Physics Department Colloquium, University of Alberta, April 6, 2018

Spontaneous Symmetry Breaking in Superfluid ^{3}He

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Northwestern University

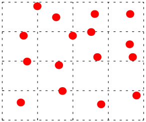
• Supported by National Science Foundation Grant DMR-1508730

• Oleksii Shevtsov • Joshua Wiman • Hao Wu

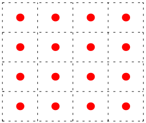
• Takeshi Mizushima (Osaka University)

- Spontaneous Symmetry Breaking
- Nambu-Goldstone & Higgs Modes
- Bosonic Spectrum of Superfluid ^{3}He-B
- Topological Order in Superfluid ^{3}He

Liquid

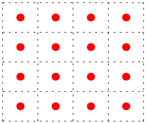


Broken Symmetry, Phase Transitions and Long-Range Order



Solid

Broken Symmetry, Phase Transitions and Long-Range Order



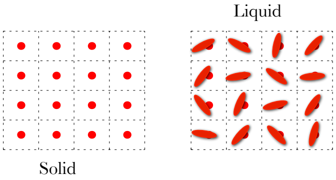
Solid

Translations

$\mathsf{G}_{\mathsf{trans}}$

$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r})$$

Broken Symmetry, Phase Transitions and Long-Range Order



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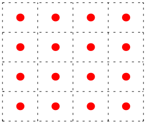
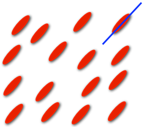


Solid



Nematic

Translations

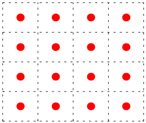
G_{trans}

Space Rotations

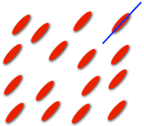
$SO(3)_L$

$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r}) \qquad Q_{ij} = Q(T)\left(\mathbf{n}_i\mathbf{n}_j - \frac{1}{3}\delta_{ij}\right)$$

Broken Symmetry, Phase Transitions and Long-Range Order

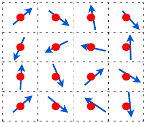


Solid



Nematic

Paramagnet



Translations

G_{trans}

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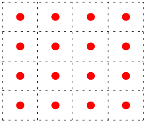
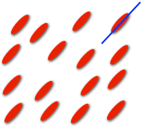
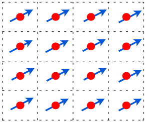


Solid



Nematic

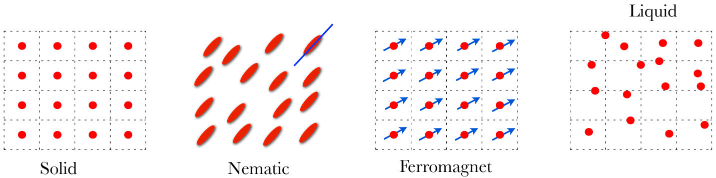


Ferromagnet

Translations	Space Rotations	Spin Rotation
G_{trans}	$SO(3)_{L}$	$SO(3)_{S}$

$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r}) \qquad Q_{ij} = Q(T)\left(\mathbf{n}_i \mathbf{n}_j - \frac{1}{3}\delta_{ij}\right) \qquad \mathbf{M} = \gamma \langle \mathbf{S} \rangle$$

Broken Symmetry, Phase Transitions and Long-Range Order



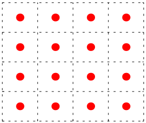
Translations — G_{trans}

Space Rotations — $SO(3)_{\mathrm{L}}$

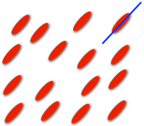
Spin Rotation — $SO(3)_{\mathrm{S}}$

$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r}) \qquad Q_{ij} = Q(T)\left(\mathbf{n}_i \mathbf{n}_j - \frac{1}{3}\delta_{ij}\right) \qquad \mathbf{M} = \gamma \left\langle \mathbf{S} \right\rangle$$

Broken Symmetry, Phase Transitions and Long-Range Order



Solid



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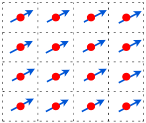
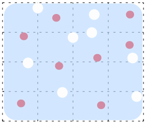


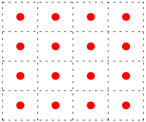
Ferromagnet

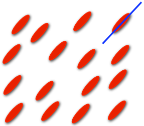


Super-liquid

Translations	Space Rotations	Spin Rotation	Gauge
G_{trans}	$SO(3)_{\mathrm{L}}$	$SO(3)_{\mathrm{S}}$	$U(1)_{\mathrm{N}}$
$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r})$	$Q_{ij} = Q(T)\left(\mathbf{n}_i \mathbf{n}_j - \frac{1}{3}\delta_{ij}\right)$	$\mathbf{M} = \gamma \langle \mathbf{S} \rangle$	$\Psi = \langle \psi(\mathbf{r}) \rangle \simeq \sqrt{N/V}$

Broken Symmetry, Phase Transitions and Long-Range Order


Solid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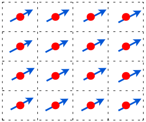
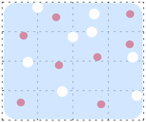

Ferromagnet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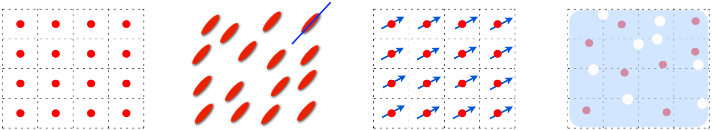
Broken Symmetry, Phase Transitions and Long-Range Order



Solid	Nematic	Ferromagnet	Super-liquid
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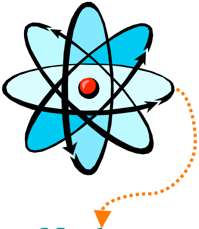
- Superfluid Phases of Superfluid ^{3}He Exhibit *all* of these Broken Symmetries!

Helium



$(1s)^2$ - closed electronic shell

- chemically inert
- $S_{electronic} = 0$

Nucleus

^4He	2 protons, 2 neutrons	$\rightarrow$ $S_{nuclear} = 0$	**Boson**
^3He	2 protons, 1 neutron	$\rightarrow$ $S_{nuclear} = \frac{1}{2}$	**Fermion**

Quantum Statistics Important for **T < T * ~ 1 K**

Helium Liquids

- Indistinguishability of identical particles becomes important …

$$\lambda = \frac{\hbar}{p} \approx \frac{\hbar}{\sqrt{2\,\mathbf{m}\,k_B\,T}} \quad > \quad \mathbf{a} = \sqrt[3]{\frac{V}{N}} \approx \text{Å}$$

$$\boxed{T < \mathbf{T}^* = \frac{\hbar^2}{2\,\mathbf{m}\,k_B\,\mathbf{a}^2} \approx 3\,\mathrm{K}}$$

^{3}He

Fermi Liquid

BCS Superfluid

$T < T_c = 2 \times 10^{-3}$ K

^{4}He

Bose Liquid

Superfluid

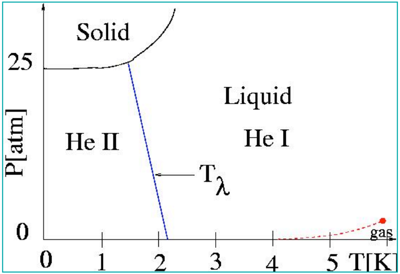
$T < T_\lambda = 2.2$ K

Phase Diagram for ^{4}He

- Permanent liquid at P < 25 atm
- He II - Superfluid
- Persistent Currents
- Origin

↓

Bose-Einstein Condensation



- S. Bose, Z. für Phy 26: 178 (1924)
- A. Einstein, Proc. Prussian Acad. Sci. 1: 3 (1925)

- **Macroscopic occupation of a single quantum state**

Fritz London

$$\Phi(\mathbf{r}_1, \ldots, \mathbf{r}_N) = \overbrace{\varphi(\mathbf{r}_1)\,\varphi(\mathbf{r}_2)\,\ldots\varphi(\mathbf{r}_N)}$$

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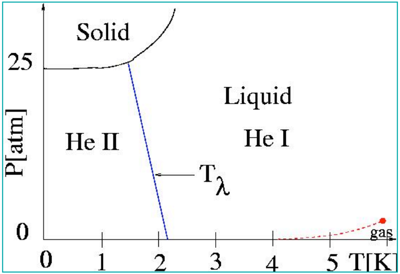

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$$|\,\Phi_N\,\rangle = \left[\int d\mathbf{r}\ \varphi(\mathbf{r})\ \psi^\dagger(\mathbf{r})\right]^N |\,\mathrm{vac}\,\rangle$$

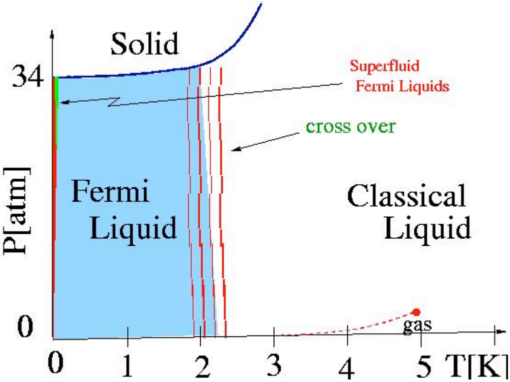
Phase Diagram for ^{3}He

- Permanent liquid at P < 34 atm
- Smooth crossover near $T^* = E_f \sim 2$ K
- … superfluidity below
- $T_c \sim$ **2 x 10^{-3} K**

D. Osheroff, R. Richardson, D. Lee (1972)

A. J. Leggett (1973)

2 Nobel Prizes: 1996 & 2006



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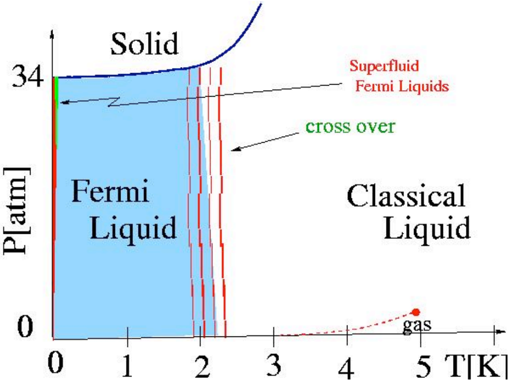

- **Macroscopic occupation of a *2-particle* quantum state**

$$|\,\Phi_N\,\rangle = \left[\iint d\mathbf{r}_1 d\mathbf{r}_2\, \varphi_{s_1 s_2}(\mathbf{r}_1,\mathbf{r}_2)\, \psi^\dagger_{s_1}(\mathbf{r}_1)\psi^\dagger_{s_2}(\mathbf{r}_2)\right]^{N/2} |\,\mathrm{vac}\,\rangle$$

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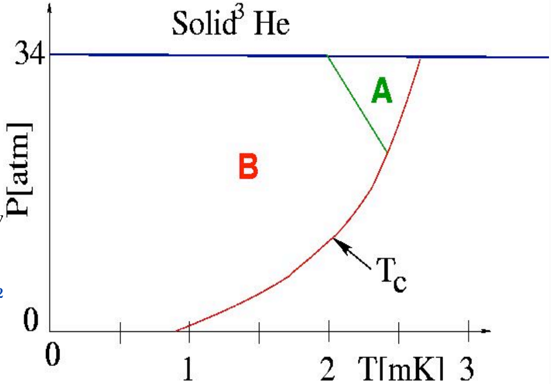

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$L = 1$ **"p-wave"** $S = 1$ **"spin triplet"**

Discovery - NMR Shift in Liquid ^{3}He-A

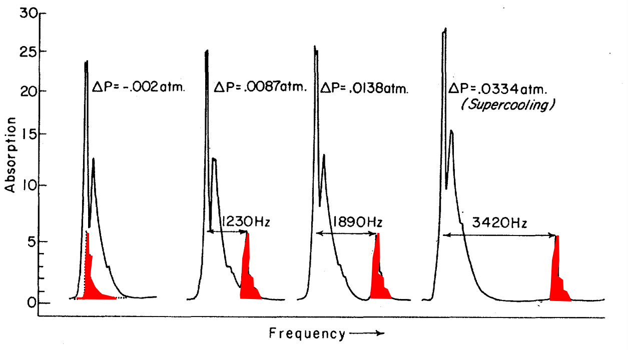
VOLUME 29, NUMBER 14 PHYSICAL REVIEW LETTERS 2 OCTOBER 1972

New Magnetic Phenomena in Liquid He^3 below 3 mK*

D. D. Osheroff,† W. J. Gully, R. C. Richardson, and D. M. Lee
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14850

(Received 7 July 1972)

Magnetic measurements have been made on a sample of He^3 in a Pomeranchuk cell. Below about 2.7 mK, the NMR line apparently associated with the liquid portion of the sample shifts continuously to higher frequencies during cooling. Below about 2 mK the frequency shift vanishes, and the magnitude of the liquid absorption drops abruptly to approximately $\frac{1}{2}$ its previous value. These measurements are related to the pressure phenomena reported by Osheroff, Richardson, and Lee.



NMR Shift $\rightsquigarrow$ Broken Spin-Orbit Symmetry

VOLUME 29, NUMBER 18 PHYSICAL REVIEW LETTERS 30 OCTOBER 1972

Interpretation of Recent Results on He^3 below 3 mK: A New Liquid Phase?

A. J. Leggett

School of Mathematical and Physical Sciences, University of Sussex, England

(Received 5 September 1972)

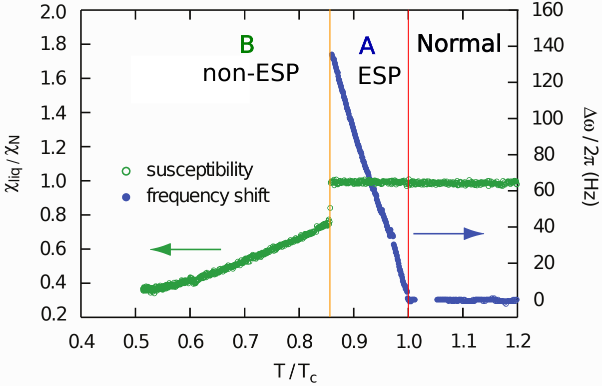
It is demonstrated that recent NMR results in ^3He indicate that at 2.65 mK, the liquid makes a second-order transition to a phase in which the "spin-orbit" symmetry is spontaneously broken. The hypothesis that this phase is a BCS-type phase in which pairs form with l odd, $S=1$, $S_z=\pm 1$ leads to reasonable agreement with both NMR and thermodynamic data, but involves some difficulties as to stability.

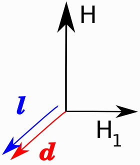
$$\omega^2 = (\gamma H)^2 + \Omega^2(T) \quad \longrightarrow \quad \omega \simeq \gamma H + \frac{\Omega^2(T)}{2\gamma H} \propto (1 - T/T_c)$$

$$\Omega^2 = -\frac{2\gamma^2}{\chi} \langle \mathcal{H}_D \rangle \qquad \Omega \neq 0 \implies \text{Broken Spin-Orbit Symmetry}$$

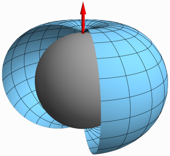
NMR frequency shift and Magnetic Susceptibility

J. Pollanen et al. PRL 107, 195301 (2011)





Chiral AM State $\vec{l}$



$$|\Psi_A\rangle = \Delta \left\{ \overbrace{(p_y + ip_z)}^{L_x=+1} \overbrace{(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)}^{S_z=\pm 1} \right\}$$

orbital FM — spin AFM

The ^{3}He Paradigm: Maximal Symmetry $\mathtt{G} = \mathtt{SO(3)}_\mathsf{S} \times \mathtt{SO(3)}_\mathsf{L} \times \mathtt{U(1)}_\mathsf{N} \times \mathtt{P} \times \mathtt{T}$

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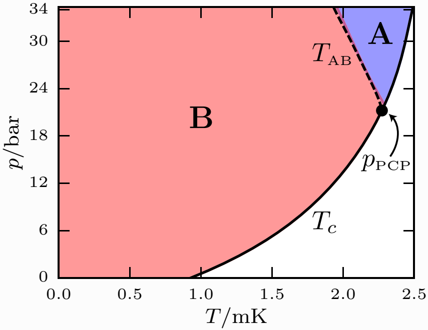
BCS Condensate of Bound Spin $1/2$ Fermions

$\Downarrow$

Cooper Pairs with Total Spin, $S = 1$ and Orbital Angular Momentum, $L = 1$

J. Wiman & J. A. Sauls, PRB 92, 144515 (2015)



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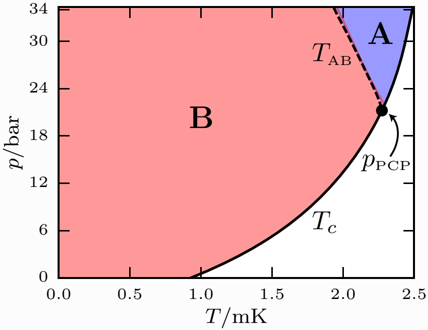
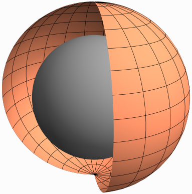


"Isotropic" BW State



$J = 0, \; J_z = 0$

$H = \mathtt{SO(3)_J} \times \mathtt{T}$

$$|\,\Psi_B\,\rangle = \Delta \left\{ \overbrace{\frac{1}{\sqrt{2}}(p_x - ip_y)}^{L_z=-1} \underbrace{|\uparrow\uparrow\rangle}_{S_z=+1} + \overbrace{\frac{1}{\sqrt{2}}(p_x + ip_y)}^{L_z=+1} \underbrace{|\downarrow\downarrow\rangle}_{S_z=-1} + \overbrace{p_z}^{L_z=0} \underbrace{\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)}_{S_z=0} \right\}$$

Is This All ?

Is This All ?

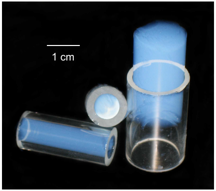
$$\Downarrow$$

Dimensionality of the Order Parameter Space

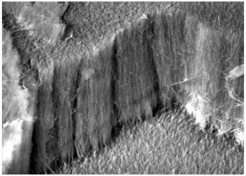
$$(2L+1)\ \times\ (2S+2)\ \times\ 2 = \boxed{18}$$

Technical Developments driving Research in Confined Quantum Fluids

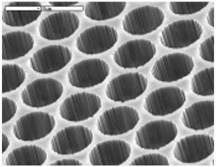
Nano-fabrication of Cavities, Mechanical Oscillators to High-Porosity Random Solids



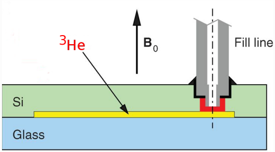
Intrinsic Anisotropy in SiO_2 Aerogels
J. Pollanen et al. Nature Physics (2012)



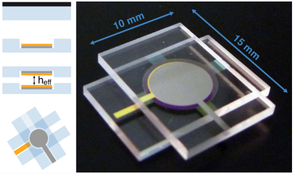
Anisotropic "Nafen" (AlO) Aerogels
Dmitriev et al PRL (2015)



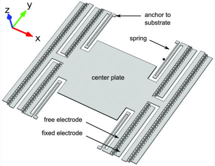
60-600 nm Pores, Synkera Inc.
A. Zimmerman et al. (2018) - NMR



100 nm Cavities & Torsional Oscillators
L. Levitin, et al., Science 340, 841 (2013)



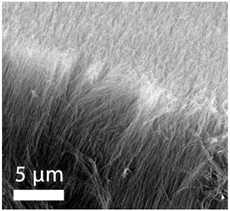
Nano-fluidic Helmholtz Resonator
X Rojas & J Davis, PRB 91, 024503 (2015)



MEMS Resonator
M. González et al., JLTP 162, 661 (2011)

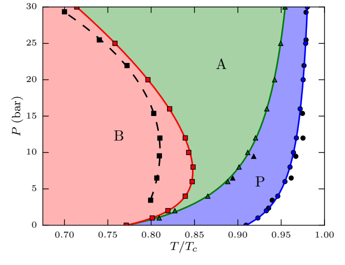
New Phases of Superfluid ^{3}He Under Strong Confinement

- V. Dmitriev et al.,PRL 115, 165304 (2015)



  - ^{3}He Confined in Nematic Aerogel

- J.J. Wiman, S. Laine, E. Thuneberg & JAS, (2018).



  - Discovery of $\frac{1}{2}$ Quantum Vortices - S. Autti et al. PRL (2016)

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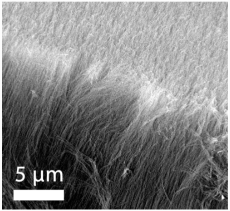

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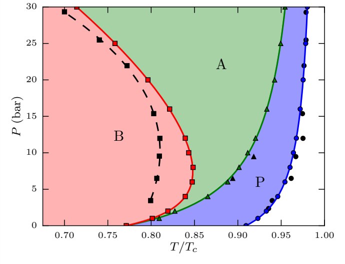
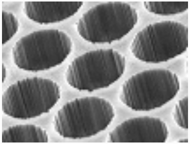
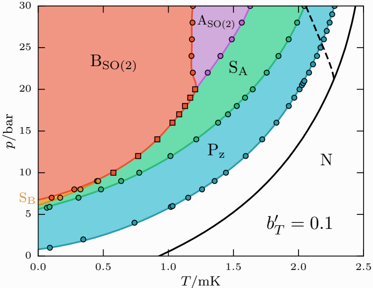


- Discovery of $\frac{1}{2}$ Quantum Vortices - S. Autti et al. PRL (2016)

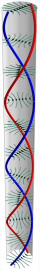
100nm Pores



- J.J. Wiman & JAS, PRB 92, 144515 (2015)



S_A - Helical Phase



Chiral Anomaly

- Six new phases of ^{3}He
- Nematic P_z Phase
- Helical Phase

Dynamical Consequences of Spontaneous Symmetry Breaking

Dynamical Consequences of Spontaneous Symmetry Breaking

New Bosonic Excitations

New Bosonic Excitations

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN)

CERN-PH-EP/2012-220
2013/01/29

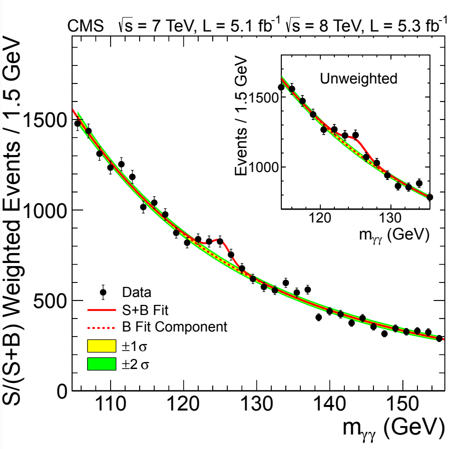
CMS-HIG-12-028

Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC

The CMS Collaboration

2013

Higgs Boson with mass $M = 125$ GeV



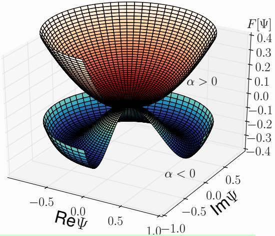
Dynamical Consequences of Spontaneous Symmetry Breaking

Scalar Higgs Boson (spin $J = 0$) [P. Higgs, PRL 13, 508 1964]

Energy Functional for the Higgs Field

$$\mathcal{U}[\Delta] = \int dV \left\{ \alpha \, |\Delta|^2 + \beta \, |\Delta|^4 + \frac{1}{2}c^2 \, |\boldsymbol{\nabla}\Delta|^2 \right\}$$

- Broken Symmetry State: $\Delta = \sqrt{|\alpha|/2\beta}$



Space-Time Fluctuations about the Broken Symmetry Vacuum State

$\Delta(\mathbf{r}, t) = \Delta + D(\mathbf{r}, t)$ ► Eigenmodes: $D^{(\pm)} = D \pm D^{*}$ (Conjugation Parity)

$$\mathcal{L} = \int d^3r \left\{ \frac{1}{2}[(\dot{D}^{(+)})^2 + (\dot{D}^{(-)})^2] - 2\Delta^2 \, (D^{(+)})^2 - \frac{1}{2}[c^2(\boldsymbol{\nabla}D^{(+)})^2 + c^2(\boldsymbol{\nabla}D^{(-)})^2] \right\}$$

- $\partial_t^2 D^{(-)} - c^2 \boldsymbol{\nabla}^2 \, D^{(-)} = 0$

  Massless Nambu-Goldstone Mode

- $\partial_t^2 D^{(+)} - c^2 \boldsymbol{\nabla}^2 \, D^{(+)} + 4\Delta^2 \, D^{(+)} = 0$

  Massive Higgs Mode: $M = 2\Delta$

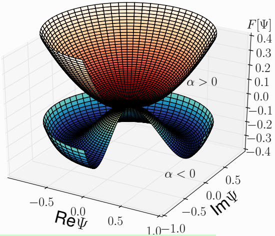
Dynamical Consequences of Spontaneous Symmetry Breaking

BCS Condensation of Spin-Singlet ($S = 0$), S-wave ($L = 0$) "Scalar" Cooper Pairs

Ginzburg-Landau Functional

$$F[\Delta] = \int dV \left\{ \alpha \, |\Delta|^2 + \beta \, |\Delta|^4 + \kappa \, |\boldsymbol{\nabla}\Delta|^2 \right\}$$

- Order Parameter: $\Delta = \sqrt{|\alpha|/2\beta}$



Space-Time Fluctuations of the Condensate Order Parameter

$\Delta(\mathbf{r}, t) = \Delta + D(\mathbf{r}, t)$ ▶ Eigenmodes: $D^{(\pm)} = D \pm D^*$ (Fermion "Charge" Parity)

$$\mathcal{L} = \int d^3r \left\{ \frac{1}{2}[(\dot{D}^{(+)})^2 + (\dot{D}^{(-)})^2] - 2\Delta^2 \, (D^{(+)})^2 - \frac{1}{2}[v^2(\boldsymbol{\nabla}D^{(+)})^2 + v^2(\boldsymbol{\nabla}D^{(-)})^2] \right\}$$

- $\partial_t^2 D^{(-)} - v^2 \boldsymbol{\nabla}^2 D^{(-)} = 0$

  Anderson-Bogoliubov Mode

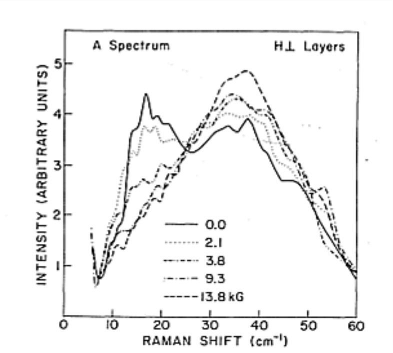
- $\partial_t^2 D^{(+)} - v^2 \boldsymbol{\nabla}^2 D^{(+)} + 4\Delta^2 \, D^{(+)} = 0$

  Amplitude Higgs Mode: $M = 2\Delta$

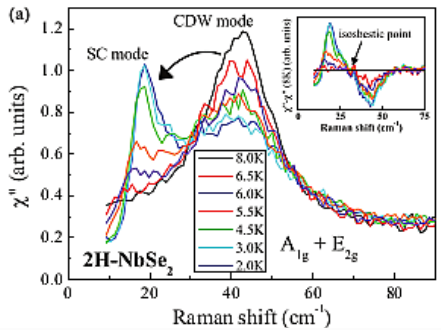
Dynamical Consequences of Spontaneous Symmetry Breaking

Higgs Mode with mass: $M = 3$ meV and spin $J = 0$ in $NbSe_2$

Raman Absorption in $NbSe_2$



R. Sooyakumar & M. Klein, PRL 45, 660 (1980)



M. Meásson et al. PRB B 89, 060503(R) (2014)

- $\hbar\omega_{\gamma_1} = \hbar\omega_{\gamma_2} + 2\Delta$
- Amplitude Higgs - CDW Phonon Coupling

- Theory: P. Littlewood & C. Varma, PRL 47, 811 (1981)

Dynamical Consequences of Spontaneous Symmetry Breaking

First Reported Observations of Higgs Bosons in BCS Condensates

VOLUME 45, NUMBER 8 PHYSICAL REVIEW LETTERS 25 AUGUST 1980

Raman Scattering by Superconducting-Gap Excitations and Their Coupling to Charge-Density Waves

R. Sooryakumar and M. V. Klein
Department of Physics and Materials Research Laboratory, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801
(Received 24 March 1980)

$2H$-$NbSe_2$ undergoes a charge-density–wave (CDW) distortion at 33 K which induces A and E Raman-active phonon modes. These are joined in the superconducting state at 2 K by new A and E Raman modes close in energy to the BCS gap 2Δ. Magnetic fields suppress the intensity of the new modes and enhance that of the CDW-induced modes, thus providing evidence of coupling between the superconducting-gap excitations and the CDW.

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VOLUME 45, NUMBER 4 PHYSICAL REVIEW LETTERS 28 JULY 1980

Measurements of High-Frequency Sound Propagation in ^{3}He-*B*

D. B. Mast, Bimal K. Sarma, J. R. Owers-Bradley, I. D. Calder, J. B. Ketterson, and W. P. Halperin

Department of Physics and Astronomy and Materials Research Center, Northwestern University, Evanston, Illinois 60201

(Received 10 April 1980)

Measurements of the attenuation and velocity of pulsed high-frequency sound have been performed up to 133 MHz in superfluid ^{3}He-*B*. A new collective mode of the order parameter was discovered at a frequency extrapolated to T_c of $\omega = (1.165 \pm 0.05)\Delta_{BCS}(T_c)$, where $\Delta_{BCS}(T)$ is the energy gap in the weak-coupling BCS theory. The group velocity has been observed to decrease by as much as $\frac{2}{3}$ of the zero-sound velocity.

Observation of a New Sound-Attenuation Peak in Superfluid ^{3}He-*B*

R. W. Giannetta,[a] A. Ahonen,[b] E. Polturak, J. Saunders, E. K. Zeise, R. C. Richardson, and D. M. Lee

Laboratory of Atomic and Solid State Physics and Materials Science Center, Cornell University, Ithaca, New York 14853

(Received 25 March 1980)

Results of zero-sound attenuation measurements in ^{3}He-*B*, at frequencies up to 60 MHz and pressures between 0 and 20 bars, are reported. At frequencies of 30 MHz and above, a new attenuation feature is observed which bears the signature of a collective mode of the superfluid.

Field Theory for Bosonic Excitations of Superfluid ^{3}He-B

$$^3\text{He-B:}\quad B_{\alpha i} = \frac{1}{\sqrt{3}}\Delta\,\delta_{\alpha i} \quad L = 1,\ S = 1 \rightsquigarrow J = 0$$

- Symmetry of ^{3}He-B: $\mathtt{H} = \mathtt{SO(3)}_{\mathtt{J}} \times \mathtt{T}$
- Fluctuations: $\mathcal{D}_{\alpha i}(\mathbf{r},t) = A_{\alpha i}(\mathbf{r},t) - B_{\alpha i} = \sum_{J,m} D_{J,m}(\mathbf{r},t)\, t^{(J,m)}_{\alpha i}$
- Lagrangian:

$$\mathcal{L} = \int d^3r \left\{ \tau\, \mathrm{Tr}\left\{\dot{\mathcal{D}}\dot{\mathcal{D}}^\dagger\right\} - \alpha\, \mathrm{Tr}\left\{\mathcal{D}\mathcal{D}^\dagger\right\} - \sum_{p=1}^{5} \beta_p\, u_p(\mathcal{D}) - \sum_{l=1}^{3} K_l\, v_l(\partial\mathcal{D}) \right\}$$

$$\partial_t^2 D^{(\mathtt{C})}_{J,m} + E^{(\mathtt{C})}_{J,m}(\mathbf{q})^2\, D^{(\mathtt{C})}_{J,m} = \frac{1}{\tau}\eta^{(\mathtt{C})}_{J,m}$$

with $\quad J = \{0,1,2\}\,, m = -J \ldots +J\,, \mathtt{C} = \pm 1$

- *Nambu's Boson-Fermion Mass Relations for Superfluid ^{3}He-B*: JAS & T. Mizushima, Phys. Rev. B 95, 094515 (2017)

Spectrum of Bosonic Modes of Superfluid ^{3}He-B : Condensate is $J^{\mathtt{C}} = 0^+$

▶ 4 Nambu-Goldstone Modes & 14 Higgs modes

$$E^{(\mathtt{C})}_{J,m}(\mathbf{q}) = \sqrt{M^2_{J,\mathtt{C}} + \left(c^{(\mathtt{C})}_{J,|m|}|\mathbf{q}|\right)^2}$$

Mode	Symmetry	Mass	Name
$D^{(+)}_{0,m}$	$J=0$, $\mathtt{C}=+1$	2Δ	Amplitude Higgs
$D^{(-)}_{0,m}$	$J=0$, $\mathtt{C}=-1$	0	NG Phase Mode
$D^{(+)}_{1,m}$	$J=1$, $\mathtt{C}=+1$	0	NG Spin-Orbit Modes
$D^{(-)}_{1,m}$	$J=1$, $\mathtt{C}=-1$	2Δ	AH Spin-Orbit Modes
$D^{(+)}_{2,m}$	$J=2$, $\mathtt{C}=+1$	$\sqrt{\frac{8}{5}}\Delta$	2^+ AH Modes
$D^{(-)}_{2,m}$	$J=2$, $\mathtt{C}=-1$	$\sqrt{\frac{12}{5}}\Delta$	2^- AH Modes

▶ Vdovin, Maki, Wölfle, Serene, Nagai, Volovik, Schopohl, JAS ...

▶ *Broken Symmetry & Nonequilibrium Superfluid ^{3}He*, Les Houches Lectures, arXiv:cond-mat/9910260 (1999), J.A. Sauls

Collective Mode Spectrum for ^{3}He-B

Bosonic Excitations of ^{3}He-B

Goldstone Mode w/ J=0$^-$ → $D^{(-)}_{00} = i|\Delta|\, \underbrace{\varphi(q,\omega)}_{\text{phase mode}}$

phase mode

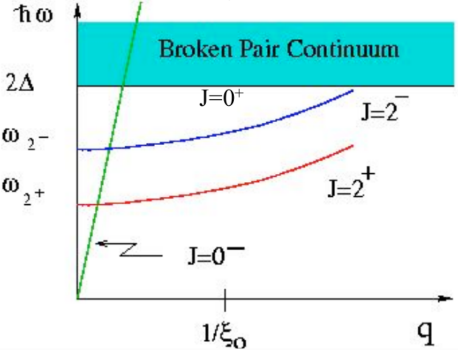
$$\left(\partial_t^2 - c_{00}^2 \nabla^2\right) D^{(-)}_{00} = \ldots$$

Pair Excitons w/ J=2$^{+/-}$

$$\left(\partial_t^2 + \Omega_{JC}^2\right) D^{(C)}_{JM} = \ldots$$

Anderson-Higgs Modes

coupling to internal & external fields



Collective Mode Spectrum for ^{3}He-B

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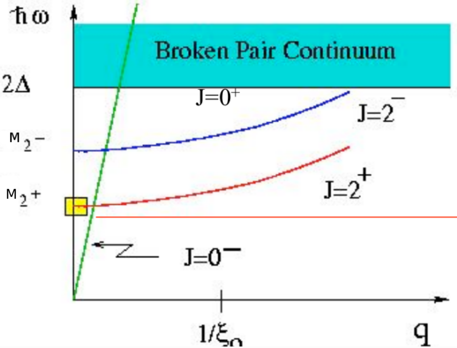
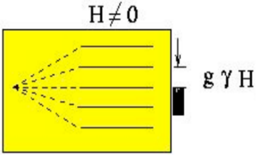
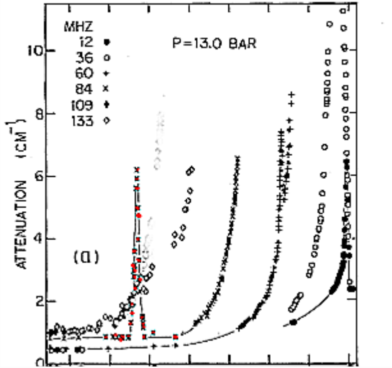


Nuclear Zeeman levels



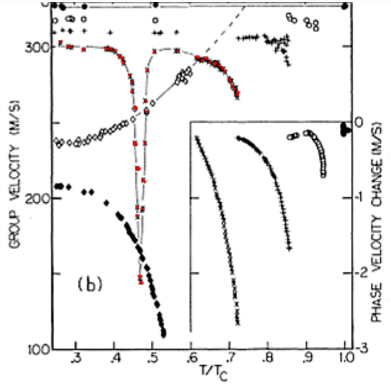
JAS & J. Serene, PRL 1982

Higgs Mode with mass: $M = 500$ neV and spin $J^{\mathrm{C}} = 2^{+}$ at ULT-Northwestern



T/T_c

Group Velocity



- D. Mast et al. Phys. Rev. Lett. 45, 266 (1980).

Superfluid ^{3}He Higgs Detector at ULT-Northwestern

^{3}He-^{4}He Dilution + Adiabatic Demagnetization Stages $\rightsquigarrow T_{\text{min}} \approx 200\mu$K

$J = 2^{-}$, $m = \pm 1$ Higgs Modes Transport Mass Currents

$J = 2^-$, $m = \pm 1$ Higgs Modes Transport Mass Currents

- "Transverse Waves in Superfluid ^{3}He-B", G. Moores and JAS, JLTP 91, 13 (1993)

$$C_t(\omega) = \sqrt{\frac{F_1^s}{15}} v_f \left[\rho_n(\omega) + \frac{2}{5}\rho_s(\omega) \underbrace{\left\{ \frac{\omega^2}{(\omega + i\Gamma)^2 - \frac{12}{5}\Delta^2 - \frac{2}{5}(q^2 v_f^2)} \right\}}_{D^{(-)}_{2,\pm 1}} \right]^{\frac{1}{2}}$$

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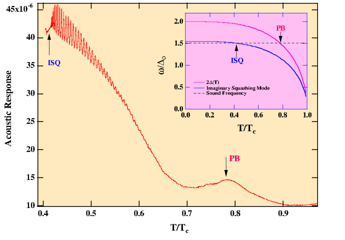
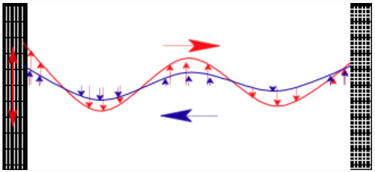
Transverse Sound in Superfluid ^{3}He-B: *Cavity Oscillations of Transverse Sound*





- Y. Lee et al. Nature 400 (1999)
- J. Davis et al. Nat. Phys. 4, 571 (2008)

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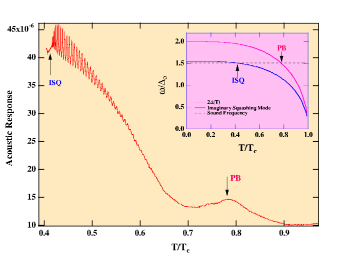
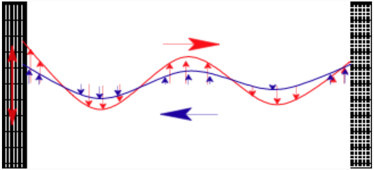


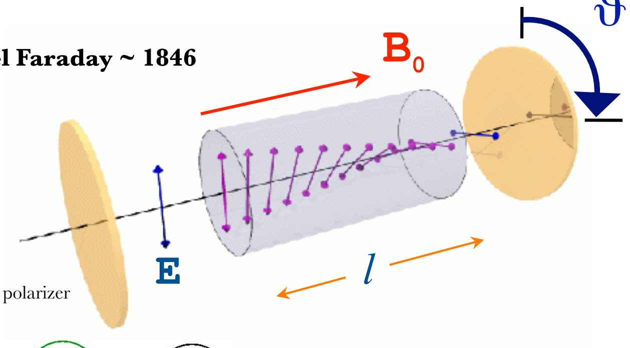


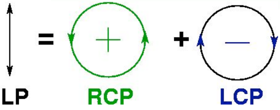
- Y. Lee et al. Nature 400 (1999)
- J. Davis et al. Nat. Phys. 4, 571 (2008)

$\mathbf{B}$ $\longrightarrow$

Faraday Effect

Michael Faraday ~ 1846





$\mathbf{B}_0 \neq 0 \rightarrow n_+ \neq n_-$

field-induced circular birefringence

Verdet's constant

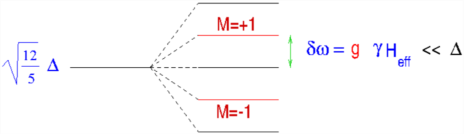
$$\vartheta = \mathcal{V}\,\mathbf{B}_0\, l$$

Faraday Rotation: Magneto-Acoustic Birefringence of Transverse Currents

- "Magneto-Acoustic Rotation of Transverse Waves in ^{3}He-B", J. A. Sauls et al., Physica B, 284,267 (2000)

$$C_{\substack{\text{RCP}\\\text{LCP}}}(\omega) = v_f \left[\frac{F_1^s}{15}\,\rho_n(\omega) + \frac{2F_1^s}{75}\,\rho_s(\omega) \underbrace{\left\{ \frac{\omega^2}{(\omega + i\Gamma)^2 - \Omega_{2,\pm}^{(-)}(\mathbf{q})} \right\}}_{D_{2,\pm 1}^{(-)}} \right]^{\frac{1}{2}}$$

$$\Omega_{2,\pm}^{(-)}(\mathbf{q}) = \sqrt{\tfrac{12}{5}}\,\Delta \pm g_{2-}\,\gamma H_{\text{eff}}$$



- Circular Birefringence $\Longrightarrow$ $C_{\text{RCP}} \neq C_{\text{LCP}}$ $\Longrightarrow$ Faraday Rotation

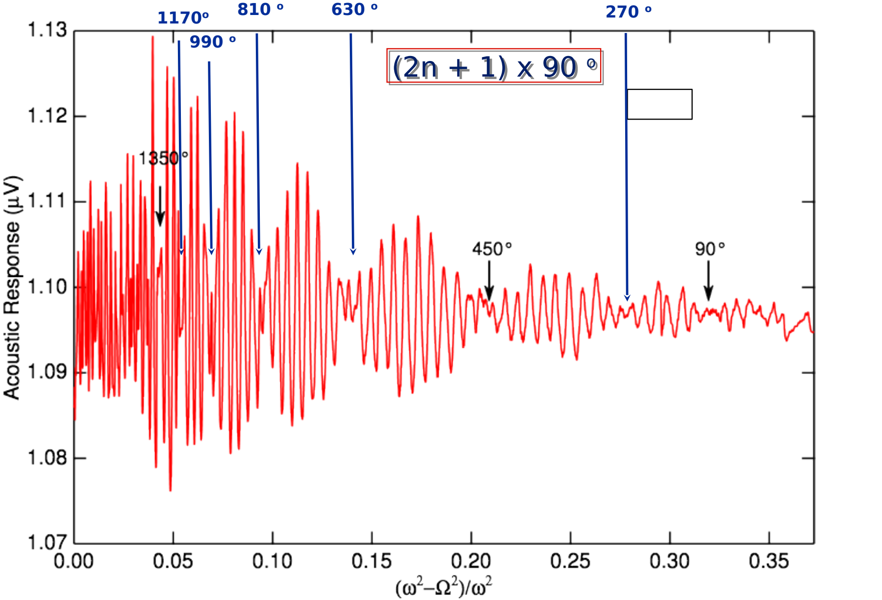
$$\left(\frac{C_{\text{RCP}} - C_{\text{LCP}}}{C_t}\right) \simeq g_{2-} \left(\frac{\gamma H_{\text{eff}}}{\omega}\right)$$

- Faraday Rotation Period $(\gamma H_{\text{eff}} \ll (\omega - \Omega_2^{(-)}))$:

$$\lambda_H \simeq \frac{4\pi C_t}{g_{2-}\gamma H} \simeq 500\,\mu m\,, \qquad H = 200\,G$$

- *Discovery of the acoustic Faraday effect in superfluid ^{3}He-B*, Y. Lee, et al. Nature 400, 431 (1999)

Large Faraday Rotations vs. ``Blue Tuning'' B = 1097 G



C. Collett et al., Phys. Rev. B 87, 024502 (2013)

Higgs Boson with mass $M = 125$ GeV - *Is this all there is?*

- *Higgs Bosons in Particle Physics and in Condensed Matter*
  G.E. Volovik & M. Zubkov, PRD 87, 075016 (2013)

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- *Boson-Fermion Relations in BCS type Theories*
  Y. Nambu, Physica D, 15, 147 (1985)

- Broken Symmetry State: $\rightsquigarrow$ Fermion mass: $m_{\mathsf{F}} = \Delta$
- Nambu's Sum Rule ("empirical observation"): $\sum_C M_{J,C}^2 = (2m_{\mathsf{F}})^2$

Mode	Symmetry	Mass	Name
$D_{0,m}^{(+)}$	$J = 0,\ \mathtt{C} = +1$	2Δ	Amplitude Higgs
$D_{0,m}^{(-)}$	$J = 0,\ \mathtt{C} = -1$	0	NG Phase Mode
$D_{1,m}^{(+)}$	$J = 1,\ \mathtt{C} = +1$	0	NG Spin-Orbit Modes
$D_{1,m}^{(-)}$	$J = 1,\ \mathtt{C} = -1$	2Δ	AH Spin-Orbit Modes
$D_{2,m}^{(+)}$	$J = 2,\ \mathtt{C} = +1$	$\sqrt{\frac{8}{5}}\Delta$	2^+ AH Modes
$D_{2,m}^{(-)}$	$J = 2,\ \mathtt{C} = -1$	$\sqrt{\frac{12}{5}}\Delta$	2^- AH Modes

- Weak Symmetry Breaking (Nuclear Dipolar & Zeeman)
  $\rightsquigarrow$ NG Modes $\rightarrow$ Pseudo-NG Modes

$J = 1^+$, $m = 0, \pm 1$ NG Modes $\rightsquigarrow$ Pseudo-NG Modes

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ARTICLE

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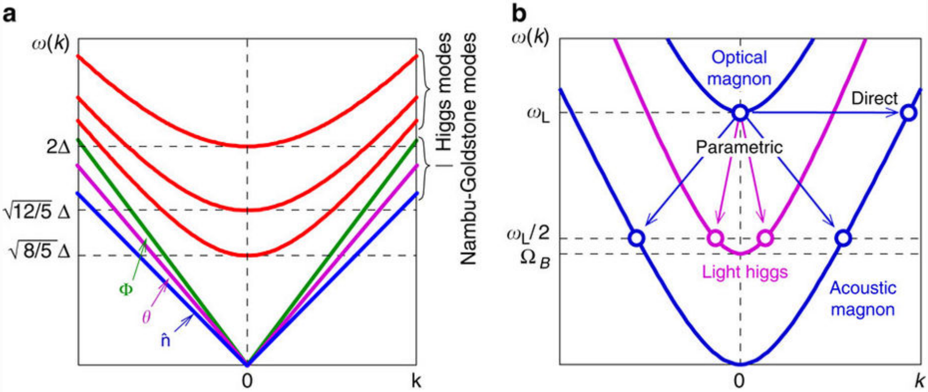

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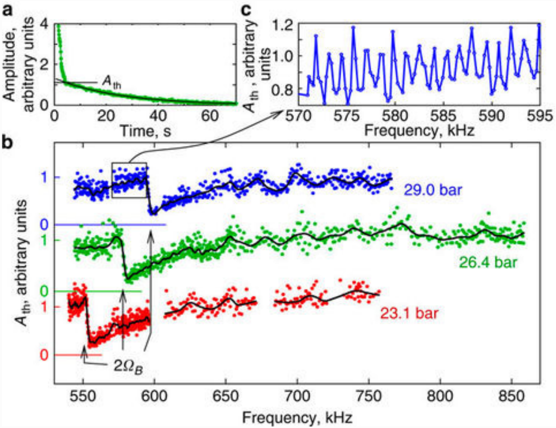
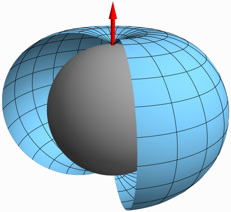


Superfluid ^{3}He as Topological Quantum Matter

Confinement, Excitations & New Phases

Bulk-Boundary Correspondence: Chiral Edge Fermions

$$\Psi(\mathbf{p}) = \Delta(p_x \pm i p_y) \sim e^{\pm i\varphi_{\mathbf{p}}}$$



Topological Invariant

$$N_{\mathrm{2D}} = \frac{1}{2\pi}\oint d\mathbf{p}\cdot \frac{1}{|\Psi(\mathbf{p})|}\mathrm{Im}[\boldsymbol{\nabla}_{\mathbf{p}}\Psi(\mathbf{p})] = \pm 1$$

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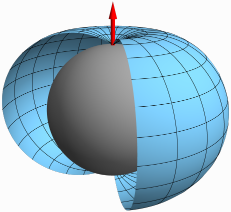

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"Vacuum" $(\Delta = 0)$ with $N_{2D} = 0$ | ^{3}He-A$(\Delta \neq 0)$ with $N_{2D} = 1$

Zero Energy Fermions ↑ Confined on the Edge

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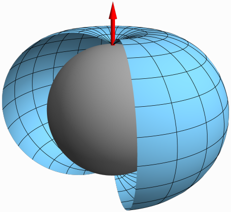

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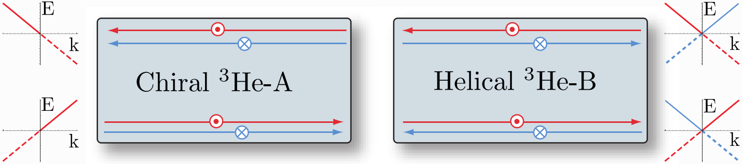
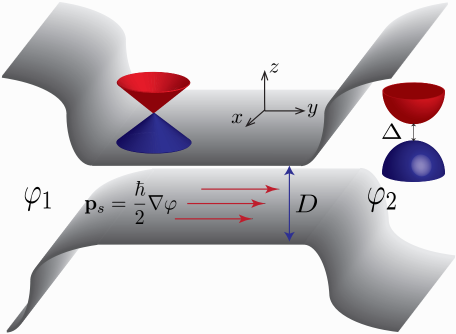


- *Surface states, edge currents, and the angular momentum of chiral superfluids*, JAS, PRB 84, 214509 (2011)

Condensate Flow and Backflow from Majorana Excitations

Condensate Flow: $\mathbf{p}_s \equiv m\mathbf{v}_s = \frac{\hbar}{2}\boldsymbol{\nabla}\varphi$

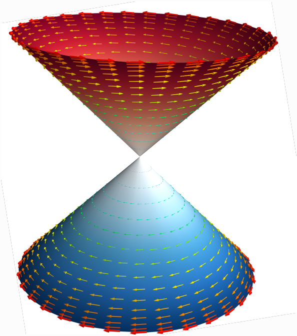


- Doppler Shifted Majorana Spectrum: $\varepsilon \to \varepsilon = c|\mathbf{p}_{||}| + \mathbf{p}_{||} \cdot \mathbf{v}_s$
- Thermal Signature: $\vec{J} = n\,\mathbf{p}_s \times \left(1 - \frac{27\pi\zeta(3)}{2}\frac{\xi_\Delta}{D}\frac{\Delta_\perp}{\Delta_{||}}\frac{m^*}{m_3}\left(\frac{T}{\Delta_{||}}\right)^3\right)$

- Majorana excitations, spin and mass currents in topological ^{3}He-B, Hao Wu, JAS, Phys. Rev. B 88, 18 184506 (2013)

Towards Spectroscopy of Helical Majorana Fermions

- Helical Majorana Exciations: $\vec{\mathbf{s}} \perp \vec{\mathbf{p}}_{||}$



- Ground State Surface Spin Current:

$$J_{xy}(0) = \frac{1}{6}\, n_{\text{2D}}\, v_f\, \frac{\hbar}{2}$$

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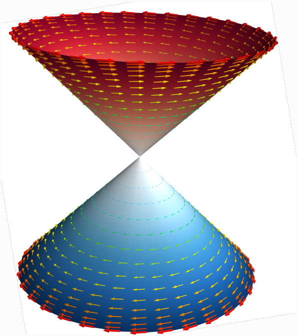

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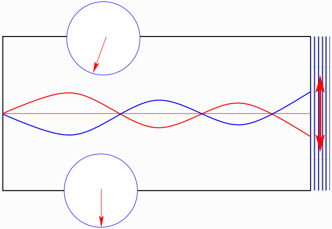
- Higgs Modes $J = 2, m = \pm 2$
- Transport Mass and Spin Current

$$\mathcal{D}^{(\pm)}_{\alpha i}(\mathbf{q},\omega) \sim \left(\mathbf{e}^{(\pm)}_{\alpha}\, \mathbf{q}_i + \mathbf{q}_{\alpha}\, \mathbf{e}^{(\pm)}_{i}\right)$$

- Generate via Transverse Sound ($J = 2,\ M = \pm 1$ Modes)
- Precision spectroscopy: dispersion, damping & Faraday rotation



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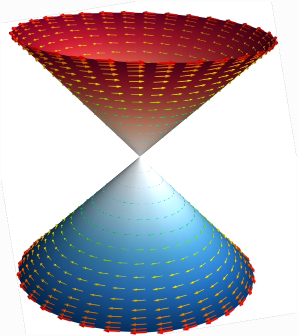

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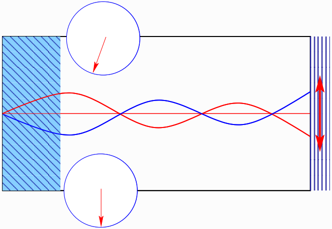
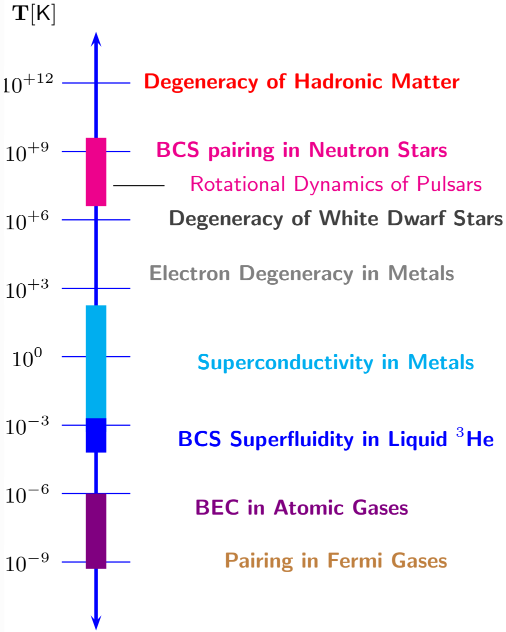


BCS Pairing from 10^{-9} K to 10^{+9} K



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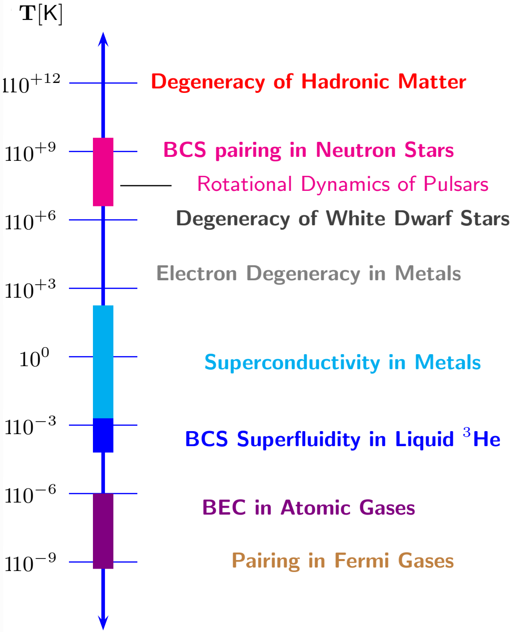


Key Discoveries

1908	Helium is liquified
1911	Superconductivity is discovered in Hg
1933	Diamagnetism - Meissner Effect
1935	London's Theory
1950	Ginzburg-Landau Theory
1956	Copper Instability
1957	BCS Pairing Theory
1957	Landau Fermi Liquid Theory
1957	Abrikosov's Theory of Type II SC
1958	Pairing in Nuclei and Nuclear Matter
1959	Gauge-Invariant Pairing Theory
1959	Field Theory formulation of BCS Theory
1961	Theory of Spin-Triplet Pairing
1962	Josephson Effect
1967	Pulsars discovered - Hewish & Bell
1969	Pulsar Glitches observed in Vela
1980	Superfluid hydrodynamics of NS
1972	Discovery of Triplet, P-wave Superfluid ^{3}He
1979	Discovery of Heavy Electron Superconductors
1982	Exotic Pairing in U-based Heavy Fermions
1986	High T_c CuO Superconductivity
1994	Exotic Pairing discovered in Sr_2RuO_4
1995	D-wave Pairing identified in $YBa_2Cu_3O_{7-x}$
2001	Co-existent Ferromagnetism & Superconductivty
2008	Fe-based Superconductors discovered
1995	Discovery of BEC in cold atomic ^{87}Rb
1998	Degeneracy of Cold Fermionic Gases: ^{6}Li
2007	BEC-BCS Condensation in ^{6}Li, ^{40}K
2008	*Topological* Superfluids & Superconductors

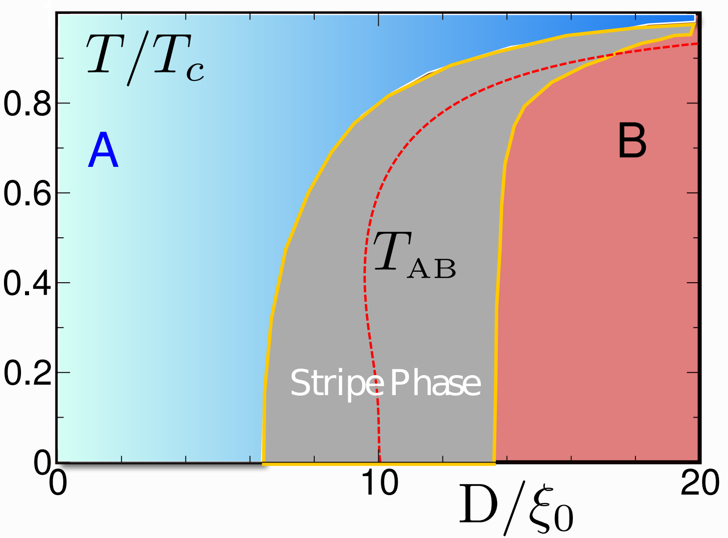
Thank You!

The End

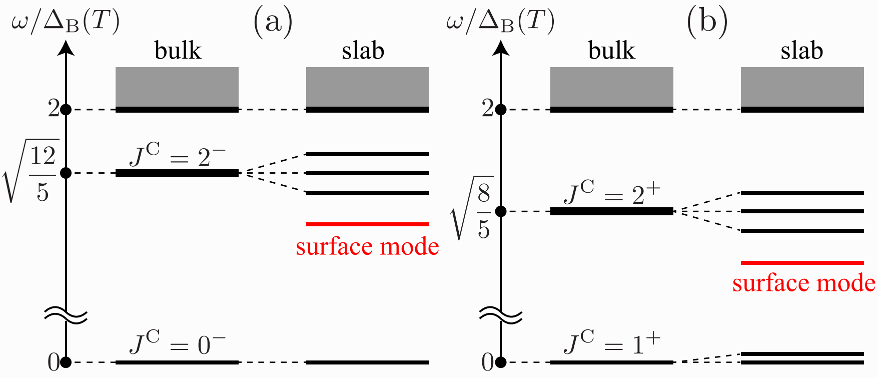
Extra Slides

Superfluid Phases of ^{3}He Films

- Length Scale for Strong Confinement: $\xi_0 = \hbar v_f / 2\pi k_B T_c \approx 20 - 80\,\mathrm{nm}$
  - A. Vorontsov & J. A. Sauls, PRL 98, 045301 (2007)



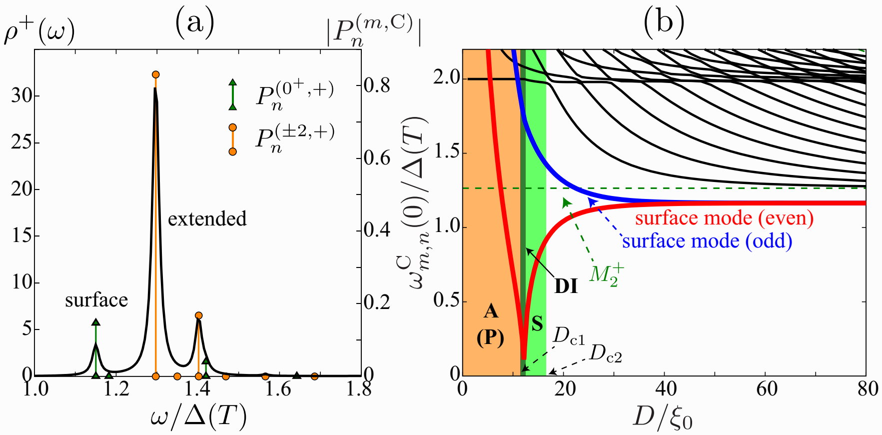
Effects of Confinement on the Bosonic Spectrum of Superfluid ^{3}He-B



- "Crystal Field" Splitting of the J^C multiplet
- $J = 1^+$ NG modes $\rightsquigarrow$ pseudo-NG mode w/ mass
- New Bosonic Modes Bound to the Surface $M < M_2^{\pm}$

- T. Mizushima & J. A. Sauls, submitted to PRL (2018)

Resonant Acoustic Excitation of the $C = +1$ Bosonic Modes of Confined Superfluid ^{3}He-B

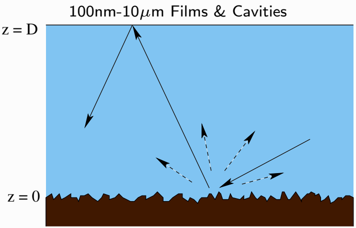


- Surface and Bulk ("extended") modes
- Evolution of the Spectrum

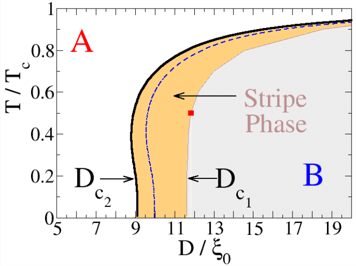
- T. Mizushima & J. A. Sauls, submitted to PRL (2018)

Superfluid ^{3}He Under Strong Confinement

New Phases with Spontaneously Broken *Translational & Time-Reversal* Symmetries



- A. Vorontsov & JAS, PRL (2007)
- J. J. Wiman & JAS JLTP (2016)



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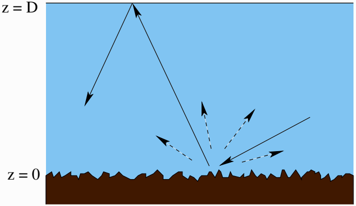
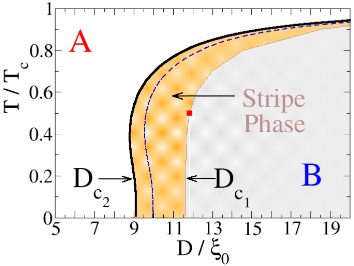
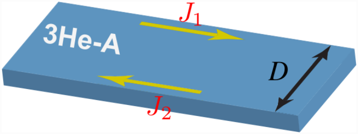
100nm-10μm Films & Cavities





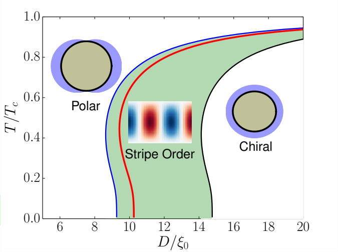
Laterally confined films of ^{3}He



Chiral Edge Currents in ^{3}He-A Films
⇝ Anomalous Hall Effects

Hybridization of Chiral Edge States ⟶

- Hao Wu, PhD Thesis (2016)



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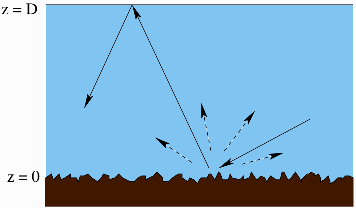

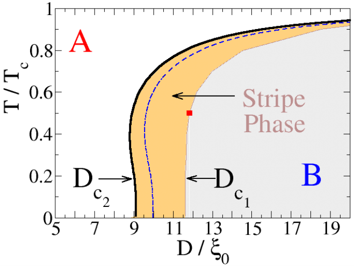

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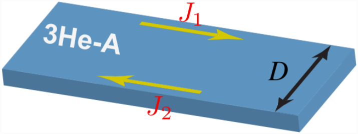

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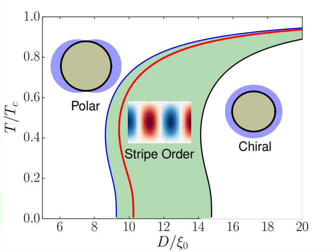
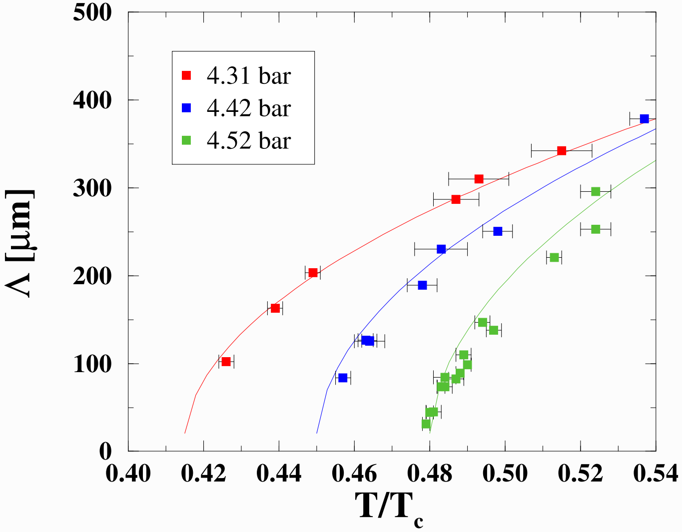


- A- B- and Polar phases - Unique Platforms for studying **Topological Quantum Matter**

Faraday Rotation: Magneto-Acoustic Birefringence of Transverse Currents



- "Broken Symmetry & Non-Equilibrium Superfluid ^{3}He", J. A. Sauls, *Lecture Notes - Les Houches 1999*

Topological Invariant for 3D Time-Reversal Invariant ^{3}He-B

- Nambu-Bogoliubov Hamiltonian for Bulk ^{3}He-B:

$$\widehat{H}_{\text{B}} = \xi(\mathbf{p})\hat{\tau}_3 + c\,\mathbf{p}\cdot\vec{\boldsymbol{\sigma}}\,\hat{\tau}_1$$

  - $E(\mathbf{p}) = \sqrt{\xi(\mathbf{p})^2 + c^2|\mathbf{p}|^2} \geq \Delta = c\,p_f$ (Gapped)
  - Emergent *spin-orbit* coupling $\rightsquigarrow$ Helicity eigenstates
  - Emergent Topology of the B-phase

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  - Emergent Topology of the B-phase

- Topology of the B-phase Bogoliubov Hamiltonian:

$$N_{\text{3D}} = \frac{\pi}{4}\int\frac{d^3p}{(2\pi)^3}\,\epsilon_{ijk}\text{Tr}\left\{\Gamma(\widehat{H}_{\text{B}}^{-1}\partial_{p_i}\widehat{H}_{\text{B}})(\widehat{H}_{\text{B}}^{-1}\partial_{p_j}\widehat{H}_{\text{B}})(\widehat{H}_{\text{B}}^{-1}\partial_{p_k}\widehat{H}_{\text{B}})\right\} = \begin{cases} 0, & \Gamma = 1 \\ 2, & \Gamma = \texttt{CT} \end{cases}$$

Zero Energy Fermions Confined on a 2D Surface $\uparrow$

Helical Majorana Modes

Protected by $\Gamma = \texttt{CT}$ symmetry: $\Gamma\widehat{H}_{\text{B}}\Gamma^{\dagger} = -\widehat{H}_{\text{B}}$

- Schnyder et al., PRB 78, 195125 (2008); - Volovik, JETP Lett. 90, 587 (2009)

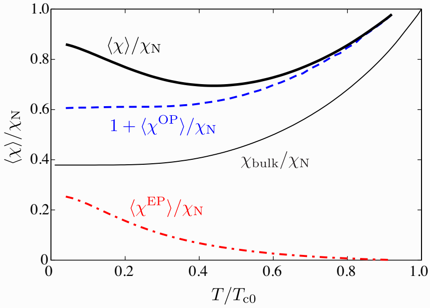
Signatures of Topological Edge and Surface Excitations

- **ESR Relaxation via Ising Majoranas:**

S-B Chung and S.C. Zhang, PRL (2008)

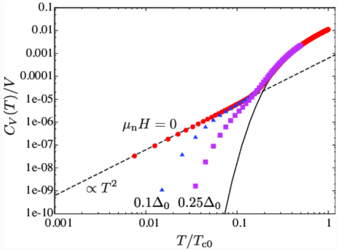
$$1/T_1 \propto |\hat{\mathbf{H}} \times \hat{\mathbf{z}}|^2$$

- **Polarization of Ising Majoranas:**
  - Y. Nagato et al., JPSJ (2009)



  - T. Mizushima, et al. arXiv:1409.6094

- **Heat Capacity of Majoranas:** $C_v \propto T^2$
  - T. Mizushima and K. Machida, JLTP 2011)



- **Thermal Hall Effect:** $J_y = -\kappa_{xy} \nabla_x T$

$$\kappa_{xy} = N_{2D} \frac{\pi^2 k_B^2}{6h} T$$

  - H. Sumiyoshi, S. Fujimoto JPSJ 82, 023602 (2013)

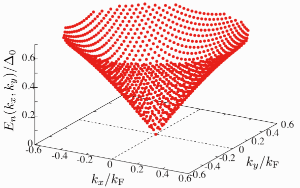
Z_2 Symmetry Breaking Topological Phase Transition in ^{3}He-B

- S-B Chung and S.C. Zhang, PRL (2008)
- Y. Nagato et al., JPSJ (2009)
- G.E. Volovik, JETP Lett. (2009)

- Majorana Ising Spins:

$$\vec{S} = \Psi^\dagger \frac{\hbar}{2} \vec{\sigma} \Psi = (0, 0, S_z)$$

- In-Plane Field: $\mathbf{H}_{\|} \perp \hat{\mathbf{z}}$



- Z_2 Symmetry: $\Gamma = U_z(\pi) \times \mathtt{T} \times \mathtt{C}$

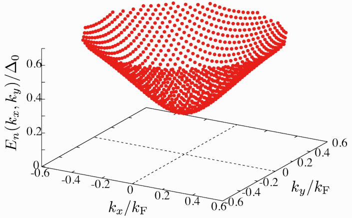
$$\Gamma^\dagger \widehat{H}(\mathbf{H}_{\|})\Gamma = -\widehat{H}(\mathbf{H}_{\|}) + \gamma\, \hat{\ell}_z \cdot \mathbf{H}_{\|}$$

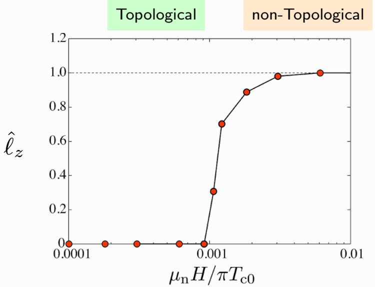
- Z_2 Topological Protection for

$$\hat{\ell}_z \equiv \hat{\mathbf{H}}_i\, R_{iz}[\hat{\mathbf{n}}, \theta] \equiv 0$$

- $\hat{\ell}_z \neq 0 \rightarrow Z_2$ Symmetry Breaking

$$\varepsilon(\mathbf{p}_{\|}) = \sqrt{c^2|\mathbf{p}_{\|}|^2 + |\mu B \hat{\ell}_z|^2}$$





- T. Mizushima, M. Sato and K. Machida, PRL 109, 165301 (2012)