TMS Intensive-Interactive Meeting, Keio University, November 17-18, 2016

Spontaneous Symmetry Breaking & Topological Order in Superfluid ^{3}He

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Supported by National Science Foundation Grant DMR-1508730

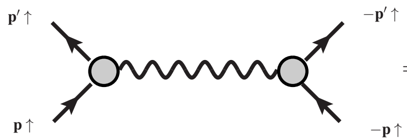
• Oleksii Shevtsov
• Hao Wu • Joshua Wiman
• Takeshi Mizushima (Osaka University)

- Spontaneous Symmetry Breaking in ^{3}He
- Nambu-Goldstone & Higgs Modes
- Nambu's Fermion-Boson Mass Relation
- Topological Order in Chiral Superfluids
- Chiral Fermions & Edge Currents
- Anomalous Hall Effect in ^{3}He-A

Ferromagnetic Spin Fluctuations $\rightsquigarrow$ Odd-Parity, Spin-Triplet Pairing for ^{3}He

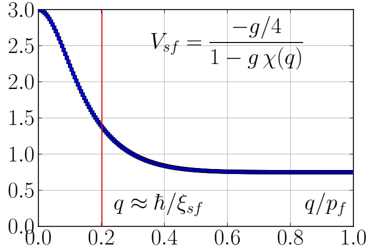
- A. Layzer and D. Fay, Int. J. Magn. 1, 135 (1971)

$V_{\text{sf}}(\mathbf{p},\mathbf{p}') =$



$$= \frac{-g/4}{1 - g\,\chi(\mathbf{p}-\mathbf{p}')}$$

$$-g_l = (2l+1)\int \frac{d\Omega_{\hat{p}}}{4\pi}\int \frac{d\Omega_{\hat{p}'}}{4\pi}\, V_{\text{sf}}(\mathbf{p},\mathbf{p}')\, P_l(\hat{p}\cdot\hat{p}')$$



- $-g_l$ is a function of $g \approx 0.75$ and $\xi_{\text{sf}} \approx 5\,\hbar/p_f$
- $l = 1$ (p-wave) is dominant pairing channel
  - p-wave basis functions:

$$\hat{p}_z \sim \cos\theta_{\hat{p}}$$

$$\hat{p}_x + i\hat{p}_y \sim \sin\theta_{\hat{p}}\ e^{+i\phi_{\hat{p}}}$$

$$\hat{p}_x - i\hat{p}_y \sim \sin\theta_{\hat{p}}\ e^{-i\phi_{\hat{p}}}$$

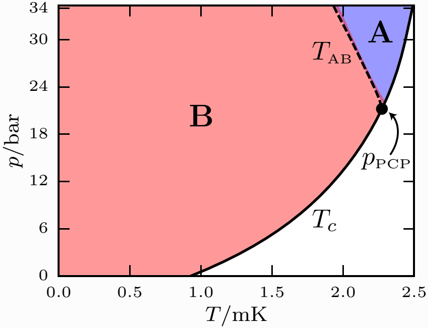
- $S = 1$ pairing fluctuations in V_{sf} $\rightsquigarrow$ Multiple P-wave Superfluid Phases

W. Brinkman, J. Serene, and P. Anderson, PRA 10, 2386 (1974)

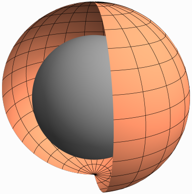
The ^{3}He Paradigm: Maximal Symmetry $\mathtt{G} = \mathtt{SO(3)_S} \times \mathtt{SO(3)_L} \times \mathtt{U(1)_N} \times \mathtt{P} \times \mathtt{T}$

BCS Condensate Amplitude: $\Psi_{\alpha\beta}(p) = \langle \psi_\alpha(p)\psi_\beta(-p)\rangle$

J. Wiman & J. A. Sauls, PRB 92, 144515 (2015)



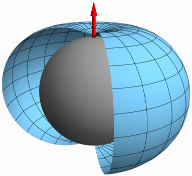
"Isotropic" BW State



$J = 0,\ J_z = 0$

$H = \mathtt{SO(3)_J} \times \mathtt{T}$

Chiral AM State $\vec{l} = \hat{\mathbf{z}}$



$L_z = 1,\ S_z = 0$

$H = \mathtt{U(1)_S} \times \mathtt{U(1)_{L_z\text{-}N}} \times \mathtt{Z_2}$

$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix}_{BW} = \begin{pmatrix} p_x - ip_y \sim e^{-i\phi} & p_z \\ p_z & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix}_{AM} = \begin{pmatrix} p_x + ip_y \sim e^{+i\phi} & 0 \\ 0 & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

Ginzburg-Landau Functional for Superfluid ^{3}He

- Maximal Symmetry of ^{3}He: $\mathtt{G} = \mathtt{SO(3)_L} \times \mathtt{SO(3)_S} \times \mathtt{U(1)_N} \times \mathtt{P} \times \mathtt{T}$

- Order Parameter for P-wave ($L = 1$), Spin-Triplet ($S = 1$) Pairing

$$\widehat{\Psi}(\hat{p}) = \overbrace{\begin{pmatrix} S_\mathbf{x} & S_\mathbf{y} & S_\mathbf{z} \end{pmatrix}}^{\text{Spin Basis}} \times \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix} \times \overbrace{\begin{pmatrix} \hat{p}_x \\ \hat{p}_y \\ \hat{p}_z \end{pmatrix}}^{\text{Orbital Basis}}$$

- GL Functional: $A_{\alpha i}$ $\rightsquigarrow$ vector under both $\mathtt{SO(3)_S}$ $[\alpha]$ and $\mathtt{SO(3)_L}$ $[i]$

$$\begin{aligned} \mathcal{U}[A] &= \int d^3r \Big[\alpha(T)\mathrm{Tr}\left\{AA^\dagger\right\} + \beta_1 \left|\mathrm{Tr}\left\{AA^{\mathrm{tr}}\right\}\right|^2 + \beta_2 \left(\mathrm{Tr}\left\{AA^\dagger\right\}\right)^2 \\ &+ \beta_3 \mathrm{Tr}\left\{AA^{\mathrm{tr}}(AA^{\mathrm{tr}})^*\right\} + \beta_4 \mathrm{Tr}\left\{(AA^\dagger)^2\right\} + \beta_5 \mathrm{Tr}\left\{AA^\dagger(AA^\dagger)^*\right\} \\ &+ \kappa_1 \partial_i A_{\alpha j}\, \partial_i A^*_{\alpha j} + \kappa_2 \partial_i A_{\alpha i}\, \partial_j A^*_{\alpha j} + \kappa_3 \partial_i A_{\alpha j}\, \partial_j A^*_{\alpha i}\Big] \end{aligned}$$

New Bosonic Excitations

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN)

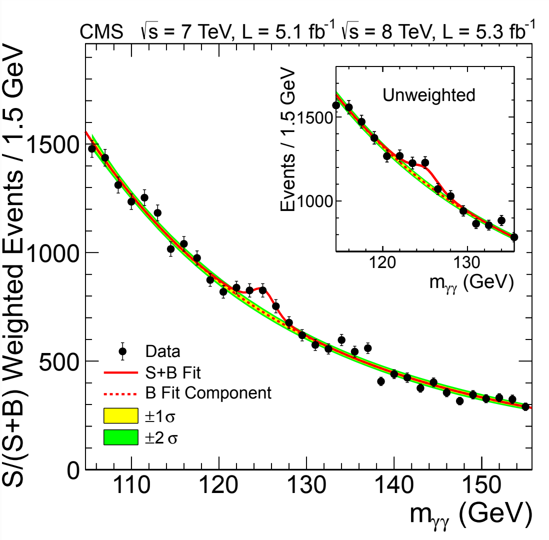
CERN-PH-EP/2012-220
2013/01/29

CMS-HIG-12-028

Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC

The CMS Collaboration

Higgs Boson with mass $M = 125$ GeV

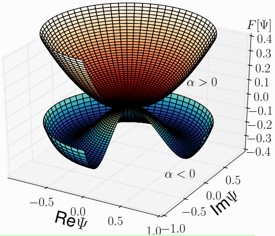


Dynamical Consequences of Spontaneous Symmetry Breaking

Scalar Higgs Boson (spin $J = 0$) [P. Higgs, PRL 13, 508 1964]

Energy Functional for the Higgs Field

$$\mathcal{U}[\Delta] = \int dV \left\{ \alpha\, |\Delta|^2 + \beta\, |\Delta|^4 + \tfrac{1}{2}c^2\, |\nabla\Delta|^2 \right\}$$



- Broken Symmetry State: $\Delta = \sqrt{|\alpha|/2\beta}$

Space-Time Fluctuations about the Broken Symmetry Vacuum State

$\Delta(\mathbf{r},t) = \Delta + D(\mathbf{r},t)$ ▸ Eigenmodes: $D^{(\pm)} = D \pm D^*$ (Conjugation Parity)

$$\mathcal{L} = \int d^3r \left\{ \frac{1}{2}[(\dot{D}^{(+)})^2 + (\dot{D}^{(-)})^2] - 2\Delta^2\,(D^{(+)})^2 - \frac{1}{2}[c^2(\nabla D^{(+)})^2 + c^2(\nabla D^{(-)})^2] \right\}$$

- $\partial_t^2 D^{(-)} - c^2\nabla^2\, D^{(-)} = 0$

Massless Nambu-Goldstone Mode

- $\partial_t^2 D^{(+)} - c^2\nabla^2\, D^{(+)} + 4\Delta^2\, D^{(+)} = 0$

Massive Higgs Mode: $M = 2\Delta$

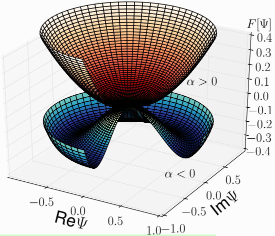
Dynamical Consequences of Spontaneous Symmetry Breaking

BCS Condensation of Spin-Singlet ($S = 0$), S-wave ($L = 0$) "Scalar" Cooper Pairs

Ginzburg-Landau Functional

$$F[\Delta] = \int dV \left\{ \alpha \, |\Delta|^2 + \beta \, |\Delta|^4 + \kappa \, |\nabla \Delta|^2 \right\}$$

- Order Parameter: $\Delta = \sqrt{|\alpha|/2\beta}$



Space-Time Fluctuations of the Condensate Order Parameter

$\Delta(\mathbf{r}, t) = \Delta + D(\mathbf{r}, t)$ ► Eigenmodes: $D^{(\pm)} = D \pm D^*$ (Fermion "Charge" Parity)

$$\mathcal{L} = \int d^3r \left\{ \frac{1}{2}[(\dot{D}^{(+)})^2 + (\dot{D}^{(-)})^2] - 2\Delta^2 \, (D^{(+)})^2 - \frac{1}{2}[v^2(\nabla D^{(+)})^2 + v^2(\nabla D^{(-)})^2] \right\}$$

- $\partial_t^2 D^{(-)} - v^2 \nabla^2 \, D^{(-)} = 0$

  Anderson-Bogoliubov Mode

- $\partial_t^2 D^{(+)} - v^2 \nabla^2 \, D^{(+)} + 4\Delta^2 \, D^{(+)} = 0$

  Amplitude Higgs Mode: $M = 2\Delta$

First Reported Observations of Higgs Bosons in BCS Condensates

Observation of a New Sound-Attenuation Peak in Superfluid ^{3}He-B

R. W. Giannetta,[a] A. Ahonen,[b] E. Polturak, J. Saunders,
E. K. Zeise, R. C. Richardson, and D. M. Lee
Laboratory of Atomic and Solid State Physics and Materials Science Center, Cornell University, Ithaca, New York 14853
(Received 25 March 1980)

Results of zero-sound attenuation measurements in ^{3}He-B, at frequencies up to 60 MHz and pressures between 0 and 20 bars, are reported. At frequencies of 30 MHz and above, a new attenuation feature is observed which bears the signature of a collective mode of the superfluid.

VOLUME 45, NUMBER 4 PHYSICAL REVIEW LETTERS 28 JULY 1980

Measurements of High-Frequency Sound Propagation in ^{3}He-B

D. B. Mast, Bimal K. Sarma, J. R. Owers-Bradley, I. D. Calder,
J. B. Ketterson, and W. P. Halperin
Department of Physics and Astronomy and Materials Research Center, Northwestern University, Evanston, Illinois 60201
(Received 10 April 1980)

Measurements of the attenuation and velocity of pulsed high-frequency sound have been performed up to 133 MHz in superfluid ^{3}He-B. A new collective mode of the order parameter was discovered at a frequency extrapolated to T_c of $\omega = (1.165 \pm 0.05)\Delta_{\rm BCS}(T_c)$, where $\Delta_{\rm BCS}(T)$ is the energy gap in the weak-coupling BCS theory. The group velocity has been observed to decrease by as much as $\frac{2}{3}$ of the zero-sound velocity.

VOLUME 45, NUMBER 8 PHYSICAL REVIEW LETTERS 25 AUGUST 1980

Raman Scattering by Superconducting-Gap Excitations and Their Coupling to Charge-Density Waves

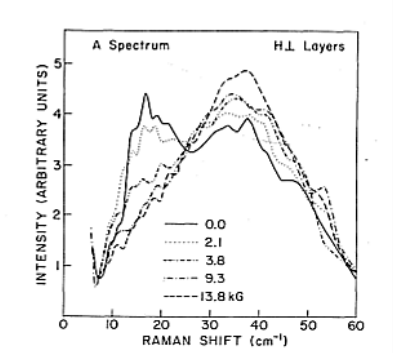
R. Sooryakumar and M. V. Klein
Department of Physics and Materials Research Laboratory, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801
(Received 24 March 1980)

$2H$-$NbSe_2$ undergoes a charge-density–wave (CDW) distortion at 33 K which induces A and E Raman-active phonon modes. These are joined in the superconducting state at 2 K by new A and E Raman modes close in energy to the BCS gap 2Δ. Magnetic fields suppress the intensity of the new modes and enhance that of the CDW-induced modes, thus providing evidence of coupling between the superconducting-gap excitations and the CDW.

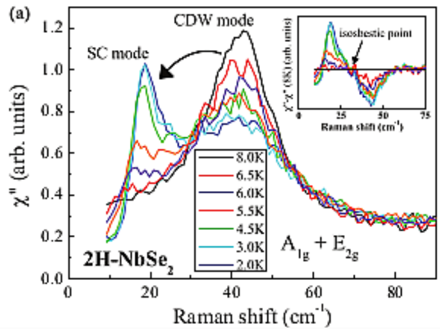
Dynamical Consequences of Spontaneous Symmetry Breaking

Higgs Mode with mass: $M = 3$ meV and spin $J = 0$ in $NbSe_2$

Raman Absorption in $NbSe_2$



R. Sooyakumar & M. Klein, PRL 45, 660 (1980)



M. Meásson et al. PRB B 89, 060503(R) (2014)

- $\hbar\omega_{\gamma_1} = \hbar\omega_{\gamma_2} + 2\Delta$
- Amplitude Higgs - CDW Phonon Coupling

- Theory: P. Littlewood & C. Varma, PRL 47, 811 (1981)

Lagrangian Field Theory for Bosonic Excitations of Superfluid ^{3}He-B

$$^3\text{He-B:}\quad B_{\alpha i} = \frac{1}{\sqrt{3}}\Delta\,\delta_{\alpha i} \quad L=1,\; S=1 \;\rightsquigarrow\; J=0$$

- Symmetry of ^{3}He-B: $\mathtt{H} = \mathtt{SO(3)}_{\mathtt{J}} \times \mathtt{T}$
- Fluctuations: $\mathcal{D}_{\alpha i}(\mathbf{r},t) = A_{\alpha i}(\mathbf{r},t) - B_{\alpha i} = \sum_{J,m} D_{J,m}(\mathbf{r},t)\, t^{(J,m)}_{\alpha i}$
- Lagrangian:

$$\mathcal{L} = \int d^3r \left\{ \tau\,\mathrm{Tr}\left\{\dot{\mathcal{D}}\dot{\mathcal{D}}^\dagger\right\} - \alpha\,\mathrm{Tr}\left\{\mathcal{D}\mathcal{D}^\dagger\right\} - \sum_{p=1}^{5}\beta_p\,u_p(\mathcal{D}) - \sum_{l=1}^{3} K_l\,v_l(\partial\mathcal{D}) \right\}$$

$$\partial_t^2 D^{(\mathtt{C})}_{J,m} + E^{(\mathtt{C})}_{J,m}(\mathbf{q})^2\, D^{(\mathtt{C})}_{J,m} = \frac{1}{\tau}\eta^{(\mathtt{C})}_{J,m}$$

with $\quad J = \{0,1,2\}\,,\, m = -J\ldots+J\,,\, \mathtt{C} = \pm 1$

- *Time-Dependent Ginzburg-Landau Theory for Superfluid ^{3}He-B*: JAS & T. Mizushima, arXiv:1611.07273 (2016)

Spectrum of Bosonic Modes of Superfluid ^{3}He-B : Condensate is $J^{\mathtt{C}} = 0^+$

- 4 Nambu-Goldstone Modes & 14 Higgs modes

$$E^{(\mathtt{C})}_{J,m}(\mathbf{q}) = \sqrt{M^2_{J,\mathtt{C}} + \left(c^{(\mathtt{C})}_{J,|m|}|\mathbf{q}|\right)^2}$$

Mode	Symmetry	Mass	Name
$D^{(+)}_{0,m}$	$J = 0$, $\mathtt{C} = +1$	2Δ	Amplitude Higgs
$D^{(-)}_{0,m}$	$J = 0$, $\mathtt{C} = -1$	0	NG Phase Mode
$D^{(+)}_{1,m}$	$J = 1$, $\mathtt{C} = +1$	0	NG Spin-Orbit Modes
$D^{(-)}_{1,m}$	$J = 1$, $\mathtt{C} = -1$	2Δ	AH Spin-Orbit Modes
$D^{(+)}_{2,m}$	$J = 2$, $\mathtt{C} = +1$	$\sqrt{\frac{8}{5}}\Delta$	2^+ AH Modes
$D^{(-)}_{2,m}$	$J = 2$, $\mathtt{C} = -1$	$\sqrt{\frac{12}{5}}\Delta$	2^- AH Modes

- Vdovin, Maki, Wölfle, Serene, Nagai, Volovik, Schopohl, McKenzie, JAS ...

Collective Mode Spectrum for ^{3}He-B

Bosonic Excitations of ^{3}He-B

Goldstone Mode w/ J=0⁻ → $D^{(-)}_{00} = i|\Delta|\,\varphi(q,\omega)$

phase mode

$$\left(\partial_t^2 - c_{00}^2 \nabla^2\right) D^{(-)}_{00} = \ldots$$

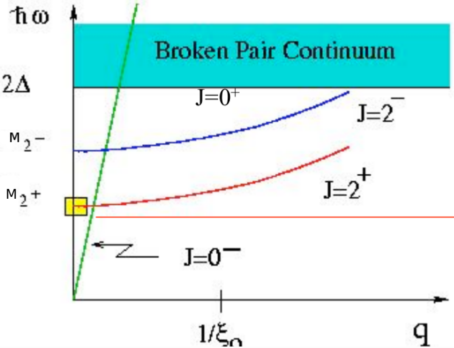
Pair Excitons w/ J=2⁺/⁻

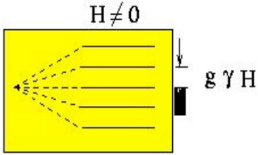
$$\left(\partial_t^2 + \Omega_{JC}^2\right) D^{(C)}_{JM} = \ldots$$

Anderson-Higgs Modes

coupling to internal & external fields

Nuclear Zeeman levels

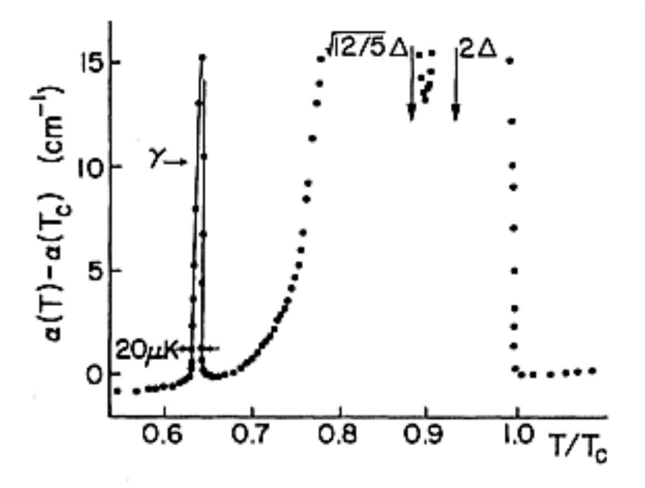




JAS & J. Serene, PRL 1982

Dynamical Consequences of Spontaneous Symmetry Breaking

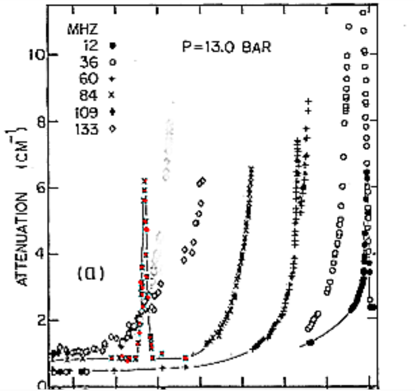
Higgs Mode with mass: $M = 500$ neV and spin $J = 2$ at LASSP-Cornell



- R. Giannetta et al., PRL 45, 262 (1980)

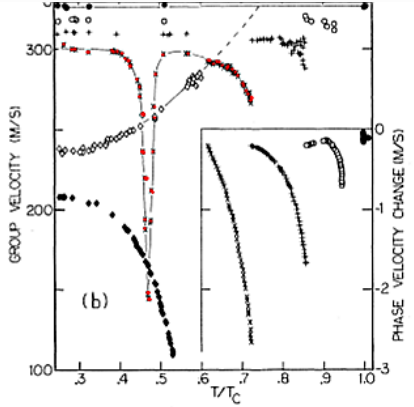
Dynamical Consequences of Spontaneous Symmetry Breaking

Higgs Mode with mass: $M = 500$ neV and spin $J^{\mathrm{C}} = 2^+$ at ULT-Northwestern



T/T_c

Group Velocity



- D. Mast et al. Phys. Rev. Lett. 45, 266 (1980).

Superfluid ^{3}He Higgs Detector at ULT-Northwestern

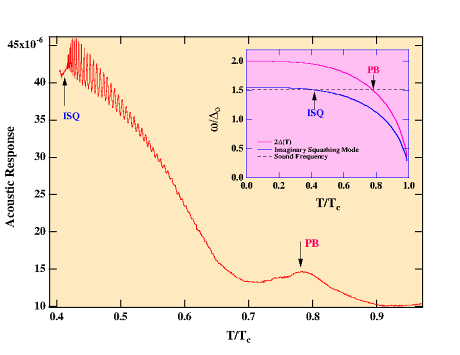
^{3}He-^{4}He Dilution + Adiabatic Demagnetization Stages $\rightsquigarrow T_{\text{min}} \approx 200\mu$K

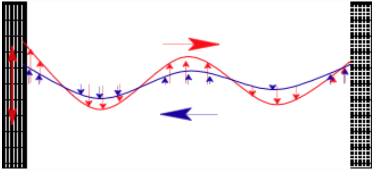
$J = 2^-$, $m = \pm 1$ Higgs Modes Transport Mass and Spin

- "Transverse Waves in Superfluid ^{3}He-B", G. Moores and JAS, JLTP 91, 13 (1993)

$$C_t(\omega) = \sqrt{\frac{F_1^s}{15}} v_f \left[\rho_n(\omega) + \frac{2}{5}\rho_s(\omega) \underbrace{\left\{\frac{\omega^2}{(\omega + i\Gamma)^2 - \frac{12}{5}\Delta^2 - \frac{2}{5}(q^2 v_f^2)}\right\}}_{D^{(-)}_{2,\pm 1}}\right]^{\frac{1}{2}}$$

Transverse Zero Sound Propagation in Superfluid ^{3}He-B: *Cavity Oscillations of TZS*





- Y. Lee et al. Nature 400 (1999)

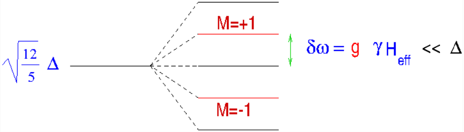
B ⟶

Faraday Rotation: Magneto-Acoustic Birefringence of Transverse Currents

- "Magneto-Acoustic Rotation of Transverse Waves in ^{3}He-B", J. A. Sauls et al., Physica B, 284,267 (2000)

$$C_{\substack{\text{RCP}\\\text{LCP}}}(\omega) = v_f \left[\frac{F_1^s}{15}\,\rho_n(\omega) + \frac{2F_1^s}{75}\,\rho_s(\omega)\,\underbrace{\left\{\frac{\omega^2}{(\omega+i\Gamma)^2 - \Omega^{(-)}_{2,\pm}(\mathbf{q})}\right\}}_{D^{(-)}_{2,\pm 1}}\right]^{\frac{1}{2}}$$

$$\Omega^{(-)}_{2,\pm}(\mathbf{q}) = \sqrt{\frac{12}{5}}\Delta \pm g_{2-}\,\gamma H_{\text{eff}}$$



- Circular Birefringence $\Longrightarrow$ $C_{\text{RCP}} \neq C_{\text{LCP}}$ $\Longrightarrow$ Faraday Rotation

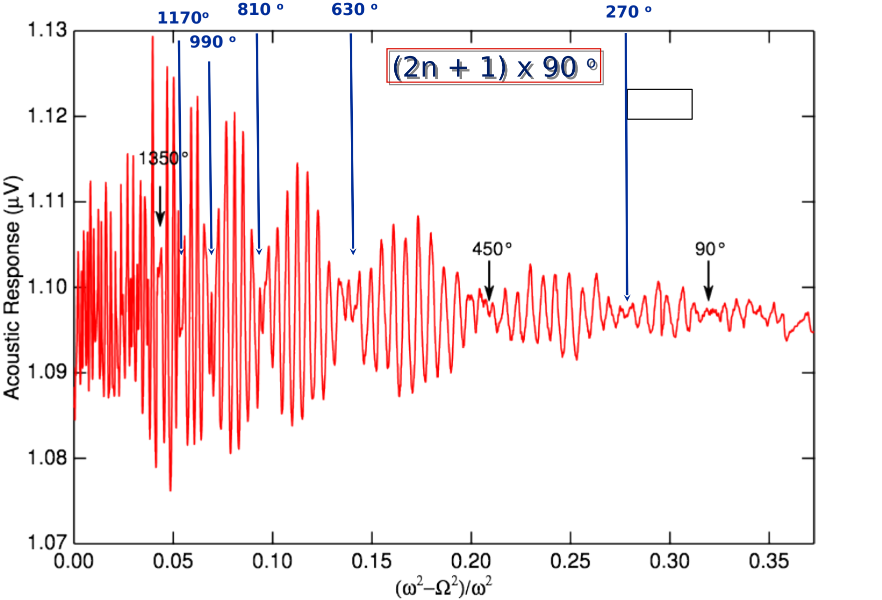
$$\left(\frac{C_{\text{RCP}} - C_{\text{LCP}}}{C_t}\right) \simeq g_{2-}\left(\frac{\gamma H_{\text{eff}}}{\omega}\right)$$

- Faraday Rotation Period ($\gamma H_{\text{eff}} \ll (\omega - \Omega^{(-)}_{2})$):

$$\lambda_H \simeq \frac{4\pi C_t}{g_{2-}\gamma H} \simeq 500\,\mu m\,, \quad H = 200\,G$$

- *Discovery of the acoustic Faraday effect in superfluid ^{3}He-B*, Y. Lee, et al. Nature 400, 431 (1999)

Large Faraday Rotations vs. ``Blue Tuning'' B = 1097 G



C. Collett et al., Phys. Rev. B 87, 024502 (2013)

Higgs Boson with mass $M = 125$ GeV - *Is this all there is?*

- *Higgs Bosons in Particle Physics and in Condensed Matter*
  G.E. Volovik & M. Zubkov, PRD 87, 075016 (2013)

- GEV & MZ: $m_{\text{top}} \approx 175\,\text{GeV}$, $M_{\text{H},-} = 125\,\text{GeV}$, $\therefore$ NSR $\rightsquigarrow$ $M_{\text{H},+} \approx 270\,\text{GeV}$

- *Boson-Fermion Relations in BCS type Theories*
  Y. Nambu, Physica D, 15, 147 (1985)

- Broken Symmetry State: $\rightsquigarrow$ Fermion mass: $m_{\text{F}} = \Delta$
- Nambu's Sum Rule ("empirical observation"): $\sum_{C} M^2_{J,C} = (2m_{\text{F}})^2$

Mode	Symmetry	Mass	Name
$D^{(+)}_{0,m}$	$J = 0$, $\mathtt{C} = +1$	2Δ	Amplitude Higgs
$D^{(-)}_{0,m}$	$J = 0$, $\mathtt{C} = -1$	0	NG Phase Mode
$D^{(+)}_{1,m}$	$J = 1$, $\mathtt{C} = +1$	0	NG Spin-Orbit Modes
$D^{(-)}_{1,m}$	$J = 1$, $\mathtt{C} = -1$	2Δ	AH Spin-Orbit Modes
$D^{(+)}_{2,m}$	$J = 2$, $\mathtt{C} = +1$	$\sqrt{\frac{8}{5}}\Delta$	2^+ AH Modes
$D^{(-)}_{2,m}$	$J = 2$, $\mathtt{C} = -1$	$\sqrt{\frac{12}{5}}\Delta$	2^- AH Modes

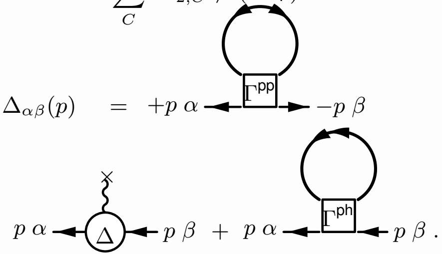
Corrections to the masses of the $J^{c} = 2^{\pm}$ Higgs in ^{3}He-B

- *Weak-Coupling BCS Pairing Theory* $\rightsquigarrow$

$$M_{2,+} = \sqrt{\frac{J}{2J+1}}\Delta = \sqrt{\frac{8}{5}}\Delta \quad \& \quad M_{2,-} = \sqrt{\frac{J+1}{2J+1}}\Delta = \sqrt{\frac{12}{5}}\Delta$$

$$\therefore \sum_C M_{J,C}^2 = (2m_{\mathsf{F}})^2$$

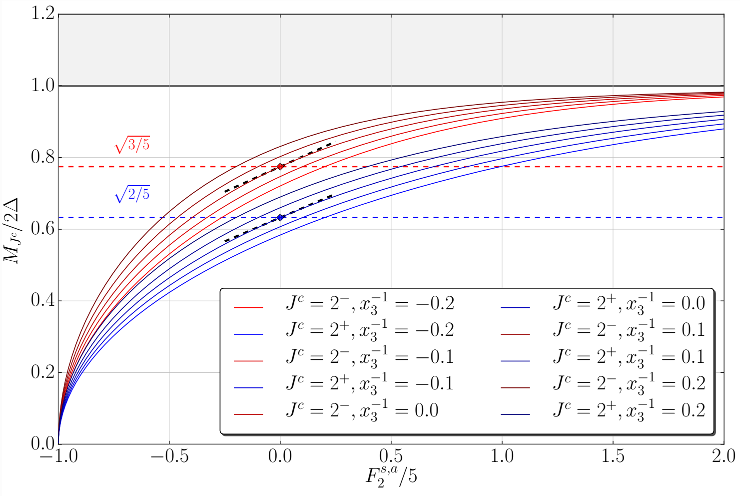
- *Interactions & Polarization of the Fermionic Vacuum*
  - Corrections to Higgs masses with $J^{c} \neq 0^{+}$ (Symmetry of the Vacuum State)
  - Violation of Nambu's Sum Rule: $\sum_C M_{2,C}^2 \neq (2m_{\mathsf{F}})^2$



(1)

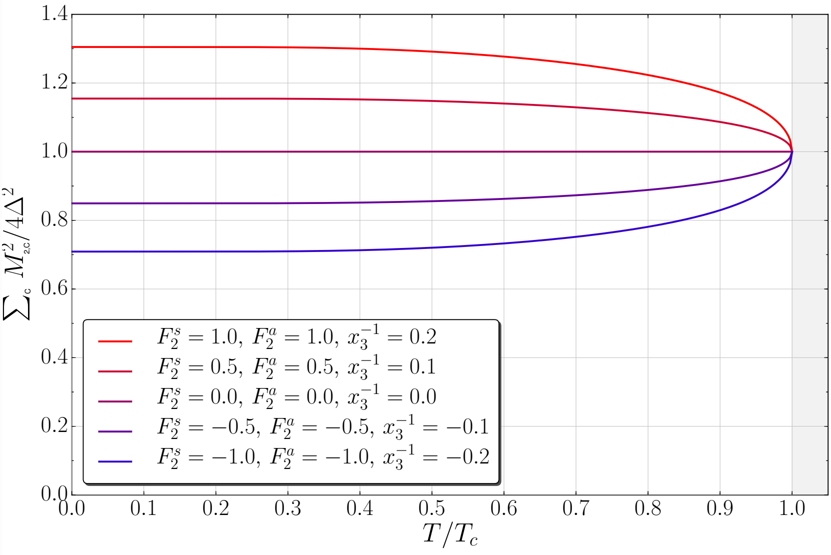
- *Nambu's Fermion-Boson Mass Relations*, JAS and T. Mizushima, for Phys. Rev. B 2016.

Vacuum polarization corrections to the masses of the $J^{\mathsf{C}} = 2^{\pm}$ Higgs in ^{3}He-B



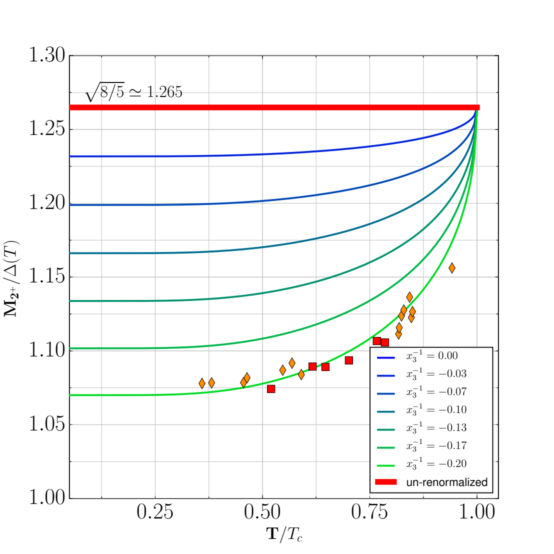
- $F_2^{s,a}$: $\ell = 2$ particle-hole interactions (scalar and spin exchange)
- x_3^{-1}: f-wave, $S = 1$ pairing (particle-particle) channel

Violation of the Nambu Sum Rule from Polarization of the Condensate in ^{3}He-B



- TDGL satisfies the NSR (Fermionic degrees of freedom “frozen”)
- p-p and p-h Interactions plus vacuum polarization $\rightsquigarrow$ violations of the NSR

Mass shift of the $J^{\mathrm{C}} = 2^+$ Higgs Mode in ^{3}He-B

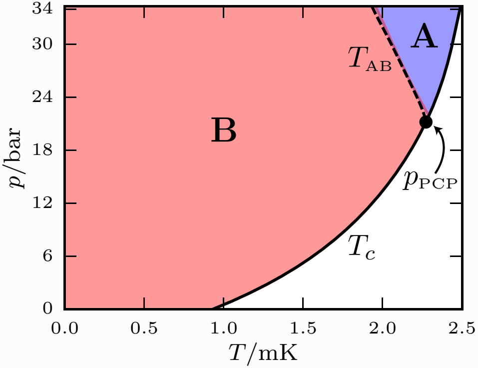


- Measurements: D. Mast et al. PRL 45, 266 (1980)
- exchange p-h channel: $F_2^a = -0.88$ (from Magnetic susceptibility of ^{3}He-B)
  - *attractive* f-wave interaction in the pp-channel $\rightsquigarrow$ New physics at $M \approx 2\Delta$!

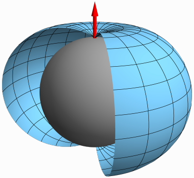
The Helium Paradigm: Superfluid Phases of ^{3}He

Symmetry of Normal Liquid ^{3}He: $G = SO(3)_S \times SO(3)_L \times U(1)_N \times P \times T$

J. Wiman & J. A. Sauls, PRB 92, 144515 (2015)



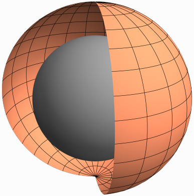
Chiral ABM State $\vec{l} = \hat{\mathbf{z}}$



$L_z = 1,\ S_z = 0$

$\mathbf{d}_z = \Delta\,(\hat{p}_x + i\hat{p}_y)$

"Isotropic" BW State



$J = 0,\ J_z = 0$

$\mathbf{d}_\alpha = \hat{p}_\alpha,\ \alpha = x, y, z$

Spin-Triplet, P-wave Order Parameter

$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix} = \begin{pmatrix} -\mathbf{d}_x + i\mathbf{d}_y & \mathbf{d}_z \\ \mathbf{d}_z & \mathbf{d}_x + i\mathbf{d}_y \end{pmatrix}$$

Signatures of Broken T and P Symmetry in ^{3}He-A

What is the Signature & Evidence for Chirality of Superfluid ^{3}He-A?

Spontaneous Symmetry Breaking $\rightsquigarrow$ Emergent Topology of ^{3}He-A

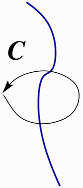
Chirality + Topology $\rightsquigarrow$ Edge States & Chiral Edge Currents

Broken T and P $\rightsquigarrow$ Anomalous Hall Effect for electrons in ^{3}He-A

Real-Space vs. Momentum-Space Topology

Topology in Real Space

$\Psi(\mathbf{r}) = |\Psi(r)|\, e^{i\vartheta(\mathbf{r})}$



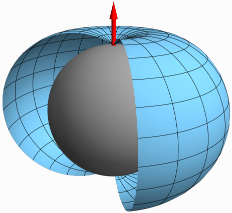
Phase Winding

$$N_C = \frac{1}{2\pi} \oint_C d\mathbf{l} \cdot \frac{1}{|\Psi|} \mathrm{Im}[\boldsymbol{\nabla}\Psi] \in \{0, \pm 1, \pm 2, \ldots$$

- Massless Fermions confined in the Vortex Core

Chiral Symmetry $\rightsquigarrow$ Topology in Momentum Space

$\Psi(\mathbf{p}) = \Delta(p_x \pm i p_y) \sim e^{\pm i\varphi_{\mathbf{p}}}$



Topological Quantum Number: $L_z = \pm 1$

$$N_{\text{2D}} = \frac{1}{2\pi} \oint d\mathbf{p} \cdot \frac{1}{|\Psi(\mathbf{p})|} \mathrm{Im}[\boldsymbol{\nabla}_{\mathbf{p}}\Psi(\mathbf{p})] = L_z$$

- Massless Chiral Fermions
  - Nodal Fermions in 3D
  - Edge Fermions in 2D

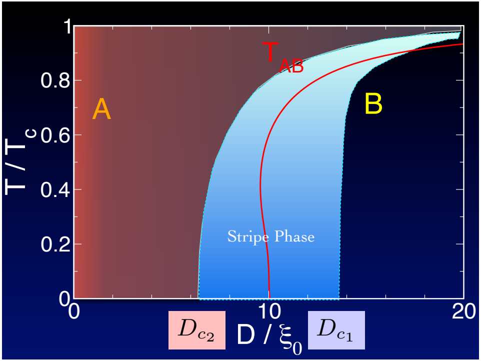
Confinement: Superfluid Phases of ^{3}He in Thin Films

Symmetry or Normal Liquid ^{3}*He*: $\mathtt{G} = \mathtt{SO(3)_S} \times \mathtt{SO(2)_L} \times \mathtt{U(1)_N} \times \mathtt{P} \times \mathtt{T}$

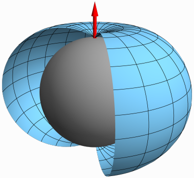
- Length Scale for Strong Confinement:

  $\xi_0 = \hbar v_f / 2\pi k_B T_c \approx 20 - 80\,\text{nm}$

  A. Vorontsov & JAS, PRL, 2007

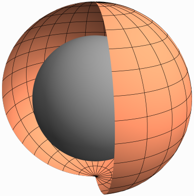


Chiral AM State $\vec{l} = \hat{\mathbf{z}}$



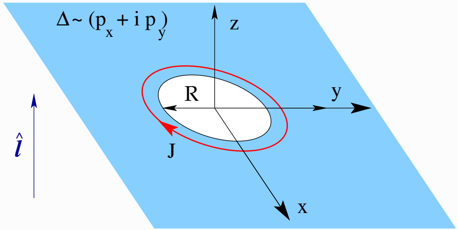
$L_z = 1,\ S_z = 0$

"Isotropic" BW State



$J = 0,\ J_z = 0$

Unbounded Film of ^{3}He-A perforated by a Hole



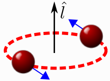
- $R \gg \xi_0 \approx 100\,\mathrm{nm}$

- Magnitude of the Sheet Current: $\frac{1}{4}\, n\, \hbar$ ($n = N/V = {}^3$He density)
- Edge Current *Counter*-Circulates: $J = -\frac{1}{4}\, n\, \hbar$ w.r.t. Chirality: $\hat{\mathbf{l}} = +\mathbf{z}$
- Angular Momentum: $L_z = 2\pi\, h\, R^2 \times (-\frac{1}{4}\, n\, \hbar) = -(N_{\text{hole}}/2)\, \hbar$

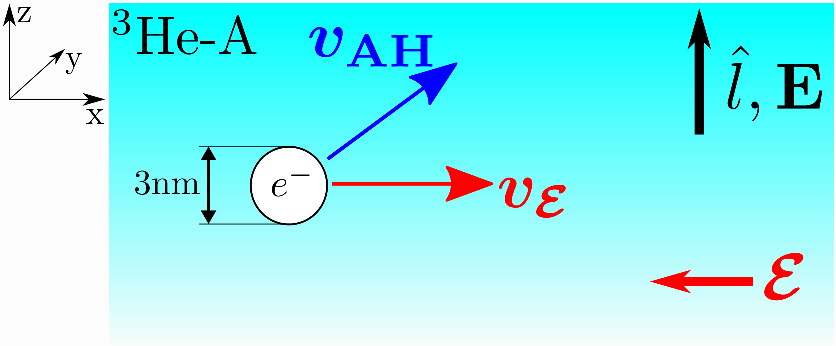
N_{hole} = Number of ^{3}He atoms excluded from the Hole

∴ An object in ^{3}He-A *inherits* angular momentum from the Condensate of Chiral Pairs!

Electron bubbles in chiral superfluid ^{3}He-A



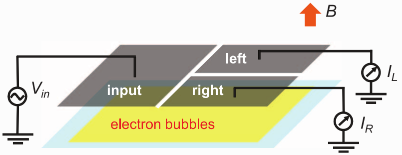
$$\Delta_{\mathrm{A}}(\hat{\mathbf{k}}) = \Delta \frac{k_x + i k_y}{k_f} = \Delta \, e^{i\phi_{\mathbf{k}}}$$

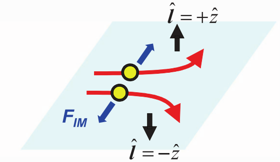


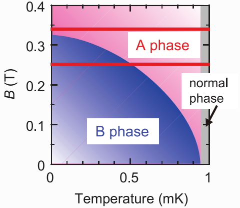
- Electric current: $\mathbf{v} = \overbrace{\mu_\perp \boldsymbol{\mathcal{E}}}^{\mathbf{v}_{\mathcal{E}}} + \overbrace{\mu_{\mathsf{AH}} \boldsymbol{\mathcal{E}} \times \hat{\mathbf{l}}}^{\mathbf{v}_{\mathsf{AH}}}$ Salmelin et al. PRL **63**, 868 (1989)
- Hall ratio: $\tan\alpha = v_{\mathsf{AH}}/v_{\mathcal{E}} = |\mu_{\mathsf{AH}}/\mu_\perp|$

Mobility of Electron Bubbles in ^{3}He-A

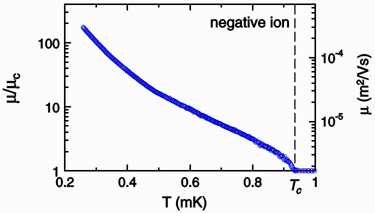
- H. Ikegami et al., Science **341**, 59 (2013); JPSJ **82**, 124607 (2013); JPSJ **84**, 044602 (2015)

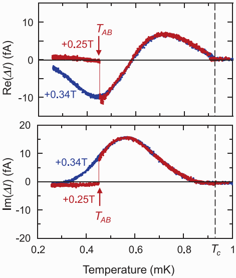


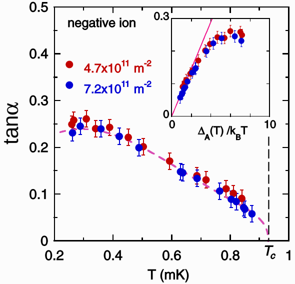




Electric current: $\mathbf{v} = \overbrace{\mu_\perp \boldsymbol{\mathcal{E}}}^{\mathbf{v}_{\mathcal{E}}} + \overbrace{\mu_{\mathrm{AH}} \boldsymbol{\mathcal{E}} \times \hat{\mathbf{l}}}^{\mathbf{v}_{\mathrm{AH}}}$ Hall ratio: $\tan\alpha = v_{\mathrm{AH}}/v_{\mathcal{E}} = |\mu_{\mathrm{AH}}/\mu_\perp|$







Forces on the Electron bubble in ^{3}He-A:

(i) $M\dfrac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} + \mathbf{F}_{\text{QP}}$, $\mathbf{F}_{QP}$ – force from quasiparticle collisions

(ii) $\mathbf{F}_{QP} = -\overleftrightarrow{\eta}\cdot\mathbf{v}$, $\overleftrightarrow{\eta}$ – generalized Stokes tensor

(iii) $\overleftrightarrow{\eta} = \begin{pmatrix} \eta_\perp & \eta_{\mathsf{AH}} & 0 \\ -\eta_{\mathsf{AH}} & \eta_\perp & 0 \\ 0 & 0 & \eta_\parallel \end{pmatrix}$ for chiral symmetry with $\hat{\mathbf{l}} \parallel \mathbf{e}_z$

(iv) $M\dfrac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} - \eta_\perp\mathbf{v} + \dfrac{e}{c}\mathbf{v}\times\mathbf{B}_{\text{eff}}$, for $\boldsymbol{\mathcal{E}} \perp \hat{\mathbf{l}}$

(v) $\mathbf{B}_{\text{eff}} = -\dfrac{c}{e}\eta_{\mathsf{AH}}\hat{\mathbf{l}} \quad B_{\text{eff}} \simeq 10^3 - 10^4\ \text{T}$!!!

(vi) $\dfrac{d\mathbf{v}}{dt} = 0 \quad \rightsquigarrow \quad \mathbf{v} = \overleftrightarrow{\mu}\boldsymbol{\mathcal{E}}$, where $\overleftrightarrow{\mu} = e\overleftrightarrow{\eta}^{-1}$

$$\mu_\parallel = \frac{e}{\eta_\parallel}\,, \quad \mu_\perp = e\,\frac{\eta_\perp}{\eta_\perp^2 + \eta_{\mathsf{AH}}^2}, \quad \mu_{\mathsf{AH}} = -e\,\frac{\eta_{\mathsf{AH}}}{\eta_\perp^2 + \eta_{\mathsf{AH}}^2}$$

▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

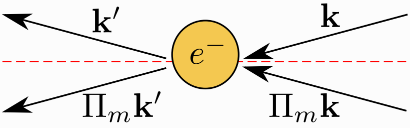
Mirror-antisymmetric scattering ⇒ transverse force

$$\mathbf{F}_{\mathsf{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^\infty dE \left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E)$$

Subdivide by mirror symmetry:

$$W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}),$$

$$\sigma_{ij}(E) = \sigma_{ij}^{(+)}(E) + \sigma_{ij}^{(-)}(E),$$



$$\sigma_{ij}^{(-)}(E) = \frac{3}{4}\int_{E\geq|\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'} \int_{E\geq|\Delta(\hat{\mathbf{k}})|}\frac{d\Omega_{\mathbf{k}}}{4\pi}\left[\epsilon_{ijk}(\hat{\mathbf{k}}'\times\hat{\mathbf{k}})_k\right]\frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E)\left[f(E)-\frac{1}{2}\right]$$

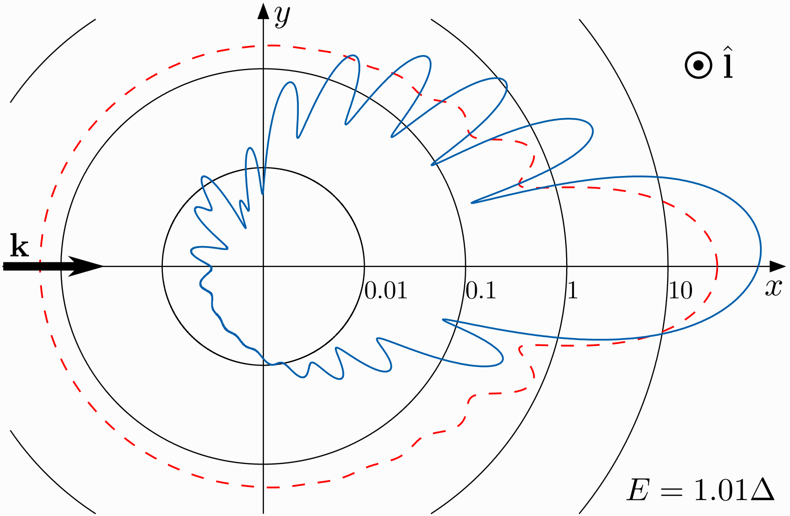
Mirror-antisymmetric cross section: $W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) - W(\hat{\mathbf{k}},\hat{\mathbf{k}}')]/2$

$$\frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2 \frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}}')|^2}}W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}})\frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}})|^2}}$$

Transverse force $\quad \eta_{xy}^{(-)} = -\eta_{yx}^{(-)} \equiv \eta_{\mathsf{AH}} \quad \Rightarrow$ anomalous Hall effect

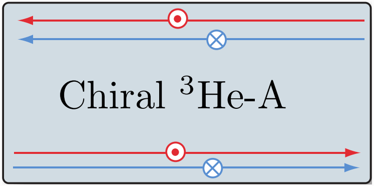
▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Differential cross section for Bogoliubov QP-Ion Scattering

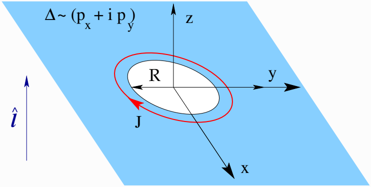


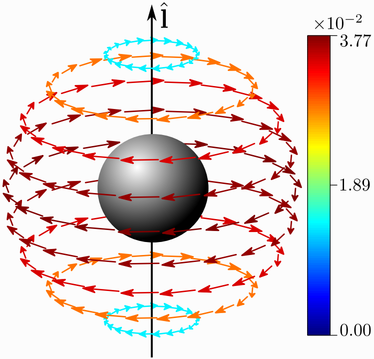
▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

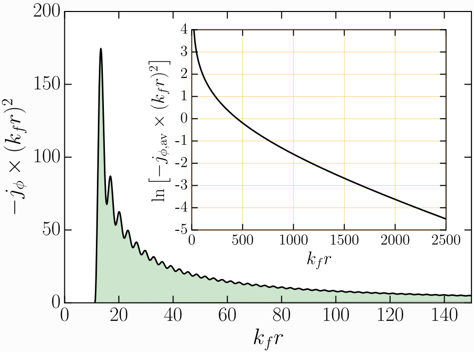
Current density bound to an electron bubble ($k_f R = 11.17$)



$\Longrightarrow$





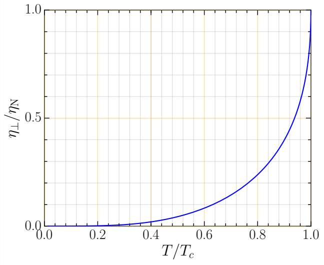


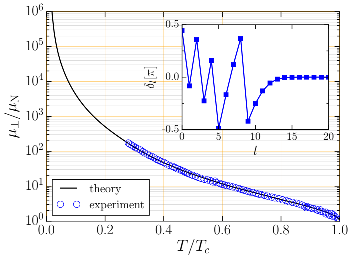
$$\mathbf{j}(\mathbf{r})/v_f N_f k_B T_c = j_\phi(\mathbf{r})\hat{\mathbf{e}}_\phi \quad \Longrightarrow \quad \mathbf{L}(T \to 0) \approx -\hbar N_{\mathrm{bubble}} \hat{\mathbf{l}}/2$$

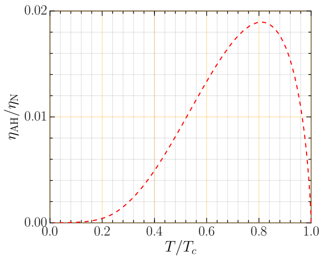
- O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

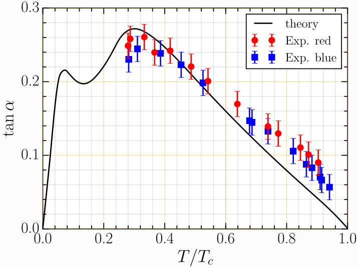
Theoretical and Experimental Comparison for the Electron Mobility in ^{3}He-A

$$\mu_\perp = e\,\frac{\eta_\perp}{\eta_\perp^2 + \eta_{AH}^2}, \quad \mu_{AH} = -e\,\frac{\eta_{AH}}{\eta_\perp^2 + \eta_{AH}^2}, \quad \tan\alpha = \left|\frac{\mu_{AH}}{\mu_\perp}\right| = \frac{\eta_{AH}}{\eta_\perp}, \quad k_f R = 11.17$$









▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Summary

- Electrons in ^{3}He-A are "dressed" by a spectrum of Weyl Fermions
- Electrons in ^{3}He-A are "Left handed" in a Right-handed Chiral Vacuum
  $\rightsquigarrow L_z \approx -(N_{bubble}/2)\hbar \approx -100\,\hbar$
- Experiment: RIKEN mobility experiments $\rightsquigarrow$ Observation an AHE in ^{3}He-A
- Scattering of Bogoliubov QPs by the dressed Ion
  $\rightsquigarrow$ Drag Force $(-\eta_\perp \mathbf{v})$ and Transverse Force $(\frac{e}{c}\mathbf{v} \times \mathbf{B}_{eff})$ on the Ion
- *Anomalous Hall Field:* $\mathbf{B}_{\text{eff}} \approx \dfrac{\Phi_0}{3\pi^2}\, k_f^2\, (k_f R)^2 \left(\dfrac{\eta_{\text{AH}}}{\eta_{\text{N}}}\right) \mathbf{l} \simeq 10^3 - 10^4\,\text{T}\,\mathbf{l}$
- Mechanism: Skew/Andreev Scattering of Bogoliubov QPs by the dressed Ion
- Origin: Broken Mirror & Time-Reversal Symmetry $\rightsquigarrow W(\mathbf{k},\mathbf{k}') \neq W(\mathbf{k}',\mathbf{k})$
- Theory: $\rightsquigarrow$ Quantitative account of RIKEN mobility experiments
- Ongoing: New directions for Novel Transport in ^{3}He-A & Chiral Superconductors