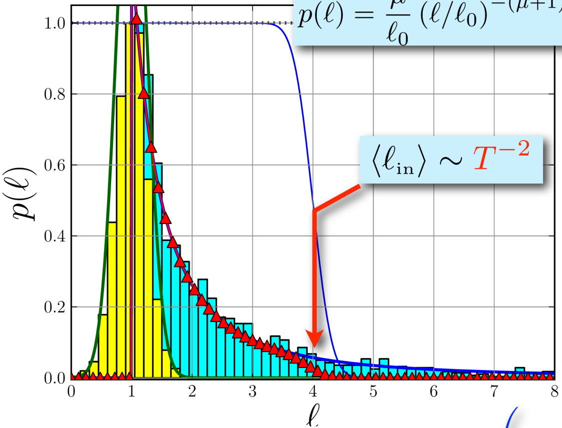
$$T \to 0$$

$$\kappa(\ell)/T = A\,\ell$$

Ensemble Average:

$$\langle\kappa(\ell)\rangle/T = A\,\langle\ell\rangle$$

Possible signature of Lévy Flights in ^{3}He-Aerogel



$$p(\ell) = \frac{\mu}{\ell_0} (\ell/\ell_0)^{-(\mu+1)}$$

$$\langle \ell_{\rm in} \rangle \sim T^{-2}$$

$$T \to 0$$

$$\kappa(\ell)/T = A\,\ell$$

Ensemble Average:

$$\langle \kappa(\ell) \rangle / T = A\,\langle \ell \rangle$$

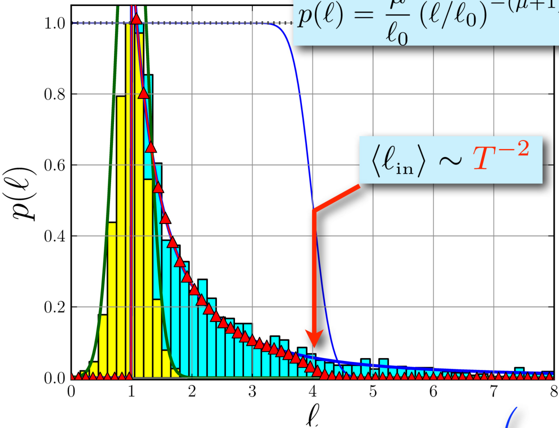
$$0 < T \ll T^\star$$

... but long paths are *interrupted by inelastic scattering*

Gaussian

$$p_{\rm Lévy}(\ell) \to p(\ell) = p_{\rm Lévy}(\ell) \times \left(1 - \int_0^\ell dx\, p_{\rm in}(x)\right)$$

Possible signature of Lévy Flights in ^{3}He-Aerogel



$$p(\ell) = \frac{\mu}{\ell_0}\,(\ell/\ell_0)^{-(\mu+1)}$$

$$\langle \ell_{\mathrm{in}} \rangle \sim T^{-2}$$

$$T \to 0$$

$$\kappa(\ell)/T = A\,\ell$$

Ensemble Average:

$$\langle \kappa(\ell) \rangle / T = A\,\langle \ell \rangle$$

$$0 < T \ll T^{\star}$$

... but long paths are *interrupted by inelastic scattering*

Gaussian

$$p_{\text{Lévy}}(\ell) \to p(\ell) = p_{\text{Lévy}}(\ell) \times \left(1 - \int_0^{\ell} dx\, p_{\mathrm{in}}(x)\right)$$

Leading order correction from interrupted Lévy flights

$$\langle \kappa(\ell) \rangle / T = A\,\langle \ell \rangle \times \left(1 - b\,T^{\beta}\right) \qquad \beta = 2\,(\mu - 1) < 2$$