Nonlinear Meissner Effects in Disordered Superconductors

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- N27-007 - NLM Intermodulation in Superconducting RF Cavities for Dark Matter Searches
- DOE NQISRC - Superconducting Quantum Materials and Systems Center (SQMS) DE-AC02-07CH11359

London's Electrodynamics

Normal electrons $\quad \mathbf{J}_n = \sigma_n \mathbf{E} \qquad \sigma_n = \dfrac{n_n e^2 \tau}{m}$

Super electrons $\quad \mathbf{J}_s = \dfrac{e}{m} n_s \mathbf{p}_s \qquad \mathbf{p}_s = \dfrac{\hbar}{2}\left(\boldsymbol{\nabla}\vartheta - \dfrac{2e}{\hbar c}\mathbf{A}\right)$

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No Vortices $\mathbf{p}_s \to -\dfrac{e}{c}\mathbf{A}$

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Dynamics ⇝ propagation for $\omega > \omega_p = \sqrt{4\pi n e^2/m}$

$$\left(\partial_t^2 - c^2\nabla^2 + \omega_p^2\right)\,\mathbf{A} = 0 \qquad \Lambda_L = \frac{c}{\omega_p}$$

⇝ Massive Gauge Bosons: $\quad E_{\mathbf{p}} = \sqrt{p^2c^2 + M_{\mathrm{A}}^2 c^4} \quad \boxed{M_{\mathrm{A}} = \hbar/c\Lambda_L}$

- P.W. Anderson, Phys. Rev. 130, 439 (1963). P. W. Higgs, Phys. Rev. Lett. 13, 508 (1964)

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Static Limit ⇝ Meissner Screening (confinement)

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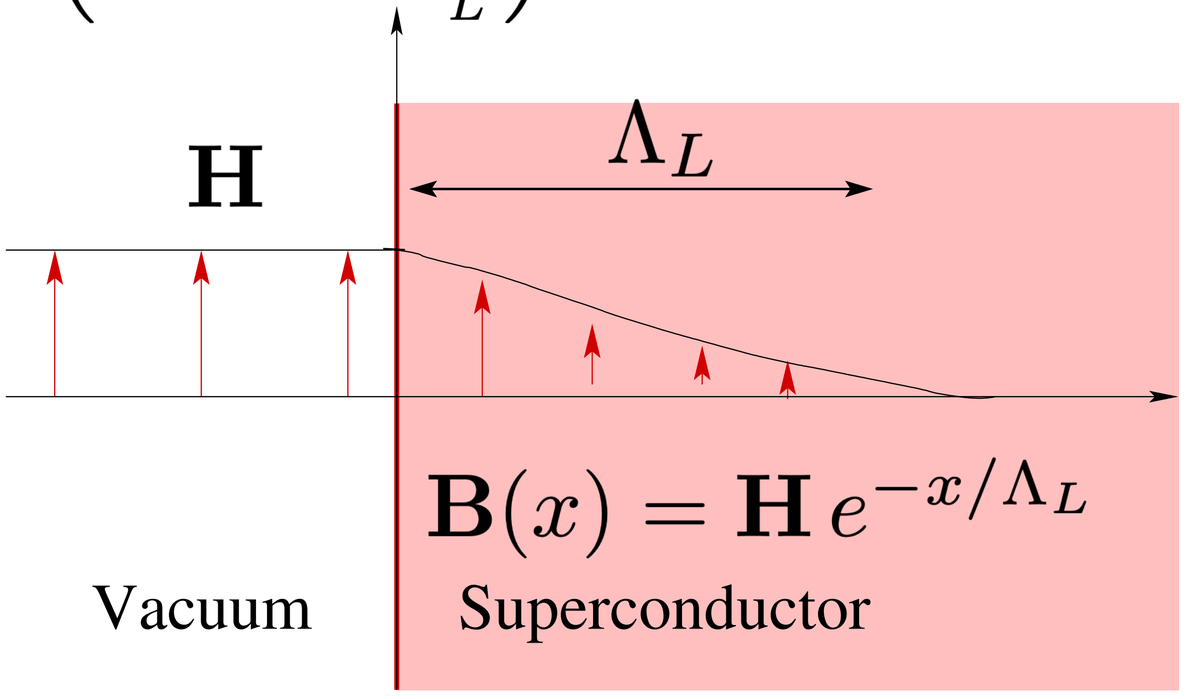
$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\mathbf{A} = \frac{4\pi}{c}\mathbf{J}_s \quad \rightsquigarrow \quad \left(-\nabla^2 + \frac{1}{\Lambda_L^2}\right)\mathbf{A} = 0$$

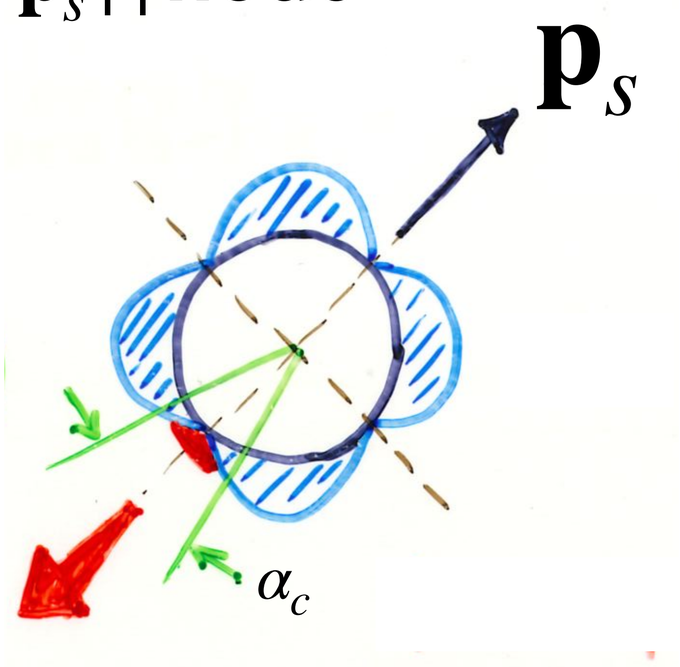
$$\left(-\nabla^2 + \frac{1}{\Lambda_L^2}\right)\mathbf{B} = 0$$

$$\Lambda_L \approx 10 - 100\text{nm}$$

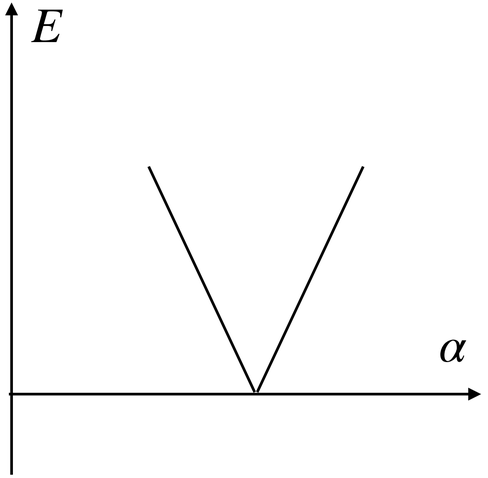


Nonlinear Meissner Current in nodal SCs - $\Delta(\mathbf{p}) = \Delta_0(\hat{p}_x^2 - \hat{p}_y^2)$

$\mathbf{p}_s \,||\, \text{node}$



$$E = \sqrt{v_f^2(p - p_f)^2 + |\Delta(\mathbf{p})|^2} \rightarrow \Delta_0 \alpha$$



❖ S.-K. Yip & JAS, Phys. Rev. Lett. 69, 2264 (1992)

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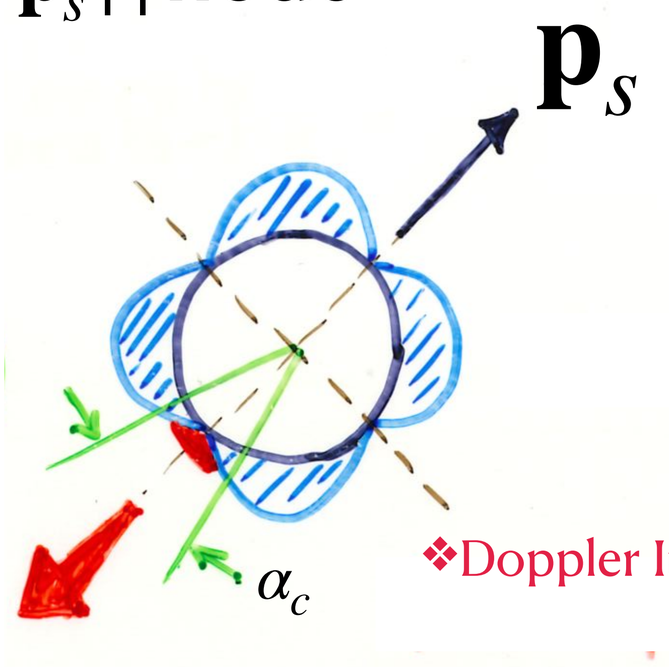

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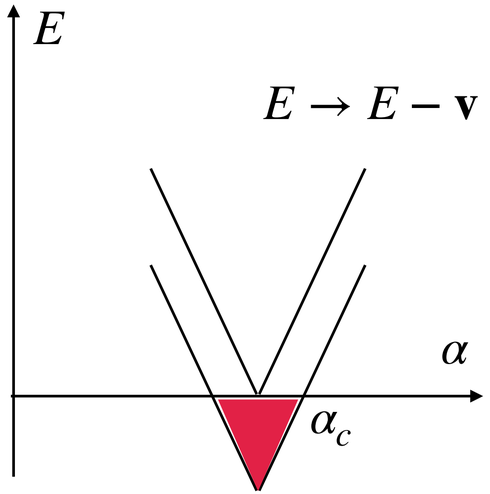

❖Doppler Induced Quasiparticle Backflow Jet at $T = 0$ K

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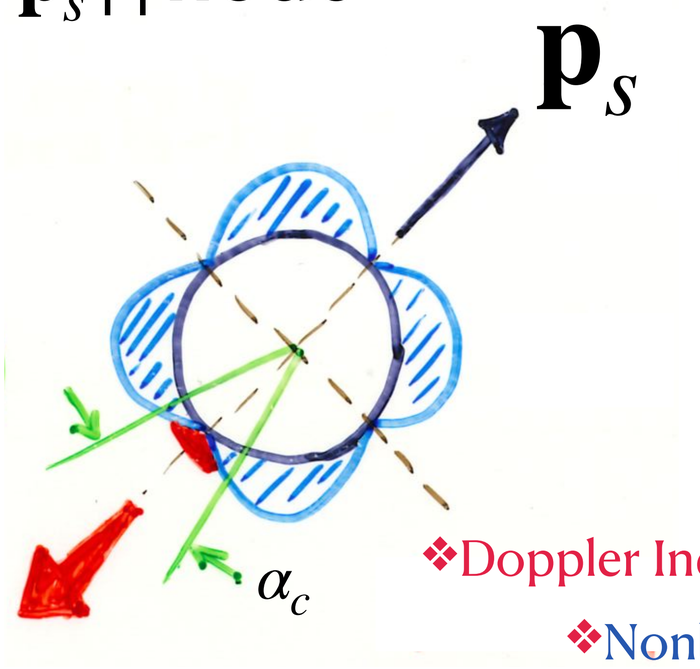

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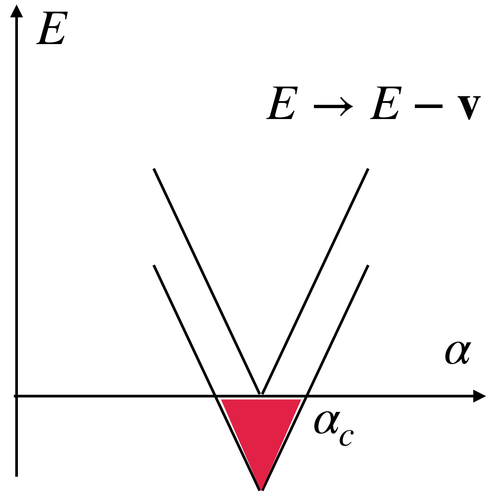

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❖Nonlinear and non-analytic Meissner Current

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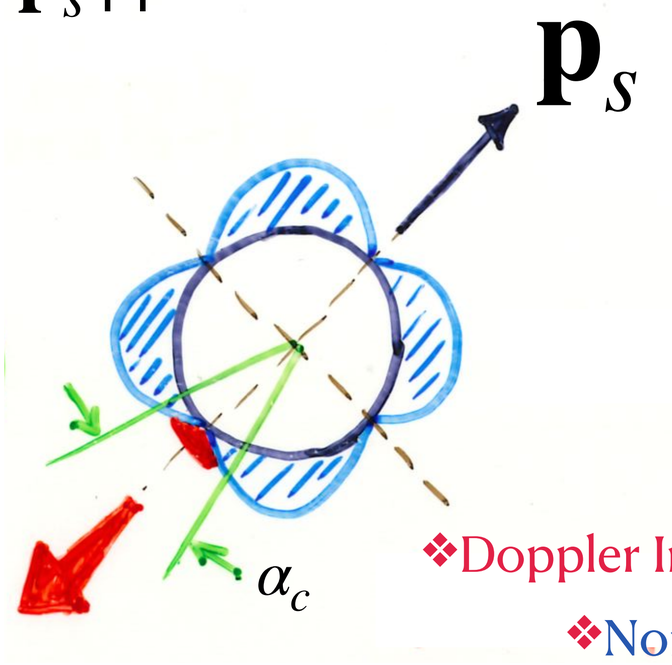

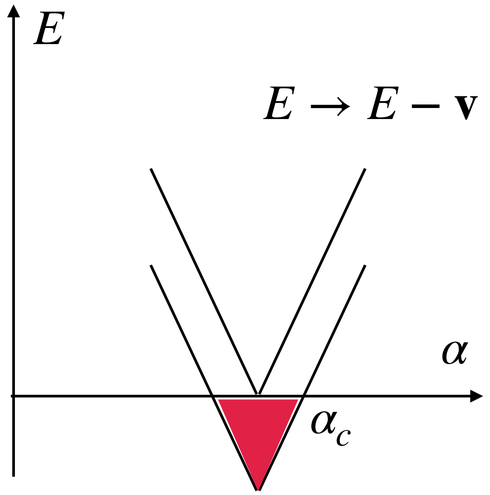

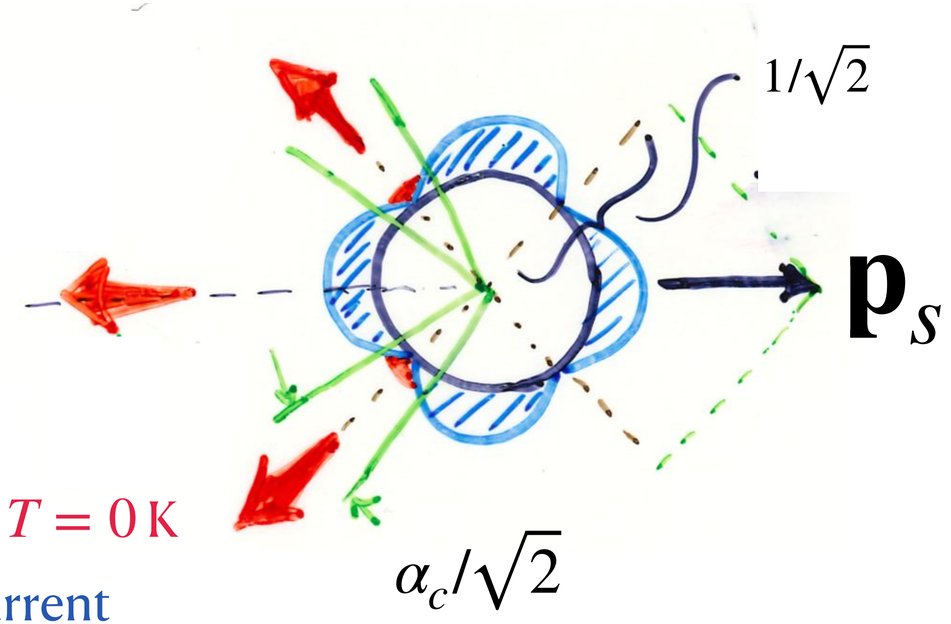

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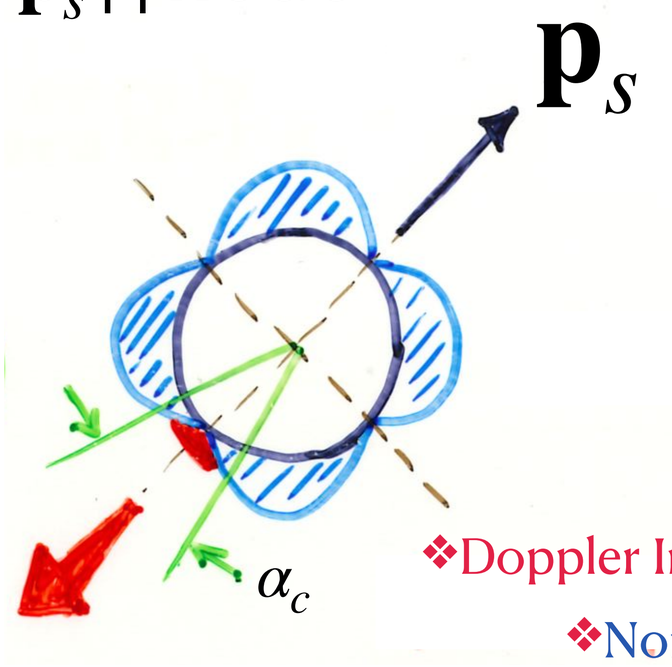

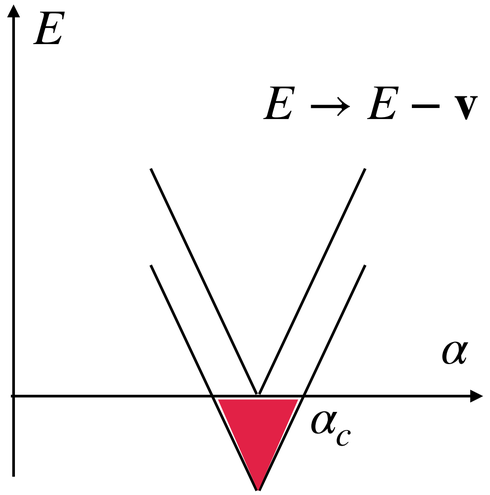

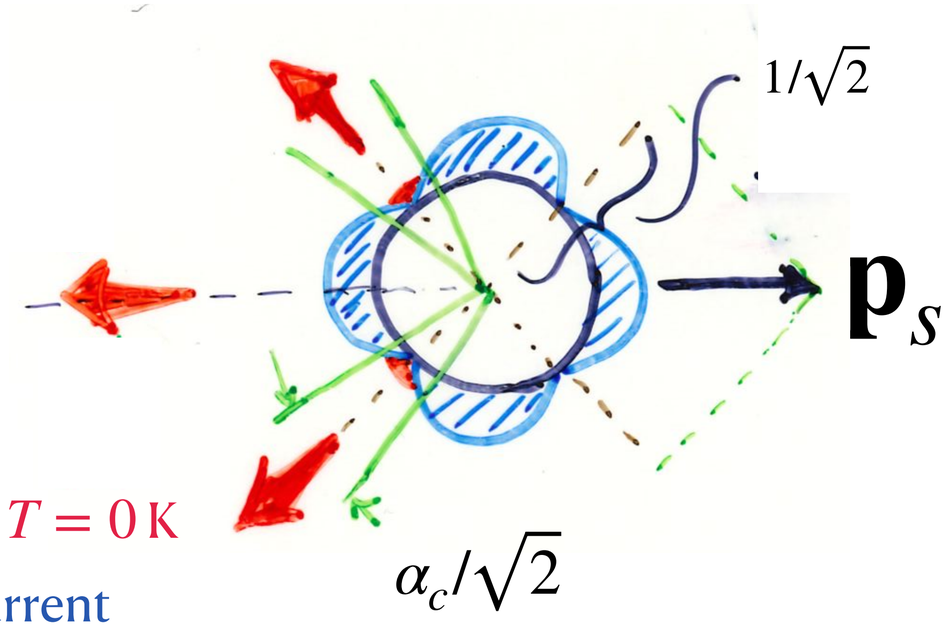

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★Finite density of QP at $T = 0$ K generates a Nonlinear Current response for any non-zero condensate flow

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Nonlinear Current Response of disordered SCs

$$\mathbf{J}_s = \frac{e}{m} n_s \mathbf{p}_s \left(1 - \theta \frac{|\mathbf{p}_s|^2}{p_c^2}\right)$$

Mechanism: condensate flow ($\mathbf{p}_s$) and disorder modify the spectrum of states contributing to the supercurrent

❖J. A. Sauls, Prog. Theor. Exp. Phys. 2022, 033I03 (2022); JAS and AG (2023)

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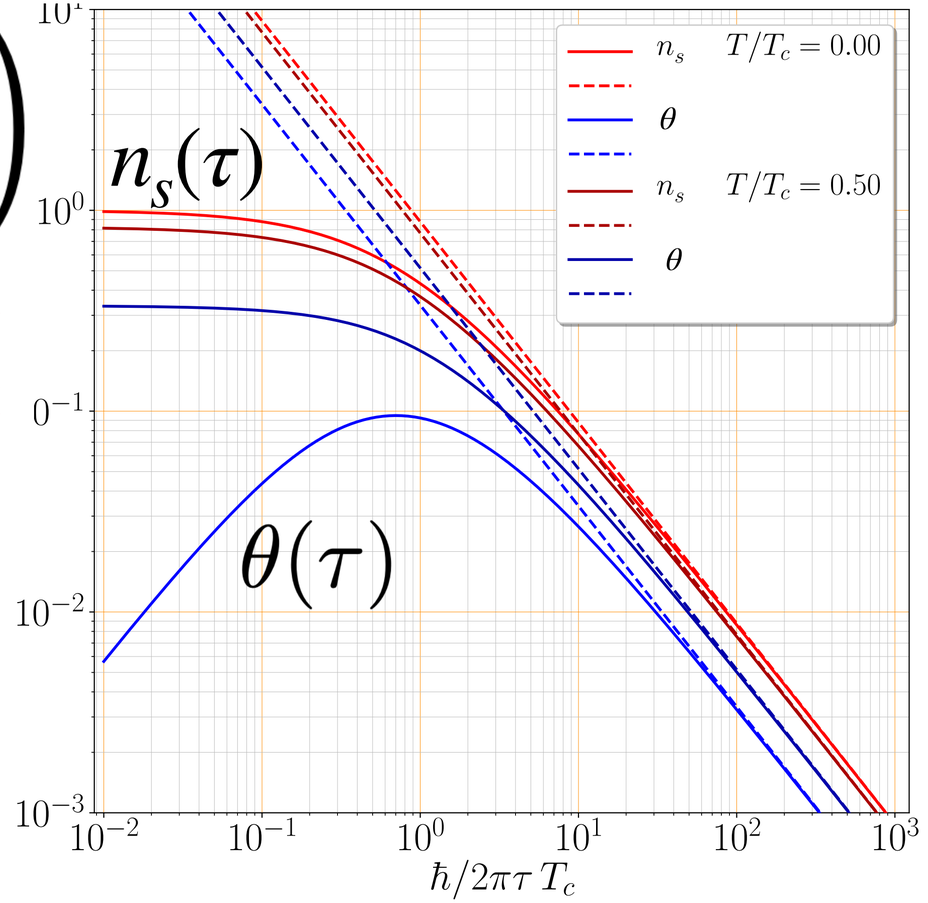

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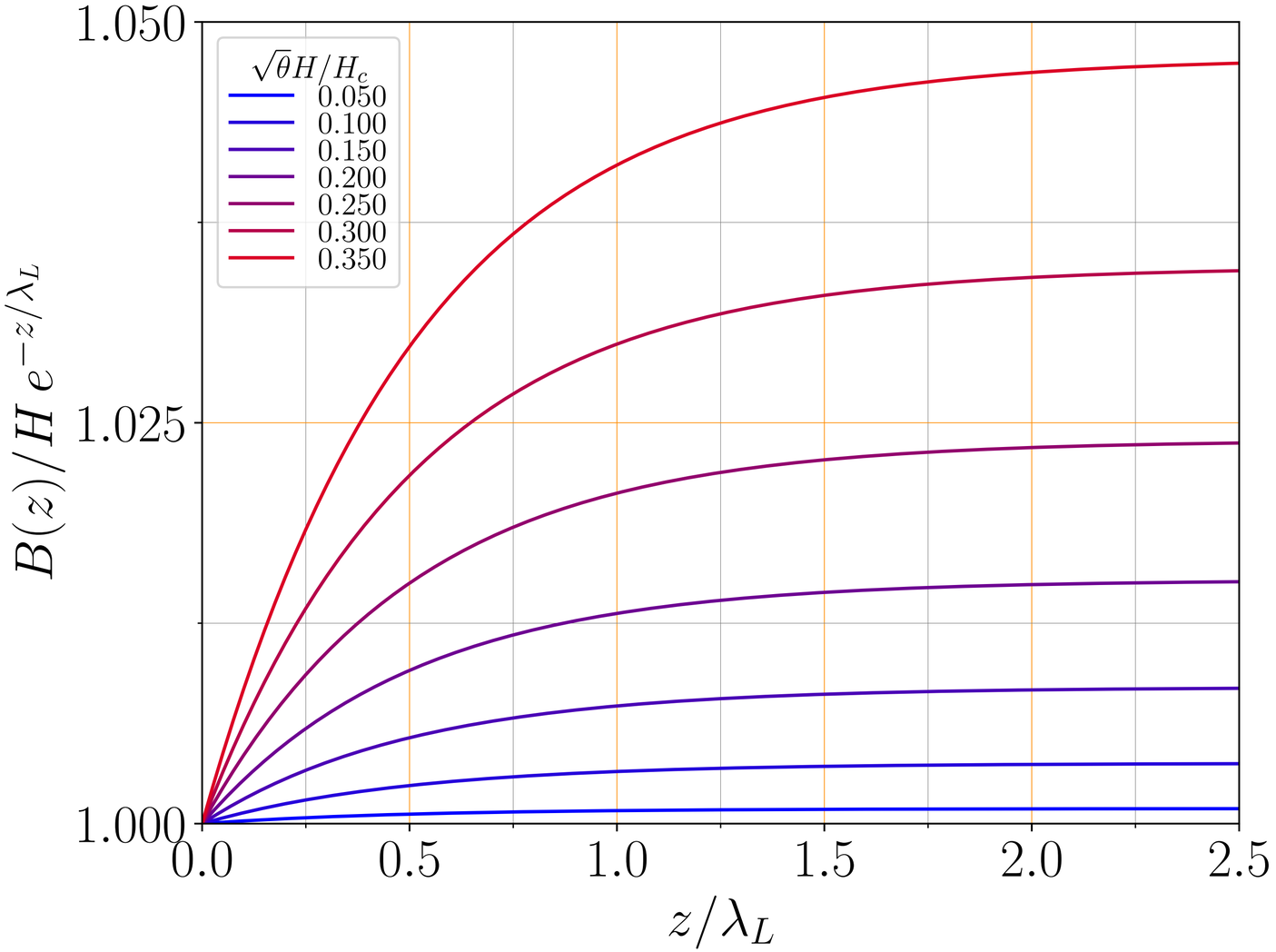
Increased Field Penetration via NLME



$$\frac{1}{\Lambda(H)} = \frac{1}{\Lambda_L}\left[1 - \frac{3}{4}\theta(T;\tau)\left(\frac{H}{H_c}\right)^2\right]$$

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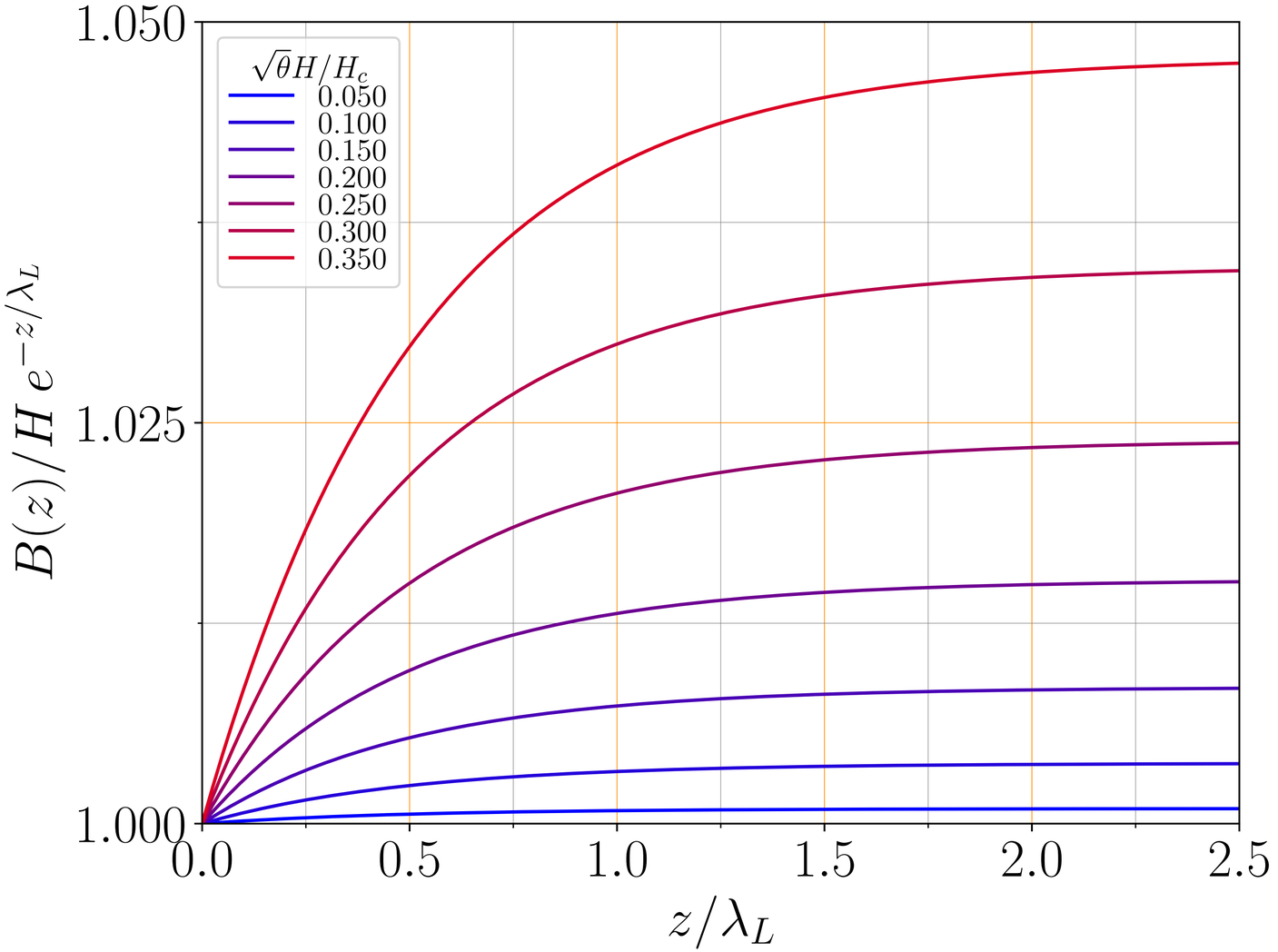

- For several reasons this is perhaps the more difficult experimental method to observe the NLM current.

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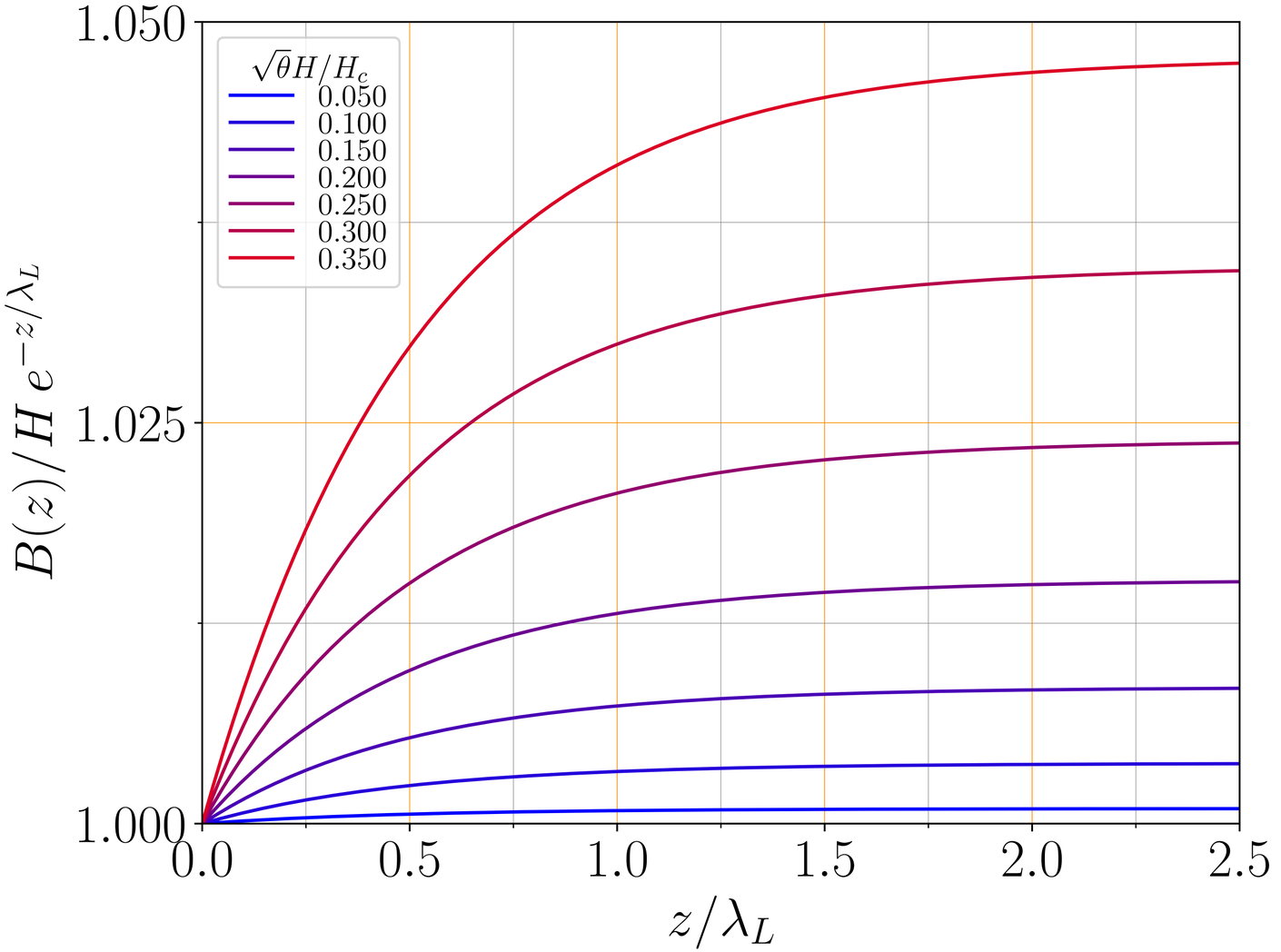

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- However, the NLME has been observed by several experimental groups, see for example:
  - ✓Groll et al. PRB **81**, 020504R (2010) [Nb films]
  - ✓Wilcox et al., Nat. Comm. **13**, 1201 (2022) [$CeCoIn_5$]

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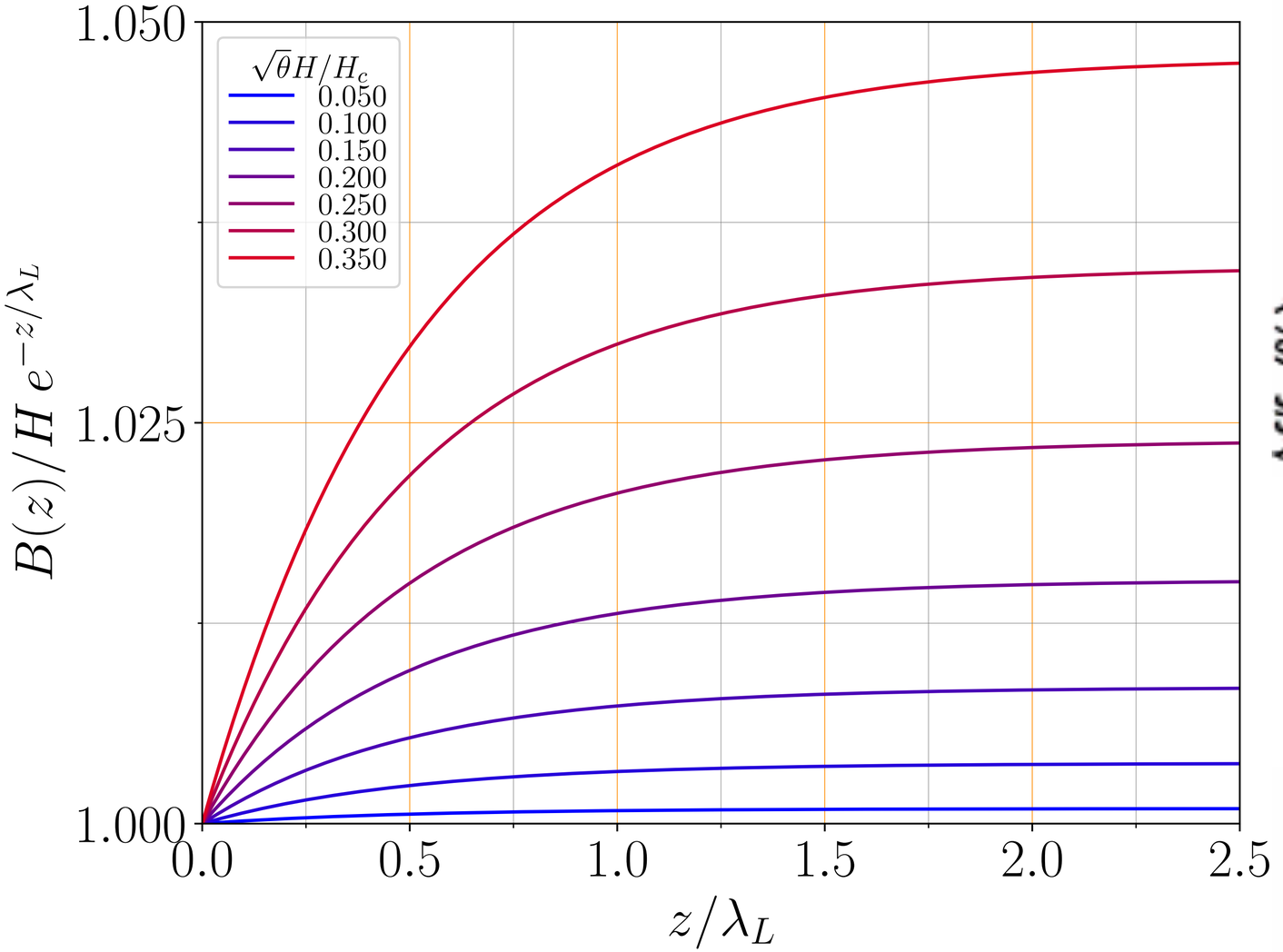

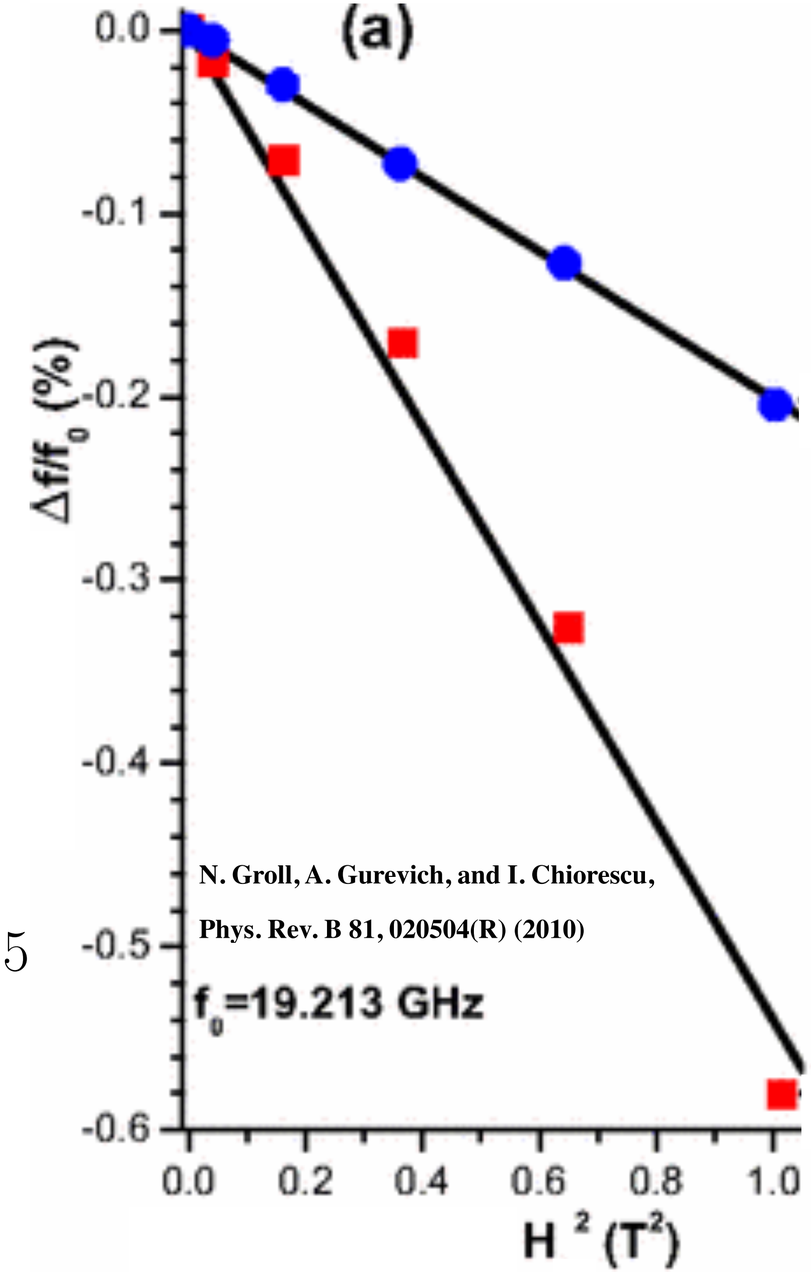

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Direct Signatures of the NLME

Generation of Photons at 3ω

$$\mathbf{J}_s = -\frac{c}{4\pi\Lambda_{\mathrm{L}}^2}\left\{1 - \frac{\theta}{A_c^2}|\mathbf{A}|^2\right\}\mathbf{A} \qquad \hbar\omega \ll 2\Delta$$

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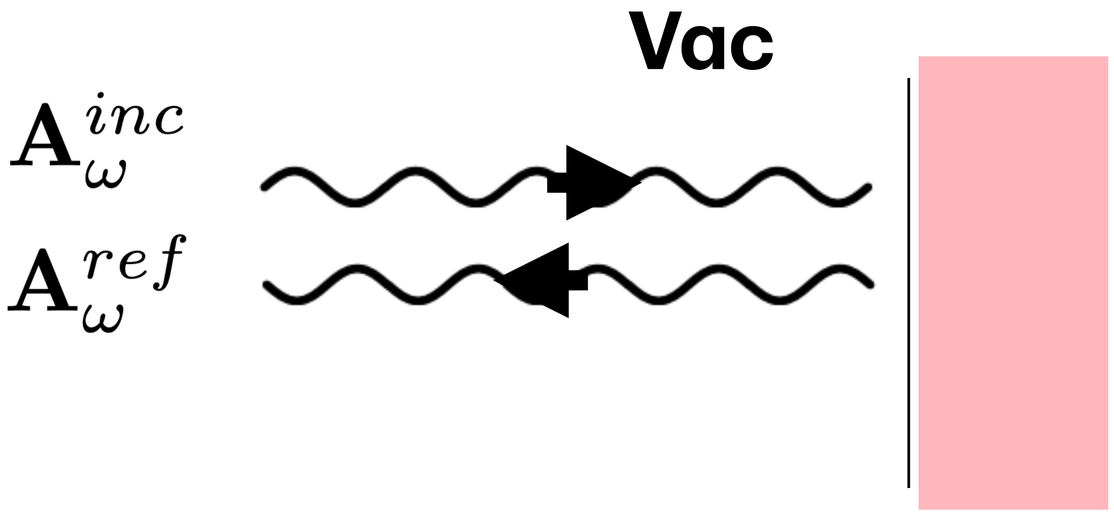
$$\mathbf{A} = A(z)\,\mathbf{e}\,\mathrm{Re}\,e^{i\omega t}$$



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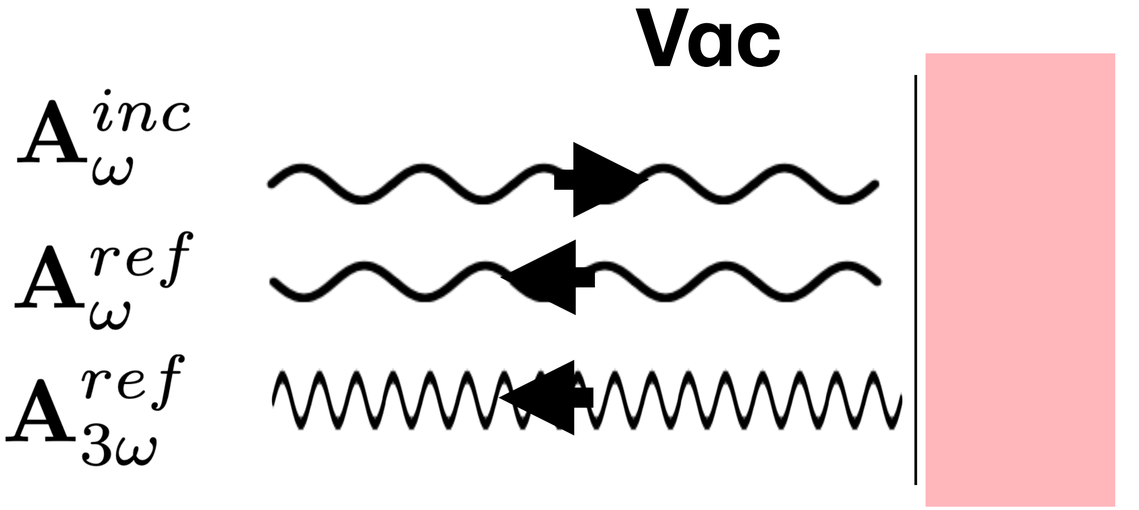


$$\mathbf{J}_{3\omega} = \frac{c}{4\pi\Lambda_{\mathrm{L}}^2}\frac{\theta}{A_c^2}|\mathbf{A}_{\omega}|^2\,\mathbf{A}_{\omega}$$

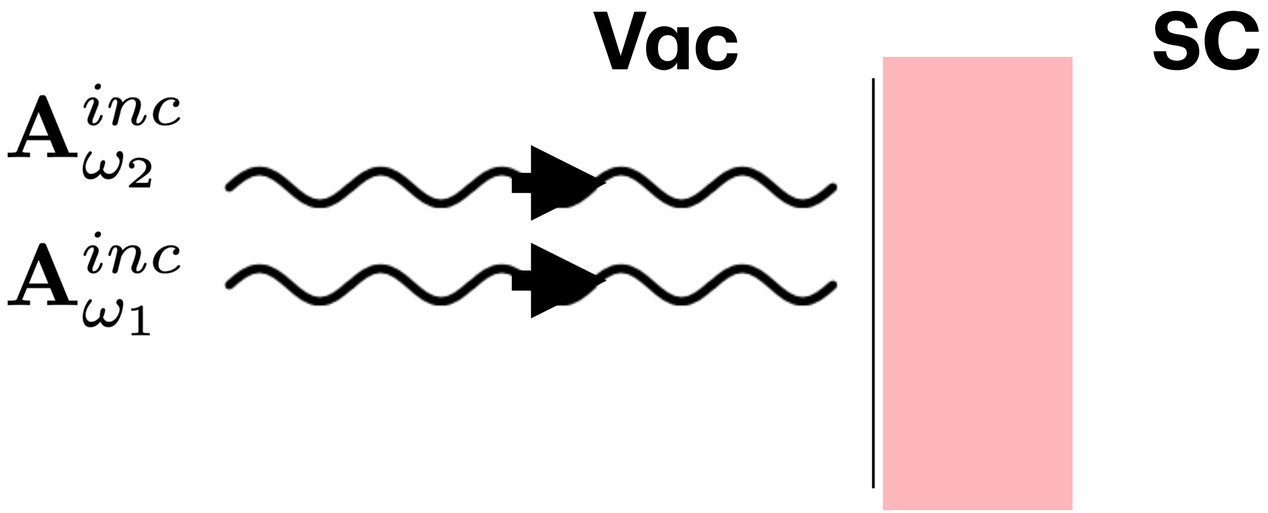
✓ **Measurement of photons @ *3ω on reflection is a direct measure of the nonlinear coupling parameter θ***

❖ Amplification ⇝ A27.005 : Nonlinear Meissner Interferometry, Alex Gurevich & JAS

Nonlinear Polar Kerr Effect

❖J. A. Sauls, Prog. Theor. Exp. Phys. 2022, 033I03 (2022)

Nonlinear Polar Kerr Effect



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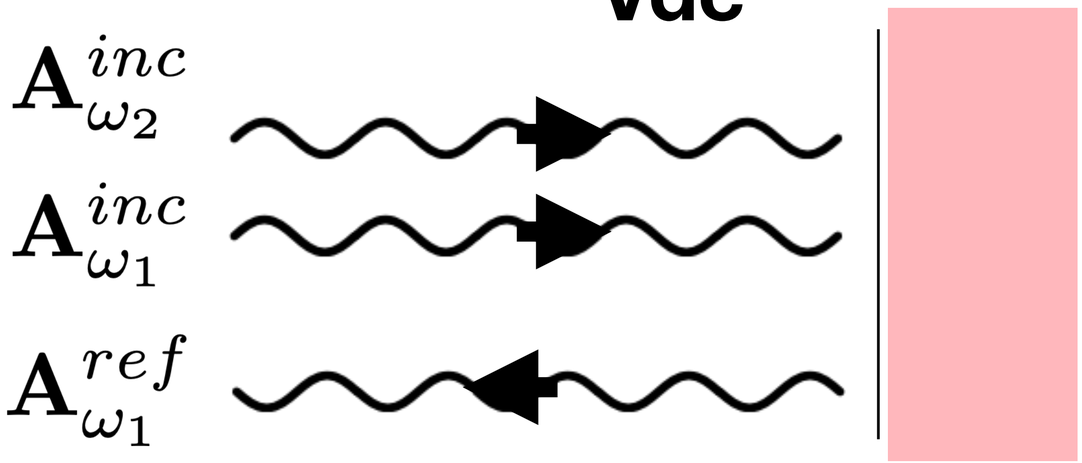
Nonlinear Polar Kerr Effect

$$\mathbf{A}_{1,2} = A_{1,2}(z)\,\mathbf{e}_{1,2}\,\mathrm{Re}\,e^{i\omega_{1,2}t}$$



$$\mathbf{J}_{\omega_a} = \frac{c}{4\pi\Lambda_{\mathrm{L}}^2}\,\frac{\theta}{A_c^2}\,\mathbf{G}(\omega_a)$$

$$\mathbf{G}(\omega_1) = \left[\frac{1}{2}\mathbf{e}_1 + (\mathbf{e}_1\cdot\mathbf{e}_2)\mathbf{e}_2\right] A_2^2\; A_1\,\mathrm{Re}\,e^{i\omega_1 t}$$

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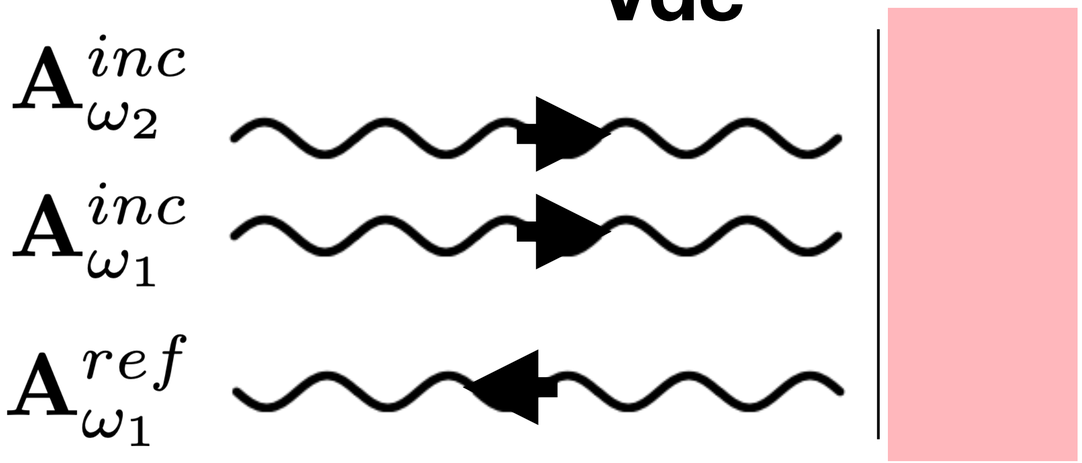

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$$\beta = \cos^{-1}(\mathbf{e}_1 \cdot \mathbf{e}_2)$$

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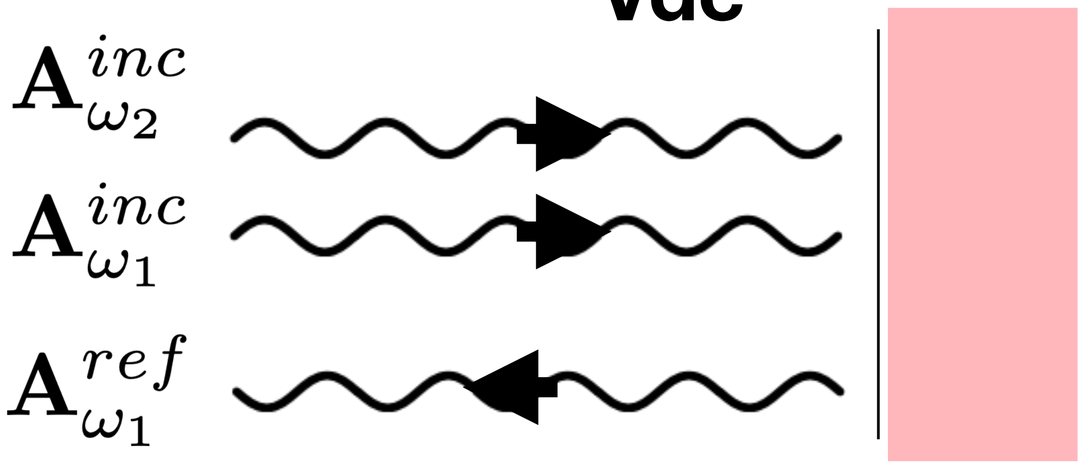
$$\Theta_K \approx \theta\,\sin(2\beta)\,\frac{A_2^2}{A_c^2}$$

❖J. A. Sauls, Prog. Theor. Exp. Phys. 2022, 033I03 (2022)

Nonlinear Polar Kerr Effect

$$\mathbf{A}_{1,2} = A_{1,2}(z)\,\mathbf{e}_{1,2}\,\mathrm{Re}\,e^{i\omega_{1,2}t}$$



$$\mathbf{J}_{\omega_a} = \frac{c}{4\pi\Lambda_{\mathrm{L}}^2}\,\frac{\theta}{A_c^2}\,\mathbf{G}(\omega_a)$$

$$\beta = \cos^{-1}(\mathbf{e}_1 \cdot \mathbf{e}_2)$$

$$\mathbf{G}(\omega_1) = \left[\frac{1}{2}\mathbf{e}_1 + (\mathbf{e}_1 \cdot \mathbf{e}_2)\mathbf{e}_2\right] A_2^2\; A_1\,\mathrm{Re}\,e^{i\omega_1 t}$$

$$\Theta_K \approx \theta\,\sin(2\beta)\,\frac{A_2^2}{A_c^2}$$

✓ **Broken P_2 & T**

under nonequilibrium conditions

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Microwave Photon Conversion/Generation

$$\mathbf{J}_s = -\frac{c}{4\pi\Lambda_{\mathrm{L}}^2}\left\{1 - \frac{\theta}{A_c^2}|\mathbf{A}|^2\right\}\mathbf{A}$$

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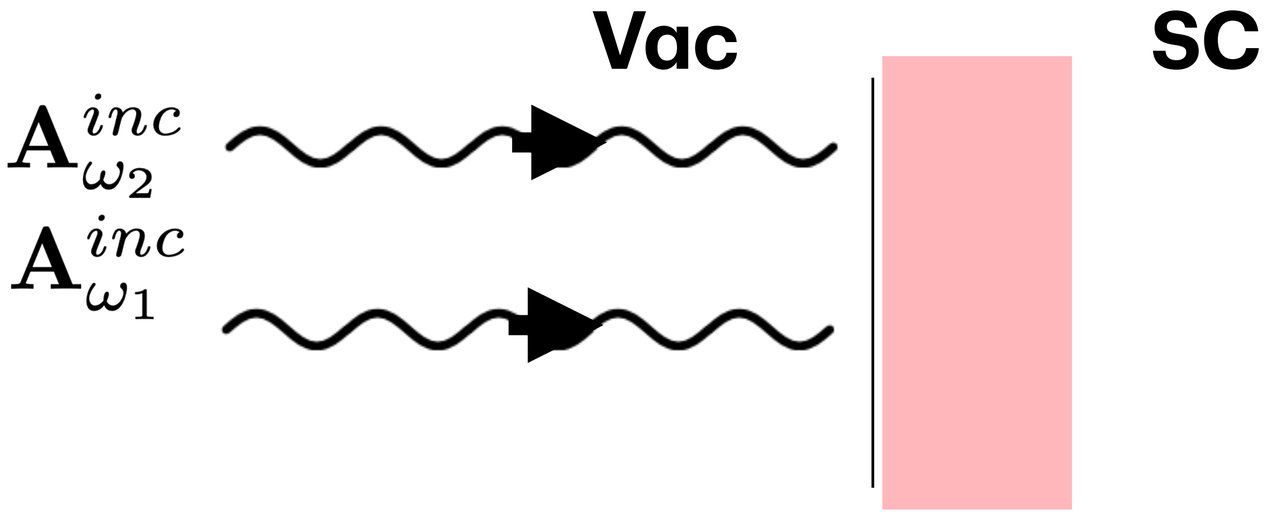

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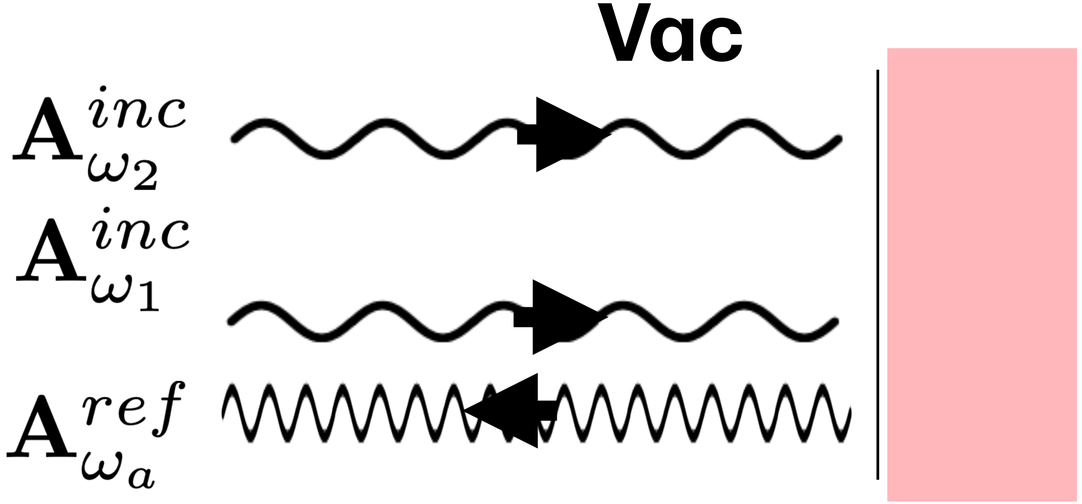

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$$\omega_a \in \{\omega_1\,,\omega_2\,,3\omega_1\,,3\omega_2\,,2\omega_1 \pm \omega_2\,,2\omega_2 \pm \omega_1\}$$

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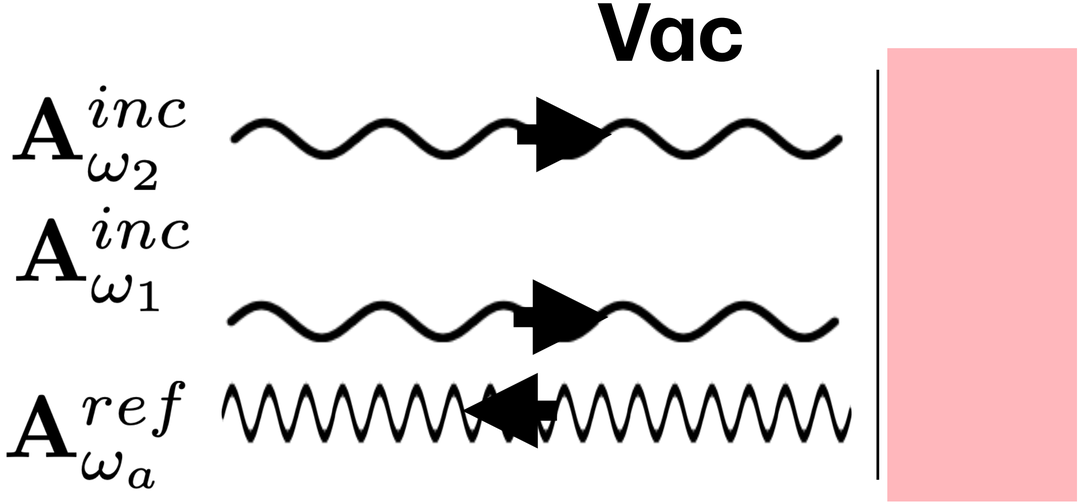

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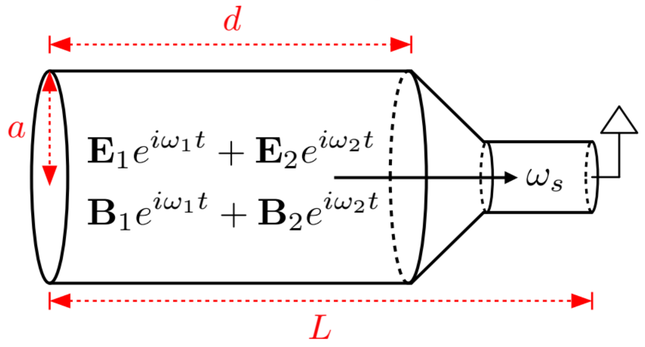
Nonlinearities in Superconducting RF Cavities

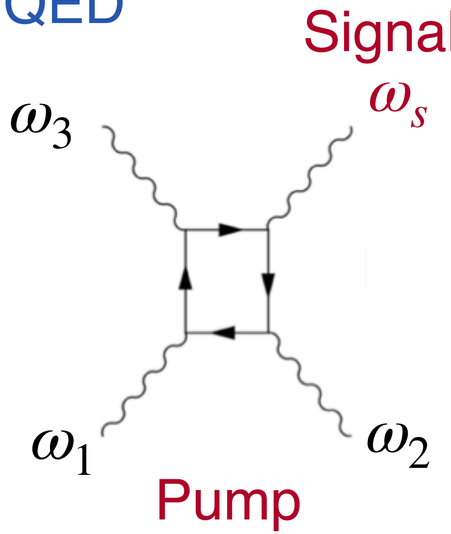
Axion Generation, QED and Nonlinear Meissner Currents

Nonlinearities in Superconducting RF Cavities
Axion Generation, QED and Nonlinear Meissner Currents

- SRF Cavity Resonators Proposed for Dark Sector Searches & New Tests of QED



$$N_s = Q_s^2 \times \frac{V E_0^6}{2\omega_s{}^3} \times f(m_a, g_{a\gamma\gamma})$$



- Z. Bogorad et al. Phys. Rev. Lett.123, 021801 (2019)

Signal photons: $\omega_s = 2\omega_1 - \omega_2$

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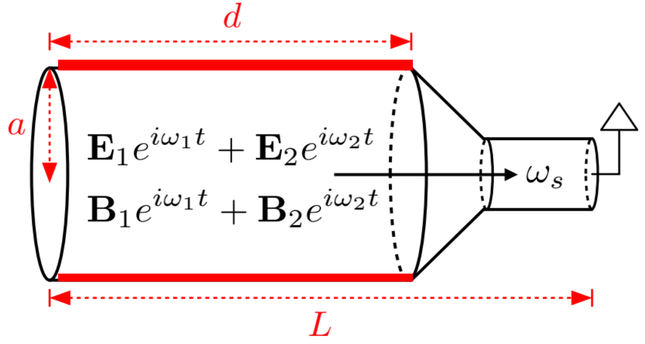

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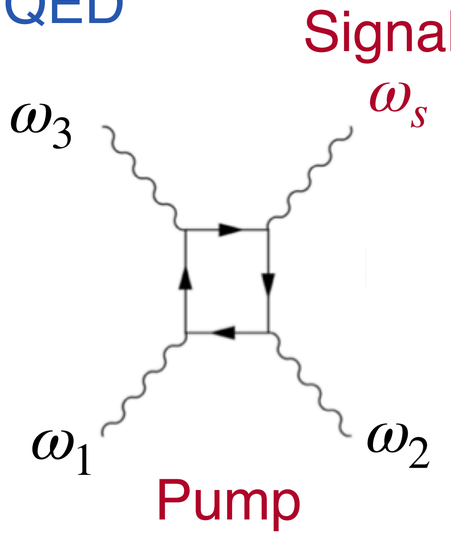


- Z. Bogorad et al. Phys. Rev. Lett.123, 021801 (2019)

Signal photons: $\omega_s = 2\omega_1 - \omega_2$

- Suppress False Positives from the NLM radiation at the signal frequency

$$N_s^{\text{false}} = Q_s^2 \times \frac{V E_0^6}{2\omega_s^3} \times S(\hbar/\tau, T, \omega_s : E_0/E_c)$$

❖ N27.00007: Photon frequency conversion in Superconducting RF cavities, Hikaru Ueki & JAS

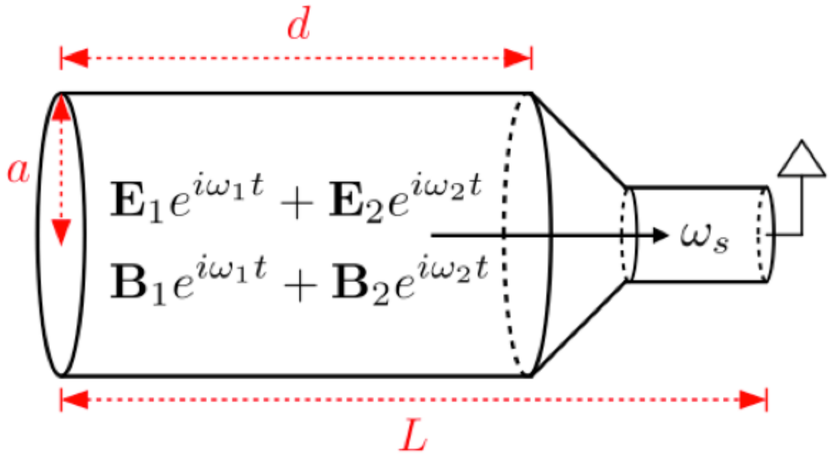
Thanks

sauls@lsu.edu

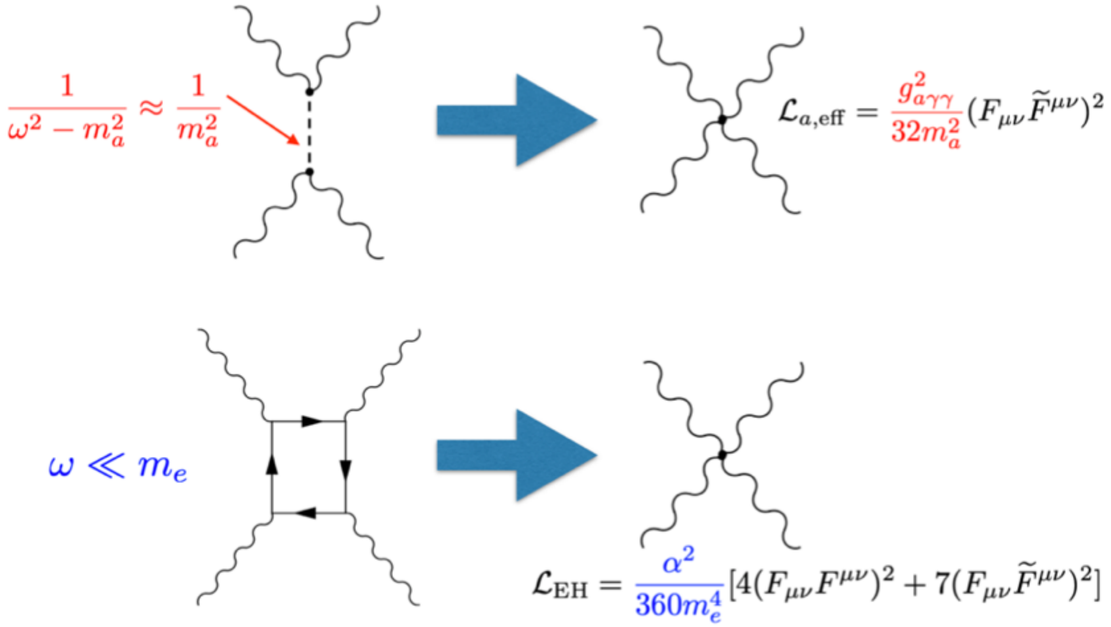
Supplementary Slides

Photon Generation by Axion Fields and QED

- High-Q cavities provide a novel technology platform for searching for evidence of axions as well as tests of photon-photon scattering via the Euler-Heisenberg (EH) term of low-energy QED [4-6].
- The sensitivity to the axion and EH signals depends on the Q of SRF cavities.



Signal at an intermodulation frequency due to the nonlinear interaction in a cavity pumped with two resonance modes



Light-by-light scattering mediated by axion and electrons!

[4] Z. Bogorad et al., Phys. Rev. Lett. 123, 021801 (2019). [5] Y. Kahn et al., Proc. SPIE 12016, 1201606 (2022). [6] A. Berlin et al., J. High Energ. Phys. 2020, 88 (2020).

Axion coupling to Maxwell's EM Field

- Modified Maxwell's equations [1,4] ⇝ a = pseudo-scalar Axion Field

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} - g_{a\gamma\gamma}\left(\mathbf{E} \times \nabla a - \mathbf{B}\frac{\partial a}{\partial t}\right)$$

$$\nabla \cdot \mathbf{E} = -g_{a\gamma\gamma}\mathbf{B} \cdot \nabla a$$

$= \mathbf{j}_a$ Axion current density

$= \rho_a$ Axion charge density

Axion coupling to Maxwell's EM Field

Key Idea: Generate the Axion Field inside the SRF Cavity[1,4]

- Modified Maxwell's equations [1,4] ⇝ a = pseudo-scalar Axion Field

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} - g_{a\gamma\gamma}\left(\mathbf{E} \times \nabla a - \mathbf{B}\frac{\partial a}{\partial t}\right) \qquad \nabla \cdot \mathbf{E} = -g_{a\gamma\gamma}\mathbf{B} \cdot \nabla a$$

$= \mathbf{j}_a$ Axion current density

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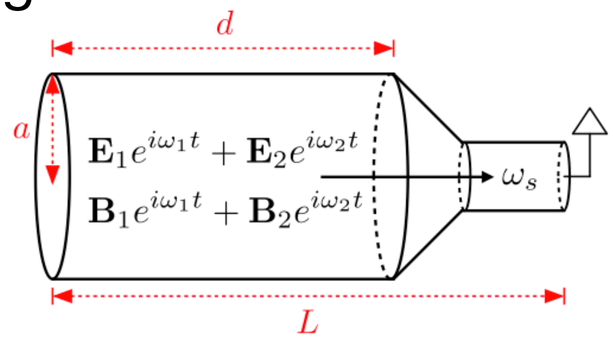
- Axion Coupling to Photons

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 + m_a^2\right) a = g_{a\gamma\gamma}\mathbf{E} \cdot \mathbf{B}$$

$g_{a\gamma\gamma}$ = axion-photon coupling

m_a = axion mass.



The axion field is sourced by $\mathbf{E} \cdot \mathbf{B}$, so two different modes are needed to generate it.

[1] Z. Bogorad et al., Phys. Rev. Lett. **123**, 021801 (2019). [4] C. Gao and R. Harnik, J. High Energ. Phys. **2021**, 53 (2021).

Summary

- ✓ High Q SRF cavities are the workhorse technology for high energy particle accelerators … and a technology platform for 3D quantum processors and sensing.
- ✓ Specially designed multi-mode SRF cavities have recently been proposed for generating and detecting pseudo-scalar Axion fields and Dark Photons
- ✓ SQMS is developing multi-mode SRF cavities for detecting the hypothesized axion field and Photon-Photon scattering at microwave frequencies (Euler-Heisenberg interaction)
- ✓ Suppressing false positive signals is essential for detection of the axion field & EH interaction
- ✓ SRF Cavity response to the EM field is intrinsically nonlinear. We calculated the number of signal photons and signal power due to the Nonlinear Meissner current at intermodulation frequency $2\omega_1 - \omega_2$ in an SRF cavity pumped with two frequencies ω_1 and ω_2 based on the microscopic theory of superconductivity and Maxwell's equations.
- ✓ For *ultra-high Q* and high power the *nonlinear frequency shift* $\Delta\omega_{\mathrm{NLM}}$ suppresses NLM radiation at the signal frequency for both EH and Axion field detection.

Thank You