Transport of electrons in superfluid $^3\mathrm{He}-\mathrm{A}$

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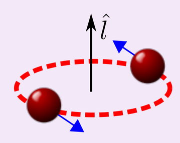
Superfluid phases of ^{3}He

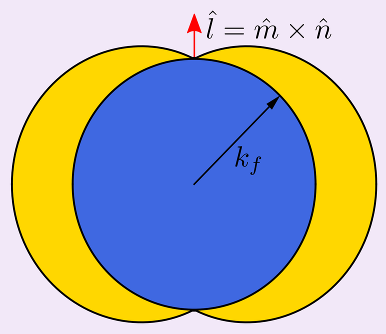
Symmetry of normal-state ^{3}He: $G = \mathrm{SO}(3)_S \times \mathrm{SO}(3)_L \times \mathrm{U}(1)_N \times \mathrm{P} \times \mathrm{T}$

Spin-triplet Cooper pairs: $S = 1, L_z = \hbar$



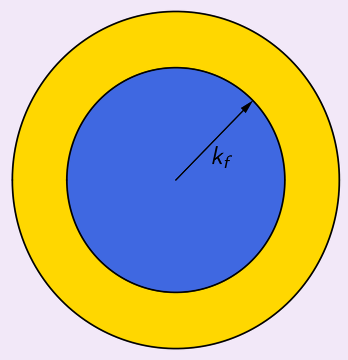
Spin-triplet p-wave order parameter: $\Delta_{\alpha\beta}(\mathbf{k}) = \vec{d}(\mathbf{k}) \cdot (i\vec{\sigma}\sigma_y)_{\alpha\beta}$, $d_\mu(\mathbf{k}) = A_{\mu j}\mathbf{k}_j$

ABM state (A-phase): $L_z = 1, S_z = 0$

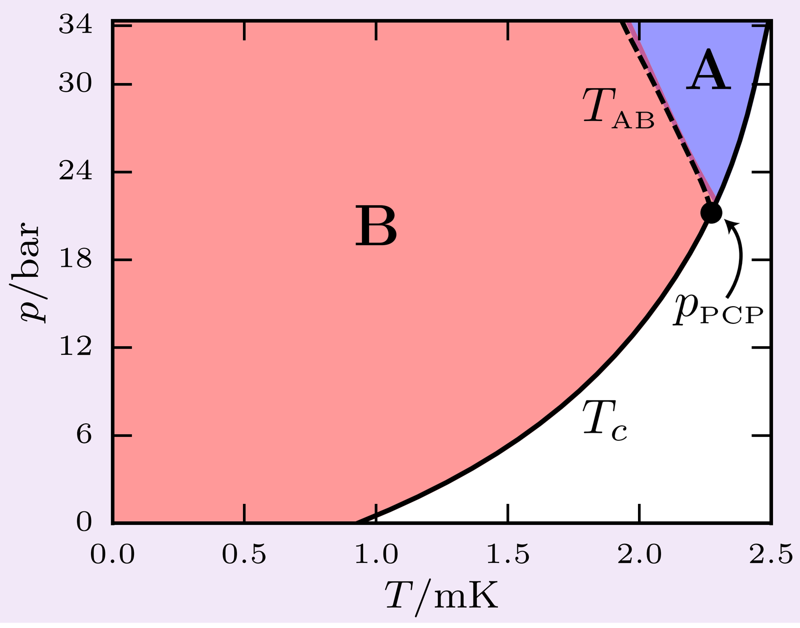


$A_{\mu j} = \Delta \hat{d}_\mu (\hat{m} + i\hat{n})_j$

BW state (B-phase): $J = 0, J_z = 0$



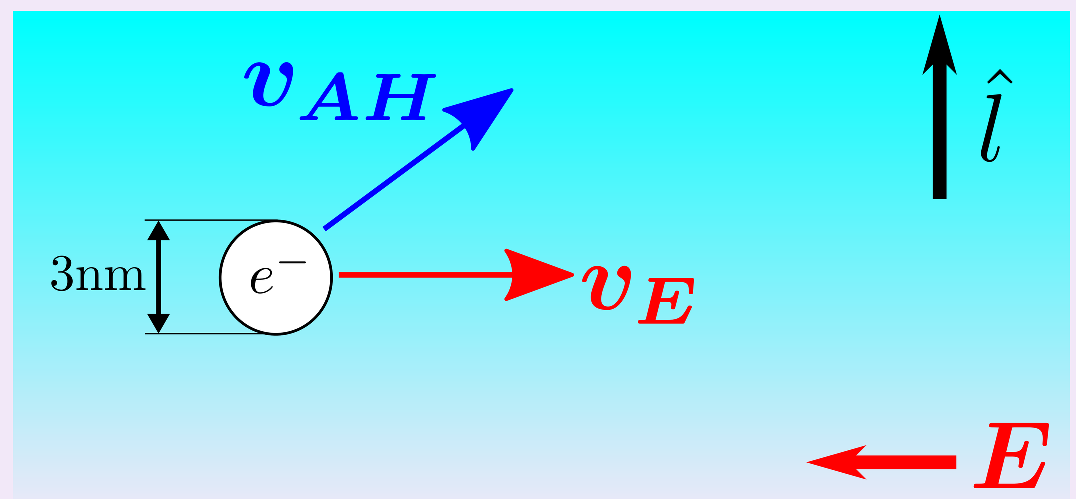
$A_{\mu j} = \Delta \delta_{\mu j}$



Motivation: experiments

RIKEN experiments on transport of electron bubbles in superfluid $^3\mathrm{He}-\mathrm{A}$ [1-2]

- electron creates a bubble in superfluid ^{3}He, $d \simeq 3$nm
- electric field $\mathbf{E}$ is applied parallel to the surface
- bubble acquires anomalous Hall component of velocity, $\mathbf{v}_{\mathrm{AH}} = \mu_{xy}\mathbf{E} \times \hat{\mathbf{l}}$



Scattering of thermal excitations determines the velocity, mobility, and drag force

$$M\frac{d\mathbf{v}}{dt} = e\mathbf{E} - \eta_\perp \mathbf{v} - \eta_{xy}\mathbf{v} \times \hat{\mathbf{l}}, \quad \mathbf{F_W} = \frac{e}{c}\mathbf{v} \times \mathbf{B_W}, \quad \mathbf{B_W} = -\frac{c}{e}\eta_{xy}\hat{\mathbf{l}}$$

Connection to: (i) mobility, $\overleftrightarrow{\eta} = e\overleftrightarrow{\mu}^{-1}$; (ii) Hall angle, $\tan\alpha = \frac{v_y}{v_x} = \frac{\eta_{xy}}{\eta_\perp}$

Theoretical description of electron bubbles

Bogoliubov-de Gennes equation for $^3\mathrm{He}-\mathrm{A}$:

$$\hat{H}_S\Psi(\mathbf{r}) = E\Psi(\mathbf{r}), \quad \hat{H}_S = \begin{pmatrix} \hat{H}_N & \Delta(\hat{\mathbf{k}}) \\ \Delta^\dagger(\hat{\mathbf{k}}) & -\hat{H}_N \end{pmatrix}, \quad \hat{H}_N = \frac{\hbar^2\mathbf{k}^2}{2m^*} - \mu, \quad \Delta(\hat{\mathbf{k}}) = \sigma_x \Delta \frac{\mathbf{k}_x + i\mathbf{k}_y}{k_f}$$

$\Psi(\mathbf{r}) = (u_\uparrow(\mathbf{r}), u_\downarrow(\mathbf{r}), v_\downarrow(\mathbf{r}), v_\uparrow(\mathbf{r}))^{\mathrm{T}}$ – Nambu spinor

Quasiparticle spectrum:

$E_\mathbf{k} = \sqrt{\xi_k^2 + |\Delta(\hat{\mathbf{k}})|^2}$, $\xi_k = \frac{\hbar^2 k^2}{2m^*} - \mu$ is the normal-state spectrum

Single-particle propagators:

$$\hat{G}_N^R(\mathbf{k}, E) = \frac{1}{\varepsilon^2 - \xi_k^2}\begin{pmatrix} \varepsilon + \xi_k & 0 \\ 0 & \varepsilon - \xi_k \end{pmatrix}, \quad \varepsilon = E + i\eta$$

$$\hat{G}_S^R(\mathbf{k}, E) = \frac{1}{\varepsilon^2 - E_\mathbf{k}^2}\begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}$$

Quasiparticle scattering is described with a t-matrix:

$$\hat{T}_S^R(\mathbf{k}', \mathbf{k}, E) = \hat{T}_N^R(\mathbf{k}', \mathbf{k}) + \int \frac{d^3k''}{(2\pi)^3}\hat{T}_N^R(\mathbf{k}', \mathbf{k}'')\left[\hat{G}_S^R(\mathbf{k}'', E) - \hat{G}_N^R(\mathbf{k}'', E)\right]\hat{T}_S^R(\mathbf{k}'', \mathbf{k}, E)$$

Normal-state scattering t-matrix:

$$\hat{T}_N^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = \begin{pmatrix} t_N^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}) & 0 \\ 0 & -[t_N^R(-\hat{\mathbf{k}}', -\hat{\mathbf{k}})]^\dagger \end{pmatrix}, \quad \hat{\mathbf{k}} = (\theta, \phi)$$

$$t_N^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = -\frac{1}{\pi N_F}\sum_{m=-\infty}^{\infty} t_N^m(u', u)e^{-im(\phi'-\phi)}, \quad t_N^m(u', u) = 4\pi\sum_{l=|m|}^{\infty} e^{i\delta_l}\sin\delta_l \Theta_l^m(u')\Theta_l^m(u)$$

$Y_l^m(\theta, \phi) = \Theta_l^m(\cos\theta)e^{im\phi}$ – spherical harmonic

Hard-sphere model for scattering phase shifts:

$\tan\delta_l = \frac{j_l(k_f R)}{n_l(k_f R)}$, j_l, n_l – spherical Bessel functions

Calculation of the generalized Stokes tensor

Generalized Stokes tensor:

$\eta_{ij} = n_3 p_F \int_0^\infty dE\left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E)$, $n_3 = \frac{k_F^3}{3\pi^2}$ – density of ^{3}He atoms

Transport scattering cross section:

$$\sigma_{ij}(E) = \sigma_{ij}^{(+)}(E) + \sigma_{ij}^{(-)}(E)$$

$$\sigma_{ij}^{(+)}(E) = \frac{3}{4}\int_{E \geq |\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'} \int_{E \geq |\Delta(\hat{\mathbf{k}})|} \frac{d\Omega_\mathbf{k}}{4\pi}[(\hat{\mathbf{k}}_i' - \hat{\mathbf{k}}_i)(\hat{\mathbf{k}}_j' - \hat{\mathbf{k}}_j)]\frac{d\sigma}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E)$$

$$\sigma_{ij}^{(-)}(E) = \frac{3}{4}\int_{E \geq |\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'} \int_{E \geq |\Delta(\hat{\mathbf{k}})|} \frac{d\Omega_\mathbf{k}}{4\pi}[\epsilon_{ijk}(\hat{\mathbf{k}}' \times \hat{\mathbf{k}})_k]3\frac{d\sigma}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E)\left[f(E) - \frac{1}{2}\right]$$

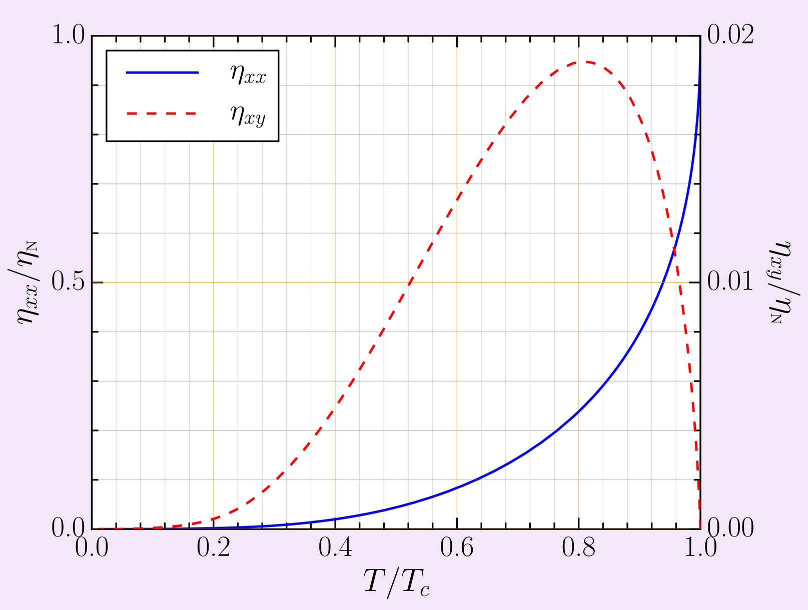
$$\frac{d\sigma}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2 \frac{E}{\sqrt{E^2 - |\Delta(\hat{\mathbf{k}}')|^2}} W(\hat{\mathbf{k}}', \hat{\mathbf{k}}) \frac{E}{\sqrt{E^2 - |\Delta(\hat{\mathbf{k}})|^2}}$$

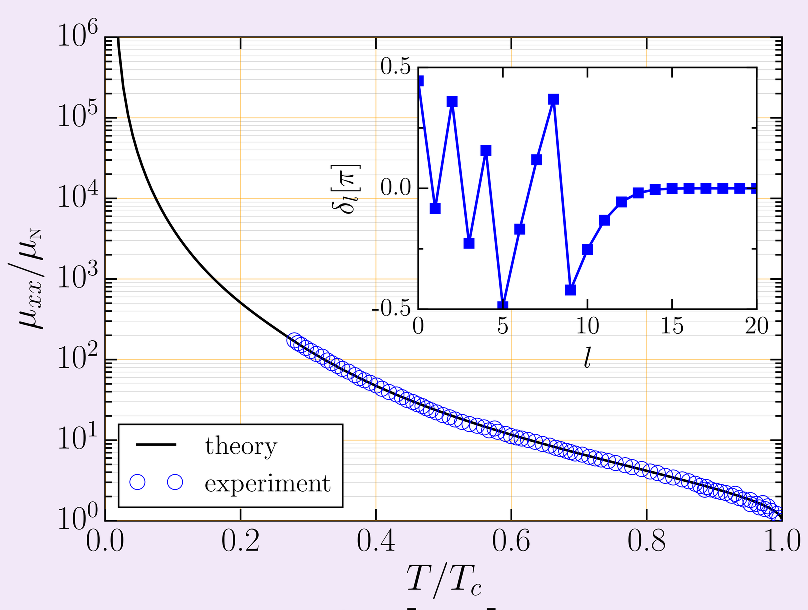
Scattering rate – Fermi's golden rule:

$\Gamma(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = \frac{2\pi}{\hbar}W(\hat{\mathbf{k}}', \hat{\mathbf{k}})\delta(E_{\mathbf{k}'} - E_\mathbf{k})$, $W(\hat{\mathbf{k}}', \hat{\mathbf{k}})$ is determined from $\hat{T}_S^R(\mathbf{k}', \mathbf{k}, E)$

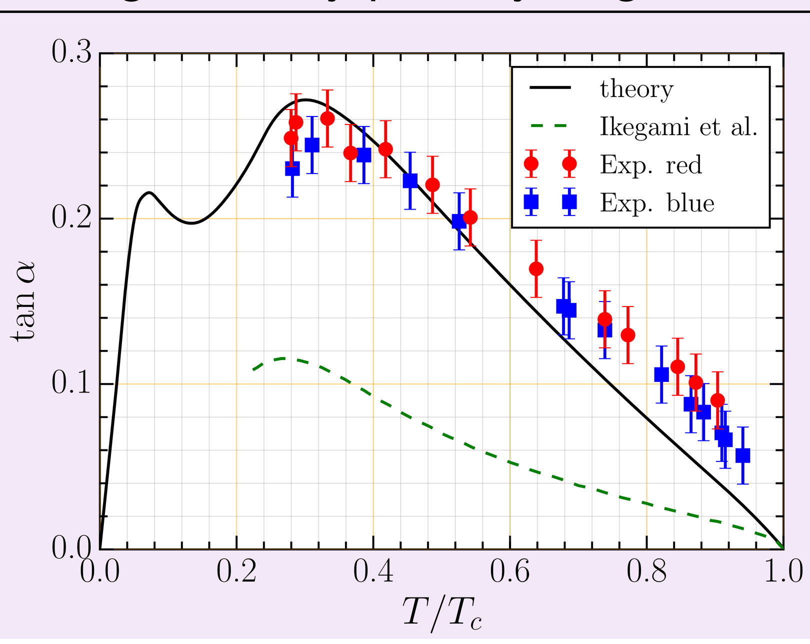
Comparison with experimental data

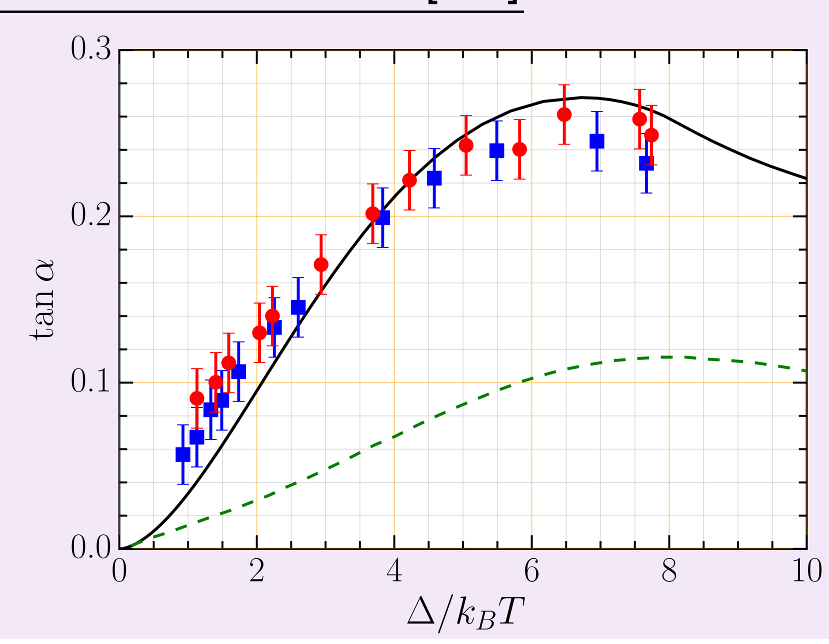
Stokes tensor and longitudinal mobility:





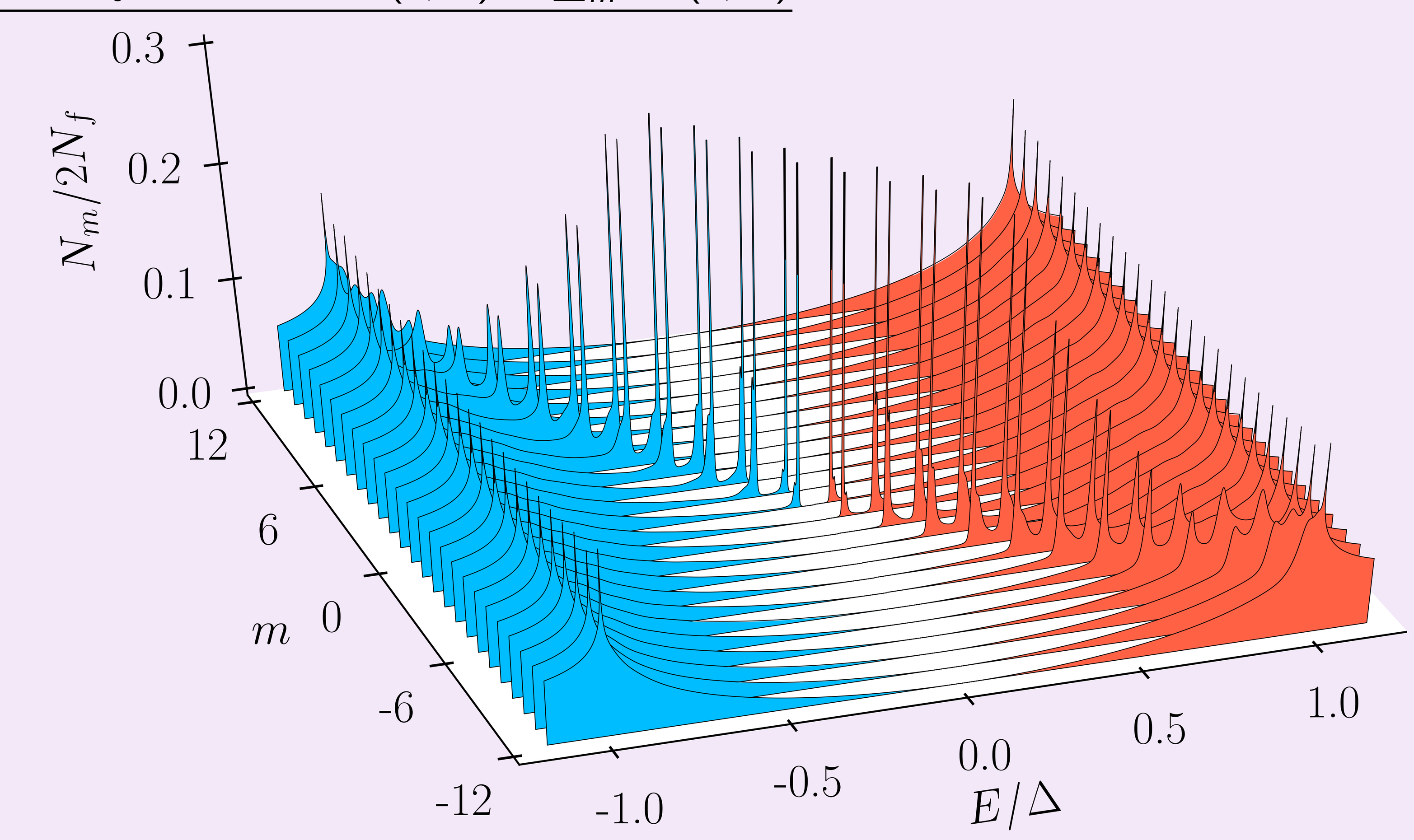
Hall angle: theory plots by Ikegami et al. are based on Refs. [3-4]



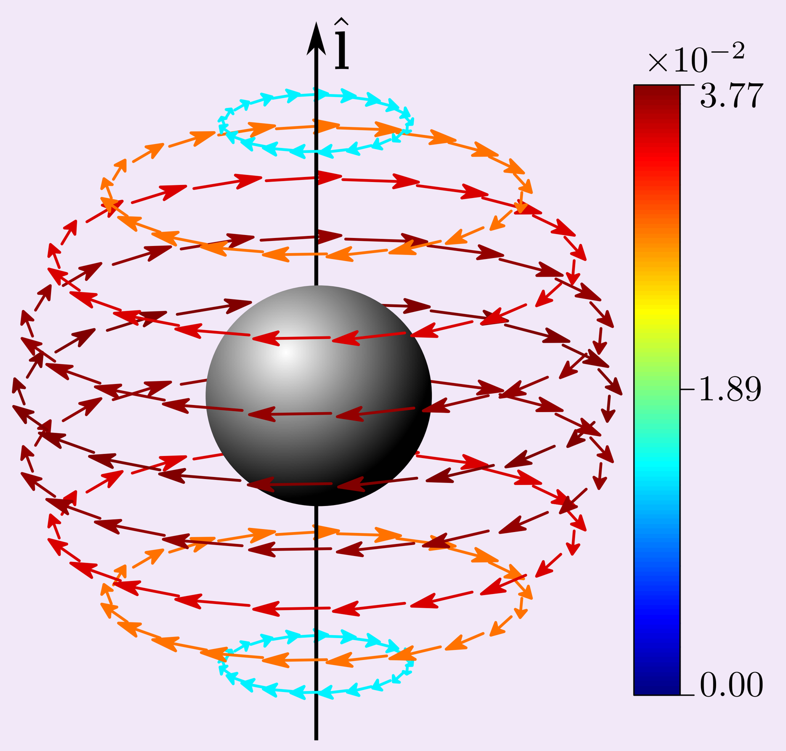


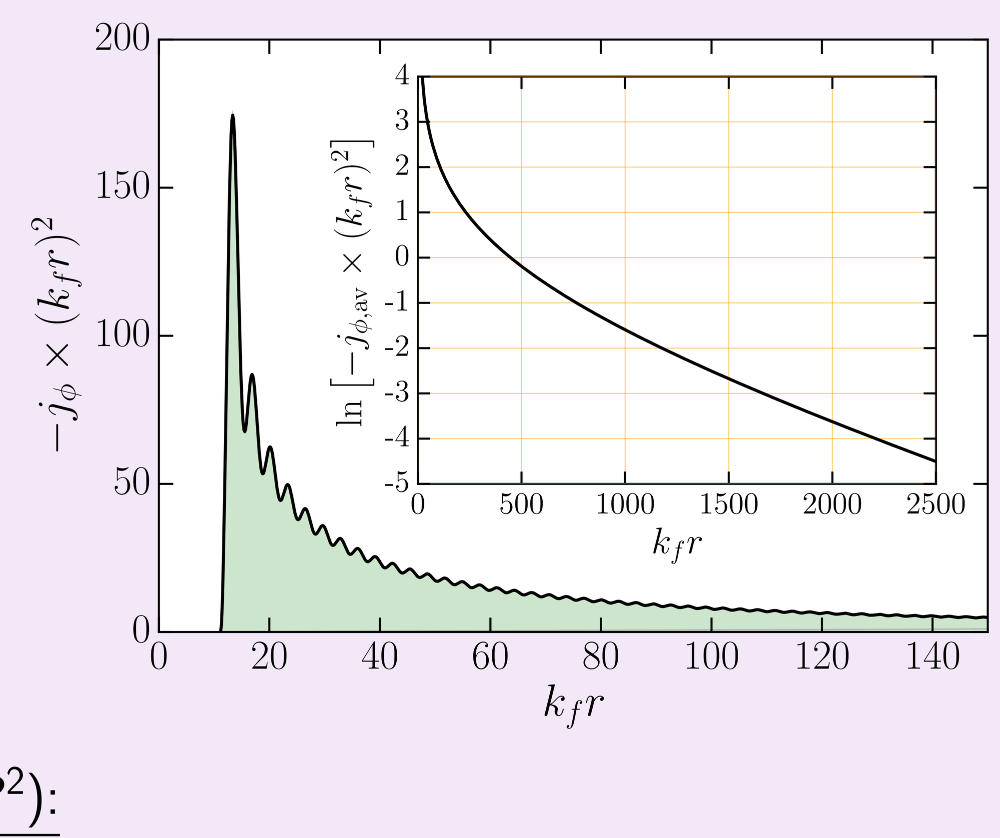
LDOS, current density, and differential cross section

Local density of states: $N(\mathbf{r}, E) = \sum_m N_m(\mathbf{r}, E)$

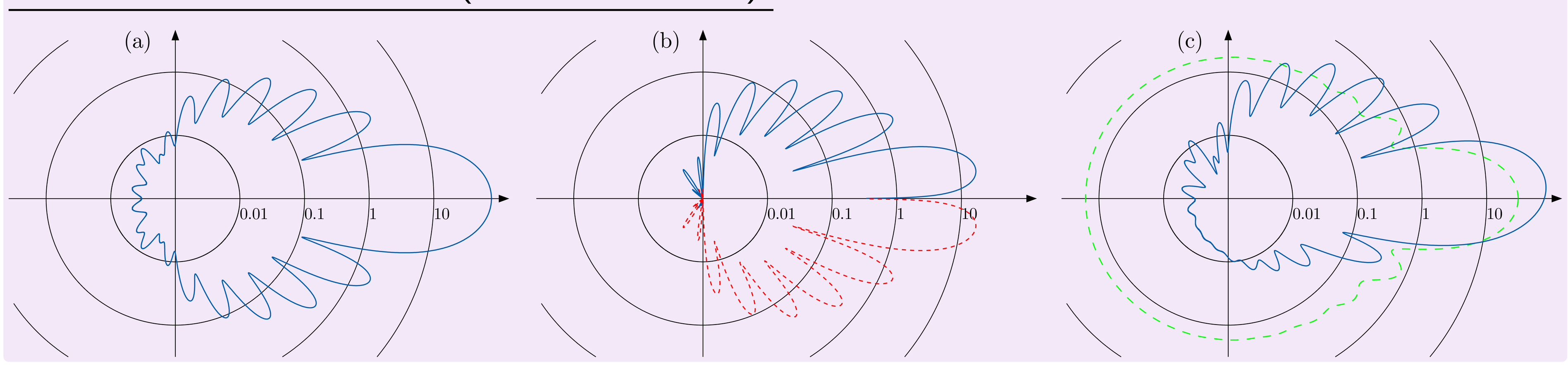


Current density (in units of $v_f N_f k_B T_c$): $\mathbf{j}(\mathbf{r}) = j_\phi(\mathbf{r})\mathbf{e}_\phi$, $\mathbf{L} = \hbar\frac{N_{\mathrm{bubble}}}{2}\hat{\mathbf{l}}$, $k_f R = 11.17$





Differential cross section (in units of πR^2):



References

[1] Ikegami et al. Science **341**, 59 (2013)
[2] Ikegami et al. JPSJ **84**, 044602 (2015)
[3] Salmelin et al. PRL **63**, 868 (1989)
[4] Salmelin and Salomaa PRB **41**, 4142 (1990)