Phase-sensitive Transport in Josephson Weak Links & Junctions

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Department of Physics & Astronomy, Hearne Institute of Theoretical Physics
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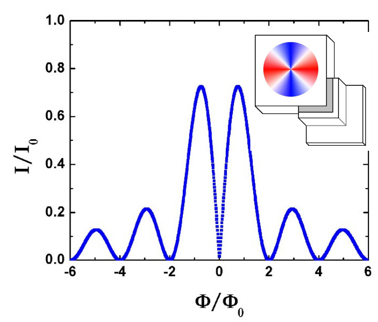

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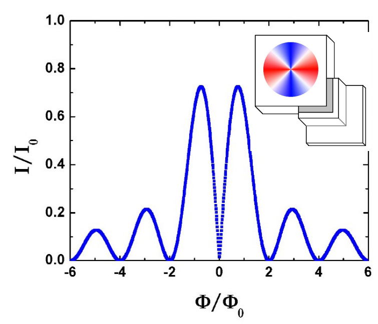

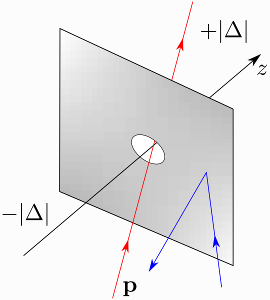

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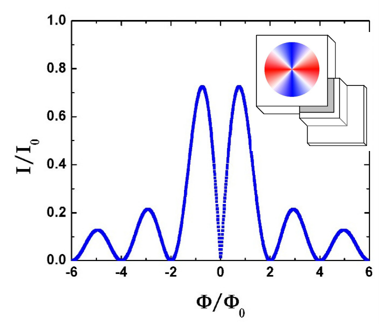

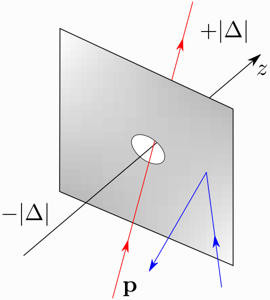

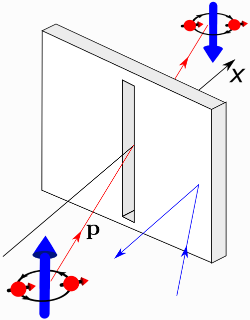

- Erhai Zhao (George Mason), Tomas Löfwander (Chalmers), Jason He (Northwestern)
- Research supported by National Science Foundation grant DMR-1508730
- *Andreev Bound States and their Signatures*, Phil. Trans. R. Soc. A 376: 20180140 (2018), J.A. Sauls

Josephson Tunneling in Superconductors

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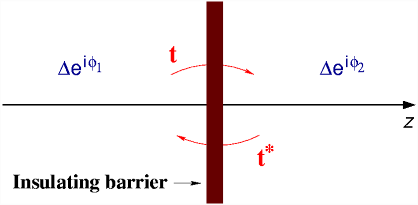

$$H = H_1 + H_2 + H_{tH}$$

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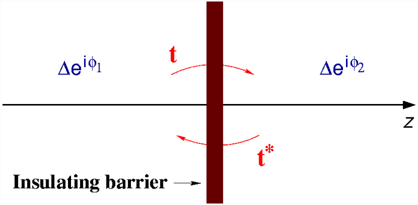

$$H = H_1 + H_2 + H_{tH}$$

$$H_1 = \sum_{k\sigma} \xi_{k\sigma} c^\dagger_{k\sigma} c_{k\sigma} + \frac{1}{2} \sum_{k\sigma} \left(\Delta_k c^\dagger_{k\sigma} c^\dagger_{-k-\sigma} + \Delta^*_k c^\dagger_{-k-\sigma} c_{k\sigma} \right)$$

$$H_2 = \sum_{p\sigma} \xi_{p\sigma} a^\dagger_{p\sigma} a_{p\sigma} + \frac{1}{2} \sum_{p\sigma} \left(\Delta_p a^\dagger_{p\sigma} a^\dagger_{-p-\sigma} + \Delta^*_p a^\dagger_{-p-\sigma} a_{p\sigma} \right)$$

$$H_{tH} = \sum_{p,k,\sigma} \left(t_{p,k}\, a^\dagger_{p\sigma} c_{k\sigma} + t^*_{p,k}\, c^\dagger_{k\sigma} a_{p\sigma} \right)$$

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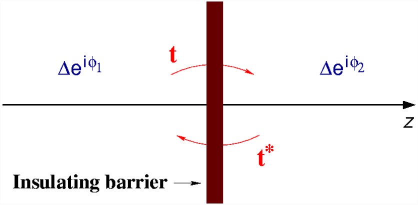

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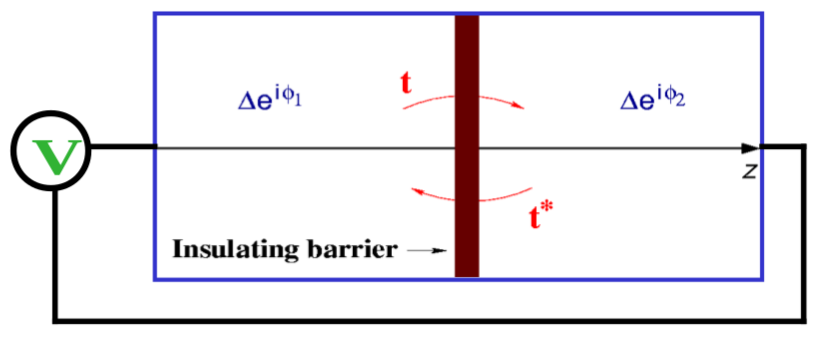
- $\langle I \rangle = I_c(T)\sin(\Delta\phi)$

$$I_c(T) = 2e \times \underbrace{\left(\pi^2 N(0)^2 |\langle |t|^2 \rangle_{FS}\right)}_{\propto \text{Transmission Probability } D \ll 1} \times \Delta \tanh\left(\frac{\Delta}{2T}\right)$$

a.c. Josephson Effects



- Supercurrent: $I_s = I_c(T)\,\sin(\phi_t)$
- *a.c.* Josephson Equation: $\phi_t = \dfrac{2e}{h}\mathbf{V}\,t$

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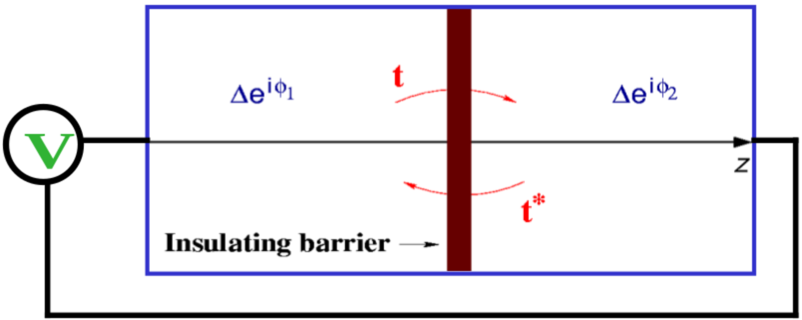
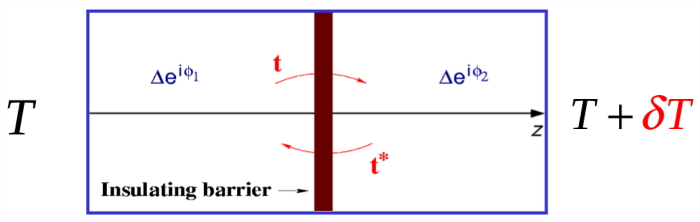


- Supercurrent: $I_s = I_c(T)\sin(\phi_t)$
- *a.c.* Josephson Equation: $\phi_t = \frac{2e}{h}\mathbf{V}\,t$
- Phase-sensitive dissipation $\rightsquigarrow$ B. Josephson, Adv. Phys. (1965).
- Dissipative Current: $I_{\text{Ohmic}} = \left(\sigma_0 + \sigma_1\cos(\phi_t)\right)\mathbf{V}$
- What is the origin of phase-modulation of the dissipative current?

Heat Transport through a Phase-Biased Josephson Junction

- Linear Response to a Thermal Bias
- Maki & Griffin, PRL (1965); Guttman et al. PRB 57, 2717 (1998)



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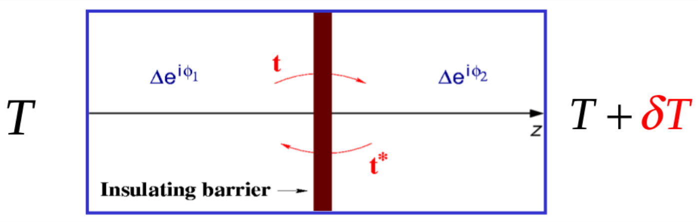

Heat Current: Tunneling Hamiltonian

▶ $$\langle I_Q \rangle = \delta T \times \underbrace{4\pi N(0)\langle |t|^2 \rangle_{FS}}_{\propto D} \underbrace{\int_{\Delta}^{\infty} d\varepsilon \left(-\frac{\partial f}{\partial T} \right)}_{\text{thermal excitations}} \underbrace{\left(\frac{\varepsilon^2 - \Delta^2 \cos(\phi)}{\varepsilon^2 - \Delta^2} \right)}$$

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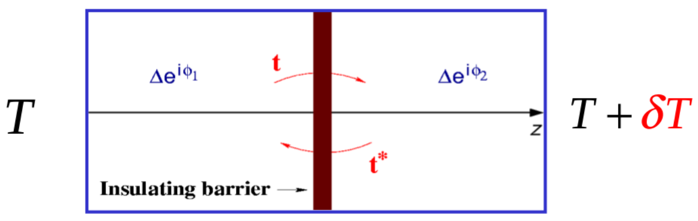

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$\rightsquigarrow$ **Failure of Linear Response Theory?**

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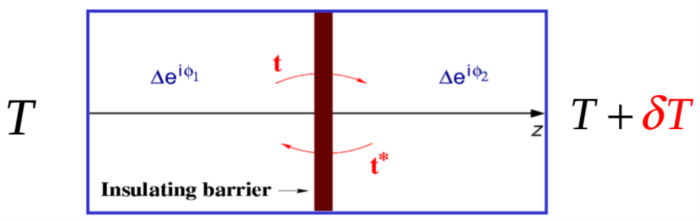

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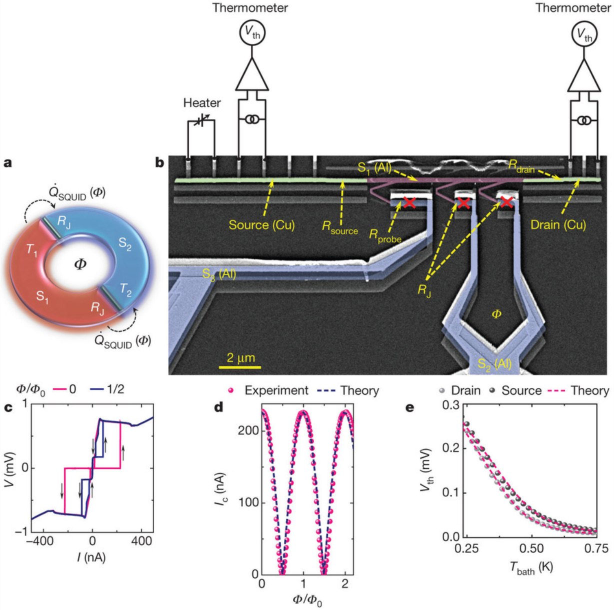
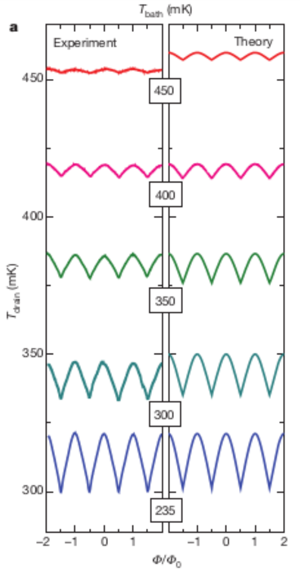
$\rightsquigarrow$ **Failure of Linear Response Theory?** No! $\rightsquigarrow$ **Failure of Perturbation Theory in H_{tH}**

- *Phase Modulated Thermal Conductance of Josephson Weak Links*, PRL 91:077003 (2003), E. Zhao, T. Löfwander, J. A. Sauls

The Josephson heat interferometer - Giazotto et al. Nature 492, 401 (2012)





Breakdown of Perturbation Theory for Transport

$\Downarrow$

Bound State Formation at the Interface
between degenerate ground states

$\Downarrow$

Linear Response Restored ... but Nonanalyticity: $\kappa_Q \sim D \ln D$

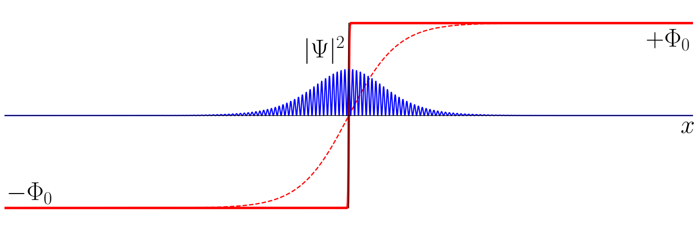
Dirac Fermions coupled to Scalar Bose Field $\rightsquigarrow$ Zero-Energy Bound State

$$i\hbar\partial_t|\psi\rangle = (-i\hbar c\vec{\alpha}\cdot\nabla + \beta g\Phi)|\psi\rangle$$

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad |\psi\rangle = \mathrm{col}(\psi_1, \psi_2, \psi_3, \psi_4)$$

▶ Broken Symmetry State: $\Phi = \Phi_0 \rightsquigarrow$ **Mass**: $Mc^2 = g\Phi_0 \rightsquigarrow E_\pm = \pm\sqrt{c^2|\mathbf{p}|^2 + (Mc^2)^2}$

▶ Degenerate Vacuum States: $\Phi(x \to \pm\infty) = \mp\Phi_0$:

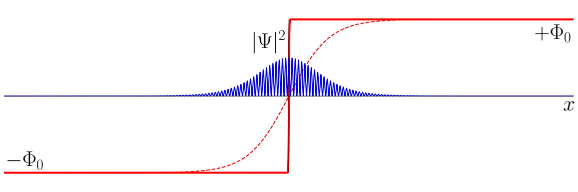


Zero-Energy Fermion Confined on the Domain Boundary

R. Jackiw and C. Rebbi, Phys. Rev. D 1976

Fermion Bound States of a Superconducting Sharvin Contact

▶ Superconducting Ground State: $\Delta(z) = g\langle\psi_\uparrow(z)\psi_\downarrow(x)\rangle = Re\Delta(z) + iIm\Delta(z)$



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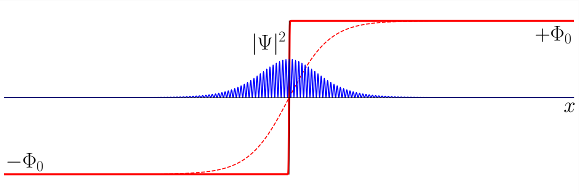
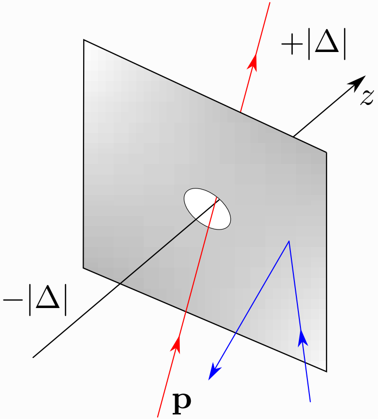


$\Longrightarrow$



$$\left(E\tau_3 - Re\Delta(z)\tau_1 + Im\Delta(z)\tau_2 + i\hbar\mathbf{v_p}\cdot\nabla\right)|\Psi\rangle = 0$$

Andreev $\leftrightarrow$ Dirac

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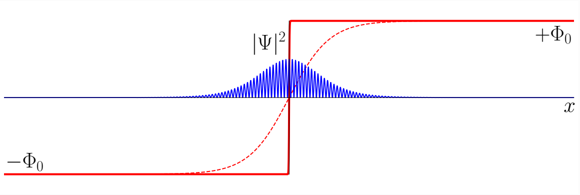

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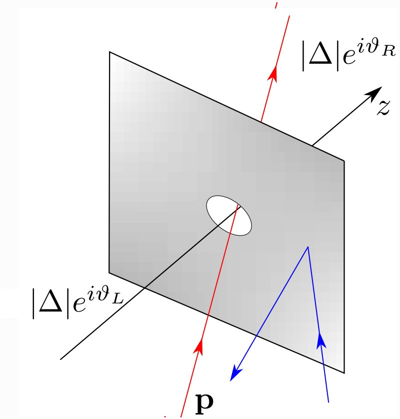

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$$\hbar/p_f \ll a \ll \xi = \hbar v_f/|\Delta|$$

▶Continuum of Degenerate Vacuum States: $\vartheta = \vartheta_R - \vartheta_L \in \{0, 2\pi\}$

▶*Andreev Bound States and their Signatures*, Phil. Trans. R. Soc. A 376: 20180140 (2018), J.A. Sauls.

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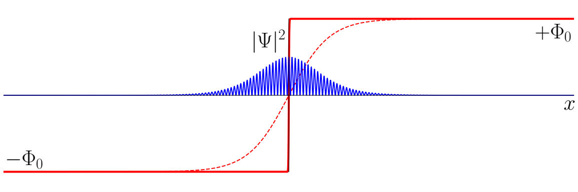

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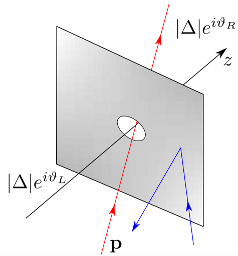

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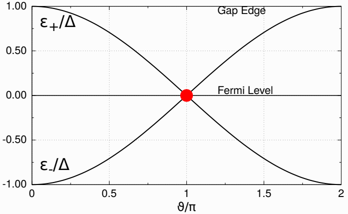
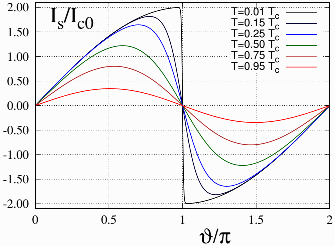


$$I_s = \frac{\pi}{eR_N}\sum_{\nu=\pm} f(\varepsilon_\nu)\,\frac{d\varepsilon_\nu}{d\vartheta}$$



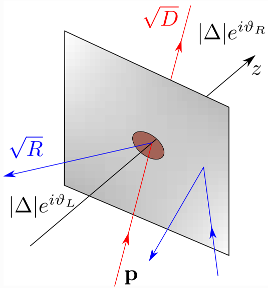
▶J. A. Sauls, Phil. Trans. R. Soc. A 376: 20180140 (2018), ▶I. Kulik and A. Omel'Yanchuk, Sov. J. Low Temp. Phys. 3, 459 (1977).

Fermion Bound States of a Pinhole + Barrier

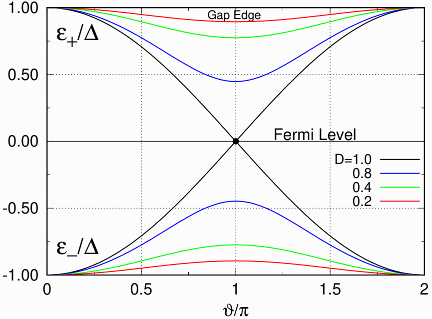
$$S = \begin{pmatrix} \sqrt{R} & i\sqrt{D} \\ i\sqrt{D} & \sqrt{R} \end{pmatrix}$$

$$\varepsilon_{\pm}(R,\phi) = \pm\Delta\sqrt{\cos^2(\phi/2) + R\sin^2(\phi/2)}$$

$$0 < D \leq 1$$



Potential + Andreev Scattering ⇝ *Gap in the Bound-State Dispersion*



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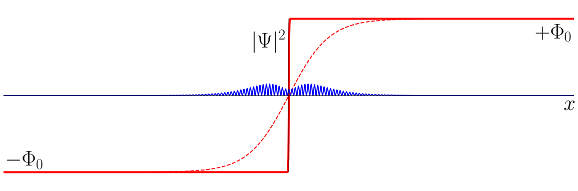

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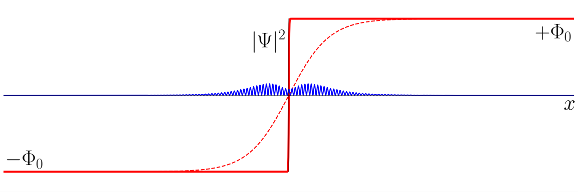

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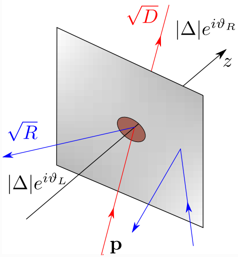

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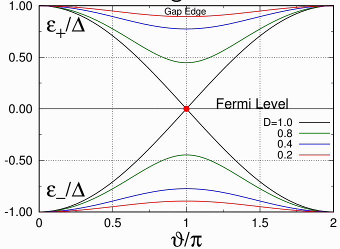

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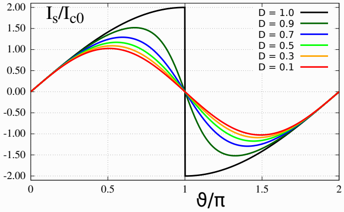

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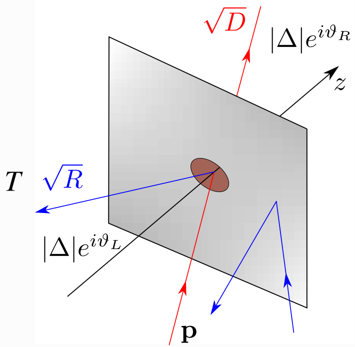
Heat Transport through a Phase-Biased Josephson Point Contact Junction



$T + \delta T$

- Heat Current $j_Q = -\kappa\,\delta T$
- Carriers = *bulk* quasiparticles
- $N_{\text{bulk}}(\varepsilon) = N(0)\,\dfrac{\varepsilon}{\sqrt{\varepsilon^2 - \Delta^2}}$
- $v_{\text{g}}(\varepsilon) = v_f\,\dfrac{\sqrt{\varepsilon^2 - \Delta^2}}{\varepsilon}$

Thermal Conductance

$$\kappa(\phi, T) = A\int_{\Delta}^{\infty} d\varepsilon\, N_{\text{bulk}}(\varepsilon)\,[\varepsilon\, v_{\text{g}}(\varepsilon)]\,\mathscr{D}(\varepsilon,\phi)\left(-\frac{\partial f}{\partial T}\right)$$

▶ $\mathscr{D}(\varepsilon,\phi)$ = Quasiparticle Transmission Probability

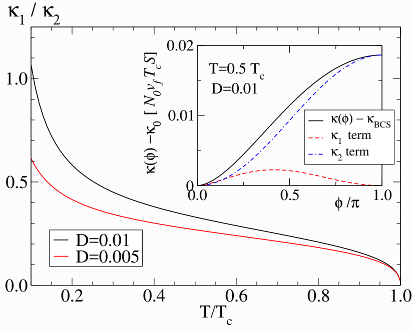
Non-analyticity of the Thermal Conductance for $D \to 0$

▶ Tunneling Hamiltonian: $\kappa^{\text{tH}} = \kappa^{\text{tH}}_{\text{BCS}} + \kappa^{\text{tH}}_2 \sin^2(\phi/2)$ ... But $\kappa^{\text{tH}}_2 \to \infty$

▶ S-matrix theory for Transport in the limit $D \ll 1$:

$$\kappa = \kappa_{\text{BCS}} - \kappa_1 \sin^2(\phi/2) \ln\left(\sin^2(\phi/2)\right) + \kappa_2 \sin^2(\phi/2)$$

▶ $\kappa_{1,2} \xrightarrow[D\to 0]{} D \ln D \Rightarrow$ Finite, but Non-Analytic in D



▶ Andreev Bound-State Formation is *non-perturbativce*

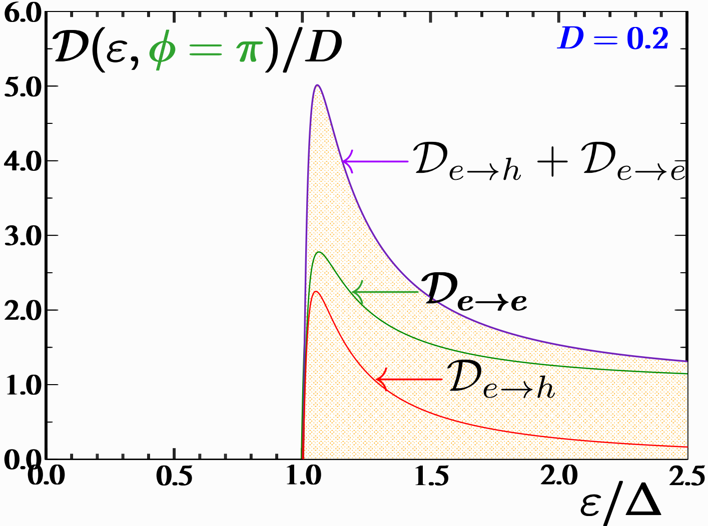
Andreev's Demon & Resonant Transmission

Direct Transmission

$$\mathscr{D}_{e\to e}(\varepsilon,\phi) = D\,\frac{(\varepsilon^2-\Delta^2)(\varepsilon^2-\Delta^2\cos^2(\phi/2))}{[\varepsilon^2-\Delta^2(1-D\sin^2(\phi/2)]^2}$$

Branch Conversion

$$\mathscr{D}_{e\to h}(\varepsilon,\phi) = DR\,\frac{(\varepsilon^2-\Delta^2)\Delta^2\sin^2(\phi/2)}{[\varepsilon^2-\Delta^2(1-D\sin^2(\phi/2)]^2}$$



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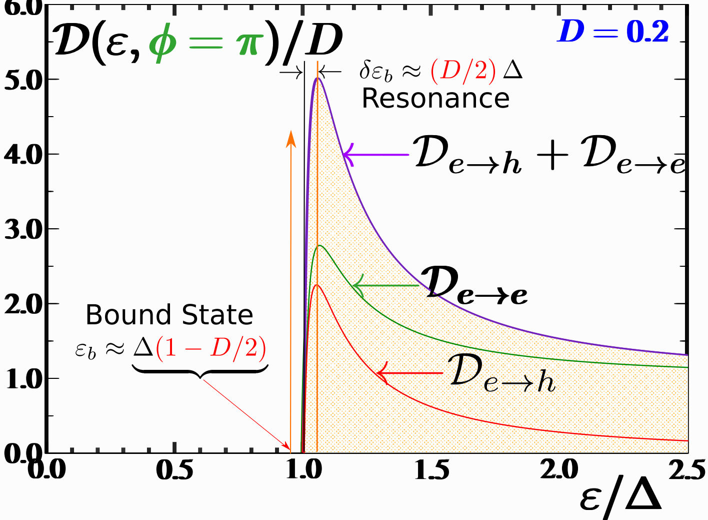
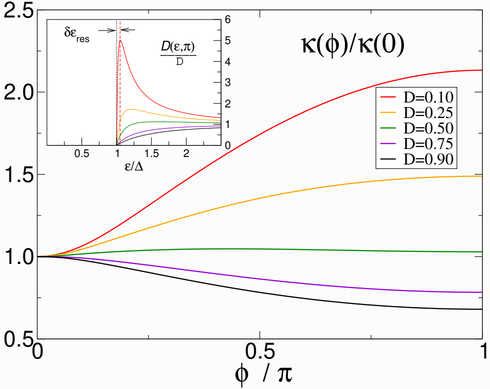


Phase-Tuneable Resonant Enhancement of the Heat Current

Andreev's Demon $\rightsquigarrow$ Fermion Bound States "control" thermal transport



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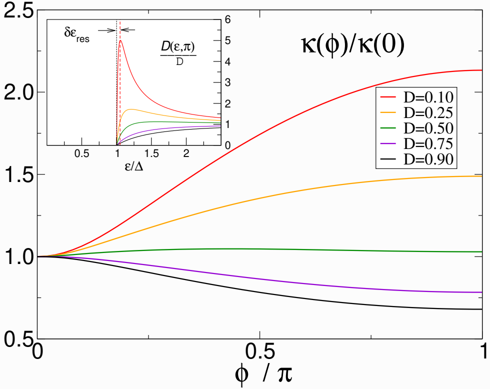
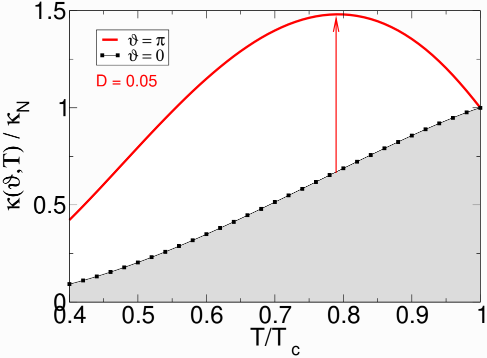


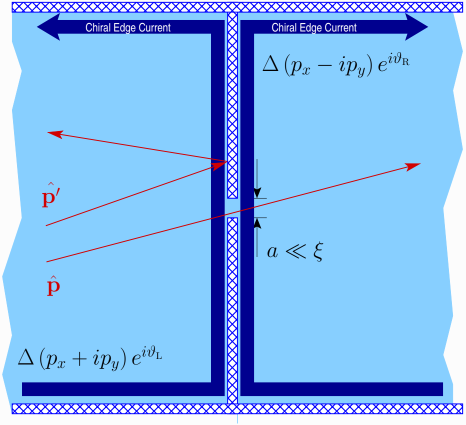


▶$D \gtrsim 0.5$: $\kappa(\phi) < \kappa(0)$ ▶$\phi = 0$: $\kappa \downarrow$ for $T < T_c$. ▶$\phi = \pi$: $D < 0.5$ $\kappa(T) \uparrow$ below T_c.

- E. Zhao, T. Löfwander and JAS (PRL 2003);
- J.A. Sauls, Proc. Roy. Soc. A (2018)

Josephson Point Contact as a Probe of Chiral Superconductivity

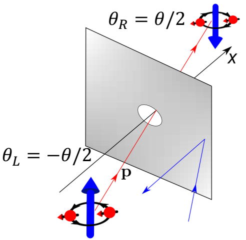
- Phase Bias: $\vartheta = \vartheta_R - \vartheta_L$
- Thermal Bias: $\delta T = T_R - T_L$

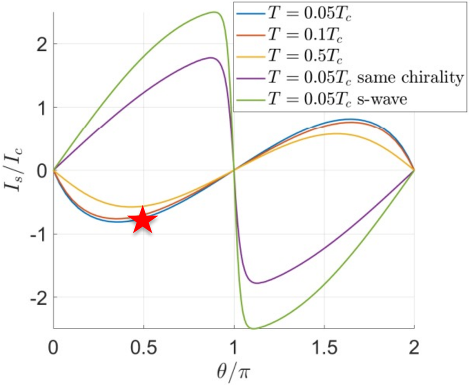


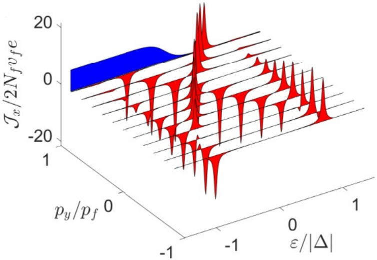
Josephson Current-Phase Relations $\rightsquigarrow$ *Spectroscopy of Chiral Edge States*

▶Jason He & JAS (unpublished)

Inhomogeneity induced negative current phase relation







$$I_s = \iint d\boldsymbol{S} \cdot \boldsymbol{j}(\boldsymbol{r})$$

$$\boldsymbol{j}(\boldsymbol{r}) = 2N_f \int_{-\pi/2}^{\pi/2} \frac{\mathrm{d}\phi_p}{2\pi} e\boldsymbol{v}_p T \sum_{\varepsilon_n} g(\hat{p}_f, \boldsymbol{r}, \varepsilon_n)$$

$$\mathcal{J}_x\left(p_y, \varepsilon \middle| r = 0, \theta = \frac{\pi}{2}\right)$$

Congratulations Dale!

Thanks for the beautiful physics you created and stimulated !