From Spontaneous Symmetry Breaking to Topological Order

Anomalous Hall Effect in Superfluid ^{3}He

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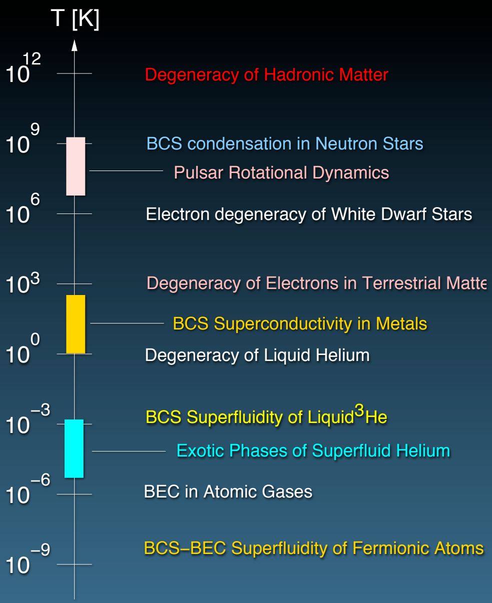
Northwestern University

• Hao Wu • Oleksii Shevtsov

- Unconventional Superconductivity
- Spin-Fluctuation Mediated Pairing
- The Helium Paradigm
- Broken T & P Symmetry
- Topological Order in ^{3}He
- Chiral Edge Currents in ^{3}He

NSF Grant DMR-1508730

BCS Pairing from 10^{-9} K to 10^{+9} K



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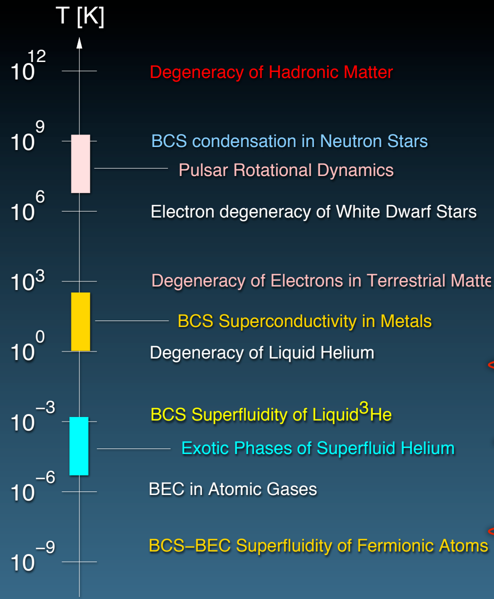


1908 Helium is liquified
1911 Superconductivity discovered in Hg
1933 Diamagnetism - Meissner Effect
1935 London Theory
1950 Ginzburg-Landau Theory

1957 BCS Theory
1957 Landau Fermi Liquid Theory
1957 Abrikosov's Theory of Type II SC
1959 Gauge-Invariant Pairing Theory
1959 Field Theory formulation of BCS Pairing
1959 Pairing in Nuclei and Nuclear Matter
1961 Theory of Spin-Triplet Pairing
1962 Josephson Effect

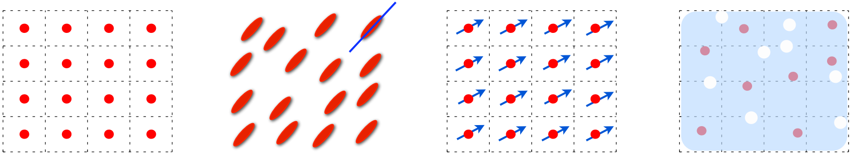
1967 Pulsars discovered - Hewish & Bell
1969 Pulsar Glitches observed in Vela
1971 - 1985 - Superfluid Hydrodynamics NS

1972 Discovery of Triplet, P-wave, Superfluid ^{3}He Phases
1979 Discovery of Heavy Electron Superconductors
1982 Exotic Pairing in U-based Heavy Fermions
1986 High Tc Superconductivity in Oxides
1994 Exotic Pairing in Sr_2RuO_4
1995 D-wave Pairing Discovered in YBCO
2001 Coexistent Ferromagnetism & Superconductivity
2008 Superconductivity in Fe-based Materials

1992 - 2008 Topological Superfluids and Superconductors

1995 Discovery of Bose-Einstein Condensation of Rb
1998 Discovery Quantized Vortices in BEC
2003 Degeneracy of Cold Fermionic Atoms - ^{6}Li, ^{40}K
2007 BEC-BCS Condensation in ^{6}Li, ^{40}K

Broken Symmetry, Phase Transitions and Long-Range Order



Solid — Nematic — Ferromagnet — Super-liquid

Translations	Space Rotations	Spin Rotation	Gauge
G_{trans}	$SO(3)_L$	$SO(3)_S$	$U(1)_N$

$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r}) \qquad Q_{ij} = Q(T)\left(\mathbf{n}_i\mathbf{n}_j - \frac{1}{3}\delta_{ij}\right) \qquad \mathbf{M} = \gamma\,\langle \mathbf{S}\rangle$$

$$\Psi = \langle \psi(\mathbf{r})\rangle \simeq \sqrt{N/V}\, e^{i\vartheta}$$

- Break one or more spin/space-group symmetries in conjunction with $U(1)_N$

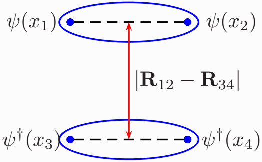
Superconductivity with Unconventional BCS Pairing

- Phases of liquid ^{3}He exhibit *all* of these broken symmetries!

- Two particle correlations for $s = 1/2$ Fermions: $x_1 = (\mathbf{r}_1, \alpha_1)$ etc.

$$G_2(x_1, x_2; x_3, x_4) = \langle \psi(x_1)\psi(x_2)\psi^\dagger(x_3)\psi^\dagger(x_4) \rangle$$

- Cooper Instability $\rightsquigarrow$ Long-range Order of Bound Fermion Pairs



- $G_2(x_1, x_2; x_3, x_4) \xrightarrow[|R_{12}-R_{34}|\to\infty]{} \Psi(x_1, x_2)\, \bar{\Psi}(x_3, x_4)$
- Macroscopic 2-particle state

$$\Psi_{\alpha_1\alpha_2}(\mathbf{r}_1, \mathbf{r}_2) \to \begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\downarrow\uparrow} & \Psi_{\downarrow\downarrow} \end{pmatrix}$$

- Total Spin $S = 0, 1$ and Orbital Angular Momentum $L = 0, 1, 2, 3, \ldots$

- Internal Structure of Cooper Pairs $\rightsquigarrow$ "Unconventional Superconductivity"

Pairing Symmetry Classes for Isotropic Normal Fermi Systems

- Total Spin, $S = 0, 1$ and ▸ relative Angular Momentum $L = 0, 1, 2, 3, \ldots$

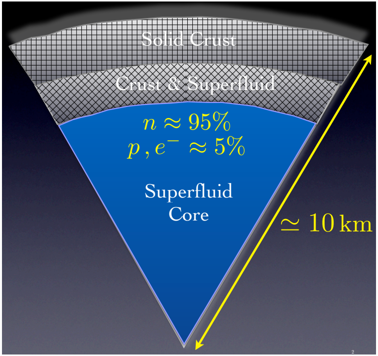
Name	Spin S	Orbital L
Singlet Pairing	0	even
Triplet Pairing	1	odd
S-wave Pairing	0	0
D-wave Pairing	0	2
P-wave Pairing	1	1
F-wave Pairing	1	3

- S-wave Pairing (only $\mathtt{U}(1)_{\mathrm{N}}$ is broken) ["Conventional" Superconductors, $NbSe_2$]
- D-wave Pairing ($\mathtt{U}(1)_{\mathrm{N}}$ and $\mathtt{SO}(3)_{\mathrm{L}}$ are broken) [$YBa_2Cu_3O_{7-x}$,$CeCoIn_5$,URu_2Si_2]
- P-wave Pairing ($\mathtt{U}(1)_{\mathrm{N}}$, $\mathtt{SO}(3)_{\mathrm{S}}$ and $\mathtt{SO}(3)_{\mathrm{L}}$ are broken) [^{3}He, Sr_2RuO_4 ,UGe_2]
- F-wave Pairing ($\mathtt{U}(1)_{\mathrm{N}}$, $\mathtt{SO}(3)_{\mathrm{S}}$ and $\mathtt{SO}(3)_{\mathrm{L}}$ are broken) [UPt_3]

- *Superconducting Classes*, G. E. Volovik and L. P. Gorkov, JETP 61, 843 (1985)

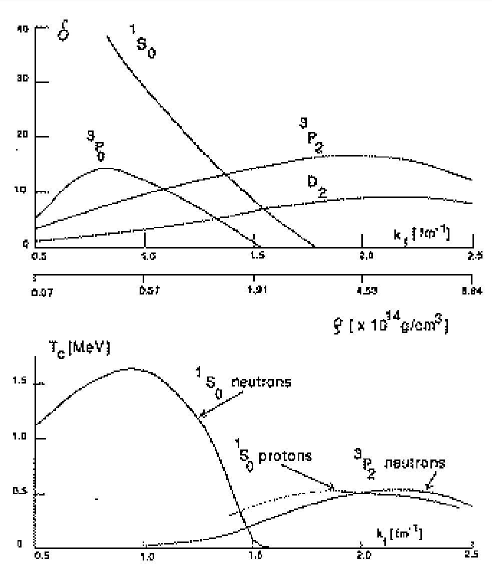
Physics at the Falls Workshop:

- ▶ “Pairing Phenomena from Neutron Stars to Cold Atoms”



- ▶ Baym and Pethick, Ann. Rev. Nucl. Phys. 25, 27 (1975)

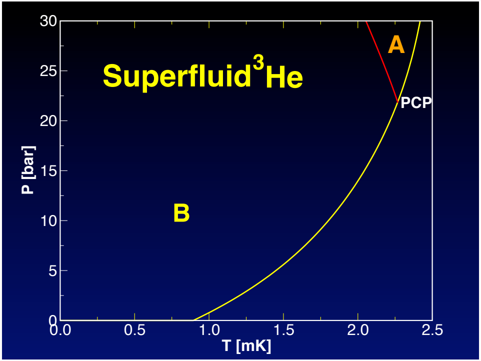
  - Superfluid Neutrons and Superconducting Protons
  - Crust: $^1S_0$ Neutron Pairs
  - Core: $^3P_2$ Neutron Pairs & $^1S_0$ Proton Pairs
    - ▶ JAS et al., Phys. Rev. D 17, 1524 (1978)



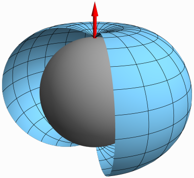
- ▶ Pairing Gaps: R. Tamagaki, Prog. Theor. Phys. 44, 905 (1970)

Symmetry of Normal Liquid ^{3}He: $\mathtt{G} = \mathtt{SO(3)_S} \times \mathtt{SO(3)_L} \times \mathtt{U(1)_N} \times \mathtt{P} \times \mathtt{T}$

Phase Diagram of Bulk ^{3}He



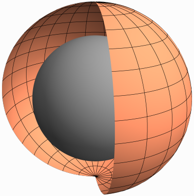
Chiral ABM State $\vec{l} = \hat{\mathbf{z}}$



$L_z = 1,\ S_z = 0$

$\mathbf{d}_z = \Delta\,(\hat{p}_x + i\hat{p}_y)$

“Isotropic” BW State



$J = 0,\ J_z = 0$

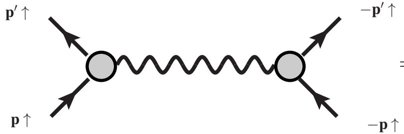
$\mathbf{d}_\alpha = \hat{p}_\alpha,\ \alpha = x, y, z$

Spin-Triplet, P-wave Order Parameter

$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix} = \begin{pmatrix} -\mathbf{d}_x + i\mathbf{d}_y & \mathbf{d}_z \\ \mathbf{d}_z & \mathbf{d}_x + i\mathbf{d}_y \end{pmatrix}$$

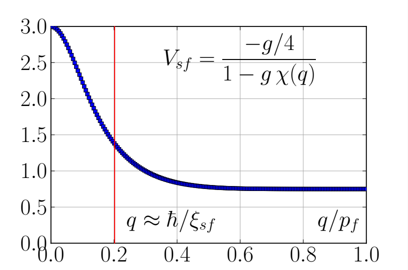
- A. Layzer and D. Fay, Int. J. Magn. 1, 135 (1971)

$V_{\text{sf}}(\mathbf{p},\mathbf{p}') =$



$$= \frac{-g/4}{1-g\,\chi(\mathbf{p}-\mathbf{p}')}$$

$$-g_l = (2l+1)\int\frac{d\Omega_{\hat{p}}}{4\pi}\int\frac{d\Omega_{\hat{p}'}}{4\pi}\,V_{\text{sf}}(\mathbf{p},\mathbf{p}')\,P_l(\hat{p}\cdot\hat{p}')$$



- $-g_l$ is a function of $g \approx 0.75$ and $\xi_{\text{sf}} \approx 5\,\hbar/p_f$
- $l = 1$ (p-wave) is dominant pairing channel
  - p-wave basis functions:

$$\hat{p}_z \sim \cos\theta_{\hat{p}}$$
$$\hat{p}_x + i\hat{p}_y \sim \sin\theta_{\hat{p}}\ e^{+i\phi_{\hat{p}}}$$
$$\hat{p}_x - i\hat{p}_y \sim \sin\theta_{\hat{p}}\ e^{-i\phi_{\hat{p}}}$$

- $l = 3$ (f-wave) is the sub-dominant channel
- $S = 1$ pairing fluctuations in V_{sf} ⇝ A-phase

W. Brinkman, J. Serene, and P. Anderson, PRA 10, 2386 (1974)

Are there electronic superconductors with broken symmetry phases analogous to ^{3}He?

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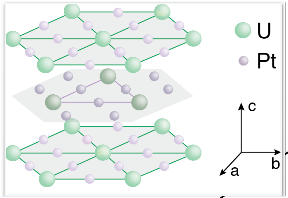
UPt_3 T_c=0.56 K

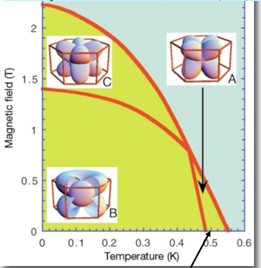


- S=1
- ``f-wave", $L_z = 2$

$$\Delta(\mathbf{p}) \sim p_z\,(p_x + i p_y)^2$$

- chiral ✔

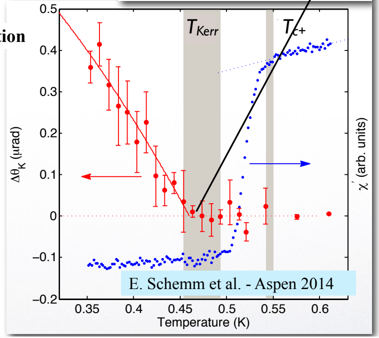
A. Huxley et al. - Nature, 406, 160 (2000)
T. Champel & V. Mineev - PRL, 86, 4903 (2001)



multiple SC phases

JAS, Adv. Phys. 43, 113(1994)

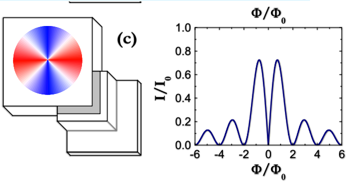
Kerr rotation



E. Schemm et al. - Aspen 2014

Josephson Interferometry

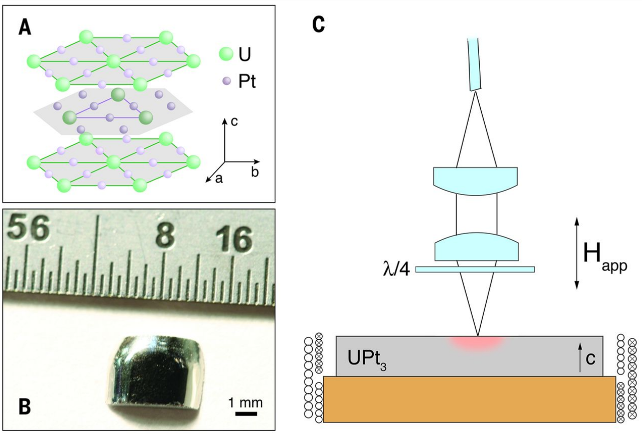
J. Strand et al. - PRL 103, 197002 (2009)



Polar Kerr Effect Measurements on UPt_3

Kerr effect - optical polarization rotation on reflection: $\theta_K \approx \frac{4\pi}{n(n^2-1)\omega}\ \sigma_{xy}(\omega)$

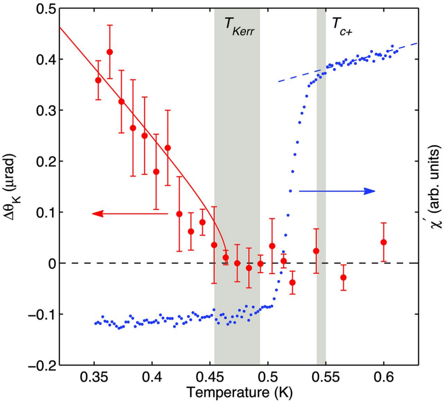
- $\theta_K \neq 0 \quad \rightsquigarrow \quad$ broken P and T
  - S.-K. Yip & JAS, J. Low Temp. Phys. 86, 257 (1992).



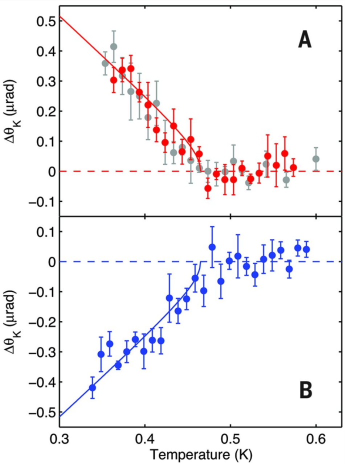
E. Schemm et al. Science (2015)
Standford (Kapitulnik) - Northwestern (Halperin)

Evidence for Broken P & T with E_{2u} Symmetry in UPt_3

Polar Kerr Effect: $\mathbf{q}||\hat{\mathbf{z}}$



E. Schemm et al. Science (2014)



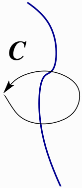
- A: $\vec{d} = \hat{\mathbf{z}}\,\Delta\hat{p}_z(\hat{p}_x^2 - \hat{p}_y^2)$ **T Symmetric**
- B: $\vec{d} = \hat{\mathbf{z}}\,\Delta\hat{p}_z(\hat{p}_x \pm i\hat{p}_y)^2$ **T Broken**
- E_{2u} Pairing: JAS, Adv. Phys. (1994)

- Polarization rotation onsets: $T \leq T_{c_2}$
- θ_K is weak field-trainable
- Single Chiral Domain

Spontaneous Symmetry Breaking $\rightsquigarrow$ Topological Order

Topology in Real Space

$\Psi(\mathbf{r}) = |\Psi(r)|\, e^{i\vartheta(\mathbf{r})}$



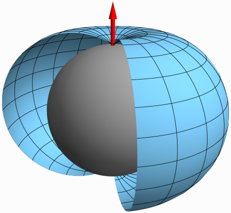
Phase Winding

$$N_C = \frac{1}{2\pi}\oint_C d\mathbf{l}\cdot \frac{1}{|\Psi|}\mathrm{Im}[\boldsymbol{\nabla}\Psi] \in \{0, \pm 1, \pm 2, \ldots$$

- Massless Fermions confined in the Vortex Core

Chiral Symmetry $\rightsquigarrow$ Topology in Momentum Space

$\Psi(\mathbf{p}) = \Delta(p_x \pm i p_y) \sim e^{\pm i\varphi_{\mathbf{p}}}$



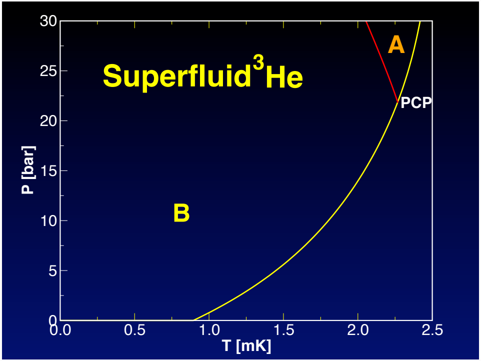
Topological Quantum Number: $L_z = \pm 1$

$$N = \frac{1}{2\pi}\oint d\mathbf{p}\cdot \frac{1}{|\Psi(\mathbf{p})|}\mathrm{Im}[\boldsymbol{\nabla}_{\mathbf{p}}\Psi(\mathbf{p})] = L_z$$

- Massless Chiral Fermions

Symmetry of Normal Liquid ^{3}*He*: $G = SO(3)_S \times SO(3)_L \times U(1)_N \times P \times T$

Phase Diagram of Bulk ^{3}He



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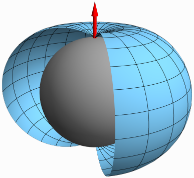

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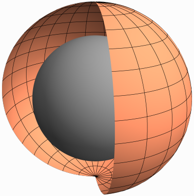
"Isotropic" BW State



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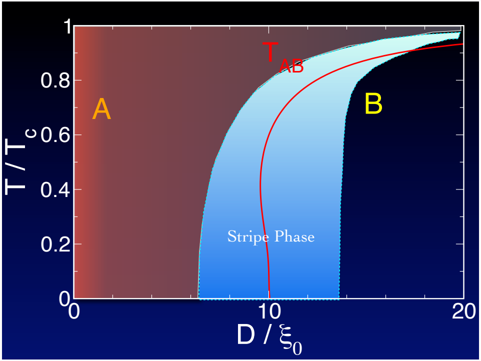
^{3}He Films: A. Vorontsov & JAS PRL 2007



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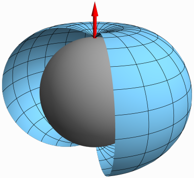

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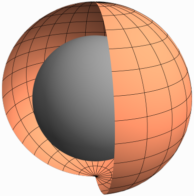

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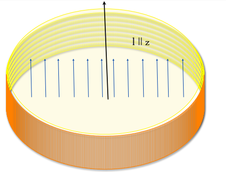
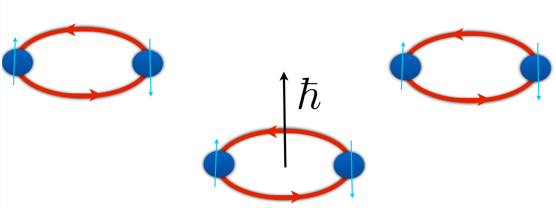
Chiral P-wave BEC Molecules or BCS Pairs (N Fermions):

$$|\,\Phi_N\,\rangle = \left[\iint d\mathbf{r}_1 d\mathbf{r}_2\; \varphi_{s_1 s_2}(\mathbf{r}_1 - \mathbf{r}_2)\; \psi^\dagger_{s_1}(\mathbf{r}_1)\psi^\dagger_{s_2}(\mathbf{r}_2)\right]^{N/2} |\,\text{vac}\,\rangle$$

- $\varphi_{s_1 s_2}(\mathbf{r}) = f(|\mathbf{r}|/\xi)\;(x + iy)\;\chi_{s_1 s_2}(S = 1, M_S = 0)$
  - BEC ($\xi < a$) vs. BCS ($\xi > a$)



$L_z = (N/2)\hbar$



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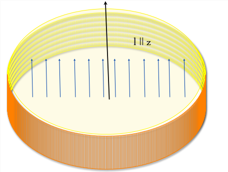

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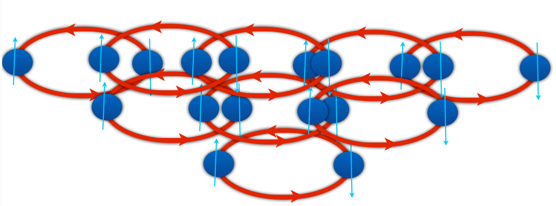
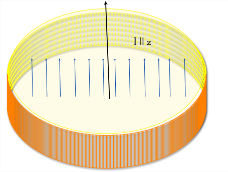


$L_z = (N/2)\hbar\,(a/\xi)^2 \ll (N/2)\hbar$? (P.W. Anderson & P. Morel, 1960, A. Leggett, 1975)

Angular Momentum of Chiral P-wave Condensates

Chiral P-wave BEC Molecules or BCS Pairs (N Fermions):

$$|\,\Phi_N\,\rangle = \left[\iint d\mathbf{r}_1 d\mathbf{r}_2\, \varphi_{s_1 s_2}(\mathbf{r}_1 - \mathbf{r}_2)\; \psi^\dagger_{s_1}(\mathbf{r}_1)\psi^\dagger_{s_2}(\mathbf{r}_2)\right]^{N/2} |\,\text{vac}\,\rangle$$



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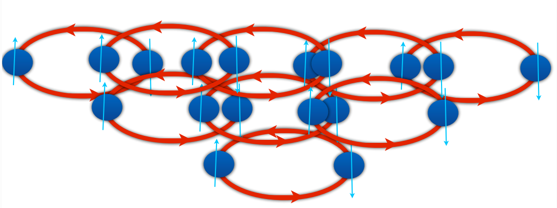
$L_z = (N/2)\hbar$



$L_z = (N/2)\hbar\,(a/\xi)^2 \ll (N/2)\hbar$? (P.W. Anderson & P. Morel, 1960, A. Leggett, 1975)

- $L_z|\,\Phi_N\,\rangle = (N/2)\hbar\,|\,\Phi_N\,\rangle$ independent of (a/ξ)! - McClure-Takagi (PRL, 1979)

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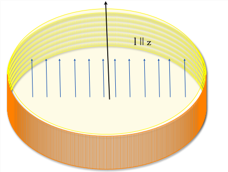

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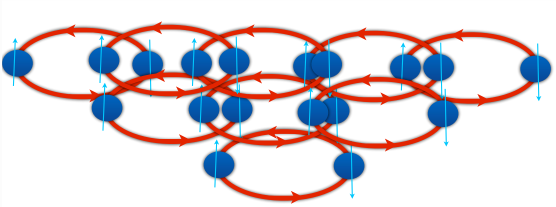
  - BEC ($\xi < a$) vs. BCS ($\xi > a$)

$L_z = (N/2)\hbar$

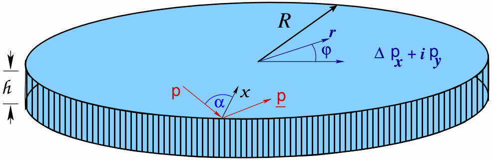


$L_z = (N/2)\hbar\,(a/\xi)^2 \ll (N/2)\hbar$? (P.W. Anderson & P. Morel, 1960, A. Leggett, 1975)

- $L_z|\,\Phi_N\,\rangle = (N/2)\hbar\,|\,\Phi_N\,\rangle$ independent of (a/ξ)! - McClure-Takagi (PRL, 1979)

BCS Limit: Currents are confined on the Edge

^{3}He-A confined in a thin cylindrical cavity - $h \ll \xi_0$ and $R \gg \xi_0$.



- 2D Chiral ABM State: $\vec{\mathbf{d}}(\mathbf{p}) = \Delta\,\hat{\mathbf{z}}\,(p_x + ip_y)/p_f \sim e^{+i\varphi_{\mathbf{p}}}$
- Fully Gapped: $|\vec{\mathbf{d}}(\mathbf{p})|^2 = \Delta^2$

Bogoliubov Equations for Fermionic Excitations: $\mathbf{p} \rightarrow \frac{\hbar}{i}\boldsymbol{\nabla}$

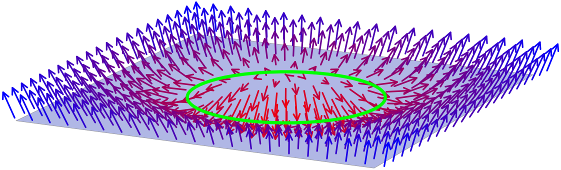
$$\begin{pmatrix} |\mathbf{p}|^2/2m^* - \mu & \Delta\,(p_x + ip_y)/p_f \\ \Delta\,(p_x - ip_y)/p_f & -|\mathbf{p}|^2/2m^* + \mu \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \varepsilon \begin{pmatrix} u \\ v \end{pmatrix}$$

Anderson's Iso-spin Representation with particle-hole (Nambu) matrices $\widehat{\vec{\tau}} = (\widehat{\tau}_1, \widehat{\tau}_2, \widehat{\tau}_3)$

$$\widehat{H} = \left(|\mathbf{p}|^2/2m - \mu\right)\widehat{\tau}_3 + \left[\Delta\, p_x\,\widehat{\tau}_1 \mp \Delta\, p_y\,\widehat{\tau}_2\right]/p_f = \vec{\mathbf{m}}(\mathbf{p}) \cdot \widehat{\vec{\tau}}$$

Nambu-Bogoliubov Hamiltonian for 2D ^{3}He-A : $\widehat{H} = \vec{\mathbf{m}}(\mathbf{p}) \cdot \widehat{\vec{\tau}}$

$\rightsquigarrow$ $\vec{\mathbf{m}} = (\, cp_x \,,\, \mp cp_y \,, \xi(\mathbf{p}))$ with $|\vec{\mathbf{m}}(\mathbf{p})|^2 = \left(|\mathbf{p}|^2/2m - \mu\right)^2 + c^2|\mathbf{p}|^2 > 0 \,, \mu \neq 0$



Topological Invariant for 2D ^{3}He-A $\leftrightarrow$ QED in d = 2+1 [G.E. Volovik, JETP 1988]:

$$N_{2D} = \pi \int \frac{d^2p}{(2\pi)^2}\, \hat{\mathbf{m}}(\mathbf{p}) \cdot \left(\frac{\partial \hat{\mathbf{m}}}{\partial p_x} \times \frac{\partial \hat{\mathbf{m}}}{\partial p_y} \right) = \begin{cases} \pm 1\,; & \mu > 0 \text{ and } \Delta \neq 0 \\ 0\,; & \mu < 0 \text{ or } \Delta = 0 \end{cases}$$

Nambu-Bogoliubov Hamiltonian for 2D ^{3}He-A : $\widehat{H} = \vec{\mathbf{m}}(\mathbf{p}) \cdot \widehat{\vec{\tau}}$

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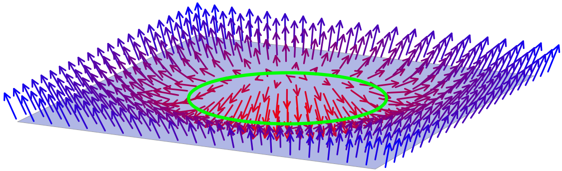

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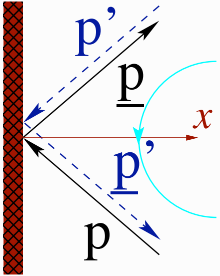
"Vacuum" $(\Delta = 0)$ with $N_{2D} = 0$ | ^{3}He-A$(\Delta \neq 0)$ with $N_{2D} = 1$

Zero Energy Fermions $\uparrow$ Confined on the Edge

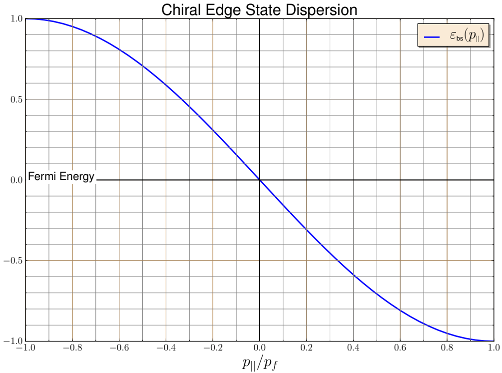
Edge Fermions: $G^{R}_{\text{edge}}(\mathbf{p},\varepsilon;x) = \frac{\pi\Delta|\mathbf{p}_x|}{\varepsilon+i\gamma-\varepsilon_{\text{bs}}(\mathbf{p}_{||})}\; e^{-x/\xi_\Delta}$

Confinement: $\xi_\Delta = \hbar v_f/2\Delta \approx 10^2 - 10^3\ \text{Å} \gg \hbar/p_f$

- $\varepsilon_{\text{bs}} = -c\,p_{||}$ with $c = \Delta/p_f \ll v_f$



- Broken P & T $\rightsquigarrow$ Edge Current



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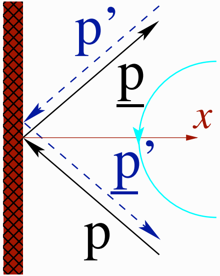

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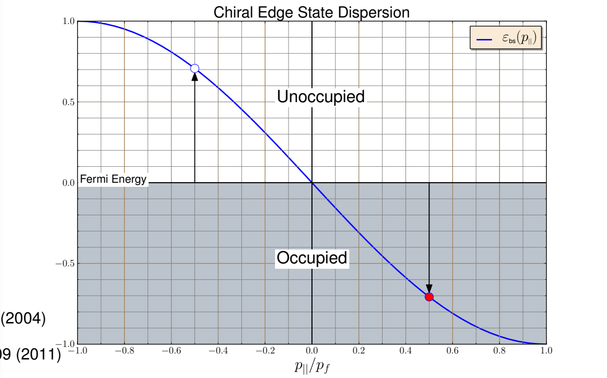
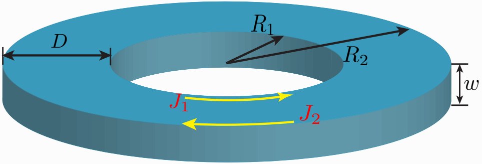


- M. Stone, R. Roy, PRB 69, 184511 (2004)
- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

^{3}He-A confined in a toroidal cavity



- $R_1, R_2, R_1 - R_2 \gg \xi_0$

- Sheet Current: $J = \frac{1}{4}\, n\, \hbar$ ($n = N/V =$ ^{3}He density)
- Counter-propagating Edge Currents: $J_1 = -J_2 = \frac{1}{4}\, n\, \hbar$
- Angular Momentum:

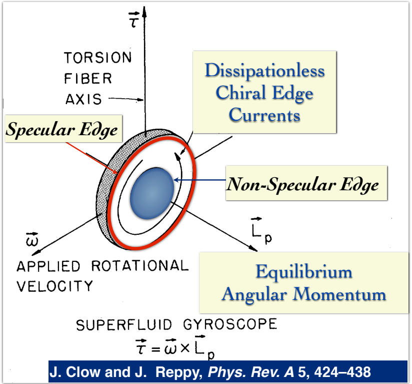
$$L_z = 2\pi\, h\, (R_1^2 - R_2^2) \times \frac{1}{4}\, n\, \hbar = (N/2)\, \hbar$$

McClure-Takagi Result

▶ J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

Possible Gyroscopic Experiment to Measure of $L_z(T)$

S. Davis, J. Parpia & J. Saunders (Cornell-RHUL)



Thermal Signature of Chiral Edge States

- Power Law for $T \lesssim 0.5 T_c$

$$L_z = (N/2)\hbar \, (1 - c\,(T/\Delta)^2)$$

Toroidal Geometry with Engineered Surfaces

- Incomplete Screening

$$L_z > (N/2)\hbar$$

Direct Signature of Edge Currents

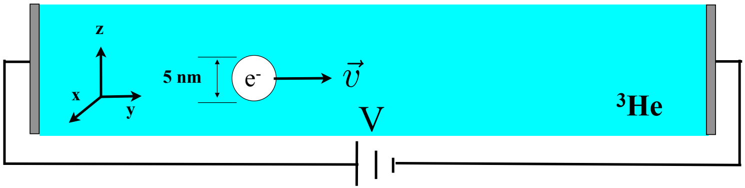
- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

Anomalous Hall Effect in ^{3}He-A

Detection of Broken *Time-Reversal* Symmetry of Cooper pairs in Superfluid ^{3}He-A

Hiroki Ikegami, Yasumasa Tsutsumi, Kimitoshi Kono, Science 341, 59-62 (2013)

RIKEN, Japan



Electron Mobility:

$$\vec{v} = \widehat{\mu} \cdot \vec{E}$$

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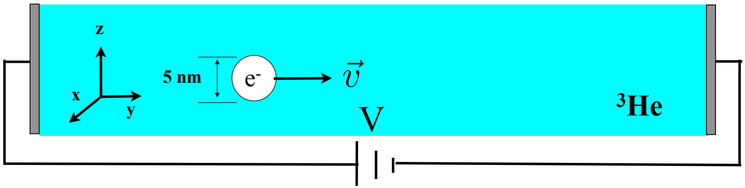

Electron Mobility:

$$\vec{v} = \widehat{\mu} \cdot \vec{E}$$

B-phase Mobility

$$\widehat{\mu} = \mu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Isotropic
Fully Gapped

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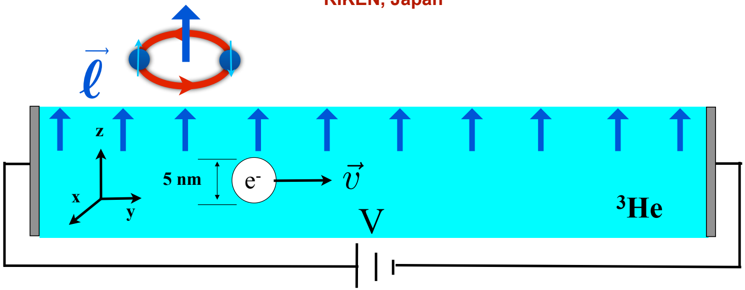

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B-phase Mobility

$$\widehat{\mu} = \mu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

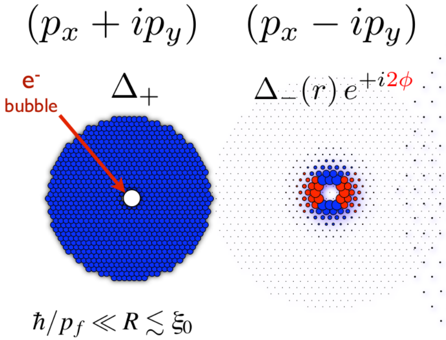
Isotropic
Fully Gapped

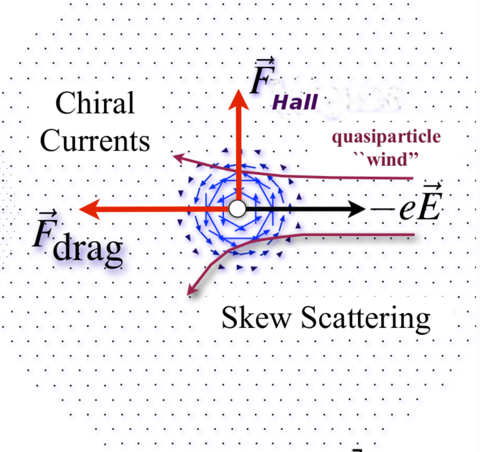
A-phase Mobility

$$\widehat{\mu} = \begin{pmatrix} \mu_\perp & \mu_{xy} & 0 \\ -\mu_{xy} & \mu_\perp & 0 \\ 0 & 0 & \mu_\parallel \end{pmatrix}$$

Anisotropic
Transverse Force

$\vec{\ell} = +\hat{\mathbf{z}}$ Structure of an Ion embedded in ^{3}He-A

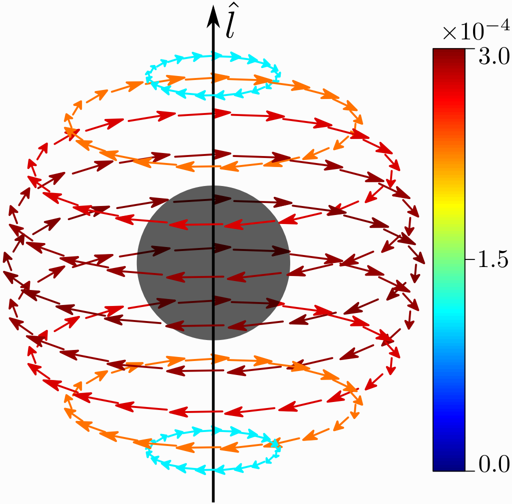




JAS & M. Eschrig, New J. Phys. 11, 075008 (2009)

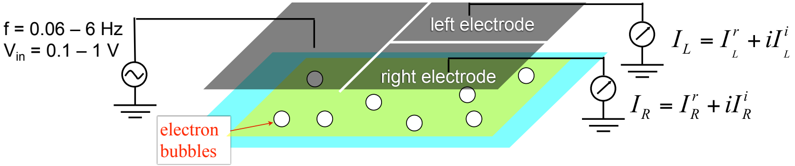
$$\vec{v} = \left[\mu_{\|} \left(\hat{\ell}\cdot\vec{E}\right)\hat{\ell} + \mu_{\perp}\,\hat{\ell}\times\left(\hat{\ell}\times\vec{E}\right) + \mu_{xy}\,\hat{\ell}\times\vec{E}\right]$$

$\frac{J_\phi(r,\phi)}{4\pi^3 v_F N_F k_B T_c}$ at $k_f r = 30$ for $k_f R = 11.17$



- Oleksii Shevtsov & JAS, 2016

Measurement of the Transverse e⁻ mobility in Superfluid ^{3}He Films



H. Ikegami, Y. Tsutsumi, K. Kono, **Science 341, 59-62 (2013)**

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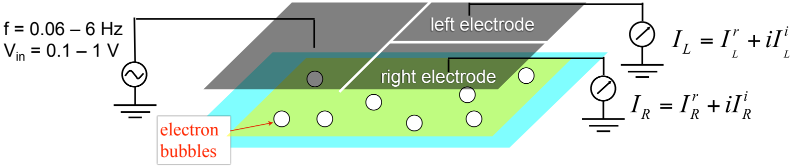
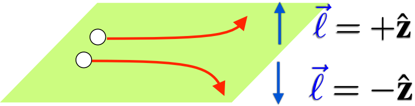


Transverse Force from *Skew* Scattering

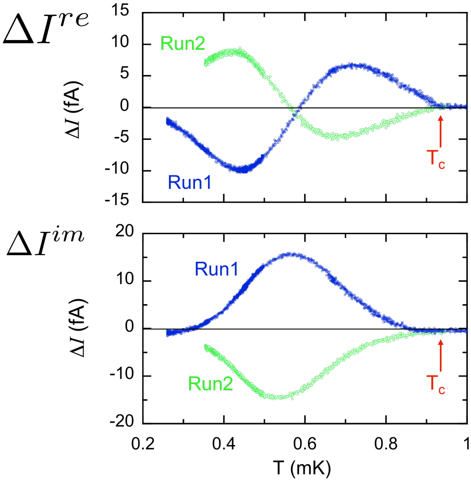
$$\rightsquigarrow \quad \Delta I = I_R - I_L \neq 0$$

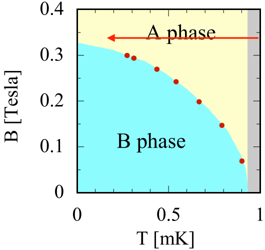
$$\vec{v} = \left[\mu_\perp \vec{E} + \mu_{xy}\,\hat{\ell} \times \vec{E}\right]$$



H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

Transverse e$^-$ bubble current in ^{3}He-A $\Delta I = I_R - I_L$





Single Domains:

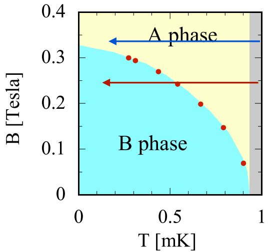
Run 1 $\vec{\ell} = +\hat{\mathbf{z}}$

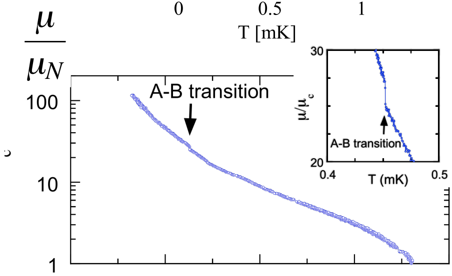
Run 2 $\vec{\ell} = -\hat{\mathbf{z}}$

$$\frac{|I_R - I_L|}{I_R + I_L} \approx 6\,\%$$

H. Ikegami, Y. Tsutsumi, K. Kono, **Science 341, 59-62 (2013)**

Zero Transverse e⁻ current in ^{3}He-B (*T - symmetric phase*)





H. Ikegami, Y. Tsutsumi, K. Kono, **Science 341, 59-62 (2013)**

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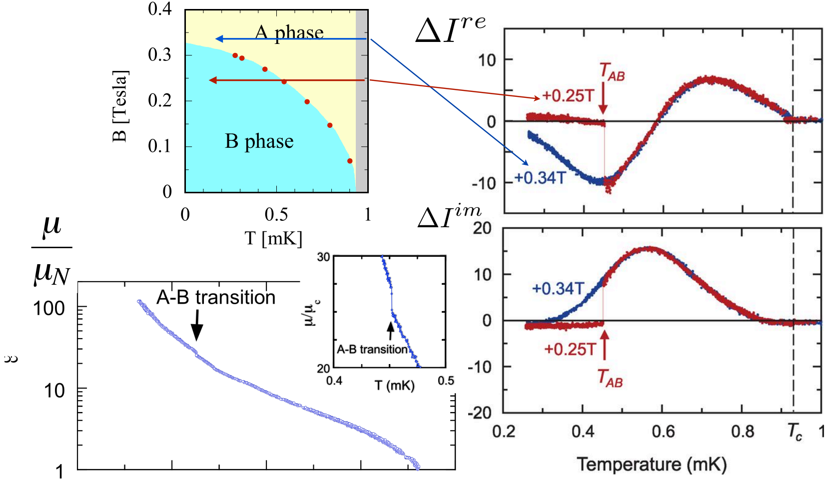
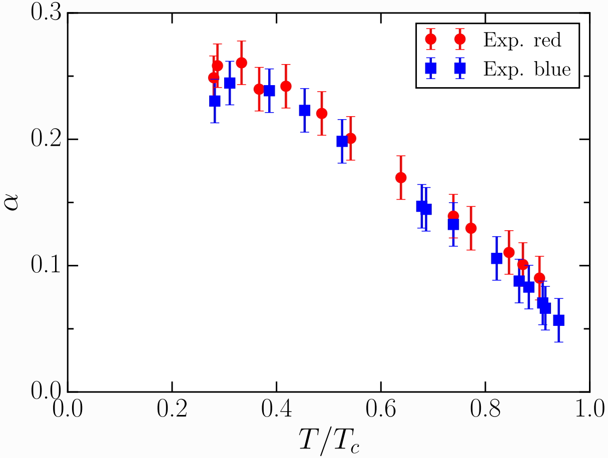


H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

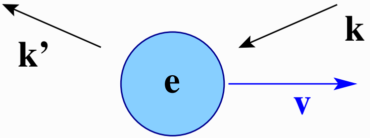
Anomalous Hall Angle: $\tan\alpha \approx \alpha = \frac{F_{xy}}{F_{xx}}$



H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59 (2013)

- Note: $\vec{F}_L = \frac{e}{c}\vec{v} \times \vec{B}$, is negligible: $|F_L|/|F_{xy}| \approx 10^{-3} - 10^{-6}$

Force on a Moving e^- Bubble from Thermally Excited Quasiparticles



$$\frac{d\mathbf{P}}{dt} = \sum_{\mathbf{k},\mathbf{k}'} \hbar(\mathbf{k}' - \mathbf{k}) f_{\mathbf{k}}(1 - f_{\mathbf{k}'})\Gamma_{\mathbf{v}}(\mathbf{k}', \mathbf{k}), \;\equiv e\,[\widehat{\mu}]^{-1}\,\mathbf{v}$$

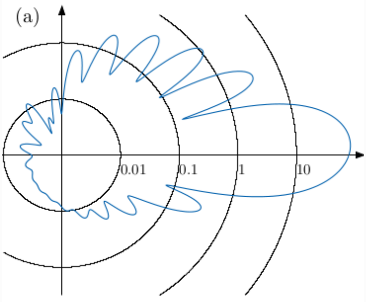
$$f_{\mathbf{k}} \to \bar{f}_{\mathbf{k}} = f_{\mathbf{k}}(E_{\mathbf{k}} - \hbar\mathbf{k}\cdot\mathbf{v}),$$

$$e(\hat{\mu}^{-1})_{ij} = n_3 p_F \int_0^\infty dE \left(-2\frac{\partial f}{\partial E}\right) [\langle\sigma_{ij}^{\text{symm}}(E)\rangle + \langle\sigma_{ij}^{\text{skew}}(E)\rangle], \quad i,j \in \{x,y,z\}, \tag{1}$$

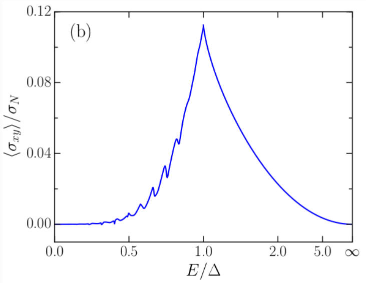
- G. Baym, C. Pethick, M. Salomaa, PRL 38, 845 (1977) - e^- Mobility in ^{3}He-B
- Oleksii Shevtsov & JAS (2016) - e^- Mobility in Chiral Superfluids

Theory of the Anomalous Hall Mobility of Negative Ions (e^- Bubbles) in ^{3}He-A

- Differential Cross-Section at $E = 1.01\Delta$
- Skew Scattering in 3D Chiral ^{3}He-A
- Hard Sphere QP-e^- Interaction with $k_f R = 11.16$ (fit μ_N)
- $l \lesssim 11$ channels dominate



- Chiral Fermions confined near the e^-
- Broadened by Nodal Fermions
- Skew Scattering Resonances

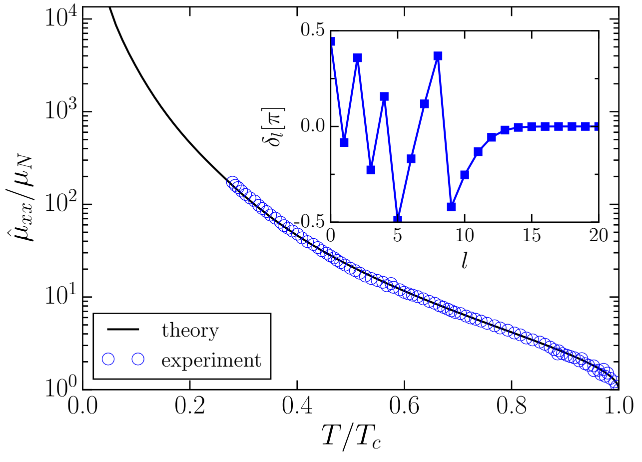


▶ Oleksii Shevtsov & JAS (2016) - e^- Mobility in Chiral Superfluids

Drag Force - Longitudinal Mobility:

- Expert: H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59 (2013)

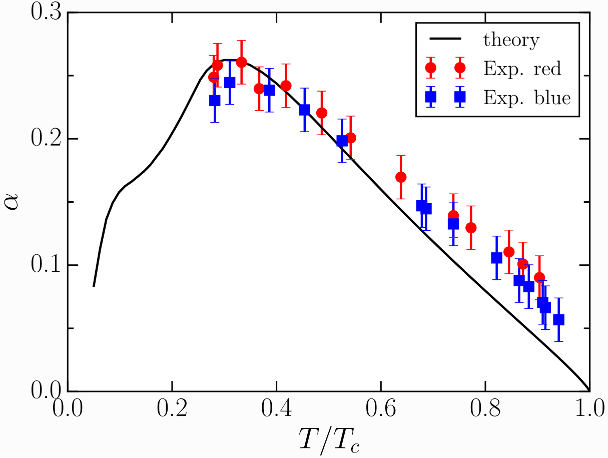
$\mu_{xx} = v_x / E_x$



^{3}He-e$^-$ Interaction: Hard-sphere with $k_f\, R = 11.16$ *fit* to normal-state mobility (μ_N)

- Oleksii Shevtsov & JAS (2016) - e^- Mobility in Chiral Superfluids

Anomalous Hall Angle: $\tan\alpha \approx \alpha = \frac{F_{xy}}{F_{xx}} = \frac{\mu_{xy}^{-1}}{\mu_{xx}^{-1}}$



- Expt: H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59 (2013)
- Theory: Oleksii Shevtsov & JAS (2016) - e^- Mobility in Chiral Superfluids

Experiment

- Detect Ground State Angular Momentum of ^{3}He-A
- Discover and Characterize New Phases of Nano-scale ^{3}He
- Local probes to detect and control Majorana states in ^{3}He-B

Theory

- Develop quantum transport theory for coupled nano-fluidic mechanical resonantors & oscillators
- Develop theory for acoustic, NMR and optical spectroscopy of topological edge/surface states of ^{3}He
- Develop theory of topological quantum matter beyond the mean-field level; strong-coupling, interactions and non-locality