Seminar, Theoretical Physics Institute, University of Alberta, April 5, 2018

The Left Hand of the Electron

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- Oleksii Shevtsov (Northwestern)

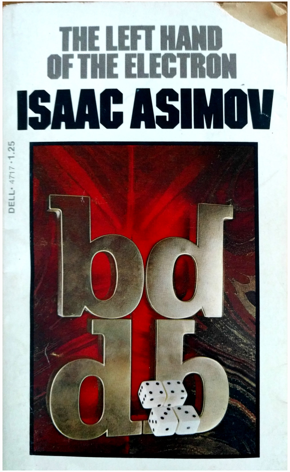
- Parity violation
- P and T violation in ^{3}He
- Left-Handed Electrons in a Chiral Vacuum
- Anomalous Hall Effect in ^{3}He-A

- NSF Grant DMR-1508730

- H. Ikegami, Y. Tsutsumi, K. Kono, Science **341**, 59 (2013)
- O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)
- O. Shevtsov and JAS, JLTP 187, 340353 (2017)

The Left Hand of the Electron, Issac Asimov, circa 1971

- An Essay on the Discovery of Parity Violation by the Weak Interaction



- ... And Reflections on Mirror Symmetry in Nature

Parity Violation in Beta Decay of ^{60}Co - Physical Review 105, 1413 (1957)

Experimental Test of Parity Conservation in Beta Decay*

C. S. Wu, *Columbia University, New York, New York*

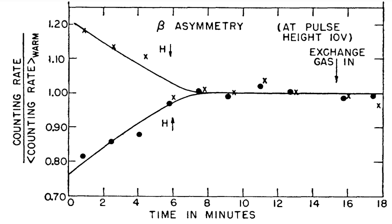
AND

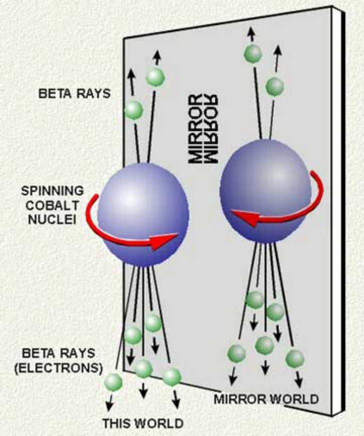
E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, *National Bureau of Standards, Washington, D. C.*

(Received January 15, 1957)

- T. D. Lee and C. N. Yang, Phys Rev 104, 204 (1956)

$$^{60}Co \rightarrow {}^{60}Ni + e^- + \bar{\nu}$$



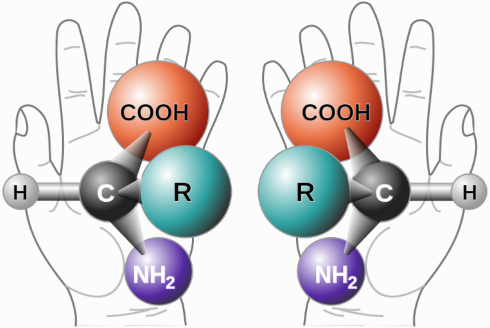


- Current of Beta electrons is (anti) correlated with the Spin of the ^{60}Co nucleus.

$$\langle \vec{S} \cdot \vec{p} \rangle \neq 0 \rightsquigarrow \text{Parity violation}$$

Chirality in Nature

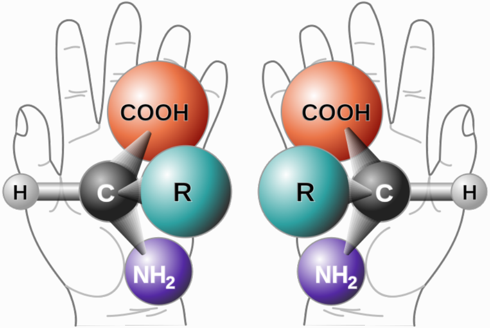
Molecular Chiral Enantiomers



Handedness: Broken Mirror Symmetry

Chirality in Nature

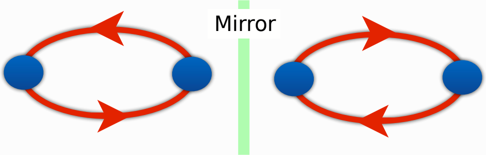
Molecular Chiral Enantiomers



Handedness: Broken Mirror Symmetry

Chiral Diatomic Molecules

$$\Psi(\mathbf{r}) = f(r)\,(x + iy)$$



Broken Mirror Symmetries

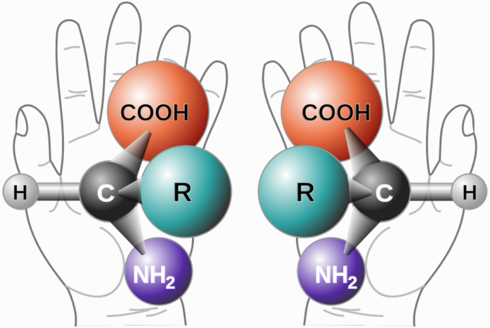
$$\Pi_{zx}\,\Psi(\mathbf{r}) = f(r)\,(x - iy)$$

Broken Time-Reversal Symmetry

$$\mathrm{T}\,\Psi(\mathbf{r}) = f(r)\,(x - iy)$$

Chirality in Nature

Molecular Chiral Enantiomers



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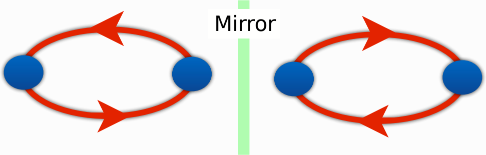


Broken Mirror Symmetries

$$\Pi_{zx}\,\Psi(\mathbf{r}) = f(r)\,(x - iy)$$

Broken Time-Reversal Symmetry

$$\mathrm{T}\,\Psi(\mathbf{r}) = f(r)\,(x - iy)$$

Realized in Quantum Condensates

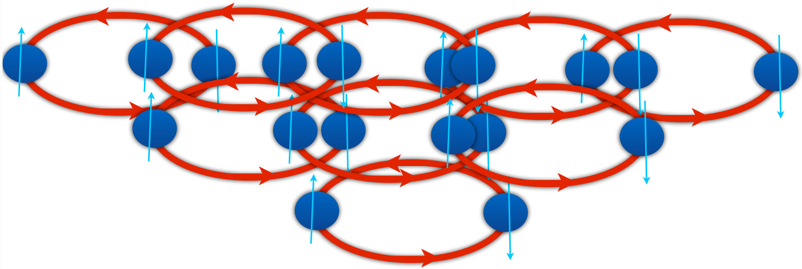
Parity Violation in a Superfluid Vacuum of Liquid ^{3}He

Chiral P-wave BCS Condensate

$$|\,\Psi_N\,\rangle = \left[\iint d\mathbf{r}_1 d\mathbf{r}_2\; \Psi_{s_1 s_2}(\mathbf{r}_1 - \mathbf{r}_2)\; \psi^\dagger_{s_1}(\mathbf{r}_1)\psi^\dagger_{s_2}(\mathbf{r}_2)\right]^{N/2} |\,\text{vac}\,\rangle$$

$$\Psi_{s_1 s_2}(\mathbf{r}) = f(|\mathbf{r}|/\xi)\;(x+iy)\;\chi^{(1,0)}_{s_1 s_2}$$

- P.W. Anderson & P. Morel, Phys. Rev. 123, 1911 (1961)



$$\mathtt{SO(3)_S \times SO(3)_L \times U(1)_N \times T \times P \longrightarrow SO(2)_S \times U(1)_{N\text{-}L_z} \times Z_2}$$

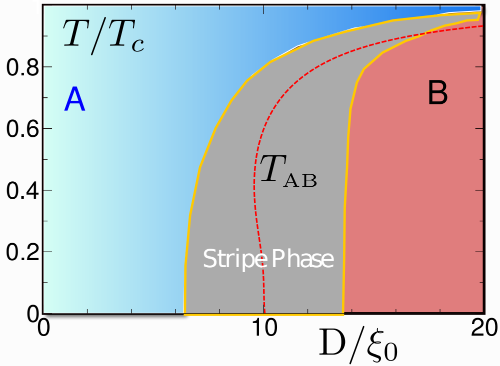
Realized as the Ground State of Superfluid ^{3}He

Realization of Broken Time-Reversal and Mirror Symmetry by the Vacuum State of ^{3}He Films

- Length Scale for Strong Confinement:

  $\xi_0 = \hbar v_f / 2\pi k_B T_c \approx 20 - 80\,\text{nm}$

  - L. Levitov et al., Science 340, 6134 (2013)
- A. Vorontsov & J. A. Sauls, PRL 98, 045301 (2007)



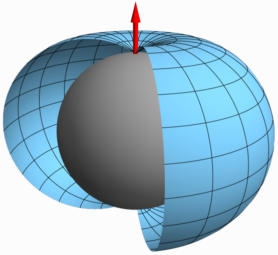
$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix}_{ABM} = \begin{pmatrix} p_x + ip_y \sim e^{+i\phi} & 0 \\ 0 & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

$$\text{SO}(3)_{\text{S}} \times \text{SO}(3)_{\text{L}} \times \text{U}(1)_{\text{N}} \times \text{T} \times \text{P}$$

$$\Downarrow$$

$$\text{SO}(2)_{\text{S}} \times \text{U}(1)_{\text{N-L}_z} \times \text{Z}_2$$

Chiral ABM State $\vec{l} = \hat{\mathbf{z}}$



$L_z = 1,\ S_z = 0$

Ground-State Angular Momentum

$\langle \widehat{L}_z \rangle = \frac{N}{2}\hbar$?

Open Question

Signatures of Broken T and P Symmetry in ^{3}He-A

What is the Evidence for Chirality of Superfluid ^{3}He-A?

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Broken T and P $\rightsquigarrow$ Anomalous Hall Effect for Electrons in ^{3}He-A

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What is the Evidence for Chirality of Superfluid ^{3}He-A?

Broken T and P $\rightsquigarrow$ Anomalous Hall Effect for Electrons in ^{3}He-A

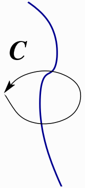
Broken Symmetries $\rightsquigarrow$ Topology of ^{3}He-A

Chirality $+$ Topology $\rightsquigarrow$ Chiral Edge States

Real-Space vs. Momentum-Space Topology

Topology in Real Space

$$\Psi(\mathbf{r}) = |\Psi(r)|\, e^{i\vartheta(\mathbf{r})}$$



Phase Winding

$$N_C = \frac{1}{2\pi} \oint_C d\vec{l} \cdot \frac{1}{|\Psi|} \mathrm{Im}[\boldsymbol{\nabla} \Psi] \in \{0, \pm 1, \pm 2, \ldots\}$$

- Massless Fermions confined in the Vortex Core

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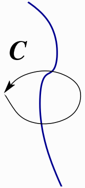

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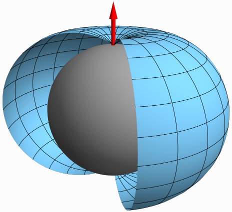
Chiral Symmetry $\rightsquigarrow$ Topology in Momentum Space

$\Psi(\mathbf{p}) = \Delta(p_x \pm i p_y) \sim e^{\pm i\varphi_{\mathbf{p}}}$



Topological Quantum Number: $L_z = \pm 1$

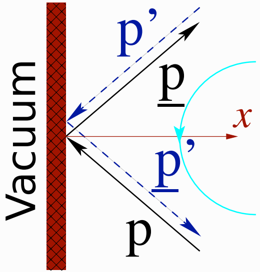
$$N_{\mathrm{2D}} = \frac{1}{2\pi}\oint d\mathbf{p}\cdot \frac{1}{|\Psi(\mathbf{p})|}\mathrm{Im}[\boldsymbol{\nabla}_{\mathbf{p}}\Psi(\mathbf{p})] = L_z$$

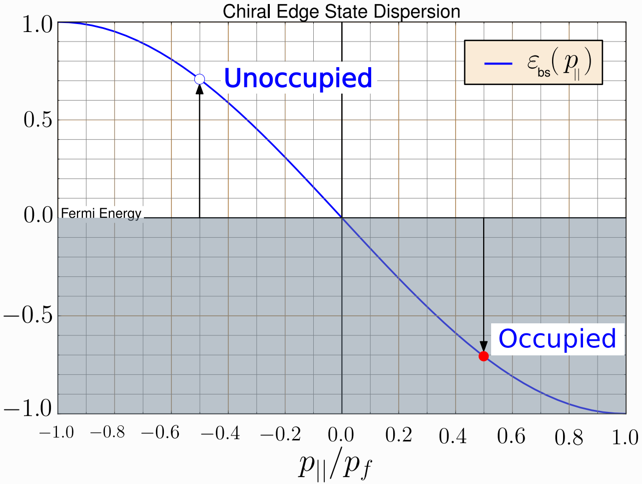
- Massless Chiral Fermions
  - Nodal Fermions in 3D
  - Edge Fermions in 2D

Massless Chiral Fermions in the 2D ^{3}He-A Films

Edge Fermions: $G^{\text{R}}_{\text{edge}}(\mathbf{p},\varepsilon;x) = \dfrac{\pi\Delta|\mathbf{p}_x|}{\varepsilon + i\gamma - \varepsilon_{\text{bs}}(\mathbf{p}_{||})}\; e^{-x/\xi_\Delta}$ $\qquad \xi_\Delta = \hbar v_f/2\Delta \approx 10^2\,\AA \gg \hbar/p_f$

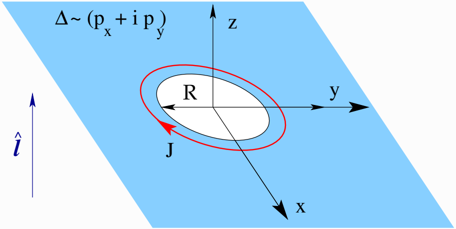
- $\varepsilon_{\text{bs}} = -c\,p_{||}$ with $c = \Delta/p_f \ll v_f$
- Broken P & T $\rightsquigarrow$ **Edge Current**





- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

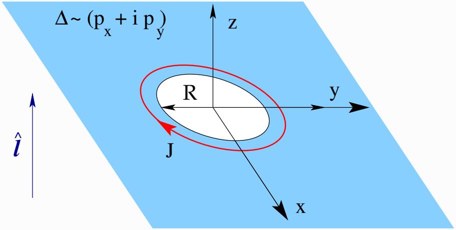
Chiral Edge Current Circulating a Hole or Defect in a Chiral Superfluid



- $R \gg \xi_0 \approx 100\,\text{nm}$
- Sheet Current :

$$J \equiv \int dx\, J_\varphi(x)$$

Chiral Edge Current Circulating a Hole or Defect in a Chiral Superfluid

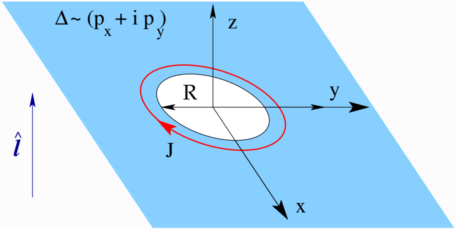


- $R \gg \xi_0 \approx 100\,\text{nm}$
- Sheet Current:

  $J \equiv \int dx\, J_\varphi(x)$

- Quantized Sheet Current: $\frac{1}{4}\, n\, \hbar$ $(n = N/V = {}^3$He density)
- Edge Current *Counter*-Circulates: $J = -\frac{1}{4}\, n\, \hbar$ w.r.t. Chirality: $\hat{\mathbf{l}} = +\mathbf{z}$

Chiral Edge Current Circulating a Hole or Defect in a Chiral Superfluid



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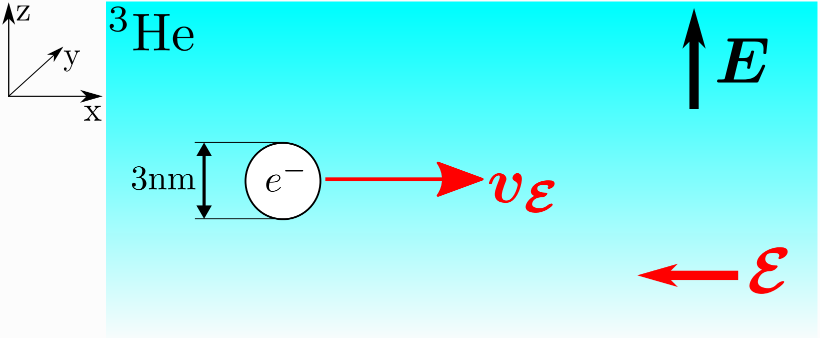
- Edge Current *Counter*-Circulates: $J = -\frac{1}{4}\, n\, \hbar$ w.r.t. Chirality: $\hat{\mathbf{l}} = +\mathbf{z}$
- Angular Momentum: $L_z = 2\pi\, h\, R^2 \times (-\frac{1}{4}\, n\, \hbar) = -(N_{\text{hole}}/2)\, \hbar$

$N_{\text{hole}}/2 =$ Number of ^{3}He Cooper Pairs excluded from the Hole

$\therefore$ An object in ^{3}He-A *inherits* angular momentum from the Condensate of Chiral Pairs!

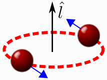
Electron bubbles in the Normal Fermi liquid phase of ^{3}He



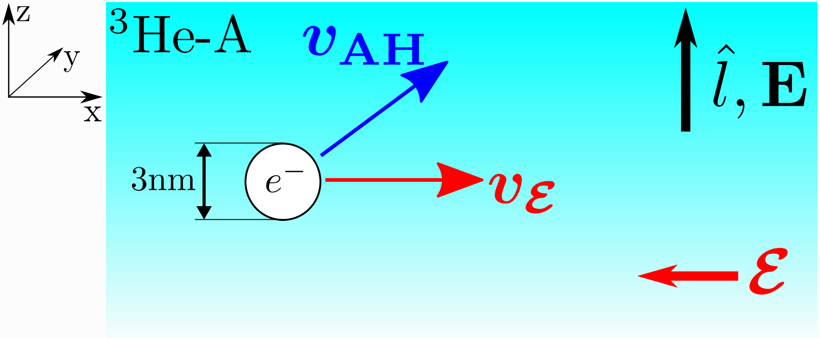
- Bubble with $R \simeq 1.5$ nm, $0.1\,\text{nm} \simeq \lambda_f \ll R \ll \xi_0 \simeq 80\,\text{nm}$
- Effective mass $M \simeq 100 m_3$ (m_3 – atomic mass of ^{3}He)
- QPs mean free path $l \gg R$
- Mobility of ^{3}He is *independent of* T for $T_c < T < 50$ mK

B. Josephson and J. Leckner, PRL 23, 111 (1969)

Electron bubbles in chiral superfluid ^{3}He-A

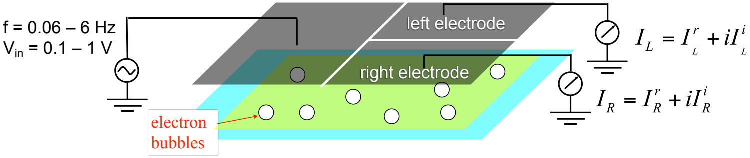


$\Delta(\hat{k}) = \Delta(\hat{k}_x + i\hat{k}_y) = \Delta\, e^{i\phi_{\mathbf{k}}}$



- Current: $\mathbf{v} = \overbrace{\mu_\perp \boldsymbol{\mathcal{E}}}^{\mathbf{v}_{\mathcal{E}}} + \overbrace{\mu_{\text{AH}} \boldsymbol{\mathcal{E}} \times \hat{\mathbf{l}}}^{\mathbf{v}_{\text{AH}}}$ R. Salmelin, M. Salomaa & V. Mineev, PRL **63**, 868 (1989)
- Hall ratio: $\tan\alpha = v_{\text{AH}}/v_{\mathcal{E}} = |\mu_{\text{AH}}/\mu_\perp|$

Measurement of the Transverse e⁻ mobility in Superfluid ^{3}He Films



H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

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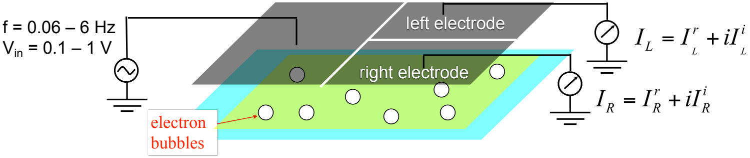
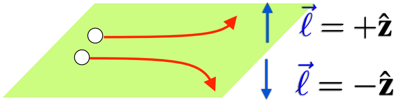


Transverse Force from ***Skew* Scattering**

$$\rightsquigarrow \quad \Delta I = I_R - I_L \neq 0$$

$$\vec{v} = \left[\mu_\perp \vec{E} + \mu_{xy}\,\hat{\ell} \times \vec{E}\right]$$

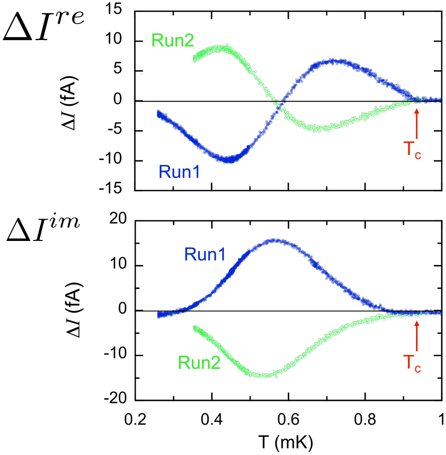


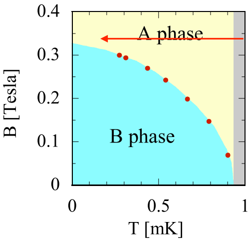
H. Ikegami, Y. Tsutsumi, K. Kono, **Science 341, 59-62 (2013)**

Detection of Broken Time-Reversal & Mirror Symmetry in ^{3}He-A

Transverse e⁻ bubble current in ^{3}He-A

$$\Delta I = I_R - I_L$$





Single Domains:

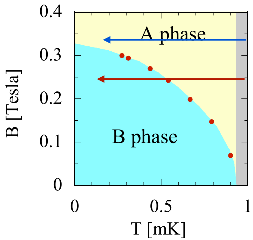
Run 1 $\vec{\ell} = +\hat{\mathbf{z}}$

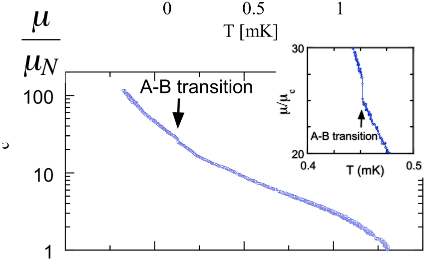
Run 2 $\vec{\ell} = -\hat{\mathbf{z}}$

$$\frac{|I_R - I_L|}{I_R + I_L} \approx 6\,\%$$

H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

Zero Transverse e$^-$ current in ^{3}He-B (*T - symmetric phase*)





H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

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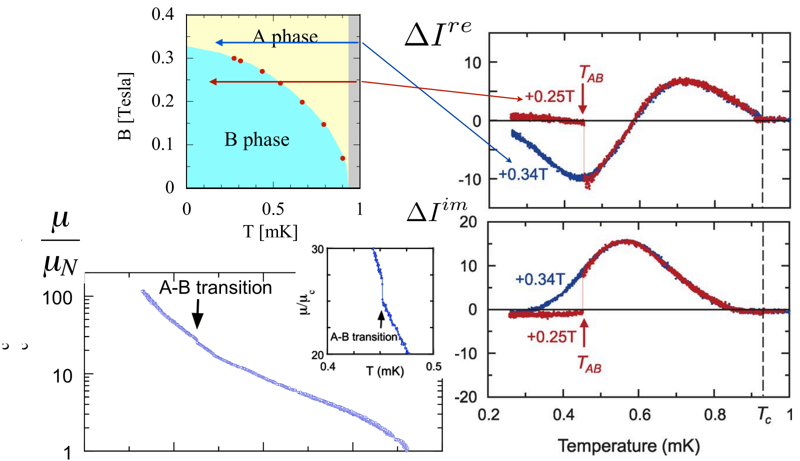
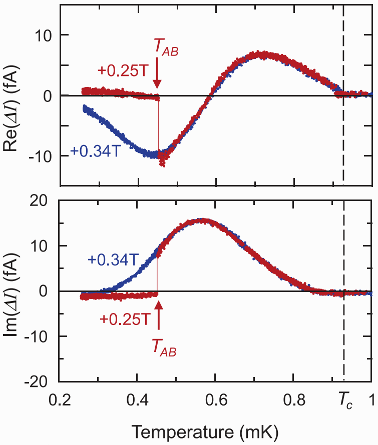
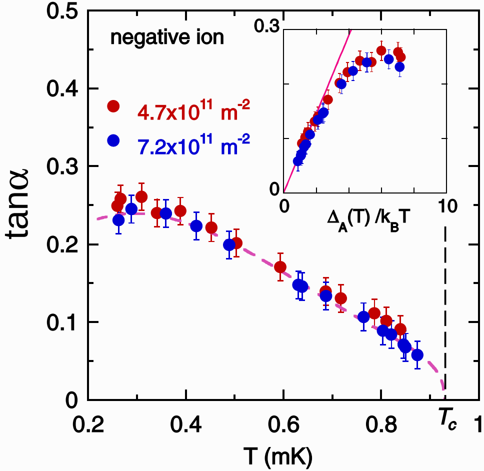


H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

e^- Mobility in ^{3}He-A - Anomalous Hall Angle

- Electric current: $\mathbf{v} = \overbrace{\mu_\perp \boldsymbol{\mathcal{E}}}^{\mathbf{v}_{\mathcal{E}}} + \overbrace{\mu_{\mathrm{AH}} \boldsymbol{\mathcal{E}} \times \hat{\mathbf{l}}}^{\mathbf{v}_{\mathrm{AH}}}$
- Hall ratio: $\tan\alpha = v_{\mathrm{AH}}/v_{\mathcal{E}} = |\mu_{\mathrm{AH}}/\mu_\perp|$





- H. Ikegami et al., Science **341**, 59 (2013); JPSJ **82**, 124607 (2013); JPSJ **84**, 044602 (2015)

- Structure of Electrons in Superfluid ^{3}He-A
- Forces of Moving Electrons in Superfluid ^{3}He-A

$\Downarrow$

- Scattering Theory of ^{3}He Quasiparticles by Electron Bubbles

Forces on the Electron bubble in ^{3}He-A:

- $M\dfrac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} + \mathbf{F}_{\text{QP}}$, $\mathbf{F}_{QP}$ – force from quasiparticle collisions

Forces on the Electron bubble in ^{3}He-A:

- $M\frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} + \mathbf{F}_{\mathrm{QP}}$, $\mathbf{F}_{QP}$ – force from quasiparticle collisions
- $\mathbf{F}_{QP} = -\overleftrightarrow{\eta} \cdot \mathbf{v}$, $\overleftrightarrow{\eta}$ – generalized Stokes tensor
- $\overleftrightarrow{\eta} = \begin{pmatrix} \eta_\perp & \eta_{\mathsf{AH}} & 0 \\ -\eta_{\mathsf{AH}} & \eta_\perp & 0 \\ 0 & 0 & \eta_\parallel \end{pmatrix}$ for broken PT symmetry with $\hat{\mathbf{l}} \parallel \mathbf{e}_z$

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- $M\frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} - \eta_\perp\mathbf{v} + \frac{e}{c}\mathbf{v}\times\mathbf{B}_{\mathrm{eff}}$, for $\boldsymbol{\mathcal{E}} \perp \hat{\mathbf{l}}$
- $\mathbf{B}_{\mathrm{eff}} = -\frac{c}{e}\eta_{\mathsf{AH}}\hat{\mathbf{l}}$ $\quad B_{\mathrm{eff}} \simeq 10^3 - 10^4\ \mathrm{T}$!!!

Forces on the Electron bubble in ^{3}He-A:

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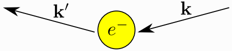
- $M\dfrac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} - \eta_\perp\mathbf{v} + \dfrac{e}{c}\mathbf{v}\times\mathbf{B}_{\text{eff}}$, for $\boldsymbol{\mathcal{E}} \perp \hat{\mathbf{l}}$
- $\mathbf{B}_{\text{eff}} = -\dfrac{c}{e}\eta_{\text{AH}}\hat{\mathbf{l}}$ $\quad B_{\text{eff}} \simeq 10^3 - 10^4\ \text{T}$!!!
- Mobility: $\dfrac{d\mathbf{v}}{dt} = 0 \quad\rightsquigarrow\quad \mathbf{v} = \overleftrightarrow{\mu}\boldsymbol{\mathcal{E}}$, where $\overleftrightarrow{\mu} = e\,\overleftrightarrow{\eta}^{-1}$

T-matrix description of Quasiparticle-Ion scattering



- Lippmann-Schwinger equation for the T-matrix ($\varepsilon = E + i\eta\,;\ \eta \to 0^+$):

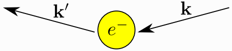
$$\hat{T}_S^R(\mathbf{k}',\mathbf{k},E) = \hat{T}_N^R(\mathbf{k}',\mathbf{k}) + \int \frac{d^3k''}{(2\pi)^3} \hat{T}_N^R(\mathbf{k}',\mathbf{k}'')\Big[\hat{G}_S^R(\mathbf{k}'',E) - \hat{G}_N^R(\mathbf{k}'',E)\Big]\hat{T}_S^R(\mathbf{k}'',\mathbf{k},E)$$

$$\hat{G}_S^R(\mathbf{k},E) = \frac{1}{\varepsilon^2 - E_{\mathbf{k}}^2}\begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad E_{\mathbf{k}} = \sqrt{\xi_k^2 + |\Delta(\hat{\mathbf{k}})|^2}, \ \ \xi_k = \frac{\hbar^2 k^2}{2m^*} - \mu$$

- Normal-state T-matrix:

$$\hat{T}_N^R(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = \begin{pmatrix} t_N^R(\hat{\mathbf{k}}',\hat{\mathbf{k}}) & 0 \\ 0 & -[t_N^R(-\hat{\mathbf{k}}',-\hat{\mathbf{k}})]^\dagger \end{pmatrix} \quad \text{in p-h (Nambu) space}$$

T-matrix description of Quasiparticle-Ion scattering



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$$t_N^R(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = -\frac{1}{\pi N_f}\sum_{l=0}^{\infty}(2l+1)e^{i\delta_l}\sin\delta_l P_l(\hat{\mathbf{k}}'\cdot\hat{\mathbf{k}}),$$ $P_l(x)$ – Legendre function

- Hard-sphere potential $\rightsquigarrow \tan\delta_l = j_l(k_f R)/n_l(k_f R)$ – spherical Bessel functions

  - $k_f R$ – determined by the Normal-State Mobility $\rightsquigarrow$ $k_f R = 11.17\ (R = 1.42\,\text{nm})$

Weyl Fermion Spectrum bound to the Electron Bubble

$$\mu_{\mathsf{N}} = \frac{e}{n_3 p_f \sigma_{\mathsf{N}}^{\mathsf{tr}}} \quad \Leftarrow \quad \mu_{\mathsf{N}}^{\mathsf{exp}} = 1.7 \times 10^{-6} \, \frac{m^2}{V\,s}$$

$$\tan \delta_l = j_l(k_f R)/n_l(k_f R) \quad \Rightarrow \quad \sigma_{\mathsf{N}}^{\mathsf{tr}} = \frac{4\pi}{k_f^2} \sum_{l=0}^{\infty} (l+1) \sin^2(\delta_{l+1} - \delta_l) \quad \rightsquigarrow \quad \boxed{k_f R = 11.17}$$

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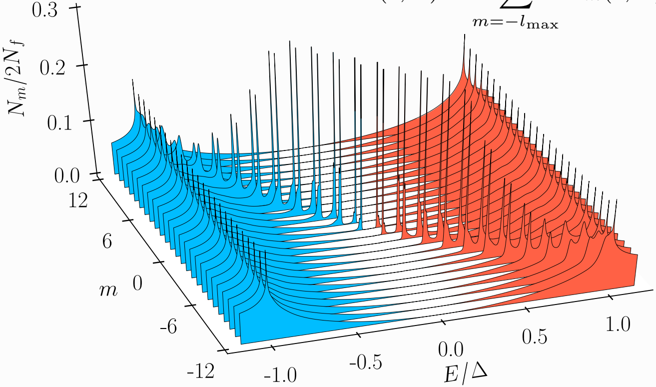
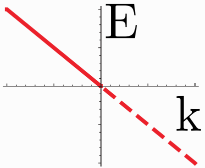
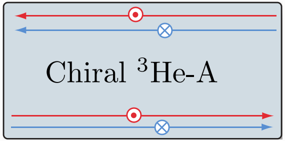
$$N(\mathbf{r},E) = \sum_{m=-l_{\max}}^{l_{\max}} N_m(\mathbf{r},E), \quad l_{\max} \simeq k_f R$$

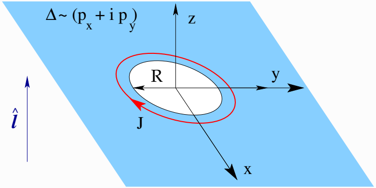


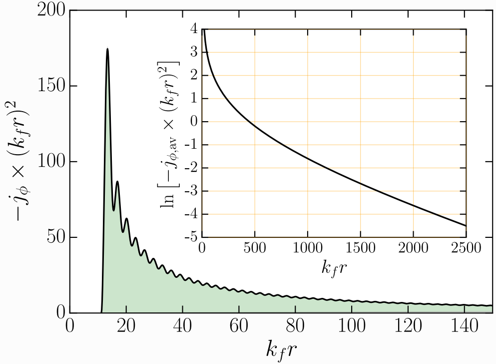


Current bound to an electron bubble ($k_f R = 11.17$)



$\Longrightarrow$





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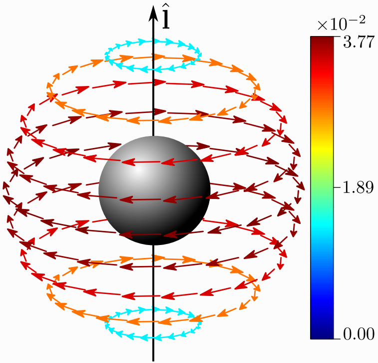

$$\mathbf{j}(\mathbf{r})/v_f N_f k_B T_c = j_\phi(\mathbf{r})\hat{\mathbf{e}}_\phi$$

▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

$$\mathbf{L}(T \to 0) \approx -\hbar N_{\mathrm{bubble}}/2\,\hat{\mathbf{l}} \approx -100\,\hbar\,\hat{\mathbf{l}}$$

Determination of the Stokes Tensor from the QP-Ion T-matrix

(i) Fermi's golden rule and the QP scattering rate:

$$\Gamma(\mathbf{k}',\mathbf{k}) = \frac{2\pi}{\hbar} W(\hat{\mathbf{k}}',\hat{\mathbf{k}})\delta(E_{\mathbf{k}'} - E_{\mathbf{k}}),$$

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(ii) Drag force from QP-ion collisions (linear in $\mathbf{v}$): ▶ Baym et al. PRL **22**, 20 (1969)

$$\mathbf{F}_{\mathsf{QP}} = -\sum_{\mathbf{k},\mathbf{k}'} \hbar(\mathbf{k}' - \mathbf{k})\left[\hbar\mathbf{k}'\mathbf{v} f_{\mathbf{k}}\left(-\frac{\partial f_{\mathbf{k}'}}{\partial E}\right) - \hbar\mathbf{k}\mathbf{v}(1 - f_{\mathbf{k}'})\left(-\frac{\partial f_{\mathbf{k}}}{\partial E}\right)\right]\Gamma(\mathbf{k}',\mathbf{k})$$

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(iii) Microscopic reversibility condition: $W(\hat{\mathbf{k}}',\hat{\mathbf{k}}:+\mathbf{l}) = W(\hat{\mathbf{k}},\hat{\mathbf{k}}':-\mathbf{l})$

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(iv) Generalized Stokes tensor:

$$\mathbf{F}_{\text{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v} \;\;\rightsquigarrow\;\; \eta_{ij} = n_3 p_f \int_0^{\infty} dE\left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E)\,,\quad \overleftrightarrow{\eta} = \begin{pmatrix} \eta_{\perp} & \eta_{\text{AH}} & 0 \\ -\eta_{\text{AH}} & \eta_{\perp} & 0 \\ 0 & 0 & \eta_{\parallel} \end{pmatrix}$$

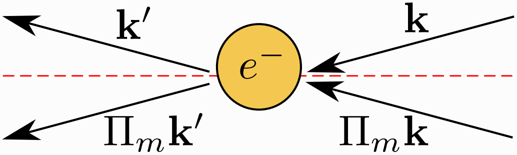
$n_3 = \dfrac{k_f^3}{3\pi^2}$ – ^{3}He particle density, $\sigma_{ij}(E)$ – transport scattering cross section,

$f(E) = [\exp(E/k_BT)+1]^{-1}$ – Fermi Distribution

Mirror-symmetric scattering $\Rightarrow$ longitudinal drag force

$$\mathbf{F}_{\mathsf{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^\infty dE\left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E)$$

Subdivide by mirror symmetry:



$$W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}),$$

$$\sigma_{ij}(E) = \sigma_{ij}^{(+)}(E) + \sigma_{ij}^{(-)}(E),$$

$$\sigma_{ij}^{(+)}(E) = \frac{3}{4}\int_{E\geq|\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'}\int_{E\geq|\Delta(\hat{\mathbf{k}})|}\frac{d\Omega_{\mathbf{k}}}{4\pi}\left[(\hat{\mathbf{k}}'_i-\hat{\mathbf{k}}_i)(\hat{\mathbf{k}}'_j-\hat{\mathbf{k}}_j)\right]\frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E)$$

Mirror-symmetric cross section: $W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W(\hat{\mathbf{k}},\hat{\mathbf{k}}')]/2$

$$\frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2 \frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}}')|^2}} W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) \frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}})|^2}}$$

$\rightsquigarrow$ Stokes Drag $\eta_{xx}^{(+)} = \eta_{yy}^{(+)} \equiv \eta_\perp,\ \eta_{zz}^{(+)} \equiv \eta_\parallel$, No transverse force $\left[\eta_{ij}^{(+)}\right]_{i\neq j} = 0$

Mirror-antisymmetric scattering ⇒ transverse force

$$\mathbf{F}_{\text{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^\infty dE\left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E)$$

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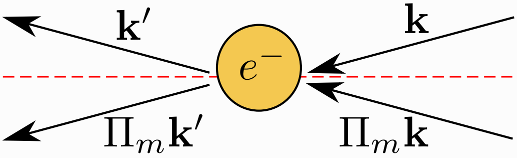


$$\sigma_{ij}^{(-)}(E) = \frac{3}{4}\int_{E\geq|\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'} \int_{E\geq|\Delta(\hat{\mathbf{k}})|}\frac{d\Omega_{\mathbf{k}}}{4\pi}\left[\epsilon_{ijk}(\hat{\mathbf{k}}'\times\hat{\mathbf{k}})_k\right]\frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E)\left[f(E)-\frac{1}{2}\right]$$

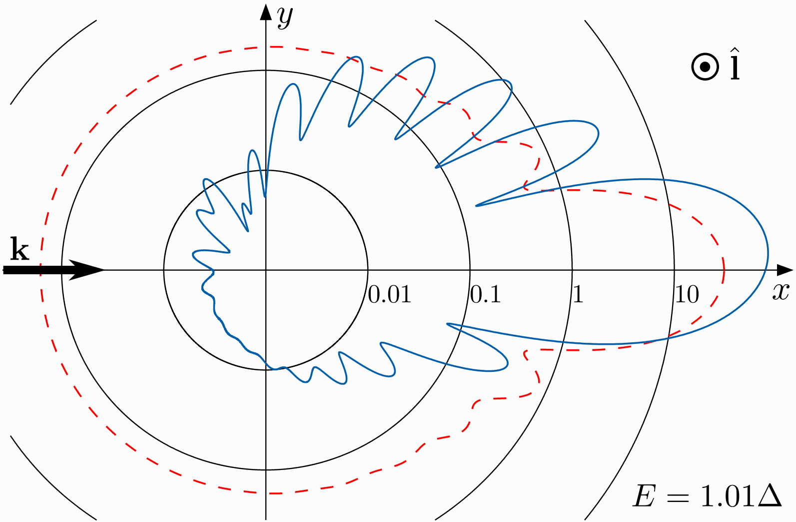
Mirror-antisymmetric cross section: $W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) - W(\hat{\mathbf{k}},\hat{\mathbf{k}}')]/2$

$$\frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2 \frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}}')|^2}}W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}})\frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}})|^2}}$$

Transverse force $\eta_{xy}^{(-)} = -\eta_{yx}^{(-)} \equiv \eta_{\text{AH}}$ ⇒ anomalous Hall effect

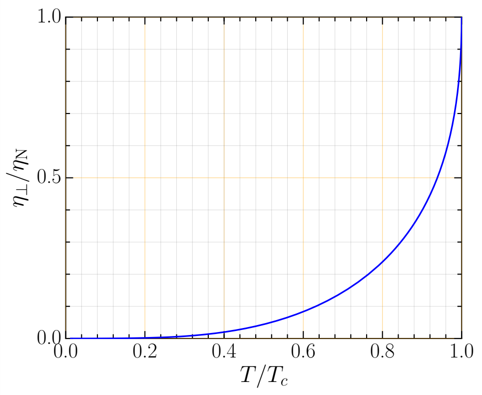
▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

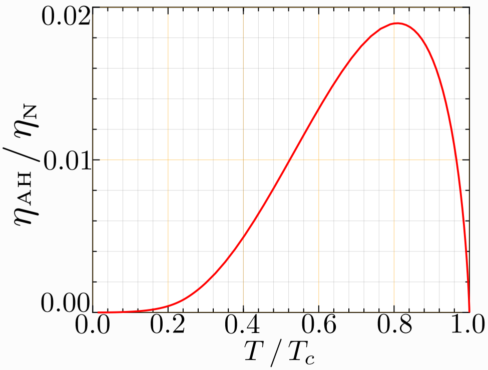
Differential cross section for Bogoliubov QP-Ion Scattering $k_f R = 11.17$



▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Theoretical Results for the Drag and Transverse Forces





- $\Delta p_x \approx p_f \qquad \sigma^{tr}_{xx} \approx \sigma^{tr}_N \approx \pi R^2$
- $F_x \approx n\, v_x\, \Delta p_x\, \sigma^{tr}_{xx}$
  $\approx n\, v_x\, p_f\, \sigma^{tr}_N$

$$|F_y/F_x| \approx \frac{\hbar}{p_f R}\,(\Delta(T)/k_B T_c)^2$$

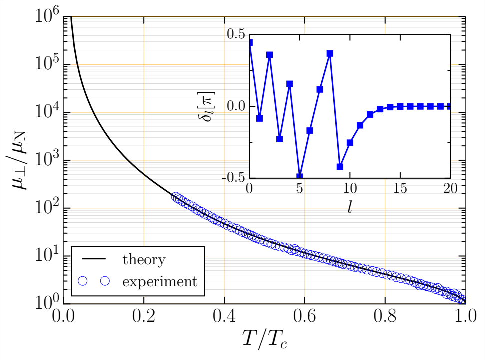
- $\Delta p_y \approx \hbar/R \; \sigma^{tr}_{xy} \approx (\Delta(T)/k_B T_c)^2 \sigma^{tr}_N$
- $F_y \approx n\, v_x\, \Delta p_y\, \sigma^{tr}_{xy}$
  $\approx n\, v_x \; (\hbar/R)\, \sigma^{tr}_N (\Delta(T)/k_B T_c)^2$

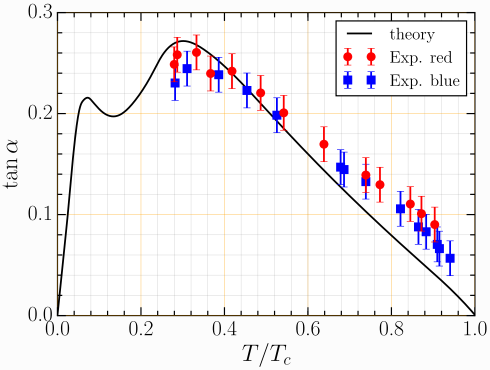
$k_f R = 11.17$

Branch Conversion Scattering

- O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Comparison between Theory and Experiment for the Drag and Transverse Forces





- $\mu_\perp = e \dfrac{\eta_\perp}{\eta_\perp^2 + \eta_{AH}^2}$
- $\mu_{AH} = -e \dfrac{\eta_{AH}}{\eta_\perp^2 + \eta_{AH}^2}$

- $\tan\alpha = \left|\dfrac{\mu_{AH}}{\mu_\perp}\right| = \dfrac{\eta_{AH}}{\eta_\perp}$
- Hard-Sphere Model: $k_f R = 11.17$

- O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)
- O. Shevtsov and JAS, JLTP 187, 340353 (2017)

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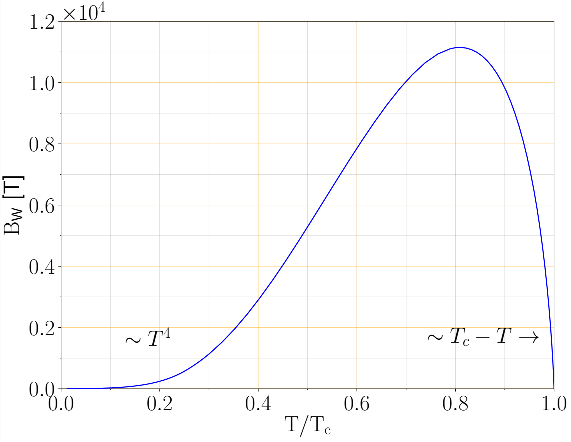
  - Drag Force $(-\eta_{\perp}\mathbf{v})$
  - Transverse Force $(\frac{e}{c}\mathbf{v}\times\mathbf{B}_{\text{eff}})$
- *Anomalous Hall Field:* $\mathbf{B}_{\text{eff}} \approx \frac{\Phi_0}{3\pi^2}\,k_f^2\,(k_f R)^2\left(\frac{\eta_{\text{AH}}}{\eta_{\text{N}}}\right)\mathbf{l} \simeq 10^3 - 10^4\,\text{T}\,\mathbf{l}$
- Mechanism: Skew/Andreev Scattering of Bogoliubov QPs by the dressed Ion
- Origin: Broken Mirror & Time-Reversal Symmetry $\rightsquigarrow W(\mathbf{k},\mathbf{k}') \neq W(\mathbf{k}',\mathbf{k})$
- Open Problem: Theory of Radiation Dominated Motion of Electrons in a Chiral Vacuum

Vanishing of the Effective Magnetic Field for $T \to 0$

Breakdown of Laminar Flow

$$B_W = 5.9 \times 10^5\ \mathsf{T} \left(\frac{\eta_{xy}}{\eta_N}\right)$$



$$\eta_{xy}/\eta_N|_{T=0.8\,T_c} \approx \frac{\hbar}{p_f R}$$

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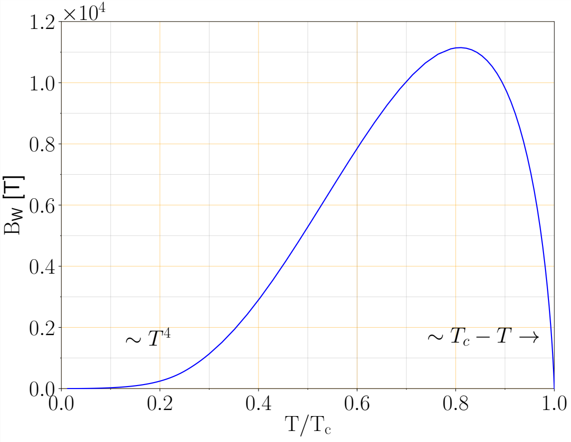
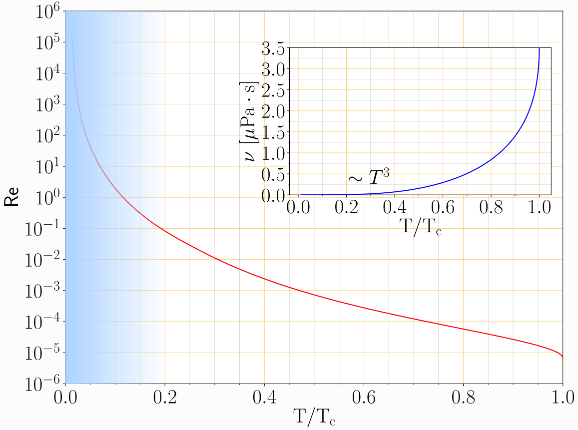


$$\eta_{xy}/\eta_N|_{T=0.8\,T_c} \approx \frac{\hbar}{p_f R}$$

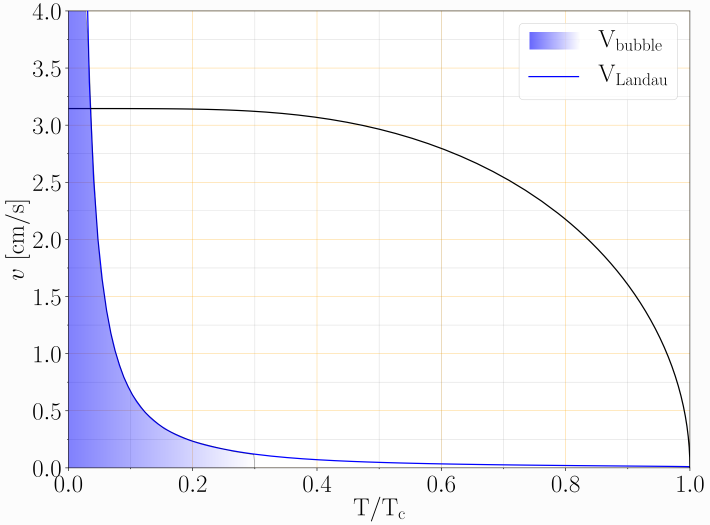
Breakdown of Laminar Flow

$$\mathrm{Re} = \mathrm{Re}_N \left(\frac{\eta_N}{\eta}\right)^{3/2} \xrightarrow[T\to 0]{} \sim \left(\frac{T_c}{T}\right)^{9/2}$$



$$\mathrm{Re}_N = 6.7 \times 10^{-6}$$

Breakdown of Scattering Theory for $T \to 0$



Electron Bubble Velocity

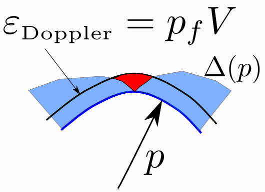
- $V_N = \mu_N E_N = 1.01 \times 10^{-4}$ m/s
- $V = \mu_N E_N \sqrt{\frac{\eta_N}{\eta}}$

Maximum Landau critical velocity

- $V_c^{max} \approx 155 \times 10^{-4}$ m/s $\frac{\Delta_A(T)}{k_b T_c}$

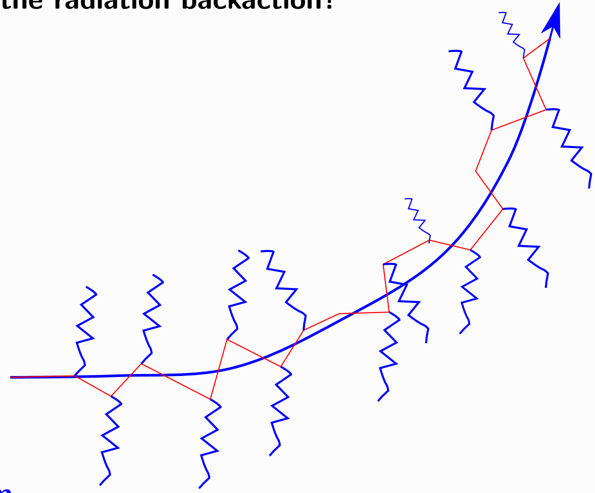
Nodal Superfluids:

- $V_c = \Delta(p)/p_f \to 0$ for $p \to p_{node}$



- Radiation Dominated Dampling for $T \lesssim 0.1 T_c$

Is their a transverse component of the radiation backaction?



Stochastic Radiative Dynamics

Fluctuations of the Chiral Vacuum

- Mesoscopic Ion coupled and driven through a Chiral "Bath"

Thank You!

The End

Feynman-Vernon Path-Integral Formulation of the Dynamics

- Ion + ^{3}He ("Bath" of quantum & thermal fluctuations) + ^{3}He-Ion Interation:

$$H = \frac{\mathbf{P}^2}{2M} - e\mathbf{E}\cdot\mathbf{R} + H_{\text{bath}} + H_{\text{int}}, \qquad H_{\text{int}}[\mathbf{R}] = \sum_{\alpha=\uparrow,\downarrow} \int d^3r \Psi_\alpha^\dagger(\mathbf{r}) V(\mathbf{r}-\mathbf{R}) \Psi_\alpha(\mathbf{r})$$

- Reduced density matrix (RDM) for the Ion: $f(\mathbf{R},\mathbf{R}',t) = \langle \mathbf{R}|\hat{f}(t)|\mathbf{R}'\rangle = \text{Tr}_{\text{bath}}\left\{\langle \mathbf{R}|\hat{\rho}(t)|\mathbf{R}'\rangle\right\}$,

- Time Evolution of the Reduced Density Matrix:

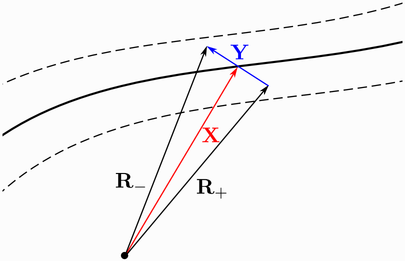
$$f(\mathbf{R},\mathbf{R}',t) = \int d^3\text{R}_i \int d^3\text{R}_i' \, J(\mathbf{R},\mathbf{R}',t;\mathbf{R}_i,\mathbf{R}_i',t_0)\, f_i(\mathbf{R}_i,\mathbf{R}_i')$$

- Path integral representation of the RDM propagator over forward ($\mathbf{R}_+$) and backward ($\mathbf{R}_-$) sub-paths:

  - $J(\mathbf{R},\mathbf{R}',t;\mathbf{R}_i,\mathbf{R}_i',t_0) = \int_{\mathbf{R}_i}^{\mathbf{R}} \mathfrak{D}\mathbf{R}_+ \int_{\mathbf{R}_i'}^{\mathbf{R}'} \mathfrak{D}\mathbf{R}_- \exp\left\{\frac{i}{\hbar}\Big(S[\mathbf{R}_+] - S[\mathbf{R}_-]\Big)\right\} F[\mathbf{R}_+,\mathbf{R}_-]$,
  - $S[\mathbf{R}] = \int_{t_0}^{t} d\tau \left[\frac{M\dot{\mathbf{R}}^2}{2} + e\mathbf{E}\cdot\mathbf{R}\right]$ – Action of a free Ion
  - $F[\mathbf{R}_+,\mathbf{R}_-] = \text{Tr}\left\{\hat{\rho}_T\, \hat{\mathcal{U}}_{\text{bath}}^\dagger[\mathbf{R}_-;t,t_0]\, \hat{\mathcal{U}}_{\text{bath}}[\mathbf{R}_+;t,t_0]\right\}$ – Feynman-Vernon influence functional

Stochastic Dynamics of an Heavy Ion in a Quantum Bath

- Forward and backward paths in RDM propagator:
  $\mathbf{X} = \frac{\mathbf{R}_+ + \mathbf{R}_-}{2} \rightsquigarrow$ "classical" trajectory,
  $\mathbf{Y} = \mathbf{R}_+ - \mathbf{R}_- \rightsquigarrow$ fluctuations around "classical" trajectory



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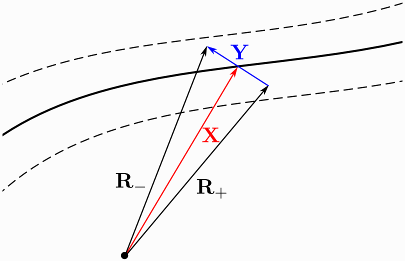
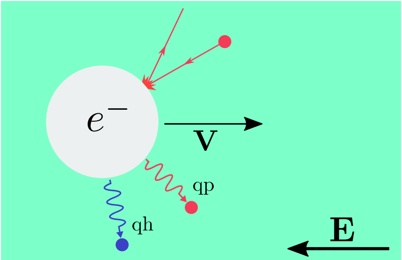
  $M \gg m$ & $|\mathbf{V}| \ll v_f \rightarrow$ trajectory fluctuations are "small"





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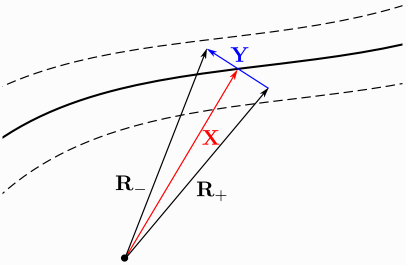
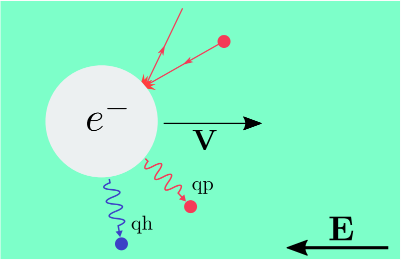
  $F[\mathbf{X},\mathbf{Y}] \approx e^{i\Phi[\mathbf{X},\mathbf{Y}]}, \quad i\Phi[\mathbf{X},\mathbf{Y}] = \underbrace{i\Phi_1[\mathbf{X},\mathbf{Y}]}_{\sim O(\mathbf{Y})} - \underbrace{\Phi_2[\mathbf{X},\mathbf{Y}]}_{\sim O(\mathbf{Y}^2)}$





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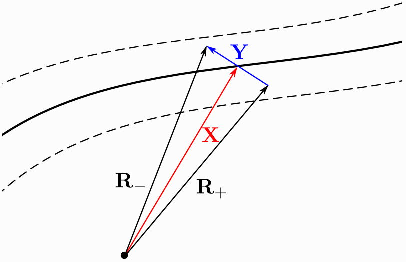

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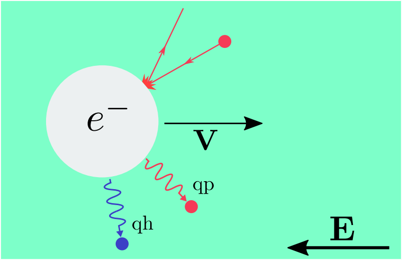
- Hubbard-Stratonovich transformation
  $$e^{-\Phi_2[\mathbf{X},\mathbf{Y}]} = \int \mathfrak{D}\boldsymbol{\xi}\, \mathfrak{F}[\boldsymbol{\xi}(t)] \exp\left[\frac{i}{\hbar}\int_{t_0}^{t} d\tau\, \mathbf{Y}(\tau)\cdot\boldsymbol{\xi}(\tau)\right],$$

  $\boldsymbol{\xi}(t) \rightsquigarrow$ Stochastic Force on the Ion



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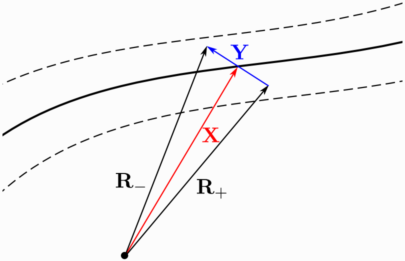

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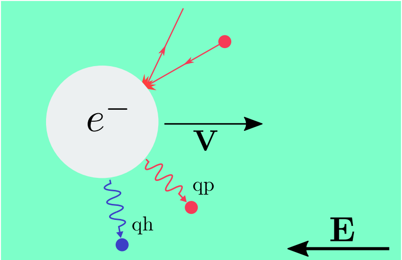


- Stochastic field correlation function:
  $$S_{ab}^{[\mathbf{X}]}(t_1,t_2) = \left\langle K_a^{[\mathbf{X}]}(t_1) K_b^{[\mathbf{X}]}(t_2)\right\rangle - \left\langle K_a^{[\mathbf{X}]}(t_1)\right\rangle \left\langle K_b^{[\mathbf{X}]}(t_2)\right\rangle,$$
  $$\mathbf{K}^{[\mathbf{X}]}(t) = \hat{\mathcal{U}}^{\dagger}_{\rm bath}[\mathbf{X};t,t_0]\left\{\boldsymbol{\nabla}_{\mathbf{X}} H_{\rm int}[\mathbf{X}]\right\}\hat{\mathcal{U}}_{\rm bath}[\mathbf{X};t,t_0],$$
  $$\langle \xi_a(t_1)\xi_b(t_2)\rangle_\xi = 2{\rm Re}S_{ab}^{[\mathbf{X}]}(t_1,t_2), \quad \langle \dots\rangle \equiv {\rm tr}\{\hat\rho_T \dots\}$$

- After Hubbard-Stratonovich transformation the RDM propagator becomes:

$$J(\mathbf{R},\mathbf{R}',t;\mathbf{R}_i,\mathbf{R}_i',t_0) \propto \int \mathfrak{D}\boldsymbol{\xi}\; \mathfrak{F}[\boldsymbol{\xi}(t)] \int \mathfrak{D}\mathbf{X} \int \mathfrak{D}\mathbf{Y}$$
$$\exp\left\{-\frac{i}{\hbar}\int_{t_0}^{t} d\tau \left[M\ddot{\mathbf{X}}\cdot\mathbf{Y} + |e|\mathbf{E}\cdot\mathbf{Y} + \mathbf{Y}\cdot\left\langle \mathbf{K}^{[\mathbf{X}]}(\tau)\right\rangle - \mathbf{Y}\cdot\boldsymbol{\xi}\right]\right\}$$

- Integrating out $\mathbf{Y}$ we arrive at the **Langevin equation**:

$$M\ddot{\mathbf{X}} + |e|\mathbf{E} + \left\langle \mathbf{K}^{[\mathbf{X}]}(t)\right\rangle = \boldsymbol{\xi}^{[\mathbf{X}]}(t)$$

  - self-consistent stochastic equation (mean force and stochastic field depend on $\mathbf{X}$)
  - describes semiclassical dynamics of the ion in presence of electric force and qp/qh scattering/emission

- Split ion's velocity into regular and fluctuating components:

$$\dot{\mathbf{X}}(t) = \mathbf{V} + \mathbf{v}(t) \quad \rightsquigarrow \quad M\dot{\mathbf{v}} + |e|\mathbf{E} + \underbrace{\left\langle \mathbf{K}^{[\mathbf{V}]}(t)\right\rangle}_{\text{static ion}} + \underbrace{\left\langle \mathbf{K}_1^{[\mathbf{v}]}(t)\right\rangle}_{\sim O(|\mathbf{v}/v_f|,\boldsymbol{\xi})} + \underbrace{\left\langle \mathbf{K}_2^{[\mathbf{v}]}(t)\right\rangle}_{\sim O(|\mathbf{v}/v_f|^2,\boldsymbol{\xi}^2)} = \boldsymbol{\xi}^{[\mathbf{X}]}(t)$$

- Approximate solution scheme:

$$\left\langle\left\langle \mathbf{K}_1^{[\mathbf{v}]}(t)\right\rangle\right\rangle_{\boldsymbol{\xi}} = 0: \quad M\dot{\mathbf{v}} + \left\langle \mathbf{K}_1^{[\mathbf{v}]}(t)\right\rangle = \boldsymbol{\xi}^{[\mathbf{V}]}(t) \quad \xrightarrow{\boldsymbol{\xi}(t)} \quad |e|\mathbf{E} + \left\langle \mathbf{K}^{[\mathbf{V}]}(t)\right\rangle + \left\langle\left\langle \mathbf{K}_2^{[\mathbf{v}]}(t)\right\rangle\right\rangle_{\boldsymbol{\xi}} = \mathbf{0}$$

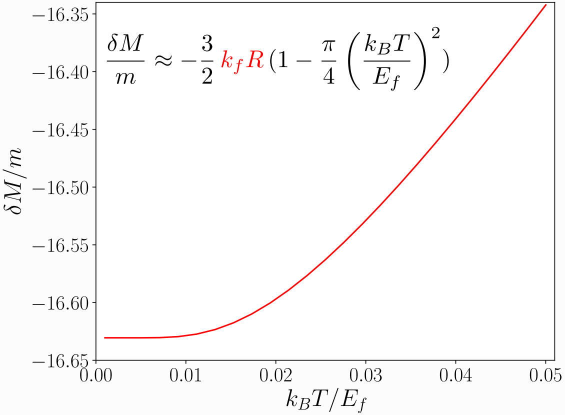
Solution to the Langevin equation allows to extract the effective mass:

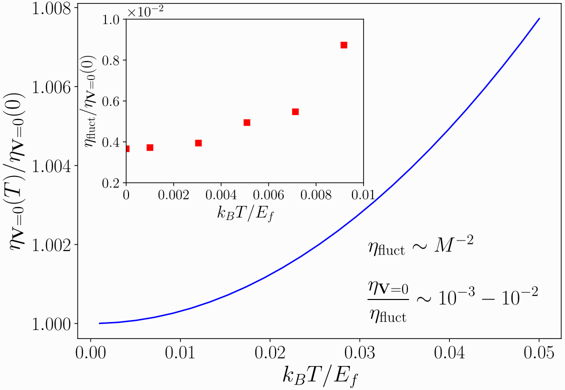
$$M\dot{v}_i + \int_{-\infty}^{t} d\tau\, \chi_{ij}(t-\tau) v_j(\tau) = \xi_i^{[\mathbf{V}]}(t), \quad \chi(\omega) \xrightarrow{\omega \to 0} -i\omega\delta M \quad \rightsquigarrow \quad M_{\text{eff}} = M + \delta M$$

Low velocity limit: i.e. small applied field $\mathbf{E}$: $\quad e\mathbf{E} - \eta_{\text{tot}}\mathbf{V} = 0,$

$$\eta_{\text{tot}}(T, M, R) = \eta_{\mathbf{V}=0}(T, R) + \eta_{\text{fluct}}(T, M, R), \qquad \eta_{\mathbf{V}=0}(T \to 0) = n_3 p_f \sigma_N^{tr}(p_f)$$

Josephson-Leckner (1969)





Hamiltonian for 2D Chiral Superfluid (^{3}He-A Thin Film & Sr_2RuO_4):

$$\widehat{H} = \begin{pmatrix} (|\mathbf{p}|^2/2m^* - \mu) & c(p_x + ip_y) \\ c(p_x - ip_y) & -(|\mathbf{p}|^2/2m^* - \mu) \end{pmatrix} = \vec{\mathbf{m}}(\mathbf{p}) \cdot \widehat{\vec{\boldsymbol{\tau}}}$$

$\vec{\mathbf{m}} = (\, cp_x \,,\, \mp cp_y \,, \xi(\mathbf{p}))$ with $|\vec{\mathbf{m}}(\mathbf{p})|^2 = \left(|\mathbf{p}|^2/2m - \mu\right)^2 + c^2|\mathbf{p}|^2 > 0 \,, \mu \neq 0$

Momentum-Space Topology of Nambu-Bogoliubov Hamiltonian

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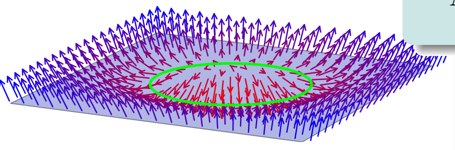
- Topological Invariant for 2D chiral SC $\leftrightarrow$ QED in d = 2+1 [G.E. Volovik, JETP 1988]:



$$N_{\mathrm{C}} = \int \frac{d^2p}{4\pi}\, \hat{\mathbf{m}}(\mathbf{p}) \cdot \left(\frac{\partial \hat{\mathbf{m}}}{\partial p_x} \times \frac{\partial \hat{\mathbf{m}}}{\partial p_y}\right) = \begin{cases} \pm 1 \,; & \mu > 0 \text{ and } \Delta \neq 0 \\ 0 \,; & \mu < 0 \text{ or } \Delta = 0 \end{cases}$$

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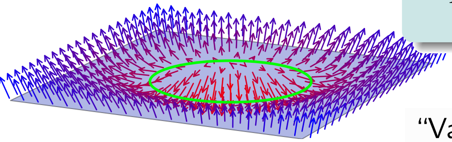
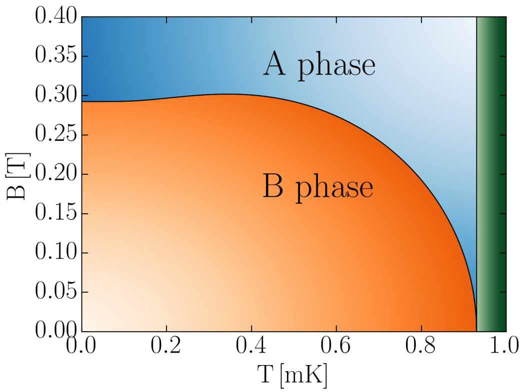


"Vacuum" $(\Delta = 0)$ & $N_{\mathsf{C}} = 0$ | ^{3}He-A $(\Delta \neq 0)$ with $N_{\mathsf{C}} = 1$

Zero Energy Fermions ↑ Confined on the Edge

Superfluid Phases of ^{3}He in a Magnetic Field for $P < P_{\text{PCP}}$

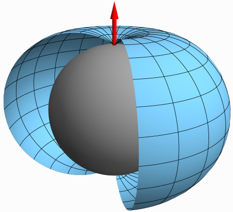
Zeeman Energy for Spin-Triplet Pairing



Spin-Triplet, P-wave Order Parameter

$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix} = \begin{pmatrix} -\mathbf{d}_x + i\mathbf{d}_y & \mathbf{d}_z \\ \mathbf{d}_z & \mathbf{d}_x + i\mathbf{d}_y \end{pmatrix}$$

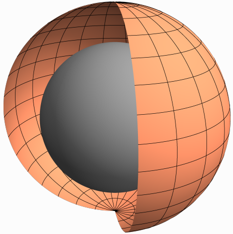
ABM State $\vec{l} = \hat{\mathbf{z}}$



$L_z = 1,\ S_y = 0$

$\Psi_{\uparrow\uparrow} = \Psi_{\downarrow\downarrow} = \hat{p}_x + i\hat{p}_y$

"Isotropic" BW State



$J = 0,\ J_z = 0$

$\Psi_{\uparrow\uparrow} = \hat{p}_x - i\hat{p}_y,\ \Psi_{\downarrow\downarrow} = \hat{p}_x + i\hat{p}_y,\ \Psi_{\uparrow\downarrow} = \hat{p}_z$

Two Fluid Motion for a moving electron bubble as $T \to 0$

- An ion moving through a fluid experiences a force originating from the scattering of excitations off the ion.
- ^{3}He-A at $T \neq 0$ ^{3}He-A: a condensate of chiral Cooper pairs & a fluid of "normal" chiral Fermions.

$$M \frac{d\mathbf{V}}{dt} = e\mathbf{E} + e\mathbf{V} \times \mathbf{B}_W - \eta\,\mathbf{V}$$

- Dynamical Effective Mass of the Ion: M ← Backflow & Virtual Excitations
- Stokes Drag Force on the Ion: $\mathbf{F}_{\text{drag}} = -\eta\mathbf{V}$ ← Dynamic Viscosity
- Chiral Effective Magnetic Field: $\mathbf{B}_W = -\frac{c}{e}\eta_{xy}\hat{\mathbf{l}}$ ← Anomalous Hall Response

- Stokes' drag for a sphere of radius R: $\eta = 6\pi\,\nu\,R \rightsquigarrow$ Reynold's Number: $\text{Re} \equiv \dfrac{2\rho V R}{\nu}$
- Normal ^{3}He: $\rho = 0.0819\,\text{g/cm}^3$ $\quad \mu_{\text{N}} = \dfrac{e}{n_3 p_f \sigma_{\text{N}}^{\text{tr}}} \simeq 1.7 \times 10^{-6}\,\text{m}^2/\text{V-s}$ $\quad \rightsquigarrow R = 1.42\,\text{nm}$ $\quad k_f R = 11.17$
- Derived Parameters: $\nu_{\text{N}} = \dfrac{\eta_{\text{N}}}{6\pi R} = 3.5 \times 10^{-6}\,\text{Pa-s}$ $\quad \text{Re}_{\text{N}} = 6.7 \times 10^{-6}$ $\quad \text{B}_{\text{N}} \equiv \dfrac{c}{e}\eta_{\text{N}} = 5.9 \times 10^5\,\text{T}$

- Reynold's Number for flow past an electron bubble in ^{3}He-A: $\text{Re} = \text{Re}_{\text{N}} \left(\dfrac{\eta_{\text{N}}}{\eta}\right)^{3/2} \xrightarrow[T\to 0]{} \sim \left(\dfrac{T_c}{T}\right)^{9/2}$!

Determination of the Electron Bubble Radius

(i) Energy required to create a bubble:

$$E(R,P) = E_0(U_0,R) + 4\pi R^2\gamma + \frac{4\pi}{3}R^3 P, \quad P \text{ – pressure}$$

(ii) For $U_0 \to \infty$: $E_0 = -U_0 + \pi^2\hbar^2/2m_e R^2$ – ground state energy

(iii) Surface Energy: hydrostatic surface tension $\rightsquigarrow \gamma = 0.15\,\mathrm{erg/cm^2}$

(iv) Minimizing E w.r.t. $R \rightsquigarrow P = \pi\hbar^2/4m_e R^5 - 2\gamma/R$

(v) For zero pressure, $P = 0$:

$$R = \left(\frac{\pi\hbar^2}{8m_e\gamma}\right)^{1/4} \approx 2.38\,\mathrm{nm} \quad \rightsquigarrow \quad k_f R = 18.67$$

Transport $\rightsquigarrow k_f R = 11.17$

- A. Ahonen et al., J. Low Temp. Phys., 30(1):205228, 1978

Mobility of an electron bubble in the Normal Fermi Liquid

- $$t^R_{\mathsf{N}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E) = \sum_{l=0}^{\infty} (2l+1) t^R_l(E) P_l(\hat{\mathbf{k}}' \cdot \hat{\mathbf{k}})$$

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- $$t^R_l(E) = -\frac{1}{\pi N_f} e^{i\delta_l(E)} \sin \delta_l(E)$$

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- $\frac{d\sigma}{d\Omega_{\mathbf{k}'}} = \left(\frac{m^*}{2\pi\hbar^2}\right)^2 |t^R_{\mathsf{N}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E)|^2$

Mobility of an electron bubble in the Normal Fermi Liquid

- $t_{\mathsf{N}}^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E) = \sum_{l=0}^{\infty} (2l+1) t_l^R(E) P_l(\hat{\mathbf{k}}' \cdot \hat{\mathbf{k}})$
- $t_l^R(E) = -\frac{1}{\pi N_f} e^{i\delta_l(E)} \sin \delta_l(E)$
- $\frac{d\sigma}{d\Omega_{\mathbf{k}'}} = \left(\frac{m^*}{2\pi\hbar^2}\right)^2 |t_{\mathsf{N}}^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E)|^2$
- Non-resonant scattering at $T \ll E_f/k_{\mathsf{B}} \approx 3\,\mathsf{K} \rightsquigarrow \delta_l(E \approx E_f)$

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- $$\sigma_{\mathsf{N}}^{\mathsf{tr}} = \int \frac{d\Omega_{\mathbf{k}'}}{4\pi} (1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}') \frac{d\sigma}{d\Omega_{\mathbf{k}'}} = \frac{4\pi}{k_f^2} \sum_{l=0}^{\infty} (l+1) \sin^2(\delta_{l+1} - \delta_l)$$

Mobility of an electron bubble in the Normal Fermi Liquid

- $t^R_{\mathsf{N}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E) = \sum_{l=0}^{\infty} (2l+1) t^R_l(E) P_l(\hat{\mathbf{k}}' \cdot \hat{\mathbf{k}})$
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- Non-resonant scattering at $T \ll E_f/k_{\mathsf{B}} \approx 3\,\mathsf{K} \rightsquigarrow \delta_l(E \approx E_f)$
- $\sigma^{\mathsf{tr}}_{\mathsf{N}} = \int \frac{d\Omega_{\mathbf{k}'}}{4\pi} (1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}') \frac{d\sigma}{d\Omega_{\mathbf{k}'}} = \frac{4\pi}{k_f^2} \sum_{l=0}^{\infty} (l+1) \sin^2(\delta_{l+1} - \delta_l)$
- $\mu_{\mathsf{N}} = \frac{e}{n_3 p_f \sigma^{\mathsf{tr}}_{\mathsf{N}}}, \quad p_f = \hbar k_f, \quad n_3 = \frac{k_f^3}{3\pi^2}$

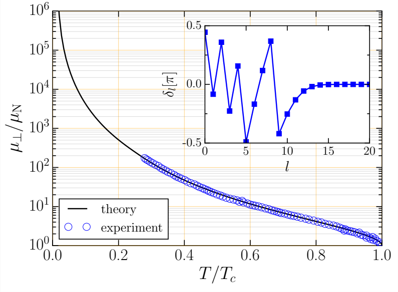
Theoretical Models for the QP-ion potential

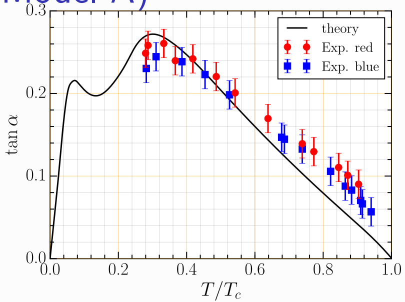
- $U(r) = \begin{cases} U_0, & r < R, \\ -U_1, & R < r < R', \\ 0, & r > R'. \end{cases}$
- $\rightsquigarrow$ Hard-Sphere Potential: $U_1 = 0$, $R' = R$, $U_0 \to \infty$
- $U(x) = U_0\,[1 - \tanh[(x-b)/c]]$, $\quad x = k_f r$
- $U(x) = U_0/\cosh^2[\alpha x^n]$, $\quad x = k_f r$ (Pöschl-Teller-like potential)
- Random phase shifts: $\{\delta_l | \, l = 1 \ldots l_{\max}\}$ are generated with δ_0 is an adjustable parameter
- Parameters for all models are chosen to fit the experimental value of the normal-state mobility, $\mu_N^{\exp} = 1.7 \times 10^{-6}\, m^2/V \cdot s$

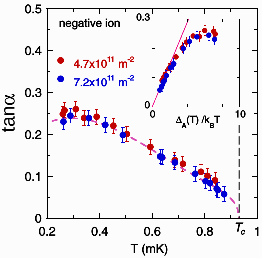
Theoretical Models for the QP-ion potential

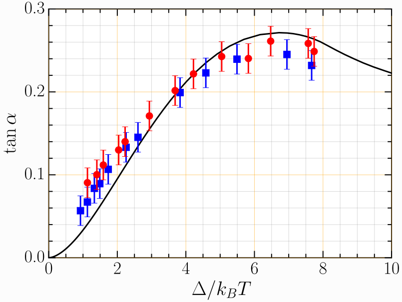
Label	Potential	Parameters
Model A	hard sphere	$k_f R = 11.17$
Model B	repulsive core & attractive well	$U_0 = 100E_f, U_1 = 10E_f, k_f R' = 11, R/R' = 0.36$
Model C	random phase shifts model 1	$l_{\max} = 11$
Model D	random phase shifts model 2	$l_{\max} = 11$
Model E	Pöschl-Teller-like	$U_0 = 1.01E_f, k_f R = 22.15, \alpha = 3 \times 10^{-5}, n = 4$
Model F	Pöschl-Teller-like	$U_0 = 2E_f, k_f R = 19.28, \alpha = 6 \times 10^{-5}, n = 4$
Model G	hyperbolic tangent	$U_0 = 1.01E_f, k_f R = 14.93, b = 12.47, c = 0.246$
Model H	hyperbolic tangent	$U_0 = 2E_f, k_f R = 14.18, b = 11.92, c = 0.226$
Model I	soft sphere 1	$U_0 = 1.01E_f, k_f R = 12.48$
Model J	soft sphere 2	$U_0 = 2E_f, k_f R = 11.95$

Hard-sphere model with $k_f R = 11.17$ (Model A)

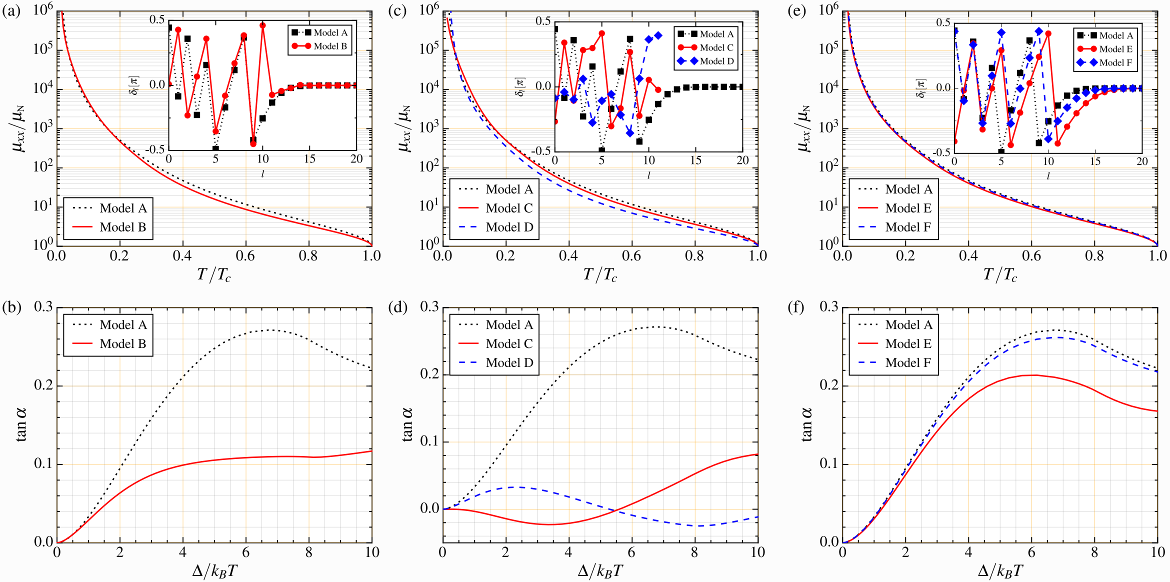




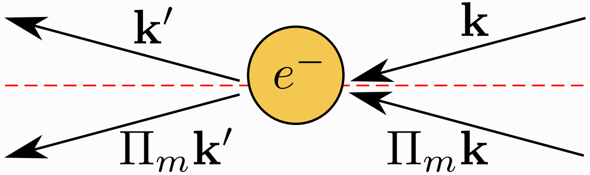




Comparison with Experiment for Models for the QP-ion potential



Broken Time-Reversal ($\mathtt{T}$) & mirror (Π_{m}) symmetries in Chiral Superfluids



- Broken TRS: $\mathtt{T}\cdot(\hat{p}_x + i\hat{p}_y) = (\hat{p}_x - i\hat{p}_y)$
- Broken mirror symmetry: $\Pi_{m}\cdot(\hat{p}_x + i\hat{p}_y) = (\hat{p}_x - i\hat{p}_y)$
- Chiral symmetry: $\mathtt{C} = \mathtt{T}\times\Pi_{m}$ $\rightsquigarrow$ $\mathtt{C}\cdot(\hat{p}_x + i\hat{p}_y) = (\hat{p}_x + i\hat{p}_y)$
- Microscopic reversibility for chiral superfluids: $W(\hat{\mathbf{k}}', \hat{\mathbf{k}}; +\hat{\mathbf{l}}) = W(\hat{\mathbf{k}}, \hat{\mathbf{k}}'; -\hat{\mathbf{l}})$
- $\therefore$ For BTRS: the chiral axis $\hat{\mathbf{l}}$ is fixed $\rightsquigarrow$ $W(\hat{\mathbf{k}}', \hat{\mathbf{k}}; \hat{\mathbf{l}}) \neq W(\hat{\mathbf{k}}, \hat{\mathbf{k}}'; \hat{\mathbf{l}})$

Calculation of LDOS and Current Density

$$\hat{\mathcal{G}}_S^R(\mathbf{r}', \mathbf{r}, E) = \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} e^{i\mathbf{k}'\mathbf{r}'} e^{-i\mathbf{k}\mathbf{r}} \hat{\mathcal{G}}_S^R(\mathbf{k}', \mathbf{k}, E)$$

$$\hat{\mathcal{G}}_S^R(\mathbf{k}', \mathbf{k}, E) = (2\pi)^3 \hat{G}_S^R(\mathbf{k}, E)\delta_{\mathbf{k}',\mathbf{k}} + \hat{G}_S^R(\mathbf{k}', E)\hat{T}_S(\mathbf{k}', \mathbf{k}, E)\hat{G}_S^R(\mathbf{k}, E)$$

$$\hat{G}_S^R(\mathbf{k}, E) = \frac{1}{\varepsilon^2 - E_{\mathbf{k}}^2} \begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad \varepsilon = E + i\eta, \quad \eta \to 0^+$$

$$N(\mathbf{r}, E) = -\frac{1}{2\pi} \mathrm{Im} \left\{ \mathrm{Tr} \left[\hat{\mathcal{G}}_S^R(\mathbf{r}, \mathbf{r}, E) \right] \right\}$$

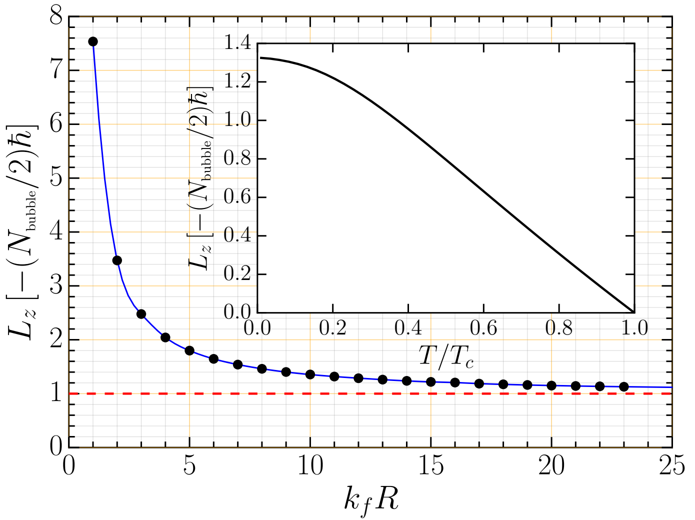
$$\mathbf{j}(\mathbf{r}) = \frac{\hbar}{4mi} k_B T \sum_{n=-\infty}^{\infty} \lim_{\mathbf{r}\to\mathbf{r}'} \mathrm{Tr} \left[(\boldsymbol{\nabla}_{\mathbf{r}'} - \boldsymbol{\nabla}_{\mathbf{r}}) \hat{\mathcal{G}}^M(\mathbf{r}', \mathbf{r}, \epsilon_n) \right]$$

$$\hat{\mathcal{G}}_S^R(\mathbf{r}', \mathbf{r}, E) = \hat{\mathcal{G}}_S^M(\mathbf{r}', \mathbf{r}, \epsilon_n) \Big|_{i\epsilon_n \to \varepsilon}, \text{ for } n \geq 0$$

$$\hat{\mathcal{G}}_S^M(\mathbf{k}, \mathbf{k}', -\epsilon_n) = \left[\hat{\mathcal{G}}_S^M(\mathbf{k}', \mathbf{k}, \epsilon_n) \right]^\dagger$$

Angular momentum of an electron bubble in ^{3}He-A ($k_f R = 11.17$)

$$\mathbf{L}(T \to 0) \approx -\hbar N_{\text{bubble}} \hat{\mathbf{l}}/2 \,; \qquad N_{\text{bubble}} = n_3 \, \frac{4\pi}{3} R^3 \approx 200 \; {}^3\text{He atoms}$$



Temperature scaling of the Stokes tensor components

- For $1 - \frac{T}{T_c} \to 0^+$:

$$\frac{\eta_\perp}{\eta_\mathsf{N}} - 1 \propto -\Delta(T) \propto \sqrt{1 - \frac{T}{T_c}}$$

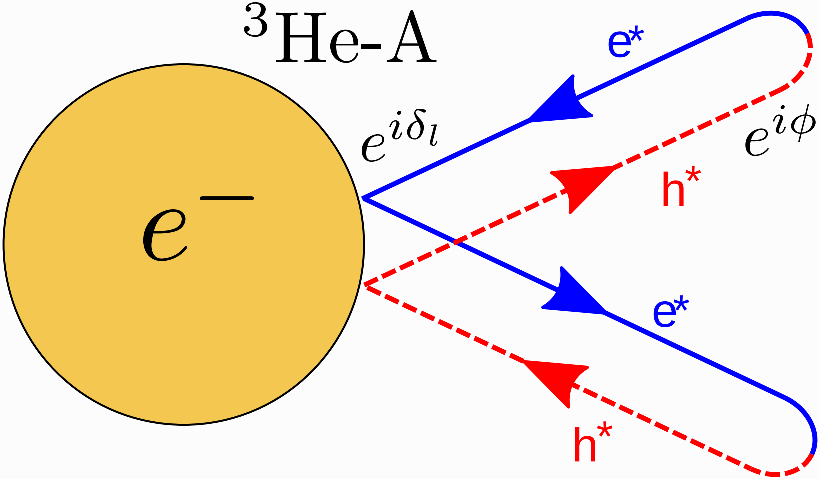
$$\frac{\eta_\mathsf{AH}}{\eta_\mathsf{N}} \propto \Delta^2(T) \propto 1 - \frac{T}{T_c}$$

- For $\frac{T}{T_c} \to 0^+$:

$$\frac{\eta_\perp}{\eta_\mathsf{N}} \propto \left(\frac{T}{T_c}\right)^2$$

$$\frac{\eta_\mathsf{AH}}{\eta_\mathsf{N}} \propto \left(\frac{T}{T_c}\right)^3$$

Multiple Andreev Scattering ⇝ Formation of Weyl fermions on e-bubbles



$$\Delta(\hat{\mathbf{k}}) = \Delta \sin\theta e^{i\phi}$$